Computer Graphics

Shape and image representation

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What is a shape?

"There can be no such thing as a mathematical theory of shape. The very notion of shape belongs to the natural sciences."[1]

[1] J. Koenderink, "Solid Shape". MIT Press 1990

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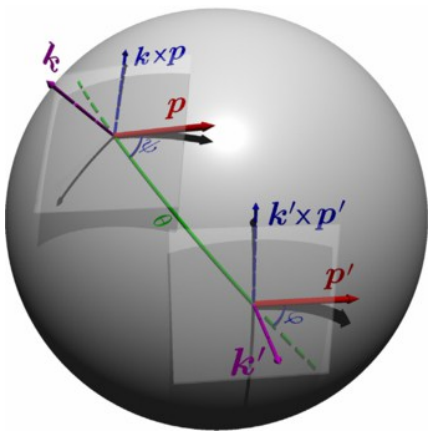
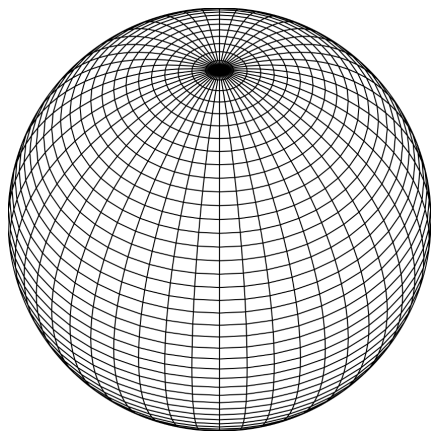
"There can be no such thing as a mathematical theory of shape. The very notion of shape belongs to the natural sciences."[1]

For us, shapes are mathematical objects (specifically, manifolds) with a precise and rigorous definition. We will model them mathematically in the continuous setting ("pen and paper") and bring them to the digital world by translating to the discrete setting.





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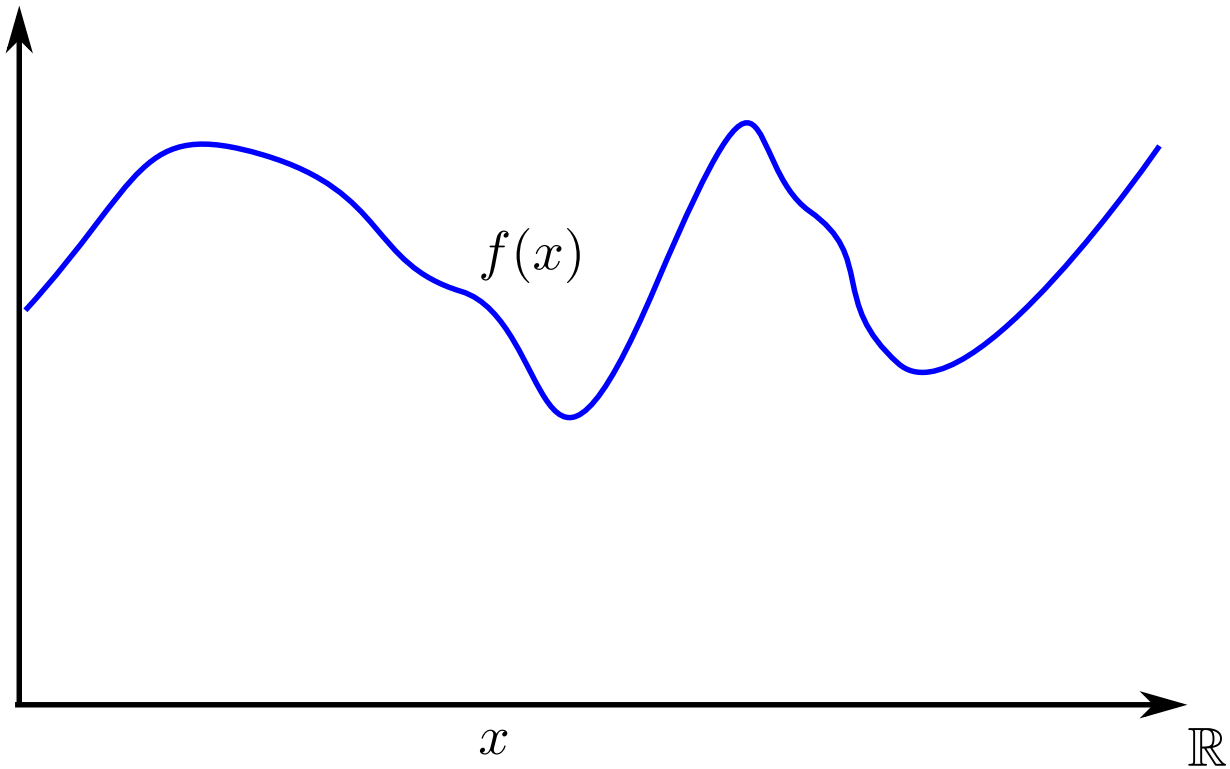
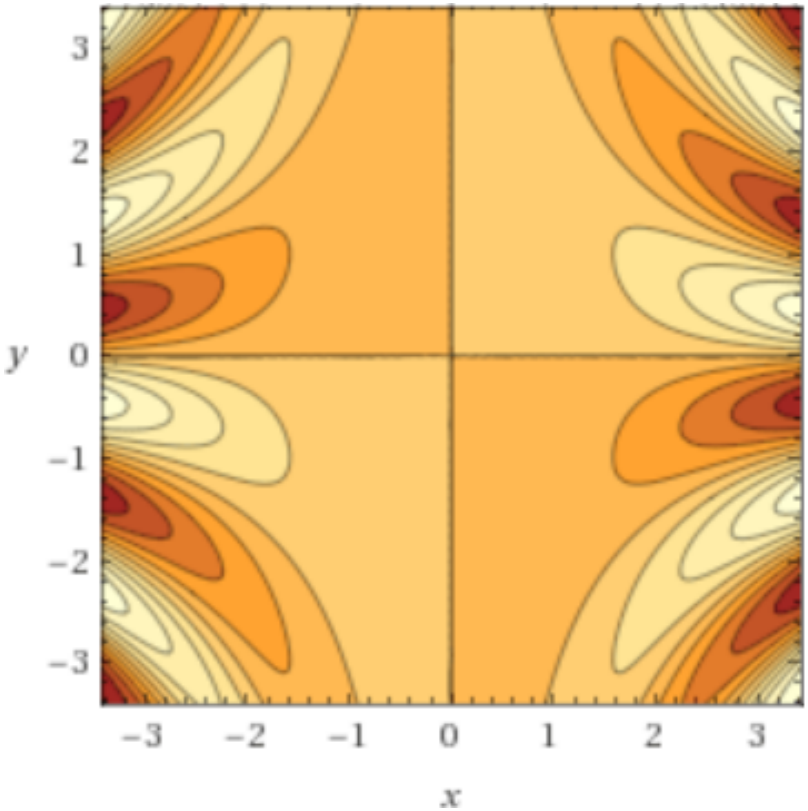
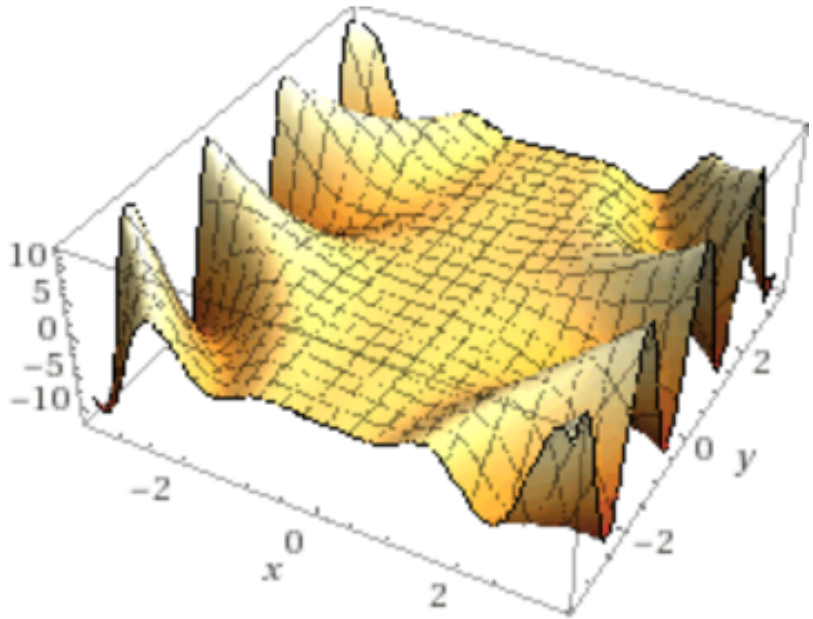
Functions over $\mathbb{R}$



