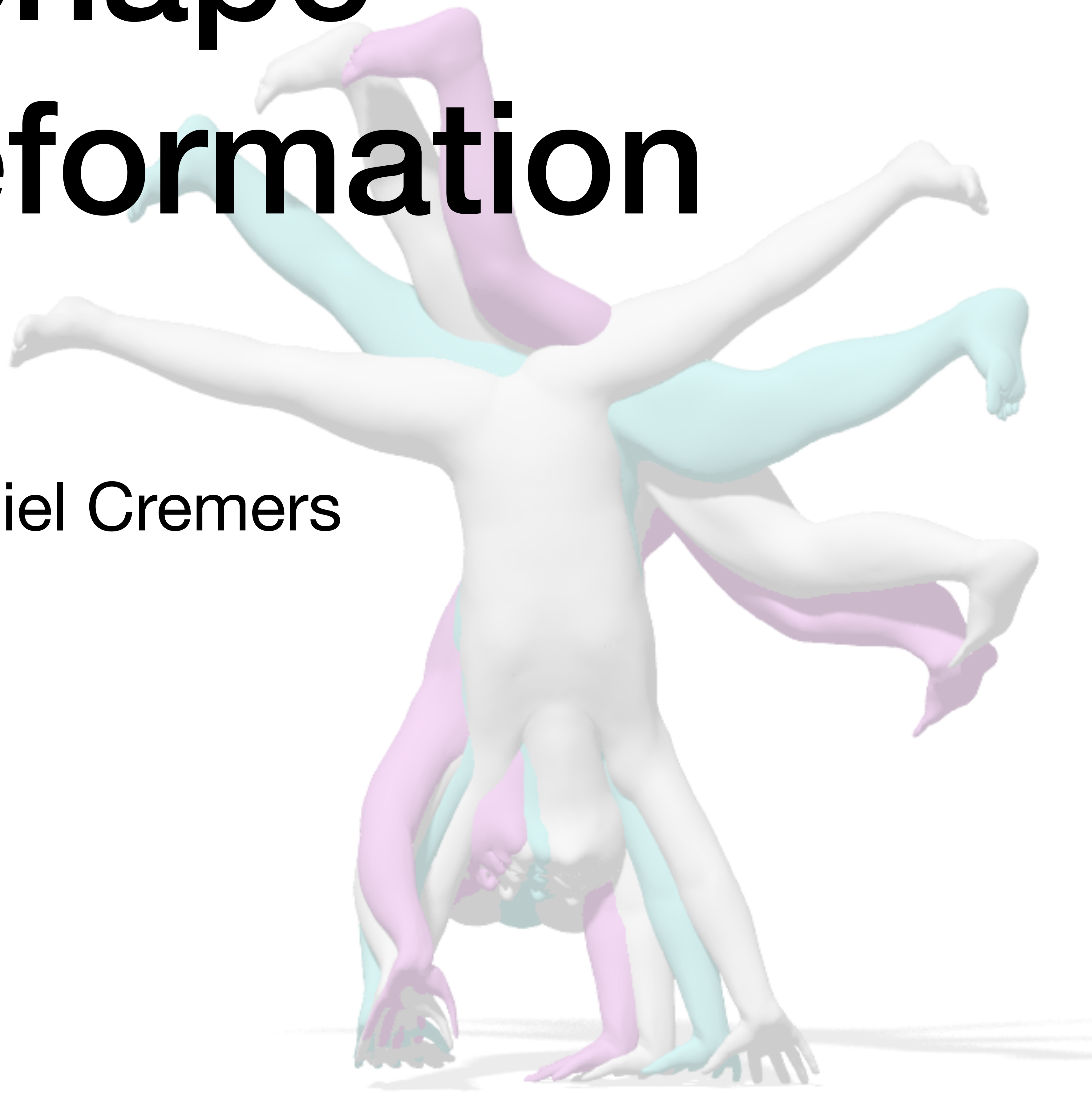
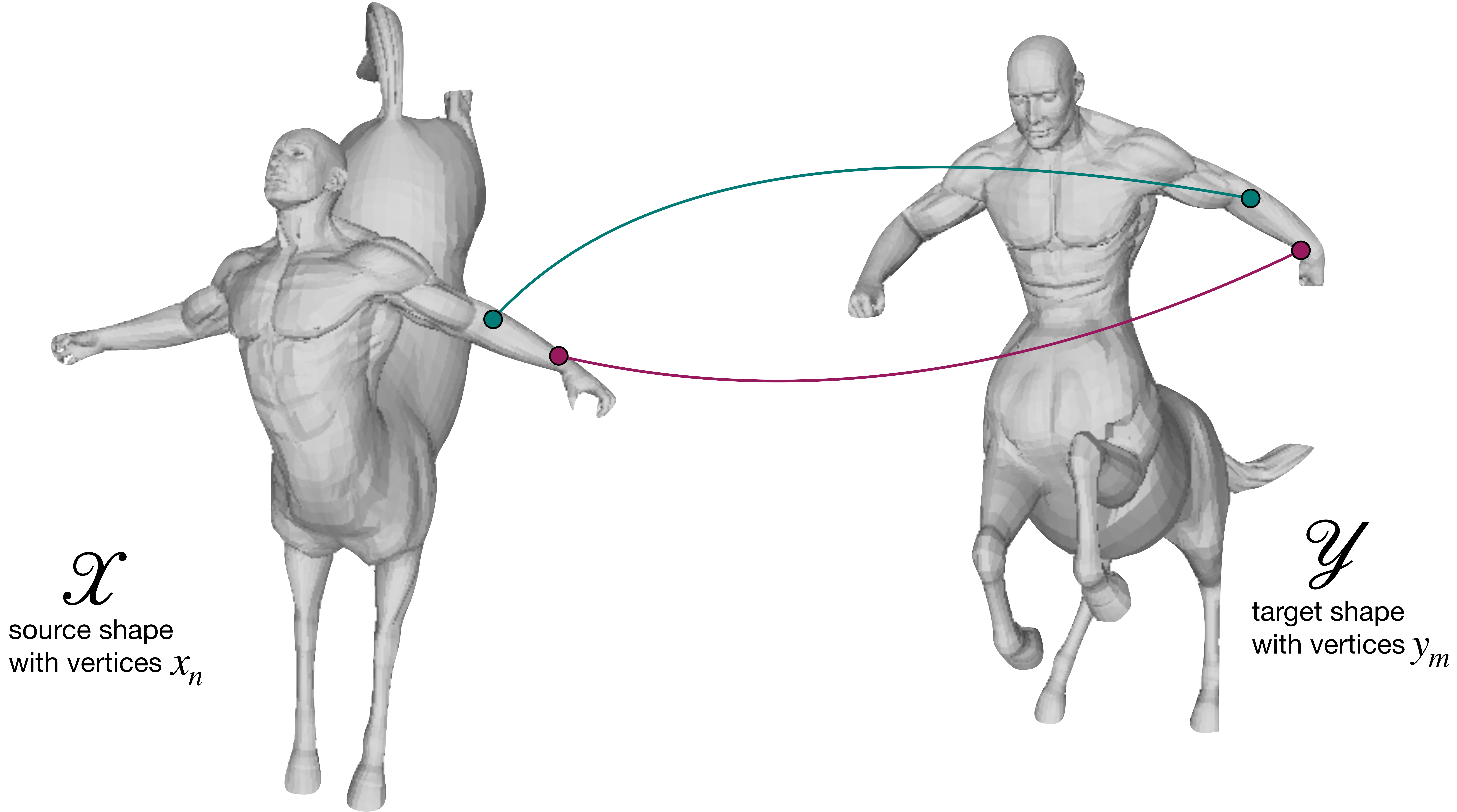
Divergence-Free Shape Correspondence by Deformation



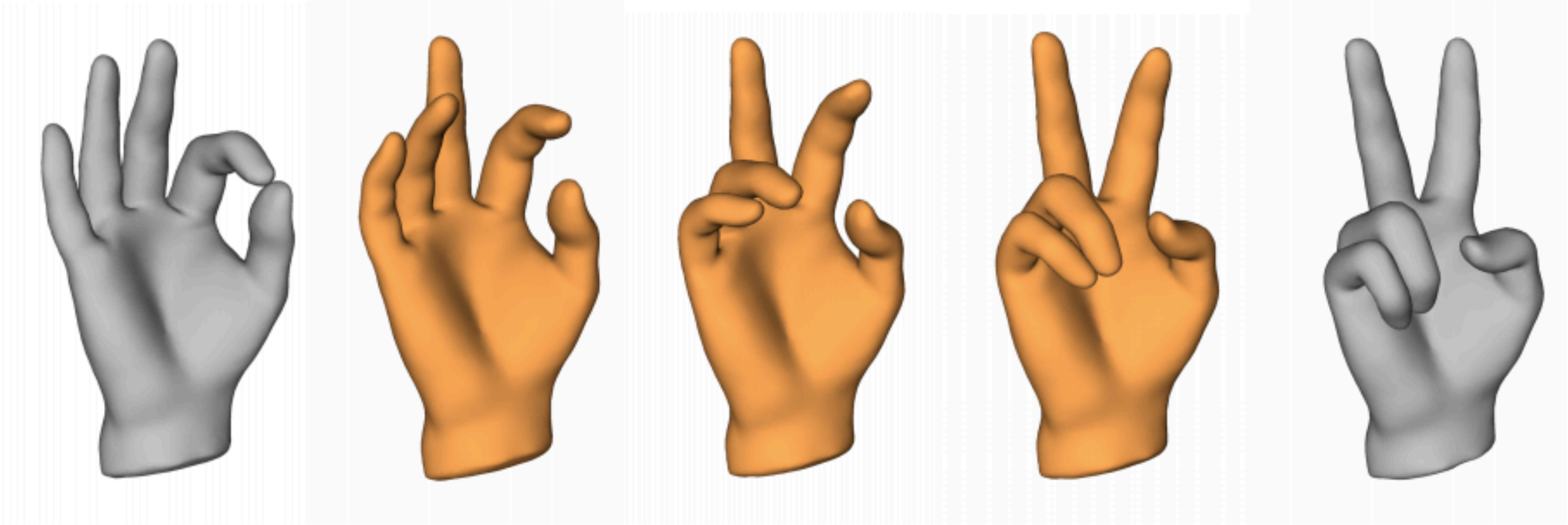
Marvin Eisenberger, **Zorah Lähner** and Daniel Cremers

