Matroids And their Graphs

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1 Matroids imply Circuits: Alternative C3

Definition 1.1. By using (I1)–(I3) , it is not difficult to show that the collection $\mathscr{C}$ of circuits of a matroid M has the following three properties:
(C1) The empty set is not in $\mathscr{C}$
(C2) No member of $\mathscr{C}$ is a proper subset of another member of $\mathscr{C}$
(C3) if C_1 and C_2 are distinct members of C and $e \in C_1 \cap C_2$, then $(C_1 \cup C_2) \setminus \{e\}$ contains a member of $\mathscr{C}$.

Theorem 1.1. *A matroid M $\implies$ all circuits in $\mathscr{C}$ satisfy (C1-C3).*

Proof: Let $A, b \in \mathscr{C}$, $A \neq B$
Let $e \in A \cap B$
Suppose(seeking a contradiction) that $(A \cap B) \setminus \{e\}$ is independent.
Let $x \in A \setminus B$, noting that $x \neq e$
$\implies A \setminus \{x\} \in \mathscr{I}$ i.e independent.
We can add elements of $(A \cup B) \setminus \{e\}$ to $A \setminus \{x\}$ retaining independence until we have $|A \cup B| - 1$ elements.
But now our new set $(A \cup B) \setminus \{x\}$ contains all of B and $B \in \mathscr{C}$. Therefore, we have a contradiction, $(A \cup B) \setminus \{e\}$ cannot be independent

$\square$