$$f : \mathbb{R} \to \mathbb{R}$$

Functions over $\mathbb{R}^2$





height map

$$f : \mathbb{R}^2 \to \mathbb{R}$$

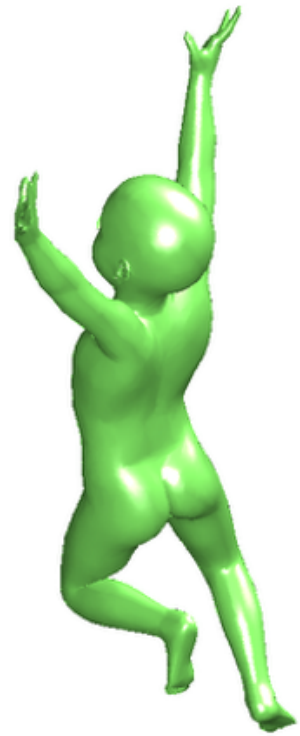
$$f : (x, y) \mapsto x^2 \sin(xy)$$

Functions over a surface

$$f : \mathcal{M} \to \mathbb{R}^3$$

Shapes vs Images: Domain

Euclidean (flat)

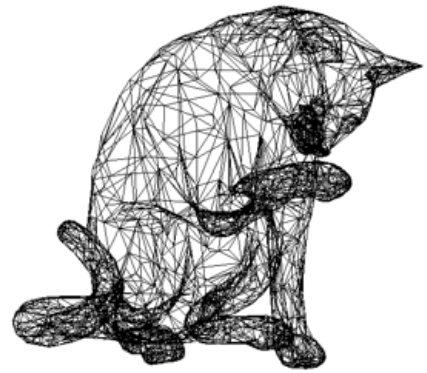


non-Euclidean (curved)

Shapes vs Images: Representation

Array of pixels (uniform grid)