Technical University of Munich

Shape Correspondence



Shape Interpolation



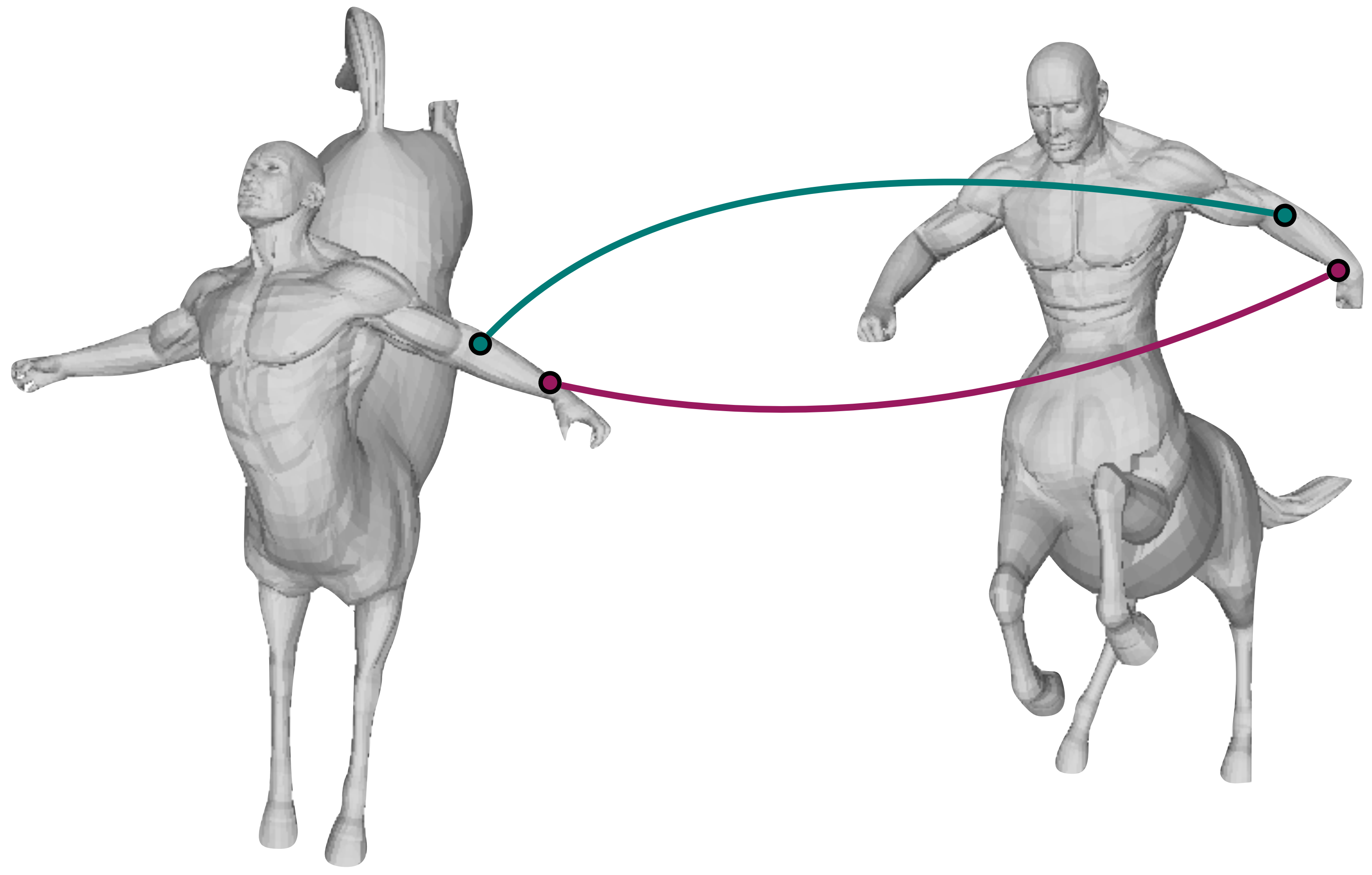
source shape | target shape

Heeren, Rumpf, Wardetzky, Wirth: Time-Discrete Geodesics in the Space of Shells, 2012.

