Theorem 0.1. Let E be the edge set of a graph G and let $\mathscr C$ be the edge sets of cycles in G.

Then $\mathscr C$ is the set of circuits of a matroid.

Proof. Let $A, B \in \mathcal{C}, A \neq B$ and let $e \in A \cap B$.

We must now construct a minimal cycle of G whose edge set is contained in $(A \cup B) \setminus \{e\}$.

For $i = 1, 2, 3, \dots$ let P_1 be a path whose edge set is $A \setminus \{e\}$.

 $A \setminus \{e\} \in \mathscr{I}$ therefore P_1 is not a cycle of G. This path will traverse from the edge a_j to a_u where u, j were the vertices connecting the edge e to $(A \cup B) \setminus \{e\}$ to make $A \cup B$.

Now perform the same procedure for a path P_2 whose edge set is $B \setminus \{e\}$.

 P_1 and P_2 should meet at the junctions u, v, where e was removed to make $(A \cup B) \setminus \{e\}$.

Therefore $P_1 \cup P_2$ should be a cycle of G.

- \implies (C3) holds.
- $\implies \mathscr{C}$ is the set of circuits in G.