Splines

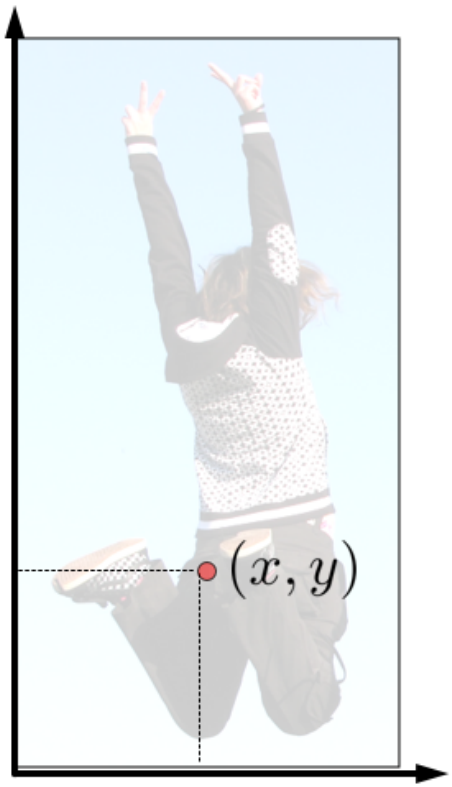


Graph

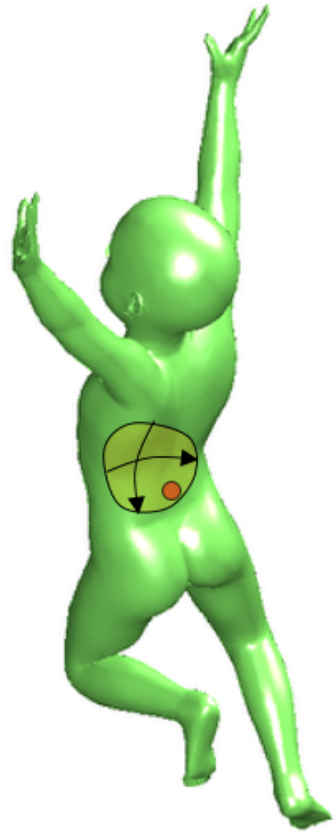
Point cloud

Triangle mesh

Shapes vs Images: Parametrization

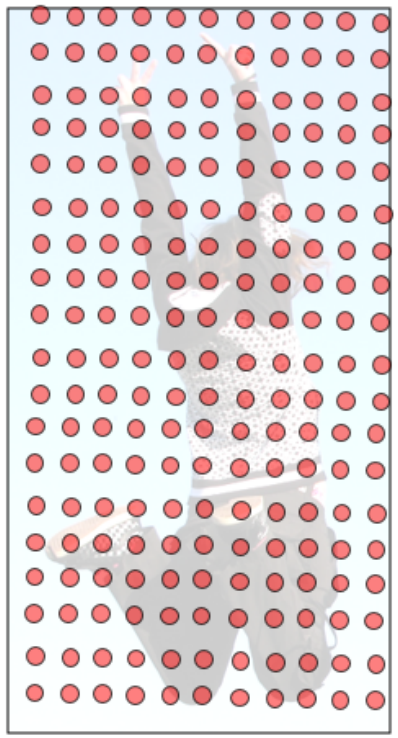


Global

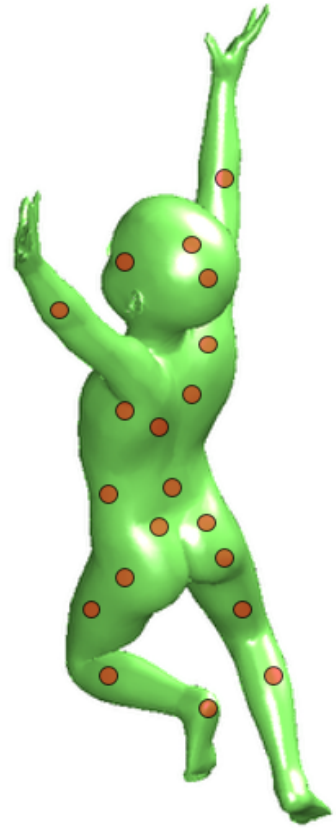


Local

Shapes vs Images: Sampling



Uniform

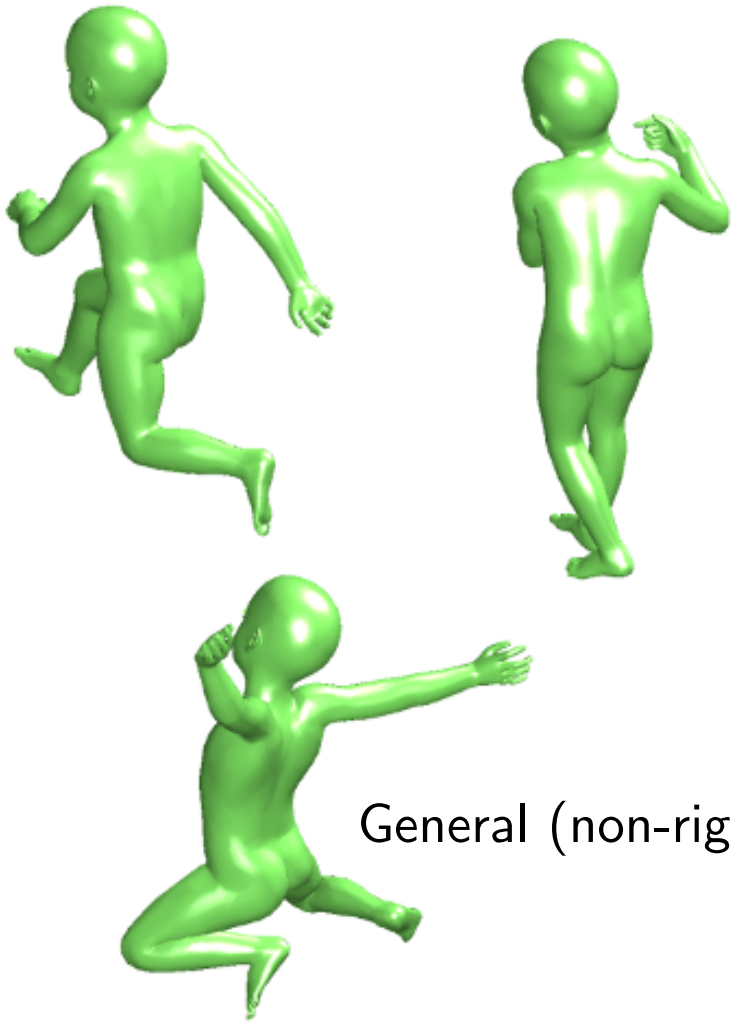


"Uniform" is not well defined

Shapes vs Images: Transformations

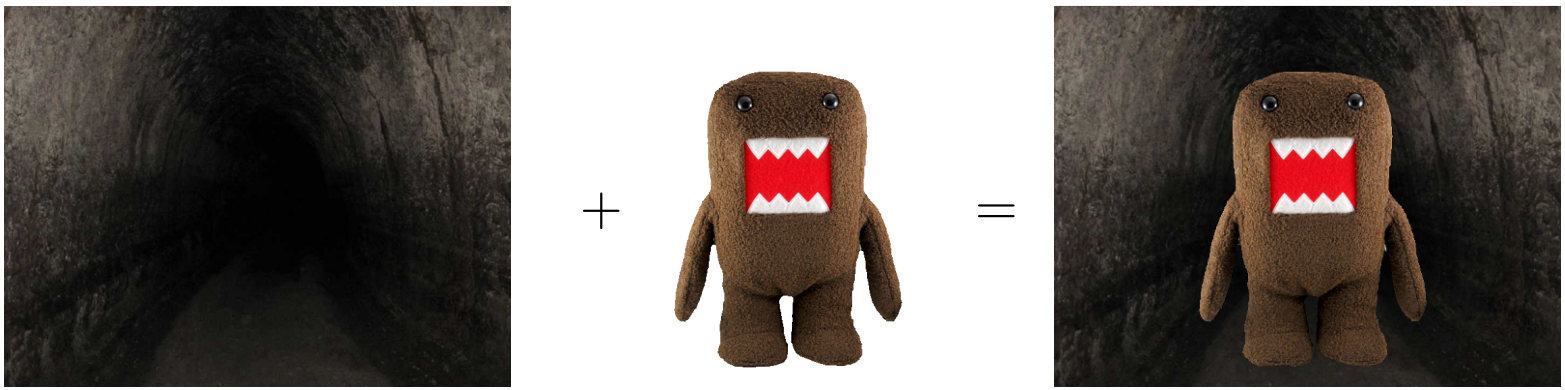
Perspective

Affine

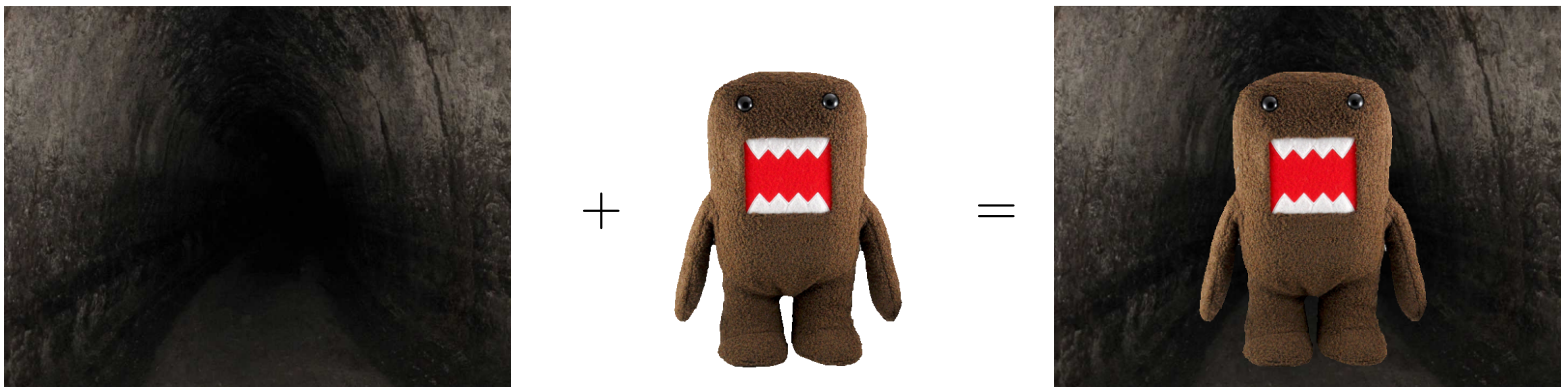


General (non-rigid)