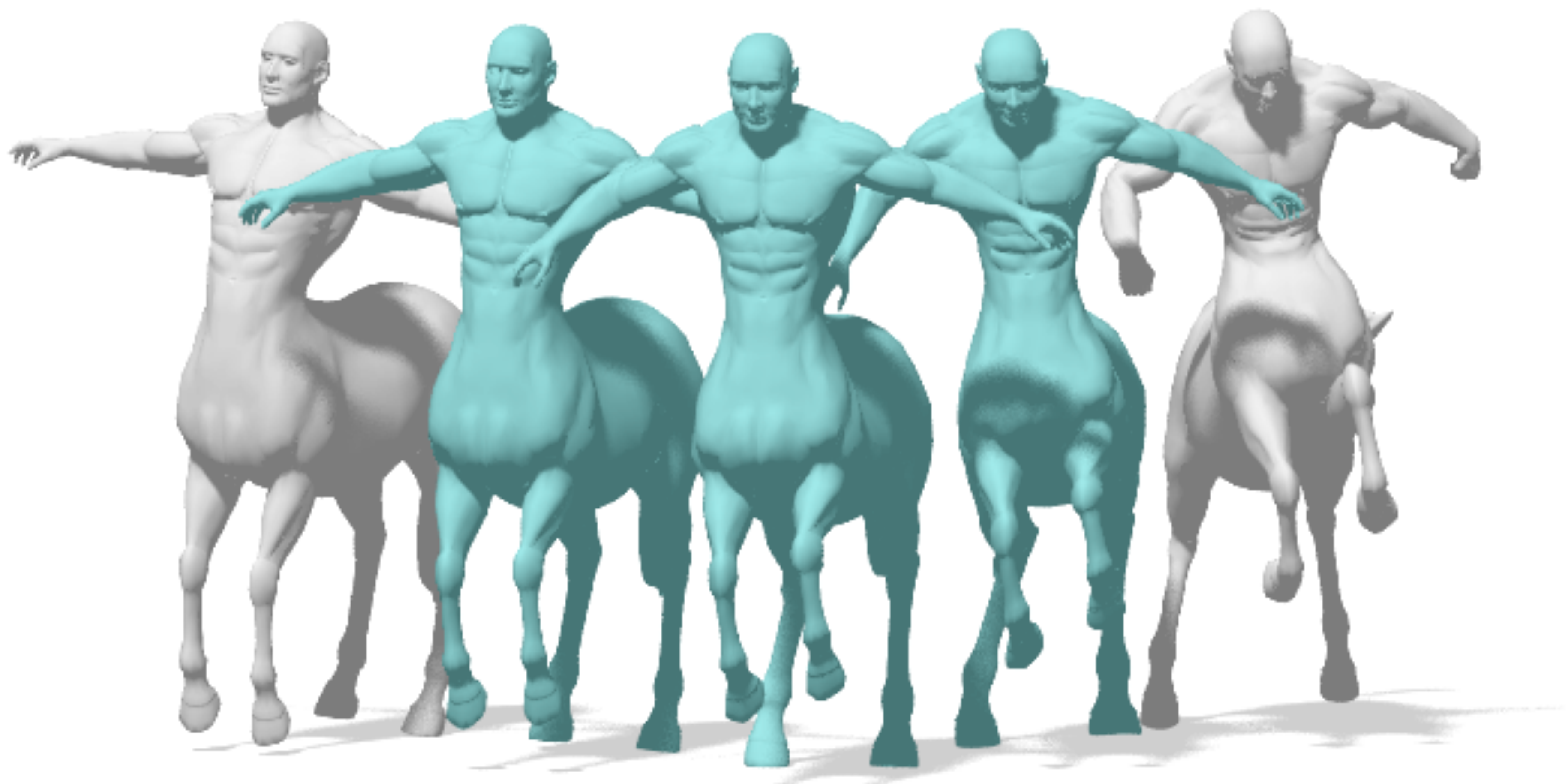
Shape Interpolation

Chair for Computer Vision
and Artificial Intelligence

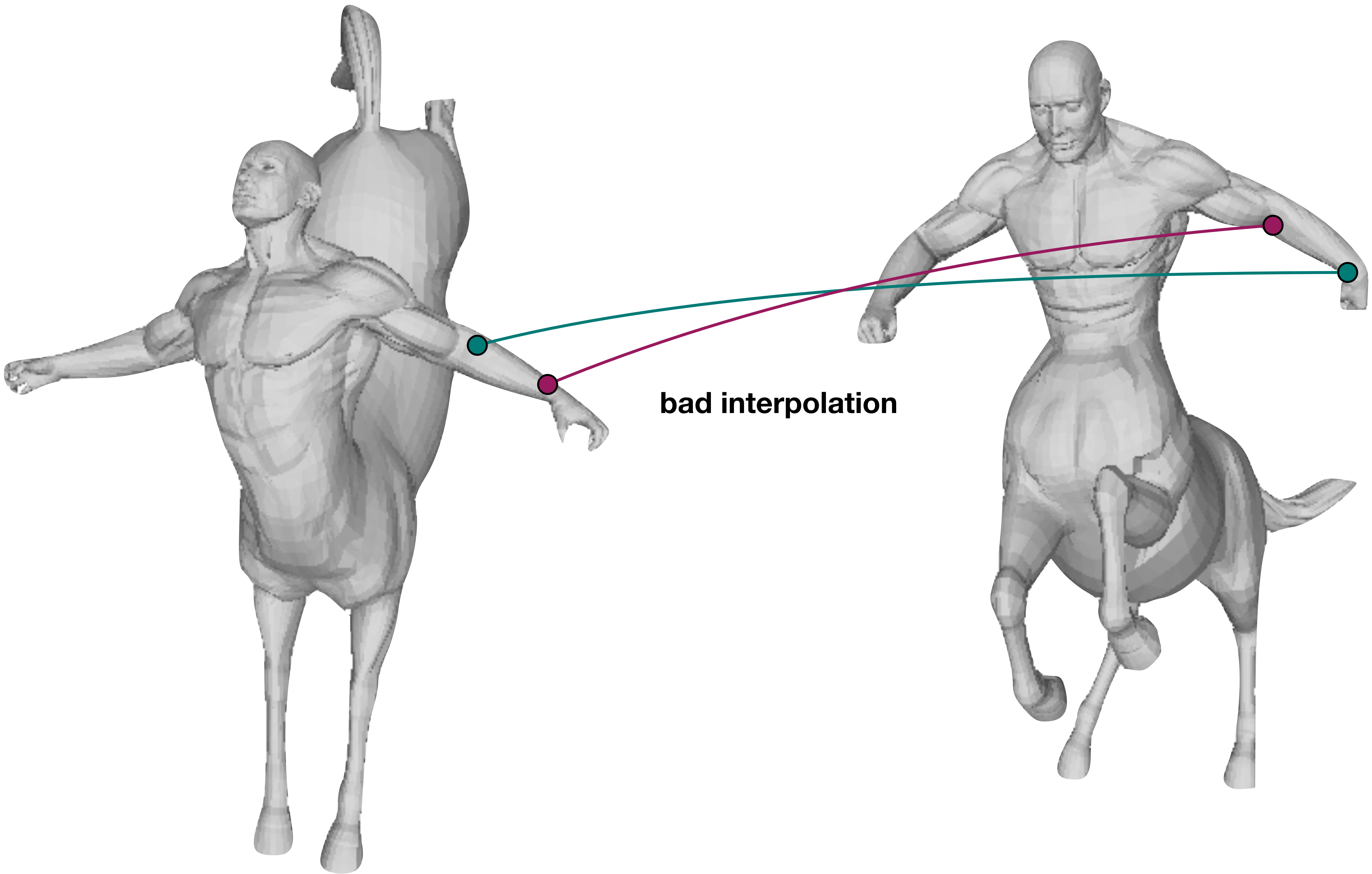
(perfect) Correspondence



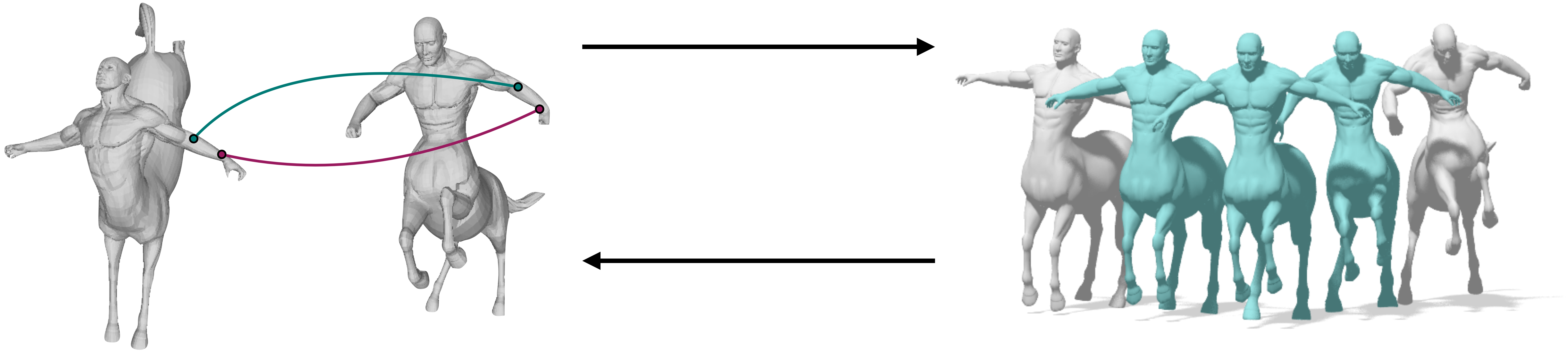
Interpolation



Noisy Correspondence



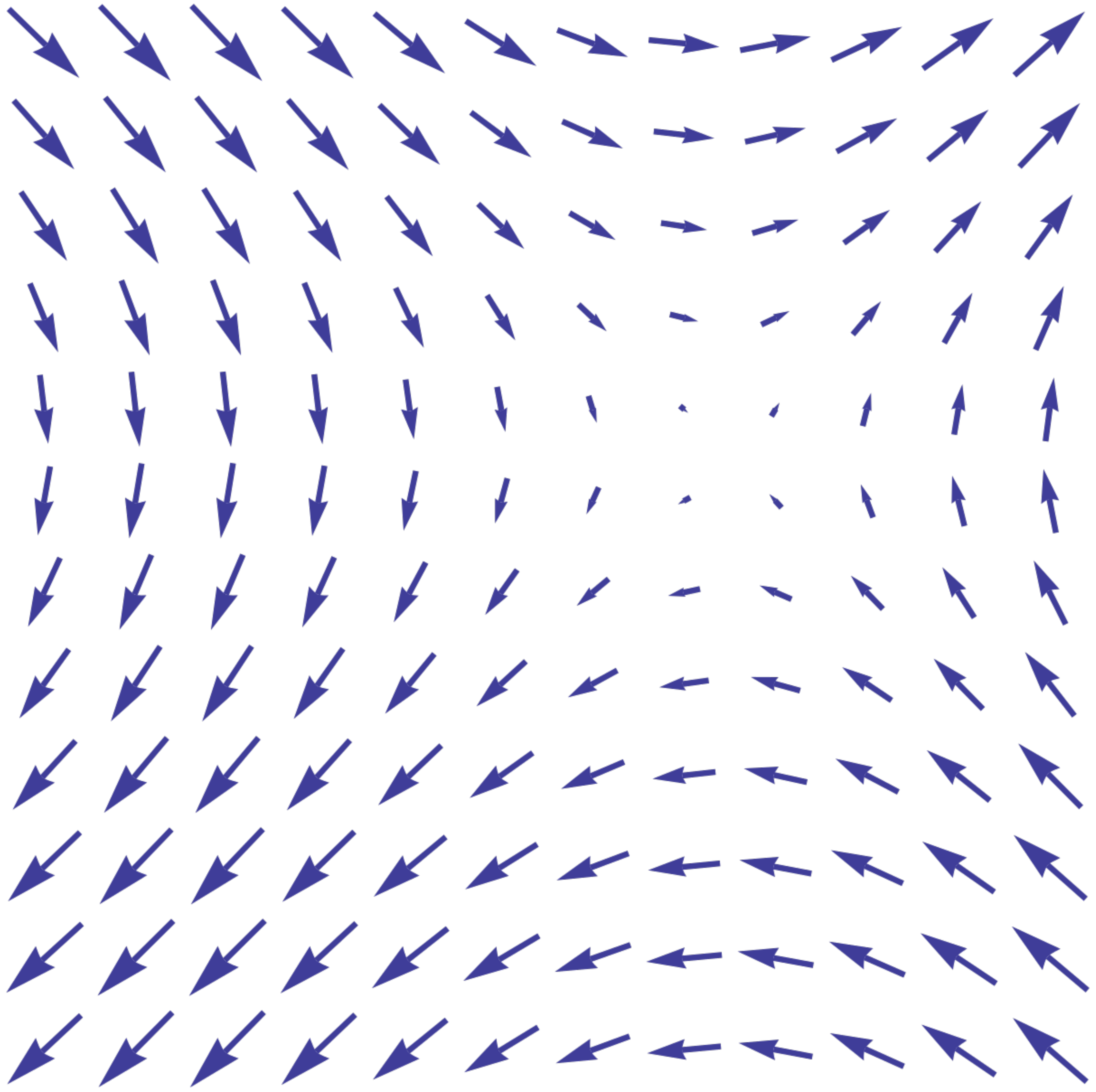
Mutual Influence



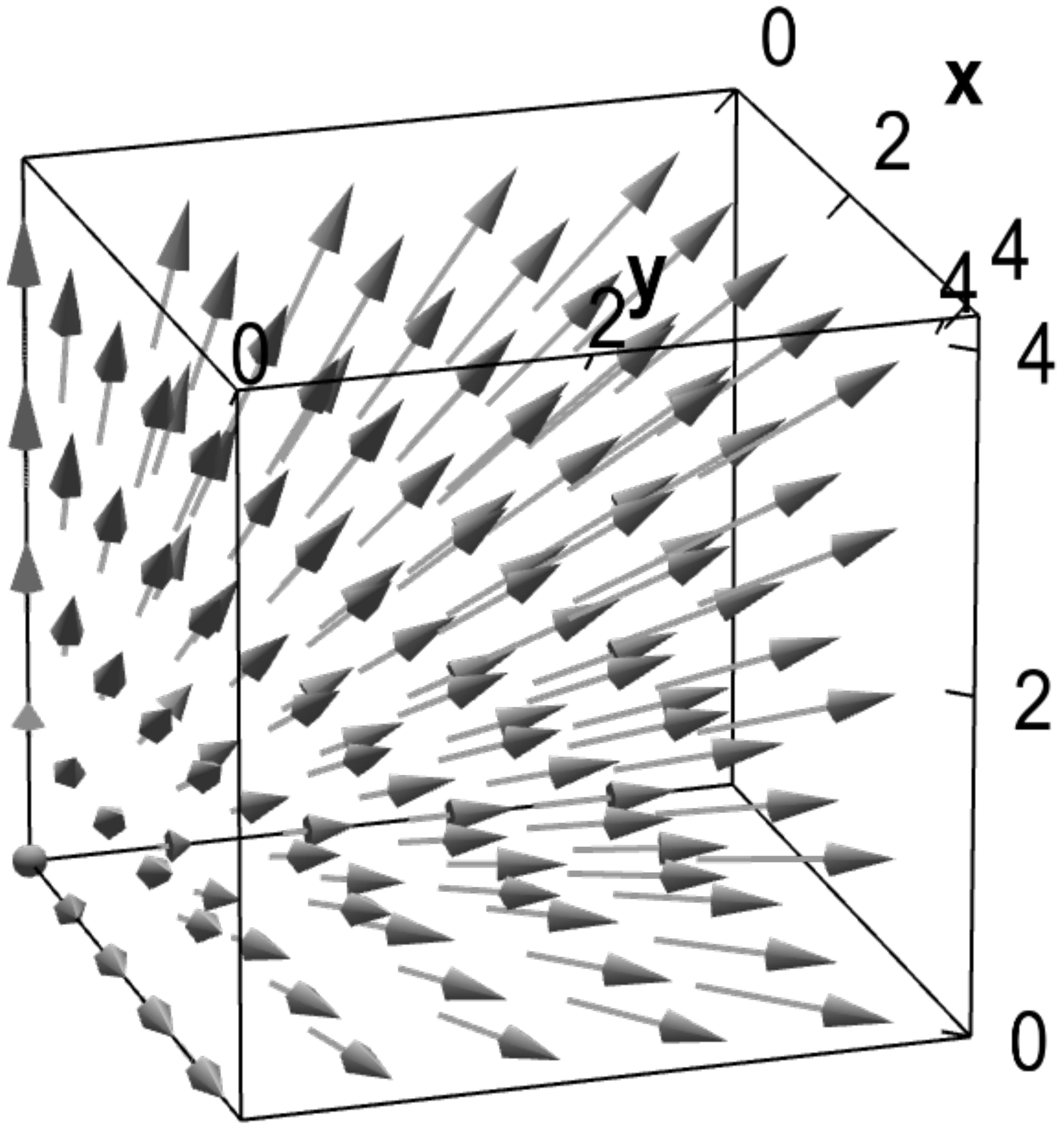
A good correspondence should lead to a good interpolation.
A good interpolation should lead to a good correspondence.

