Matroids And their Graphs

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19 January 2018

1 Circuits in a graph

Theorem 1.1. *Let E be the edge sets of a graph G and let $\mathscr{C}$ be the edge sets of cycles in G.*
Then $\mathscr{C}$ is the set of circuits of a matroid.

Proof: Let $A, B \in \mathscr{C}, A \neq B$ and let $e \in A \cap B$
We must now construct a cycle of G whose edge set is contained in $(A \cup B) \setminus \{e\}$

For $i = 1, 2, 3, ...$ let P_1 be a path whose edge set is $A \setminus \{e\}$
$A \setminus \{e\} \in \mathscr{I}$ therefore P_i is not a cycle of G. This path will traverse from the edge a_j to a_u where u, j were the vertices connecting the edge e to $(A \cup B) \setminus \{e\}$ to make $A \cup B$.

Now perform the same procedure for a path P_2 whose edge set is $B \setminus \{e\}$.

P_1 and P_2 should meet at the junctions u, v, where e was removed to make $(A \cup B) \setminus \{e\}$
Therefore $P_1 \cup P_2$ should be a cycle of G.
$\implies$ (C3) holds
$\implies$ $\mathscr{C}$ is the edge sets of cycle in G.

$\square$