Shapes vs Images: Calculus



Shapes vs Images: Calculus



Shape representation

We will use mainly two representations for shapes:

- Triangle mesh
- Point cloud

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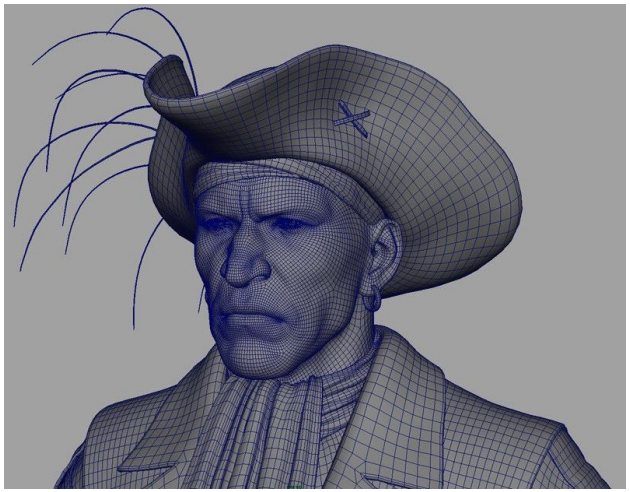
Other shape representations exist:

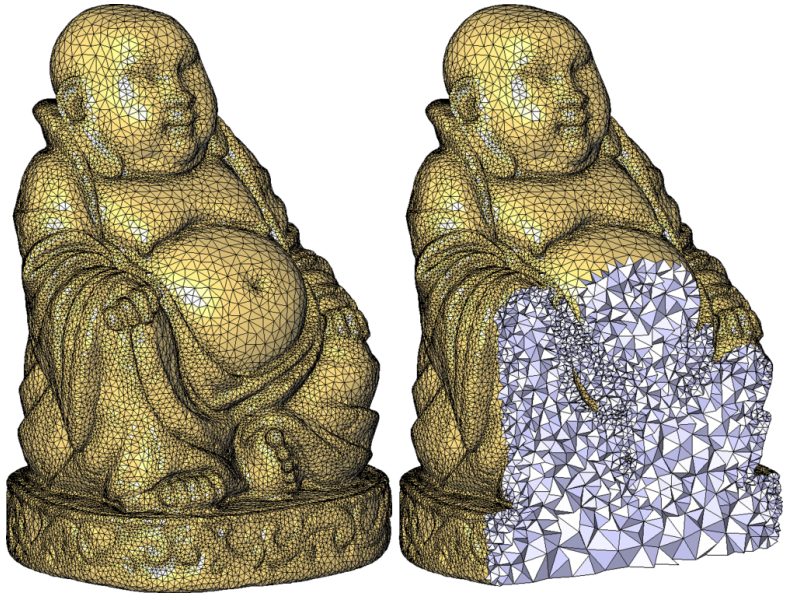
- Polygonal meshes (polygons are not restricted to triangles)
- Parametric surfaces
- Tet-meshes (volumetric)
- Voxel grids (volumetric)
- ...

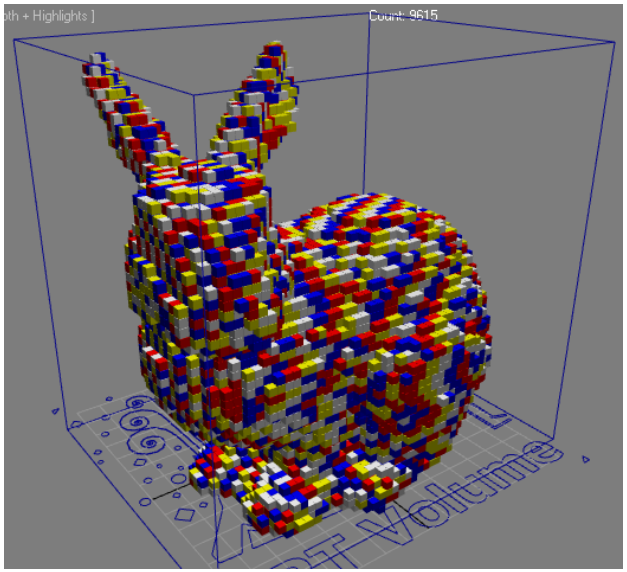
The volumetric representations model the interior of the object, whereas the shape surface is the corresponding boundary.

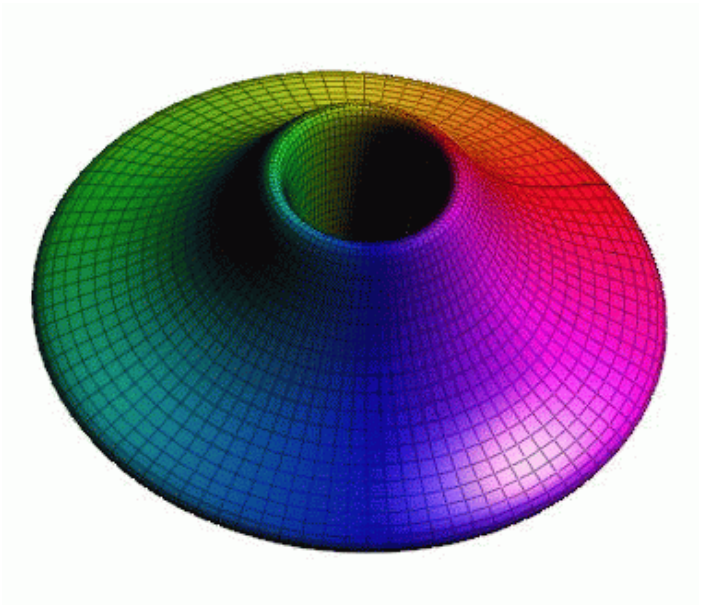
However, these alternative representations will not be considered in the remainder of this course.

Example: Other representations









Shape representation: Key ingredients

- We want to represent surfaces
- Each point can have attributes attached to it (e.g., RGB color)

Shape representation: Key ingredients

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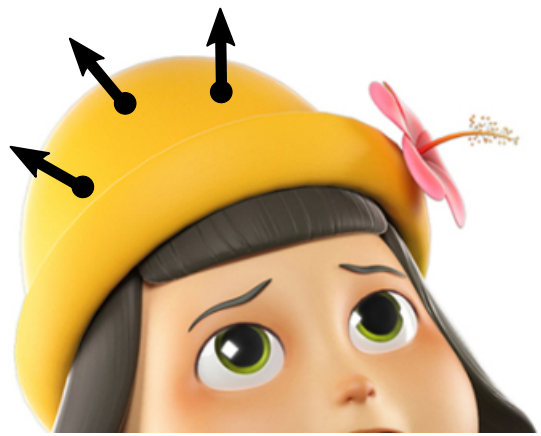
- Each point can have attributes attached to it (e.g., RGB color)
- Surfaces have an orientation defined by normal vectors



Visualization of 3 normal vectors

Shape representation: Triangle mesh

A triangle mesh is a collection of connected triangles.