Vector Fields

in 2D

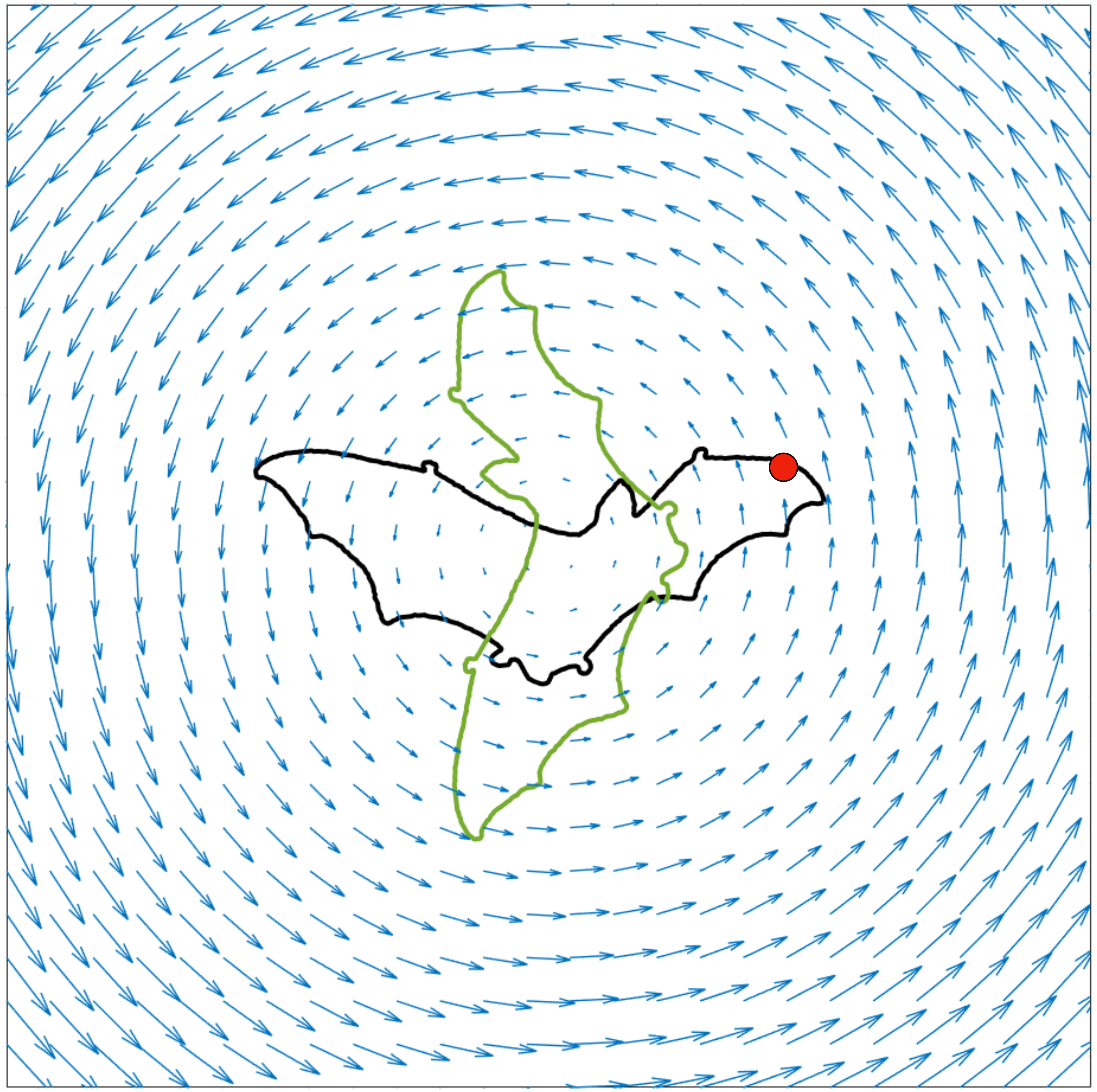


in 3D



Deformation Fields

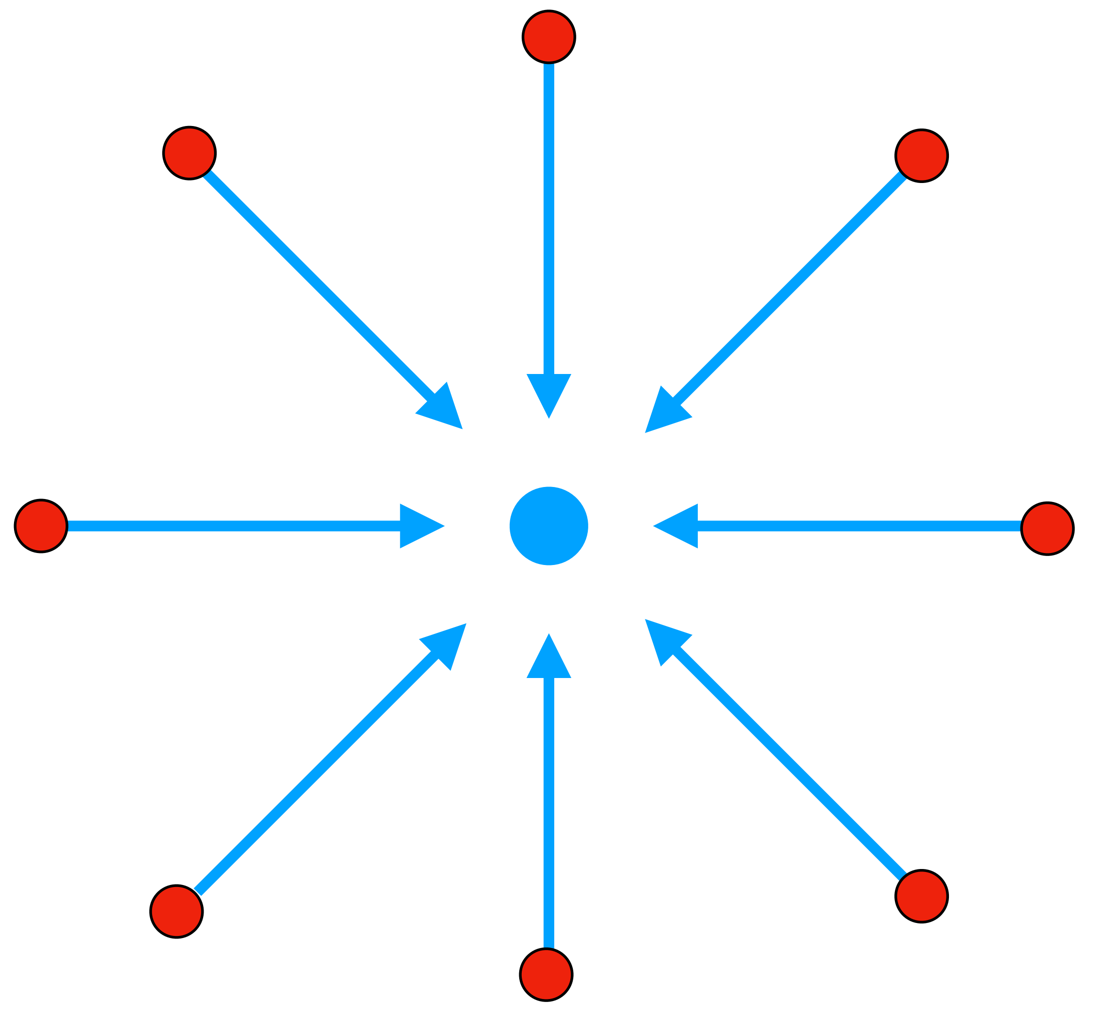
1. **Look at the point at your current position**
2. **Move an infinitesimal step in the direction indicated by the vector field at this point**
3. **Repeat**



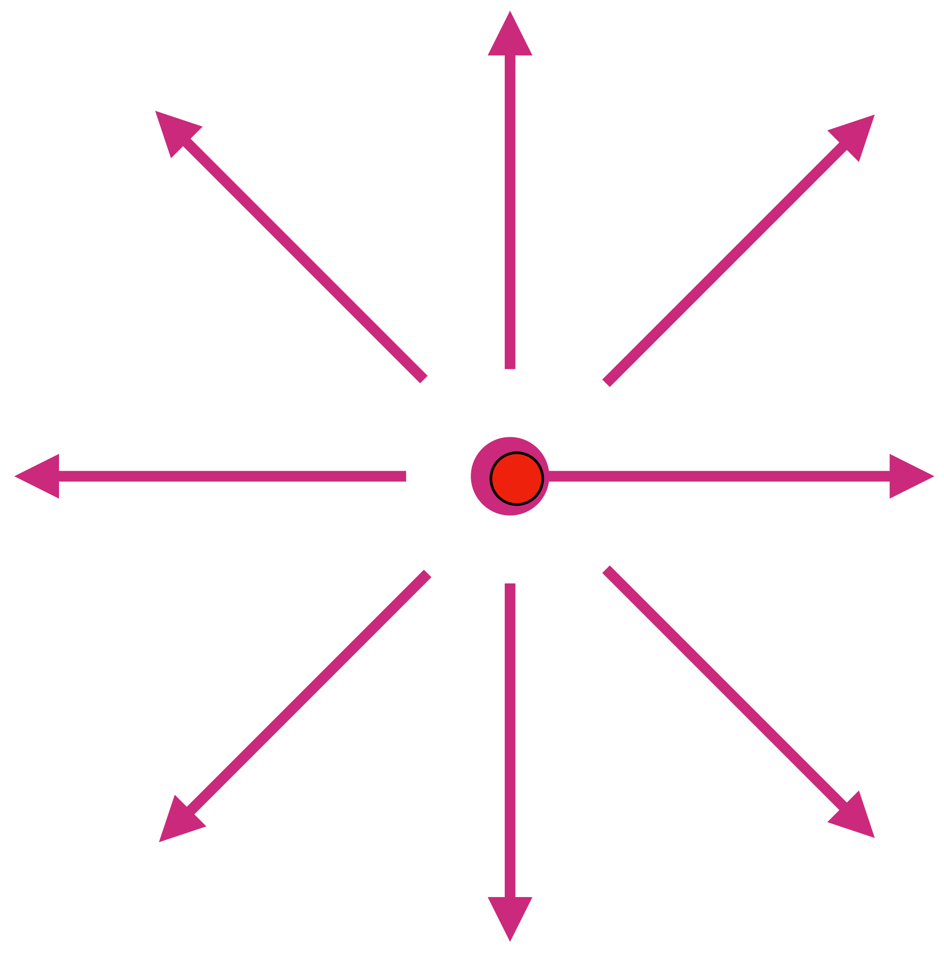
Divergence

Chair for Computer Vision and Artificial Intelligence

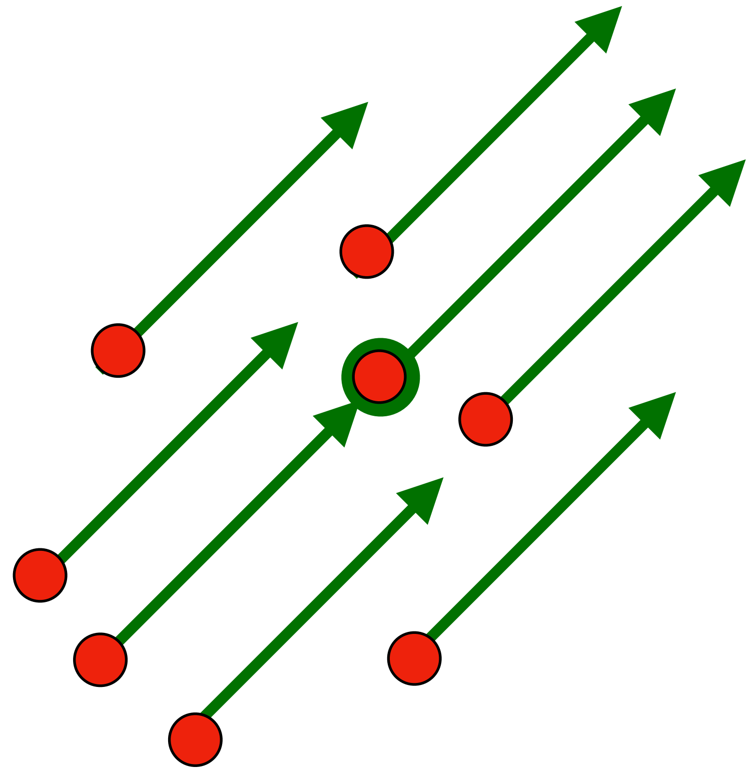
The divergence counts: 1. how many vectors point into one point
2. the magnitude of the vector pointing out of the point
3. adds both up.



