Undergraduate Project Proposal

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We plan to study matroids from scratch, following Oxley's text [1] together with [2]. A matroid is a structure that abstracts and generalizes the notion of linear independence in vector spaces. There are many equivalent ways to define a matroid, the most significant being in terms of independent sets, bases, circuits, closed sets or flats, closure operators, and rank functions.

Definition 0.1. A matroid is a pair (E,**I** with finite ground set E and **I** being a collection of independent subsets of E satisfying the following conditions

(I1): The empty set is always independent
(I2): Every subset of an independent set is independent
(I3): If A and B are two independent sets (i.e., each set is independent) of **I** and A has more elements than B, then there exists $x \in A \setminus B$ such that $B \cap \{x\}$ is in I

We will consider the motivating problems and examples from Oxley's work and using a mathematical software package attempt to reproduce some currently best-known results of the enumeration problems. Our primary target is to explain the connection between matroids and the greedy algorithm.

The Greedy algorithm is an interesting characterization of matroids in that it arises frequently in problems related to combinatorial optimzation. For example the well known graph optimization problem: finding the minimum spanning tree. Leading to a connection between Kruskal's algorithm and matroids.

References

[1] Oxley, James (1992), Matroid Theory, Oxford: Oxford University Press, ISBN 0-19-853563-5, MR 1207587, Zbl 0784.05002.

[2] James Oxley : What is a matroid? https://www.math.lsu.edu/ oxley/survey4.pdf