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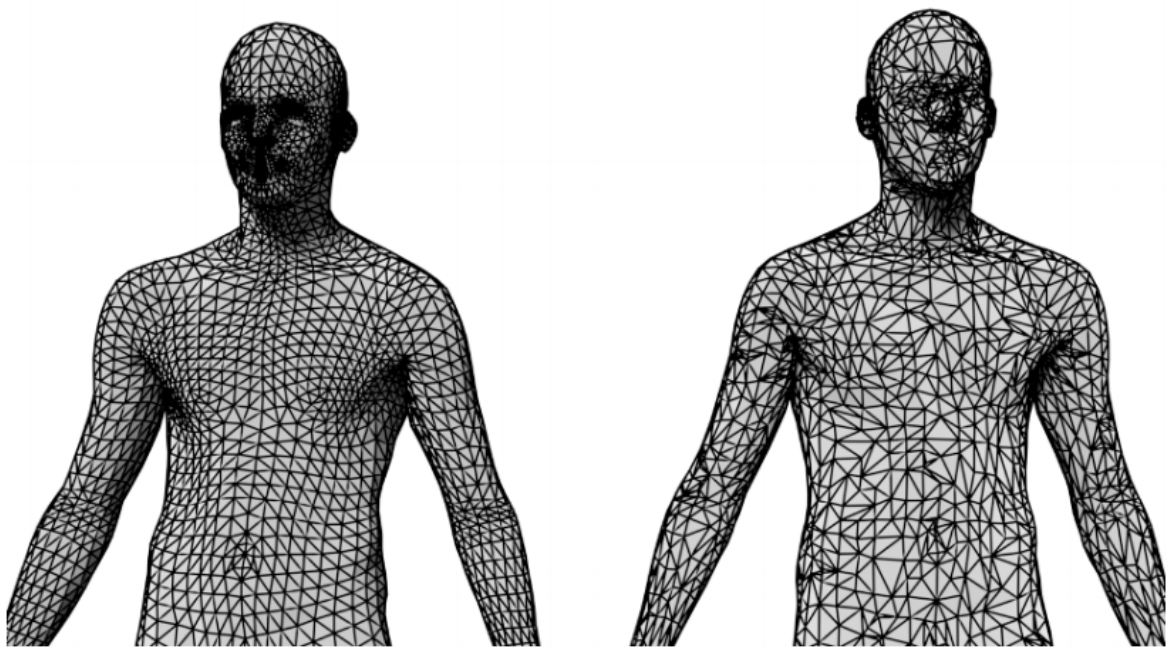
A triangle mesh is a collection of connected triangles.

The incidence relations of triangles defines the mesh connectivity (also referred to as mesh topology).

In this example, the same underlying surface is discretized with meshes having different connectivity:

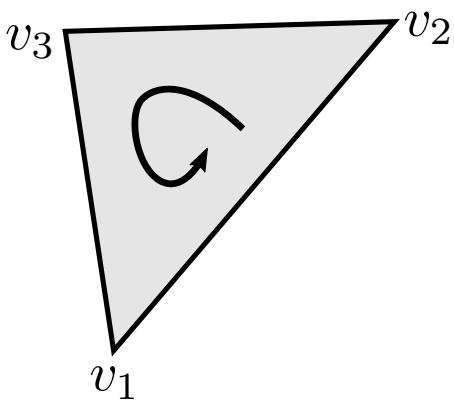


Shape representation: Triangle mesh

A triangle mesh is a collection of connected triangles.

We will only consider oriented manifold meshes.

- Each triangle has an orientation



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A triangle mesh is a collection of connected triangles.

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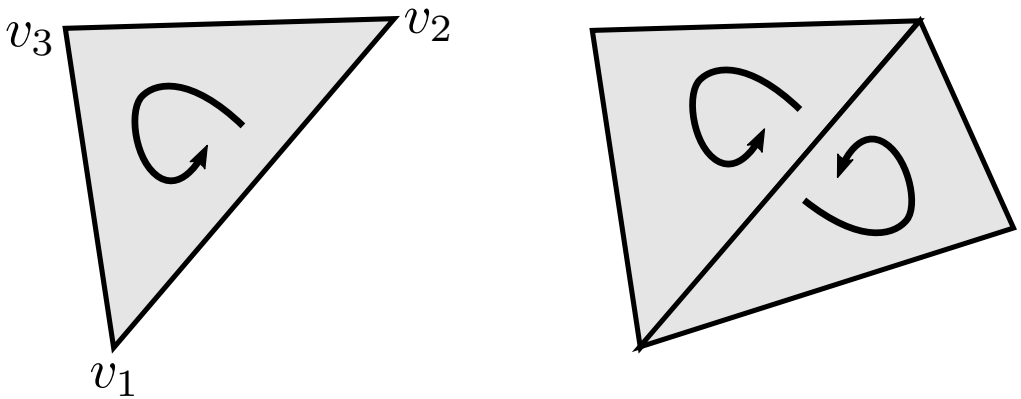
- All triangles should be consistently oriented (e.g. counter-clockwise)



Shape representation: Triangle mesh

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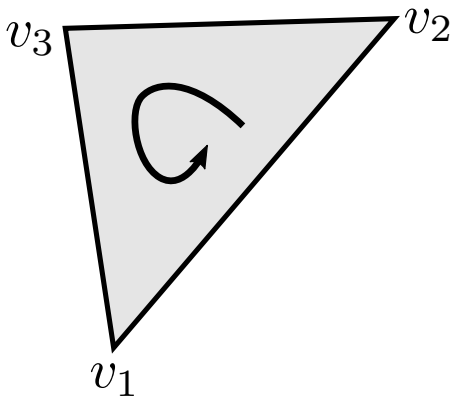
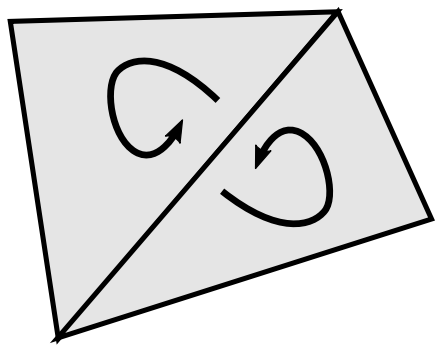
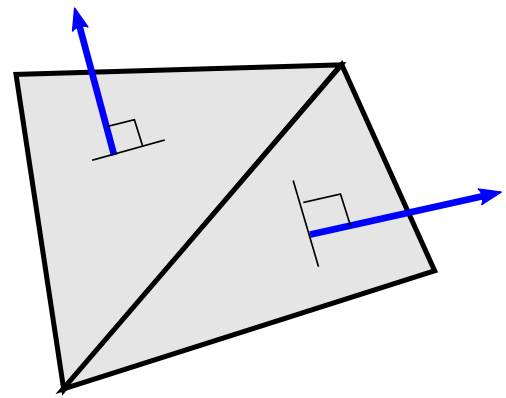
We will only consider oriented manifold meshes.