negative divergence



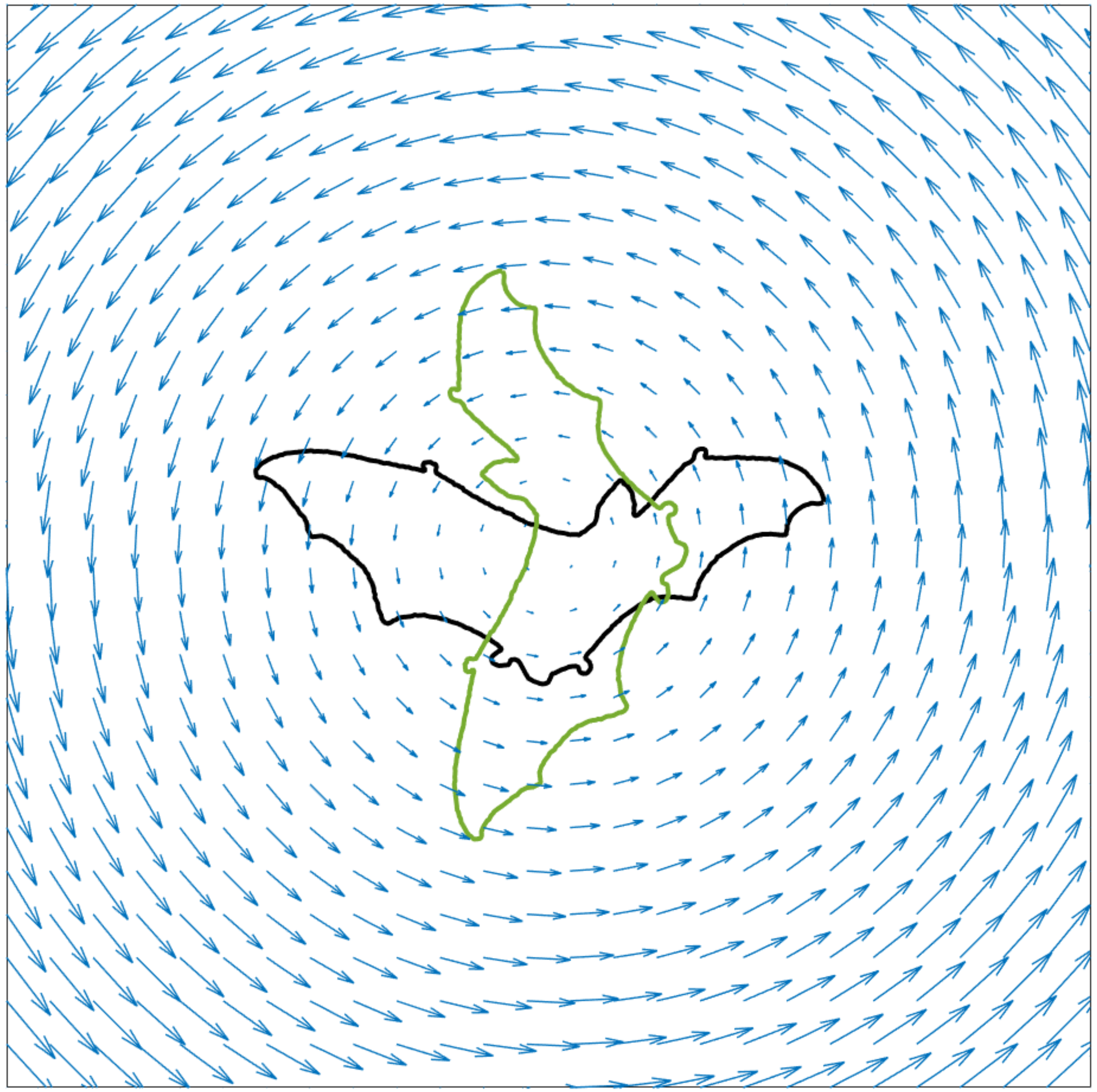
positive divergence



zero divergence

Divergence-Free Deformation Fields

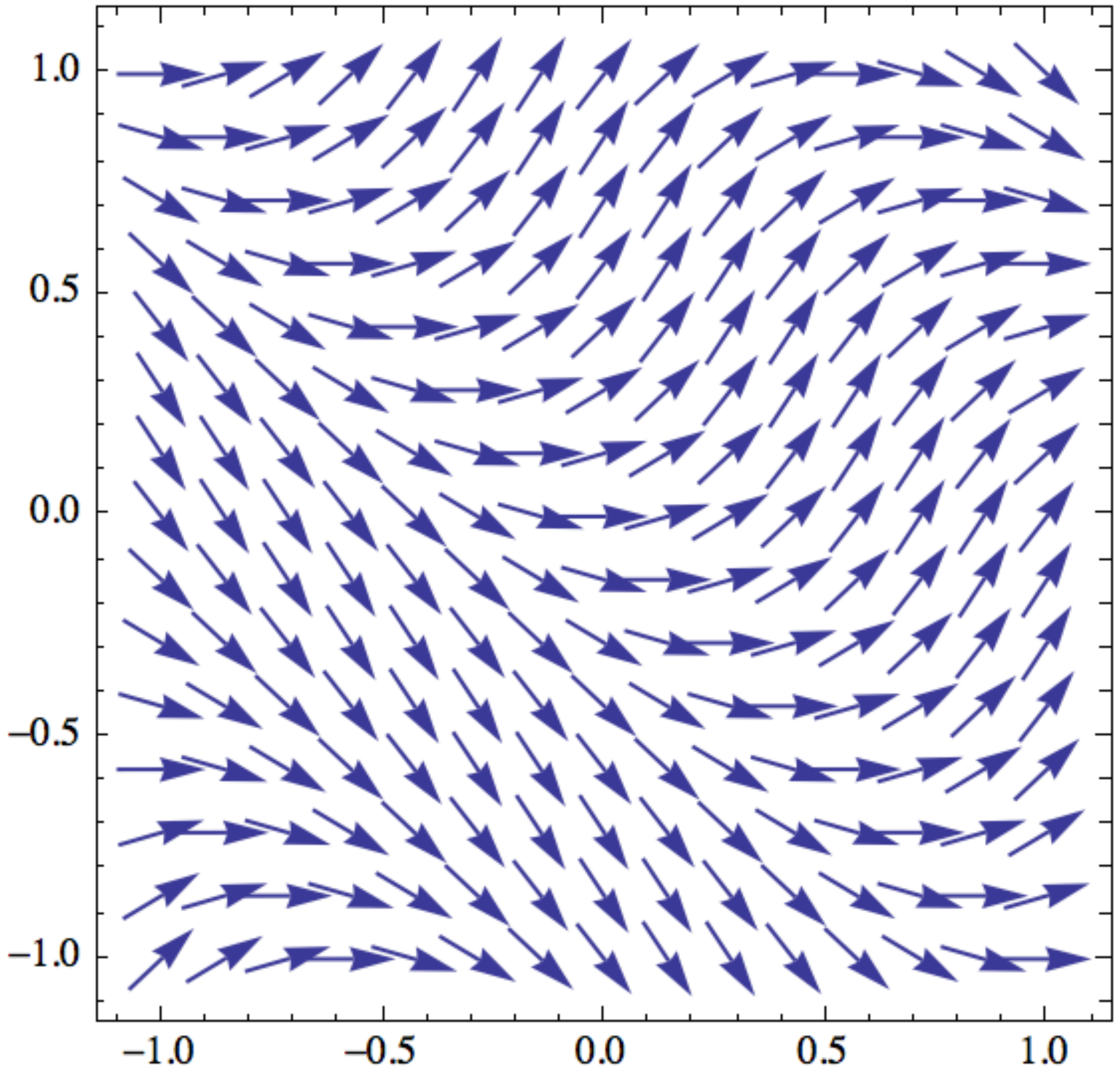
Divergence-free deformation fields preserve volume.



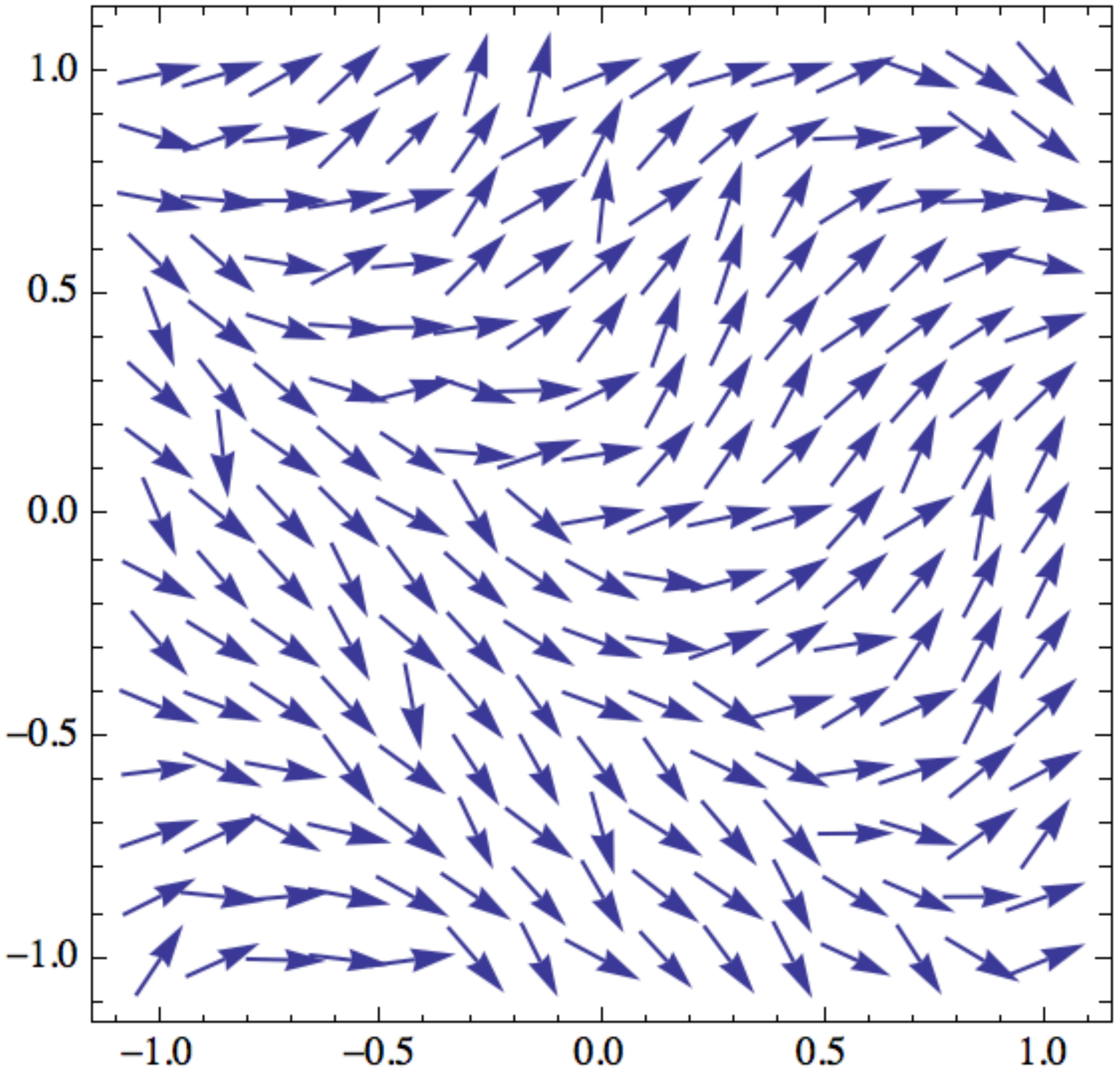
Smoothness

Chair for Computer Vision and Artificial Intelligence

smooth



not so smooth



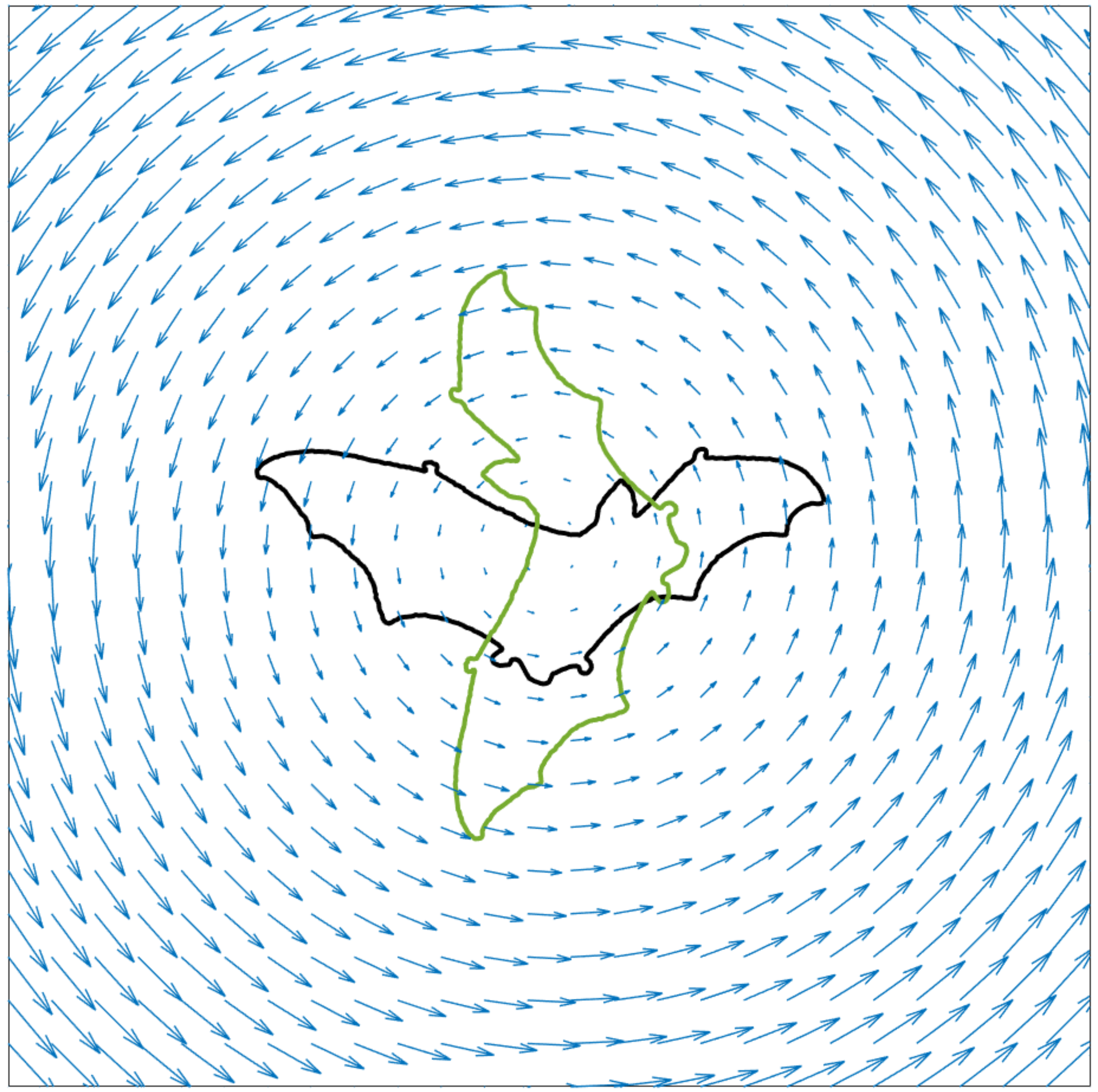
In the real world smooth movements are normally more energy efficient and therefore more likely.

Representation of Deformation Fields

1. **Discretize the space (with a grid) and assign one vector to each grid element.**

2. **Deformation Fields are just functions** $\mathbb{R}^3 \to \mathbb{R}^3$**.**
   Define a basis for those.

