Definition 0.1. If M is a matroid , then is there exists a bijection from the ground set of M_i to the ground set of M_j, such that a set is independent in the first matroid if and only if it is independent in the second matroid, then M_i and M_j are said to be isomorphic

Excercise: 2.4
Let E be a set, $\{1, 2, 3\}$
i) Show there are exactly eight non-isomorphic matroids on E.
ii) How many non-isomorphic matroids are there on a 4-element set?
Let I be the set of subsets as defined above.
Solution:
i)

$$\{\emptyset\}$$

$$\{\{\emptyset\}, \{1\}\} \cong \{\{\emptyset\}, \{2\}\} \cong \{\{\emptyset\}, \{3\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}, \{3\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}, \{1, 2\}\} \cong \{\{\emptyset\}, \{1\}, \{3\}, \{1, 3\}\} \cong \{\{\emptyset\}, \{2\}, \{3\}, \{2, 3\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$$

$$\{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

ii)

$$E = \{\emptyset\}; |I| = 2^0 = 1$$

$$E = \{1\}; |I| = 2^1 = 2$$

$$E = \{1, 2\}; |I| = 2^2 = 4$$

$$E = \{1, 2, 3\}; |I| = 2^3 = 8$$

$$E = \{1, 2, 3, 4\}; |I| = 2^4 = 16$$

So $|I|$ is bounded above by 2^n for $n \in \mathbb{N}$