- Each triangle has an orientation
- All triangles should be consistently oriented (e.g. counter-clockwise)
- Each triangle has a normal (i.e. orthogonal) vector consistent with the triangle orientation







Shape representation: Triangle mesh

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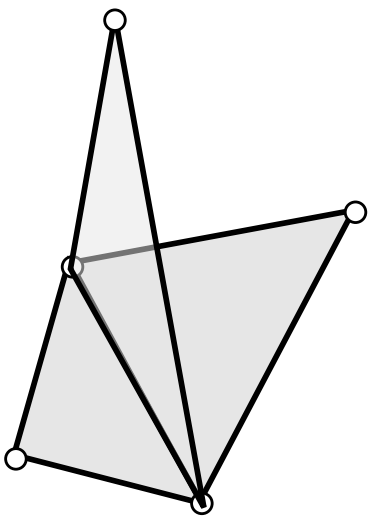
- All edges have at most two incident triangles



3 triangles incident to 1 edge
$\Rightarrow$ non-manifold

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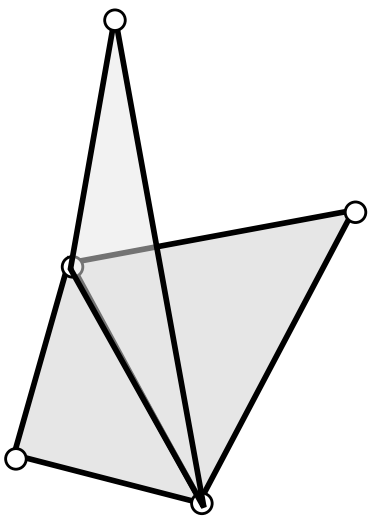
- Edges with only one incident triangle form the mesh boundary



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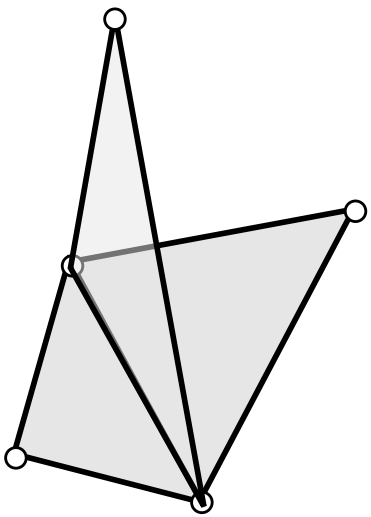
Shape representation: Triangle mesh

A triangle mesh is a collection of connected triangles.

We will only consider oriented manifold meshes.

- All edges have at most two incident triangles
- Edges with only one incident triangle form the mesh boundary
- The faces incident to a vertex form a closed or an open fan



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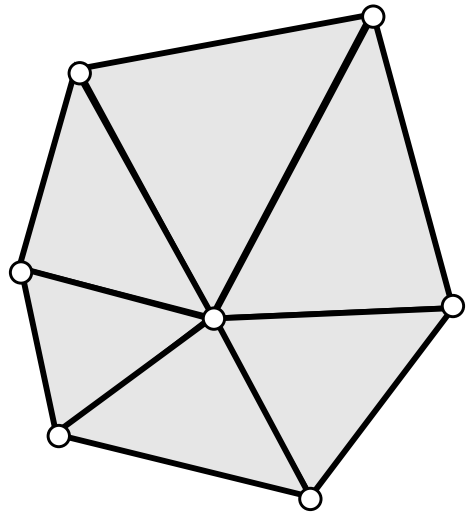
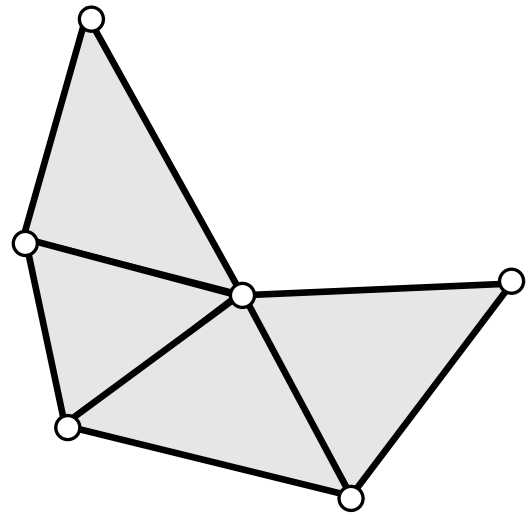
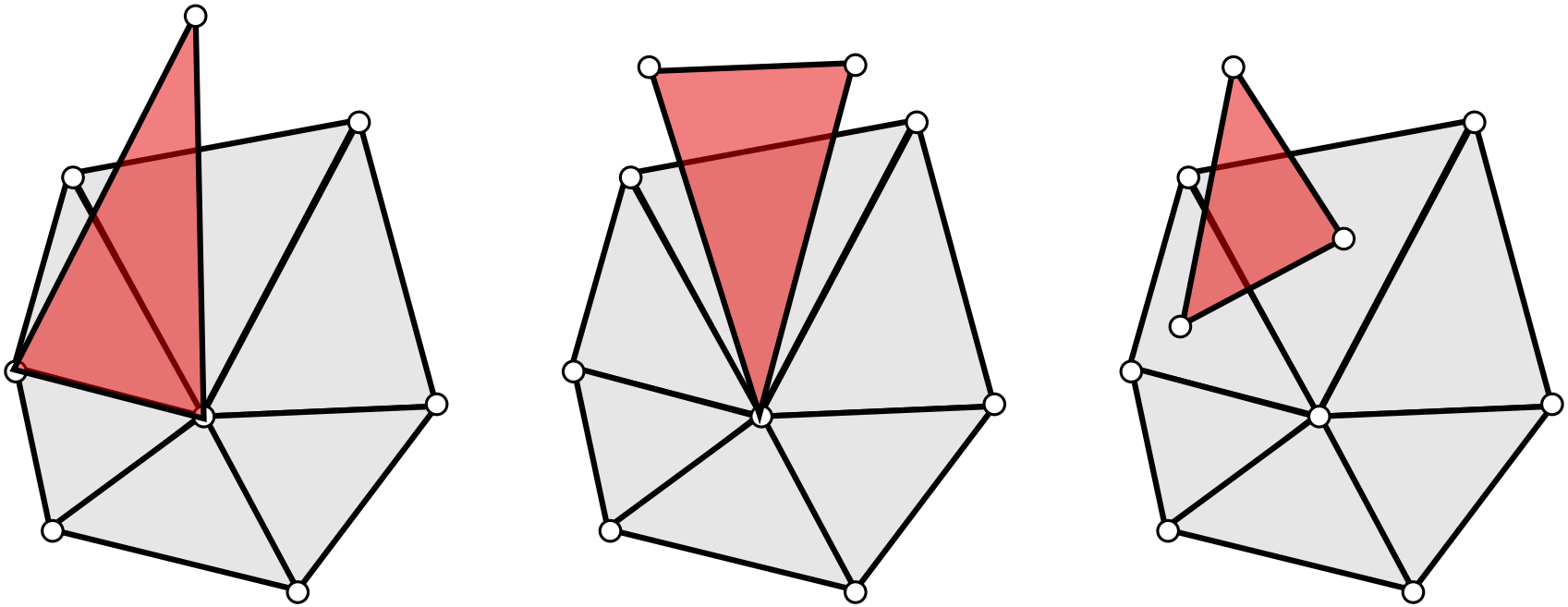