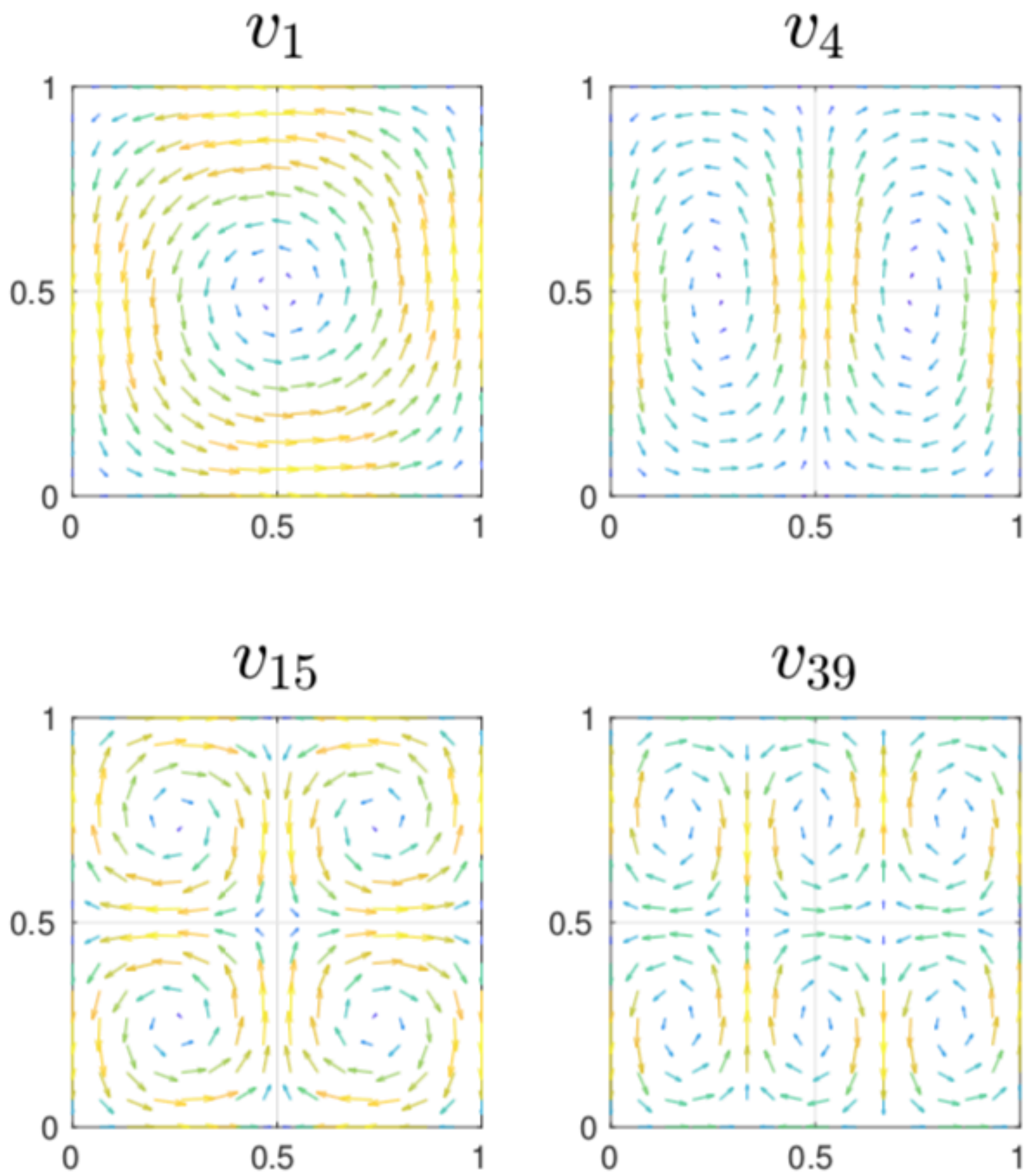
$$\sum_{k=1}^{K} a_k \cdot v_k(x)$$



Basis for Divergence-Free DFs

Chair for Computer Vision and Artificial Intelligence

There is a closed-form solution for basis functions of smooth, divergence-free DFs.

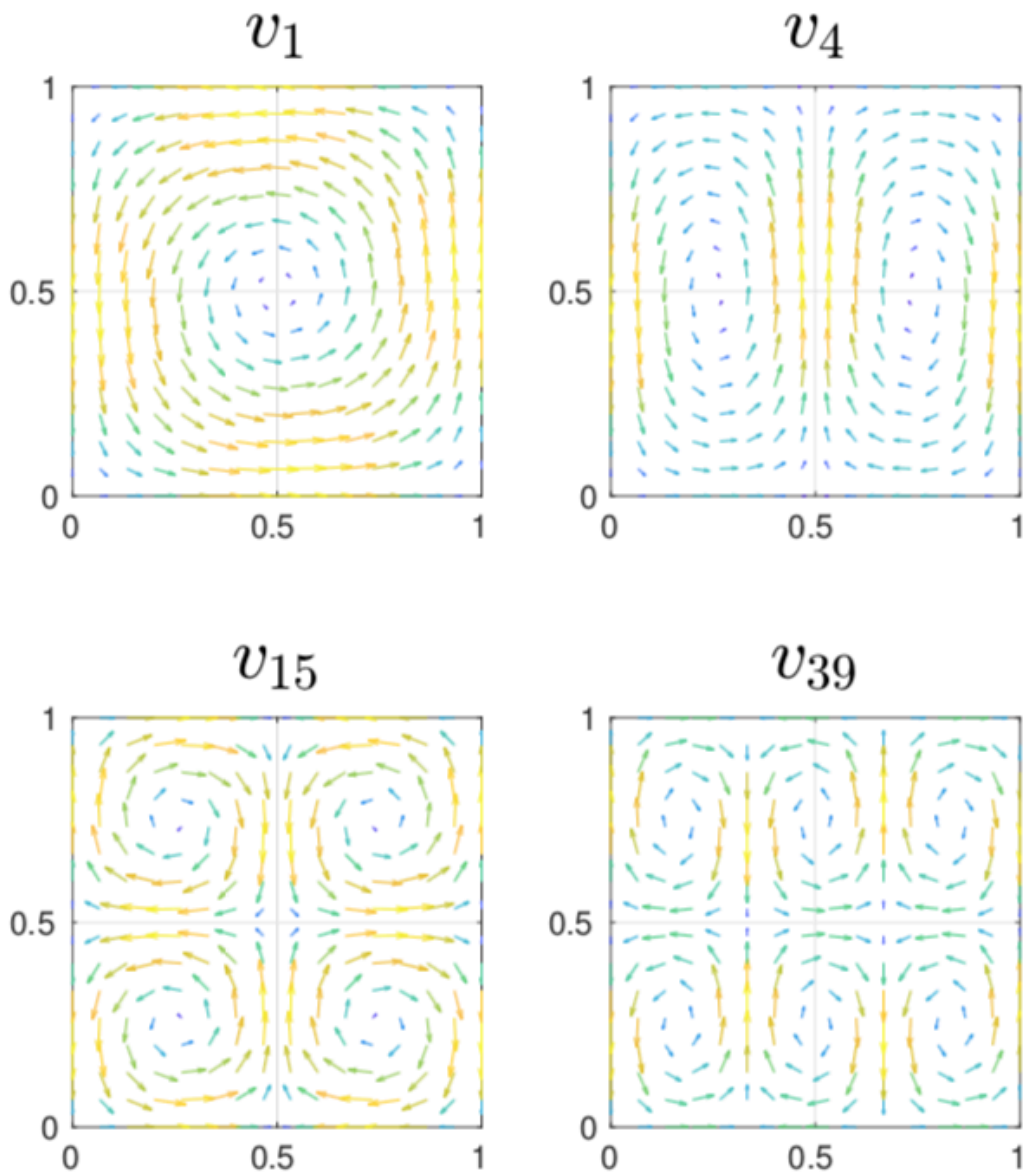


Basis for Divergence-Free DFs

Chair for Computer Vision and Artificial Intelligence

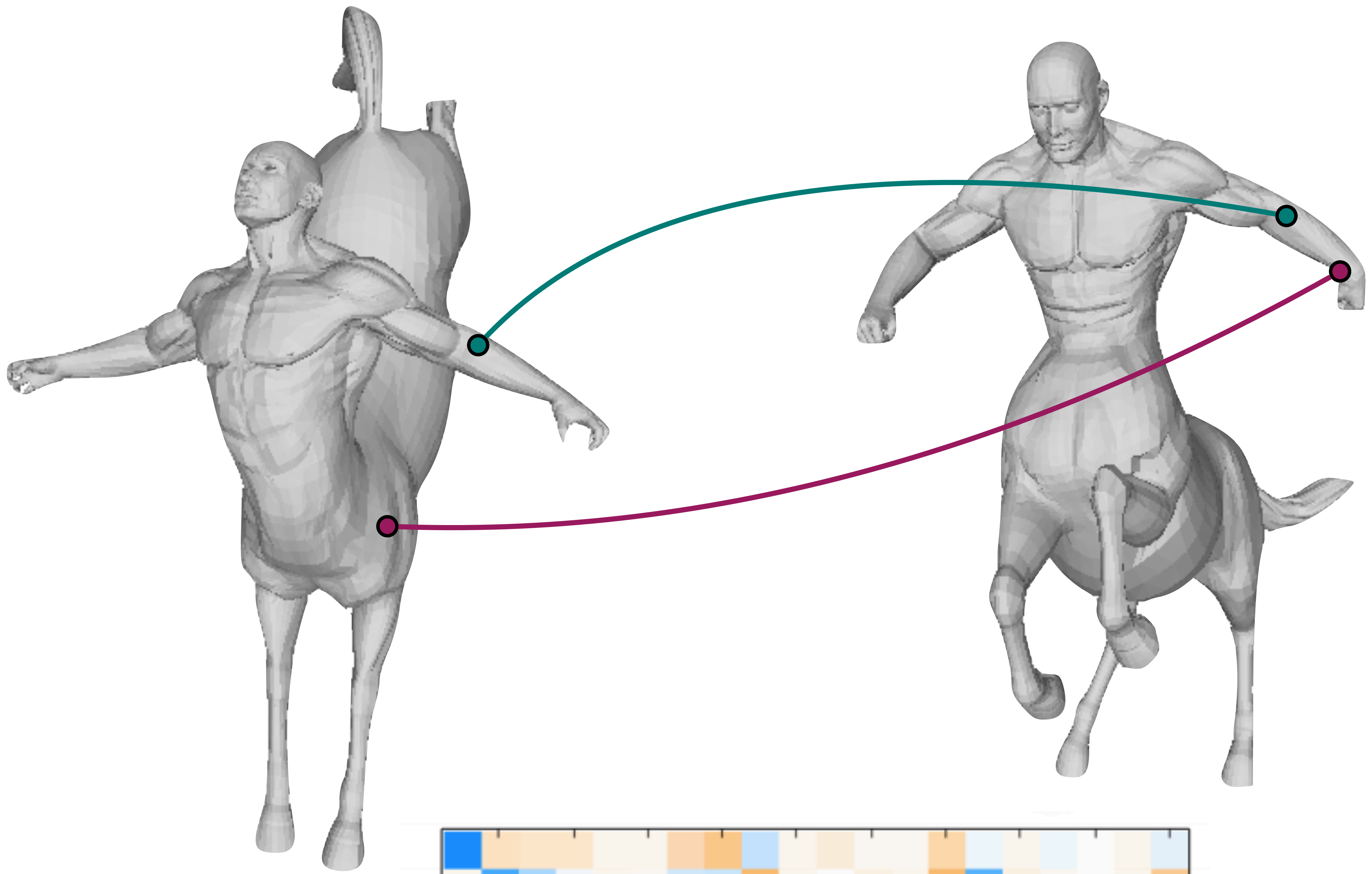
Any smooth deformation field can be written as:

$$\sum_{k=1}^{K} a_k \cdot v_k(x)$$



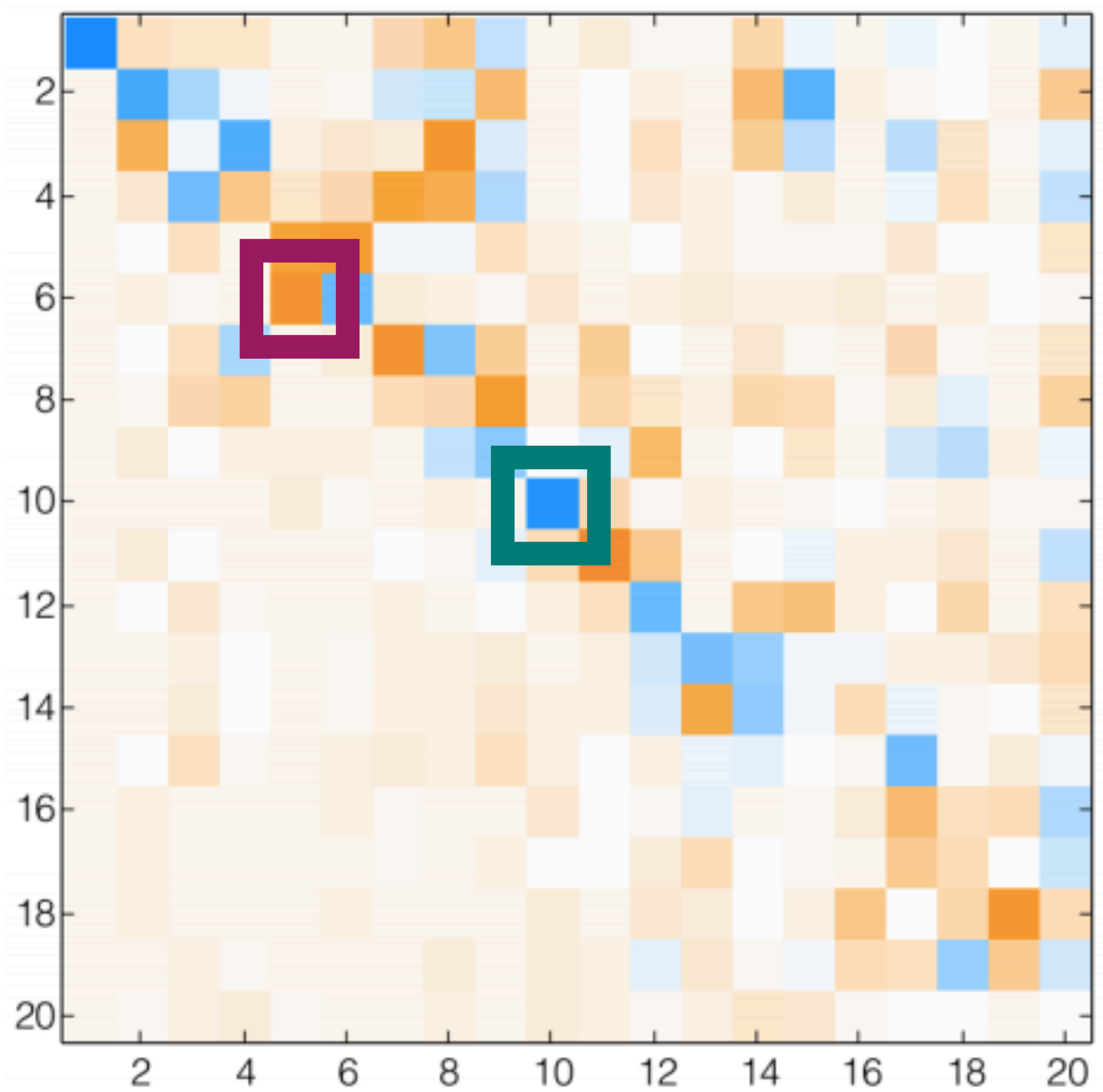
- The basis becomes more expressive for higher K.
- This is not spatially discretized and can be evaluated at any x.

Soft Correspondences



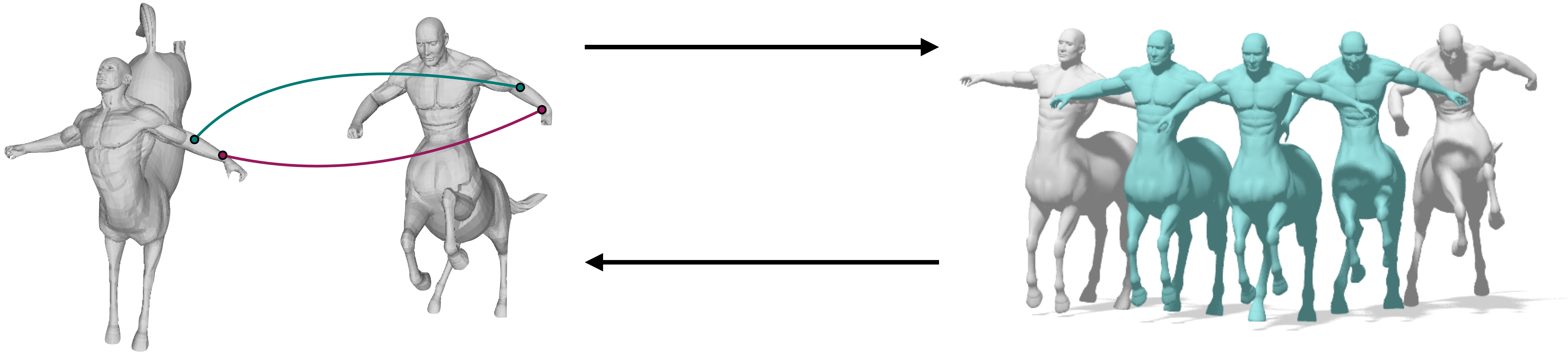
$$W \in [0,1]^{n \times m}$$

W_{ij} indicates how likely x_i and y_j are to be corresponding



Optimization

1. Given a deformation field a, finding a correspondence by applying the deformation and doing nearest neighbors is easy.



2. Given a correspondence, optimizing for the coefficients of the optimal deformation field in the basis v is moderately easy.

Expectation-Maximization

EM is mostly used when there is some observed data

1. which depends on some unknown parameters
2. and with some hidden variables

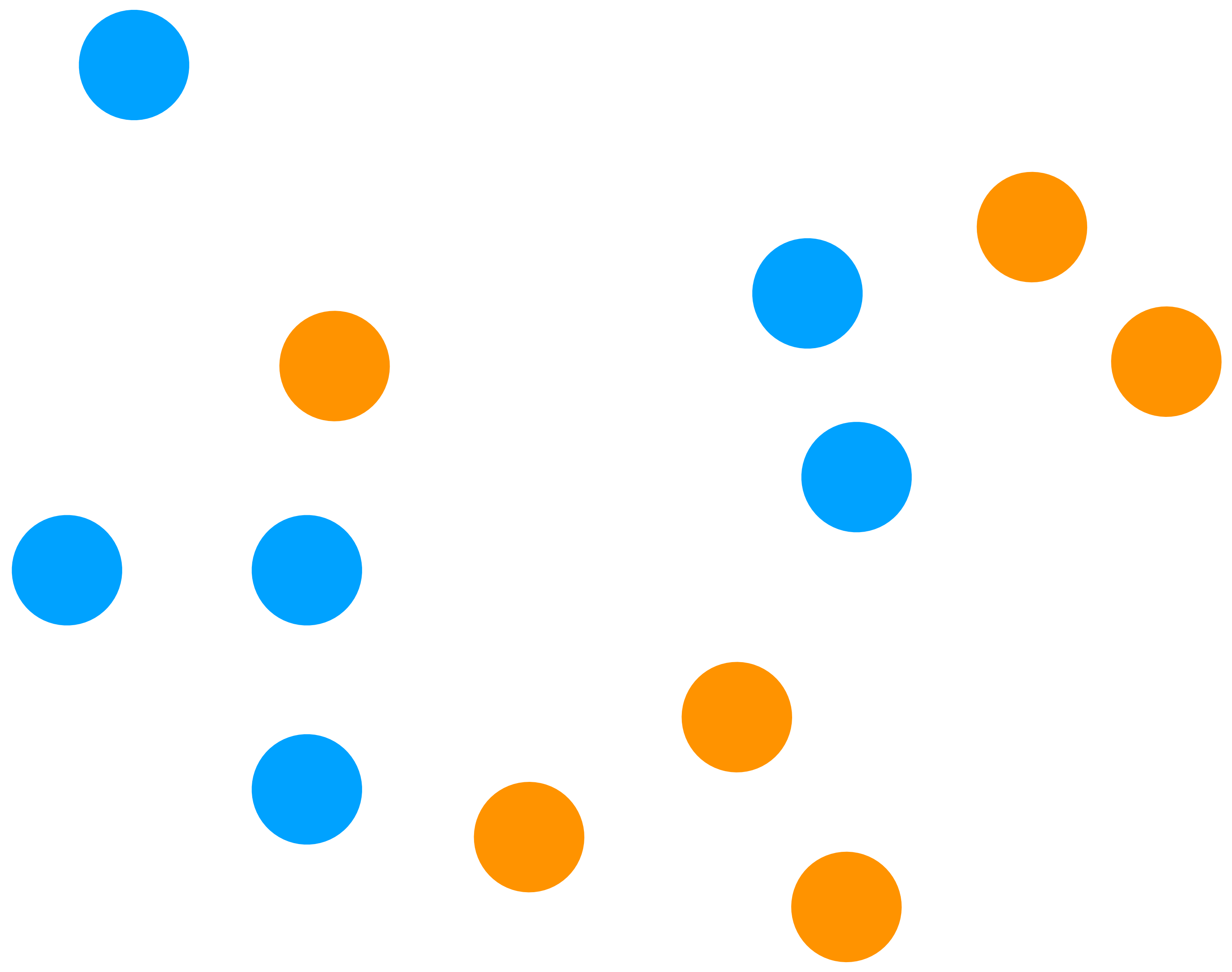
source and target point clouds

deformation field coefficients

point correspondence between source and target points

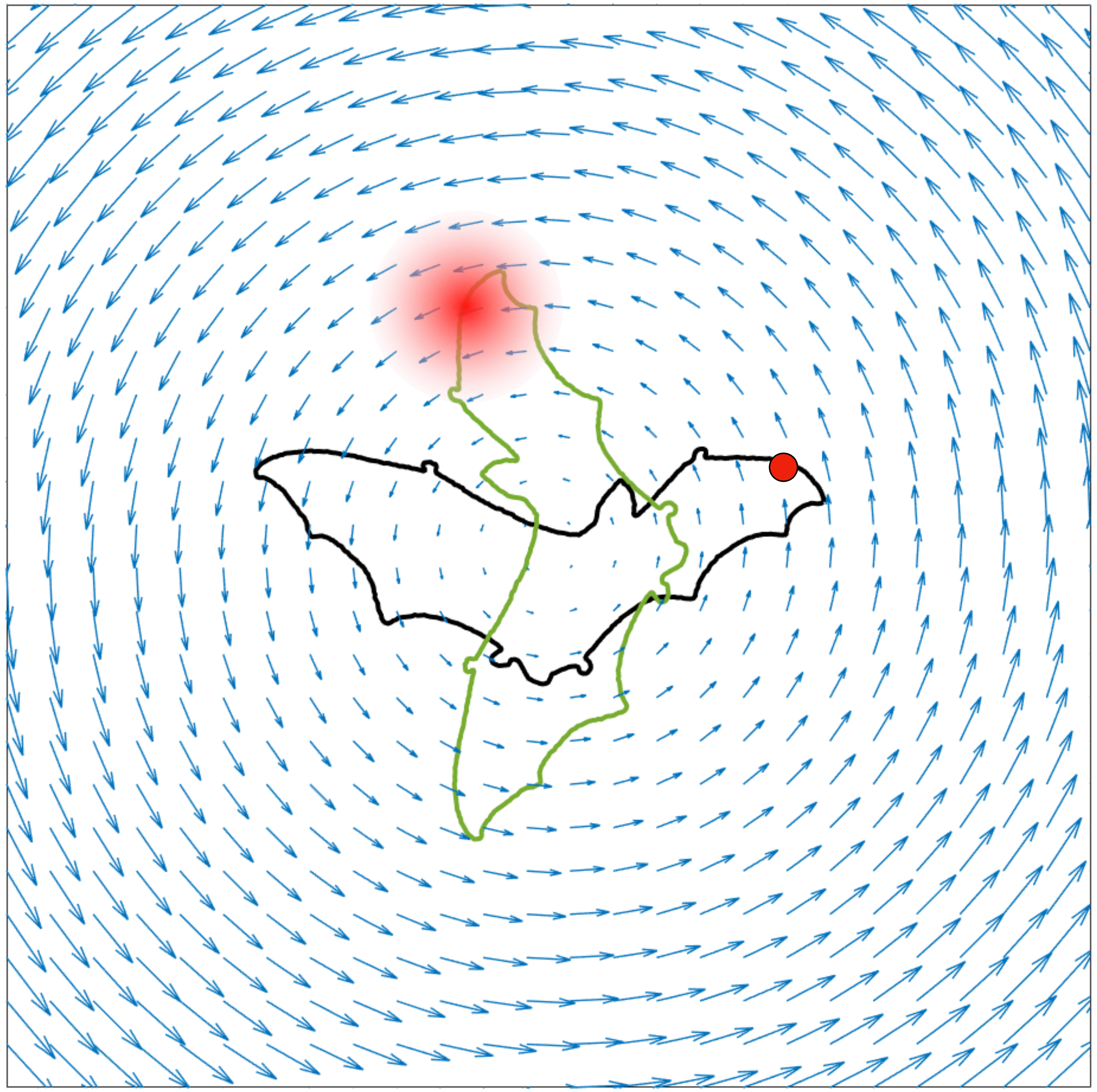
Main Idea:

Fix one of the unknowns, solve for the other.



