0.1 Proofs of correctness

Our optimisation problem described previously can now be restated as follows for mathematical clarity.
Problem: *Find a maximal member B of $\mathscr{I}$ of maximum weight.*

Note. Let B_G be a base of a matroid generated by the greedy algorithm.

Theorem 0.1. *If $(E, \mathscr{I})$ is a matroid M, then B_G is a solution to the optimization problem.*

Proof. If $r(M) = r$, then $B_G = \{e_1, e_2, ..., e_r\}$ is a basis of M. Let B be another basis of M, $B = \{f_1, f_2, ..., f_r\}$ where $\omega(f_1) \geq \omega(f_2) \geq ... \geq \omega(f_r)$. We claim that $\omega(e_j) \geq \omega(f_f)\ \forall j$, then it follows that $\omega(B_G) \geq \omega(B)$ for any other basis in $\mathscr{B}$. $\square$

Lemma 0.2. *If $1 \leq j \leq r$, then $\omega(e_j) \geq \omega(f_j)$.*

Proof. Suppose (seeking a contradiction) that k is the least integer for which $\omega(e_k) < \omega(f_k)$. Take $I_1 = \{e_1, e_2, ..., e_{k-1}\}$ and $I_2 = \{f_1, f_2, ..., f_k\}$. Since $|I_2| = |I_1| + 1$ $(I3)$ implies $I_1 \cup \{f_t\} \in \mathscr{I}$ for some $f_t \in I_2 \setminus I_1$. But this means that $\omega(f_t) \geq \omega(f_k) > \omega(e_k)$ and hence the Greedy algorithm would have chosen f_t over e_k, which gives us our contradiction. $\square$