closed fan



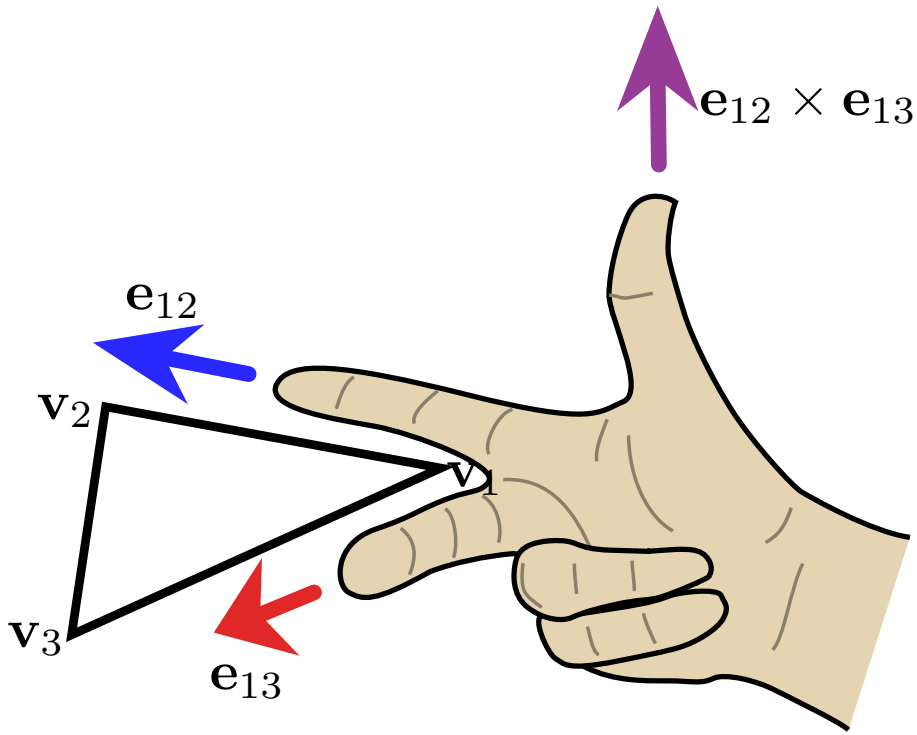
open fan

Example: Non-manifold meshes



Normals

Normals are unit vectors that are orthogonal to each face



Normals

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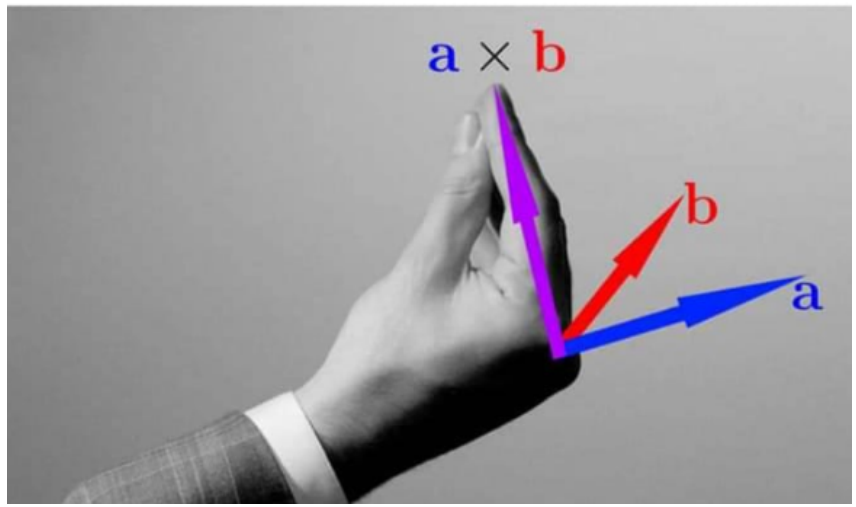
Italian right-hand rule



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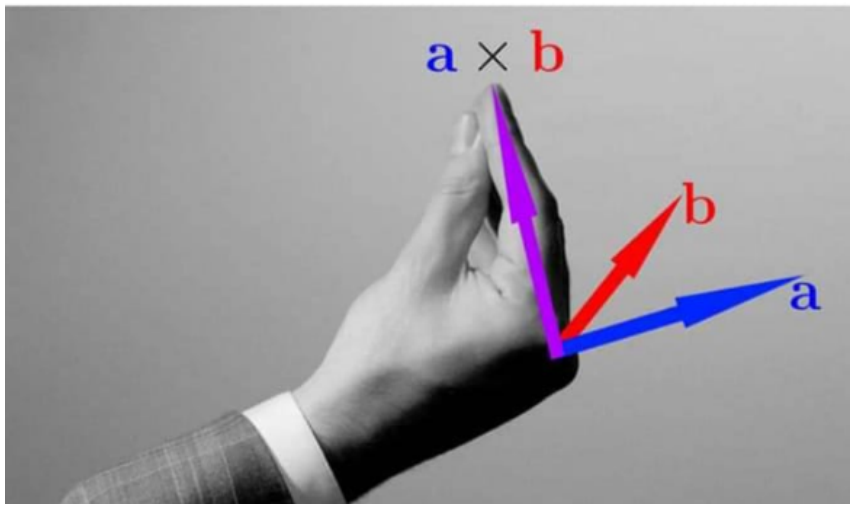

$$\hat{\mathbf{n}} = \frac{\mathbf{e}_{12} \times \mathbf{e}_{13}}{\|\mathbf{e}_{12} \times \mathbf{e}_{13}\|}$$

Shape representation: Point cloud

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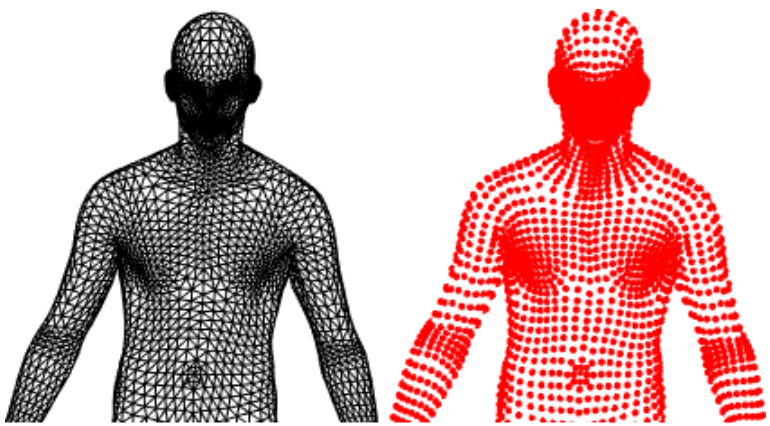
- Point clouds are interpreted as point-wise samplings of an underlying unknown continuous surface...

