

Problem: Find a maximal member B of \mathcal{I} of maximum weight.

Note. Let B_G be a base of a matroid generated by the greedy algorithm.

Theorem 0.1. If (E, \mathcal{I}) is a matroid M , then B_G is a solution to the optimization problem.

Proof. If $r(M) = r$, then $B_G = \{e_1, e_2, \dots, e_r\}$ is a basis of M . Let B be another basis of M , $B = \{f_1, f_2, \dots, f_r\}$ where $\omega(f_1) \geq \omega(f_2) \geq \dots \geq \omega(f_r)$. We claim that $\omega(e_j) \geq \omega(f_j) \forall j$, then it follows that $\omega(B_G) \geq \omega(B)$ for any other basis in \mathcal{B} . \square

Lemma 0.2. If $1 \leq j \leq r$, then $\omega(e_j) \geq \omega(f_j)$.

Proof. Suppose (seeking a contradiction) that k is the least integer for which $\omega(e_k) < \omega(f_k)$. Take $I_1 = \{e_1, e_2, \dots, e_{k-1}\}$ and $I_2 = \{f_1, f_2, \dots, f_k\}$. Since $|I_2| = |I_1| + 1$ (I3) implies $I_1 \cup \{f_t\} \in \mathcal{I}$ for some $f_t \in I_2 \setminus I_1$. But this means that $\omega(f_t) \geq \omega(f_k) > \omega(e_k)$

And hence the Greedy algorithm would have chosen f_t over e_k , which gives us our contradiction. \square