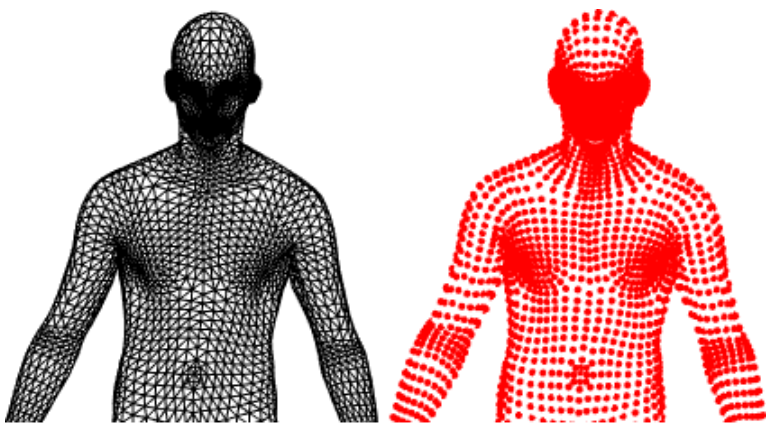


Synthetic point cloud
(obtained by removing mesh connectivity)

Shape representation: Point cloud

A point cloud is a collection of points in 3D space.

- An oriented point cloud also has a normal vector for each point
- Point clouds are interpreted as point-wise samplings of an underlying unknown continuous surface...
- ...in practice, they come from depth sensors and can be very noisy!



Synthetic point cloud
(obtained by removing mesh connectivity)

Real-world Kinect scan

Exercise: Triangle mesh data structure

Create a data structure to represent a generic triangle mesh. The data structure must contain the following information:

- A collection of (x, y, z) coordinates for all the triangle vertices;
  Example: A matrix of size $n \times 3$, where n is the total number of vertices
- A collection of triangles (v_1, v_2, v_3); each v_1, v_2, v_3 is an index to the set of vertices;
  Example: A matrix of size $m \times 3$, where m is the total number of triangles

Additionally, write the following functions:

- calc_tri_areas (): computes the area for each triangle
- calc_normals (): computes the normal vector for each triangle

Test your code by loading and visualizing cat0.off (download from the course website)

Exercise: Point cloud data structure

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- A collection of (x, y, z) coordinates for all the point vertices;
  Example: A matrix of size $n \times 3$, where n is the total number of vertices

Test your code by encoding the mesh from the previous example simply as a point cloud, and by visualizing it

Image representation

We represent images as uniform grids of pixels

Each pixel is a number or a color vector

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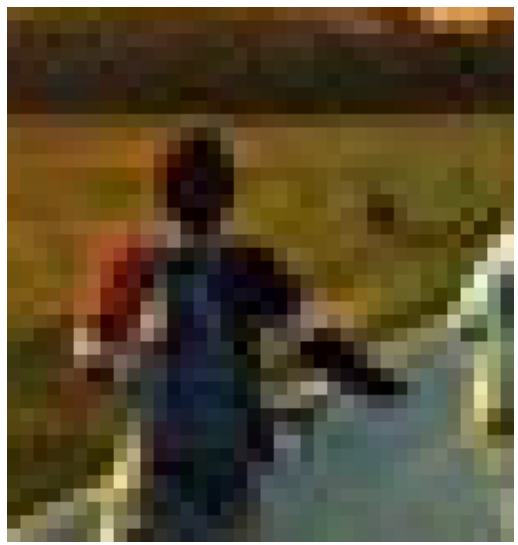
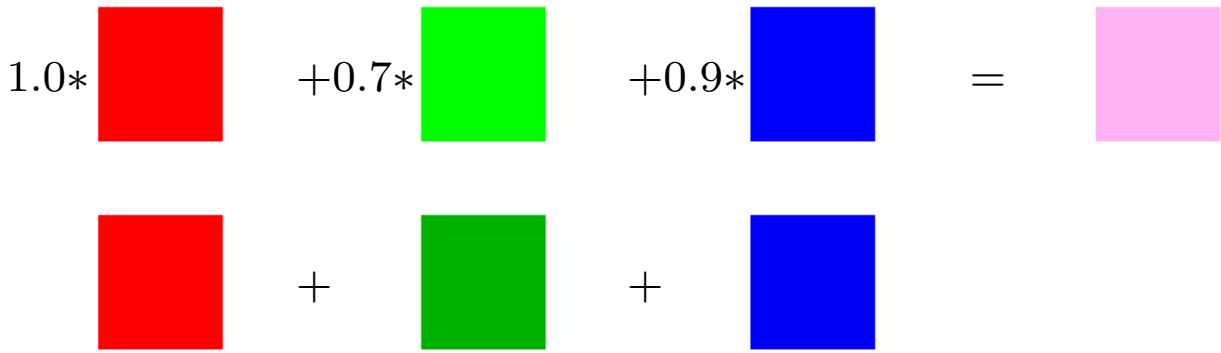


Example: RGB color space

A common way to express color is as a mixture of **R**ed, **G**reen, **B**lue



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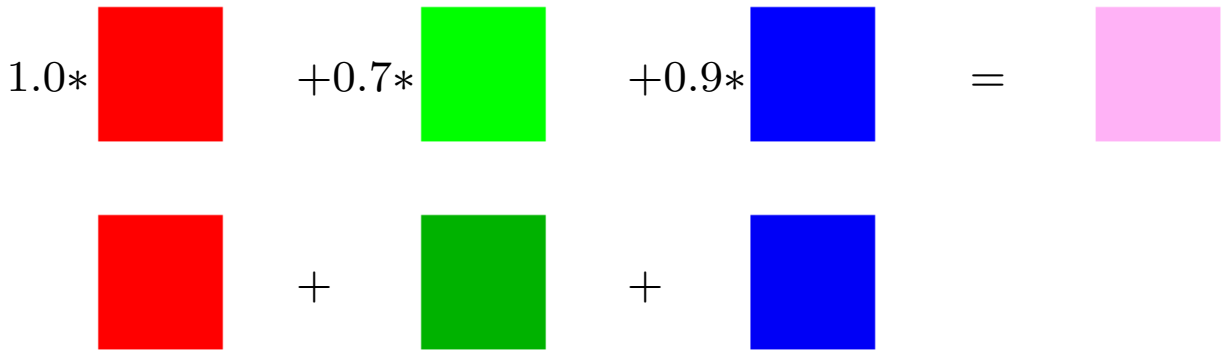

The choice of the color space depends on the task

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Example: RGB color space

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