Expectation-Step

Optimizing the correspondence given a deformation field



Expectation-Step

Chair for Computer Vision and Artificial Intelligence

Optimizing the correspondence given a deformation field

W is a soft correspondence matrix based on a Gaussian Mixture Model:

Gaussian probability that vertices m and n are corresponding according to the given deformation field (intuition from last slide)

$$W_{nm} = \frac{\boxed{exp\left(-\frac{1}{2\sigma^2} d_{nm}^2\right)}}{\boxed{(2\pi\sigma^2)^D + \sum_{\hat{n}=1}^{N} exp\left(-\frac{1}{2\sigma^2} d_{\hat{n}m}^2\right)}}$$

Normalization Term

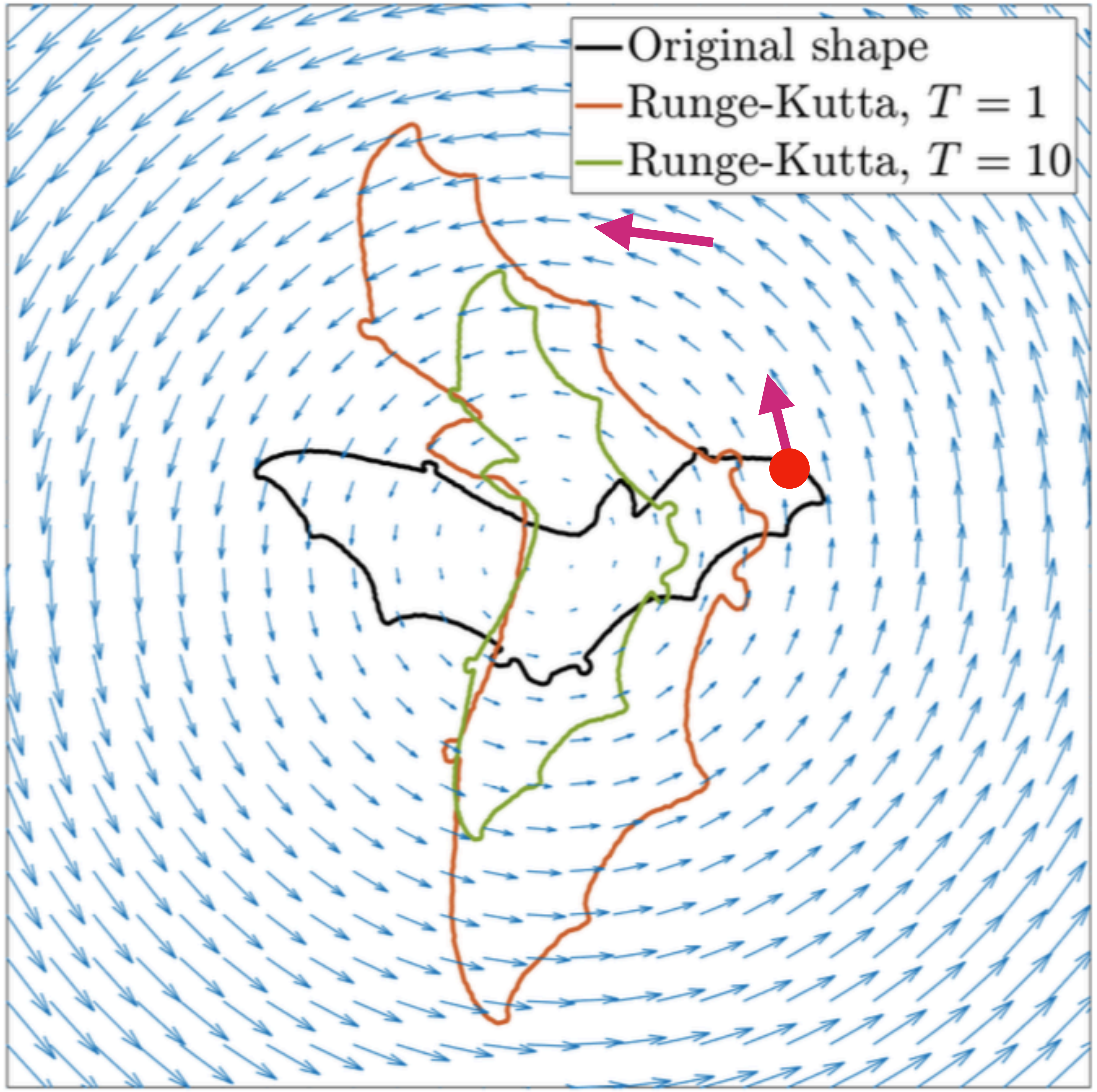
mixture of euclidean and descriptor distance between y_n and f_n dependent on deformation coefficients a

Notation Reminder

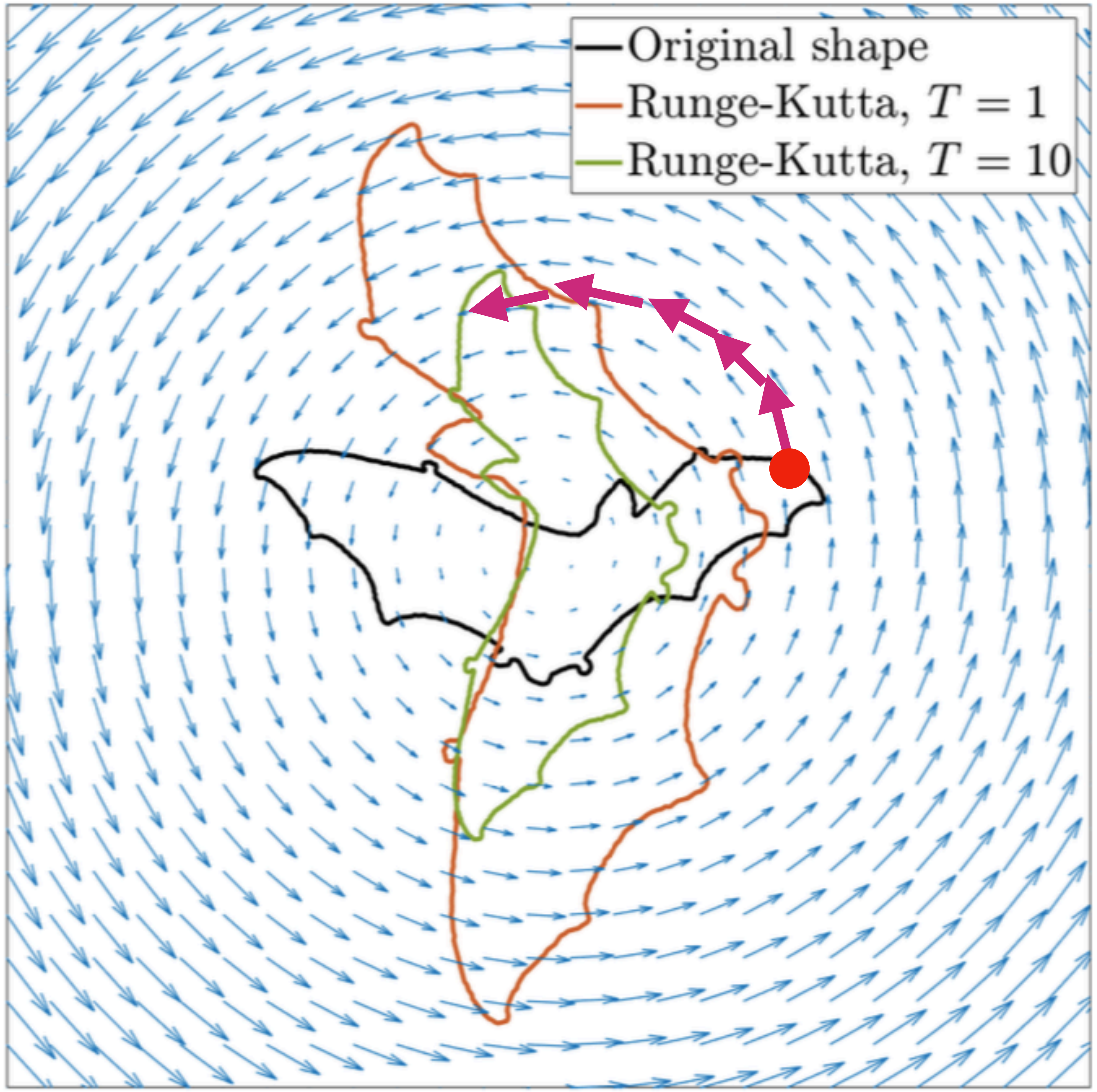
a	deformation coefficients
x_n	source vertex set
y_n	target vertex set
f_n	deformed vertex set, depends on a and x

Time Discretization

Lets move for time t
in 2 steps.



Time Discretization



Lets move for time t
in 5 steps.

less time steps
=
faster but certain properties (like volume-preservation) are violated

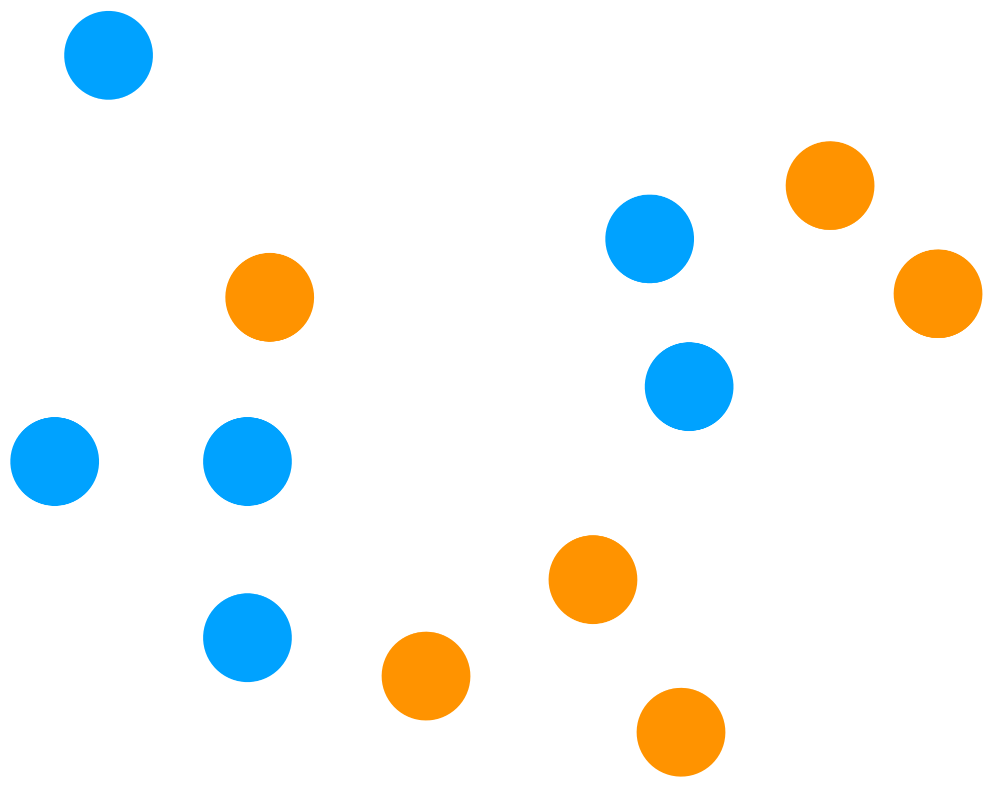
Maximization-Step

Chair for Computer Vision and Artificial Intelligence

Optimizing the deformation field given a correspondence

Goal: maximize the probability that the given correspondence comes from this particular deformation field

Unlikely: after applying the deformation field none of the corresponding points land close to each other



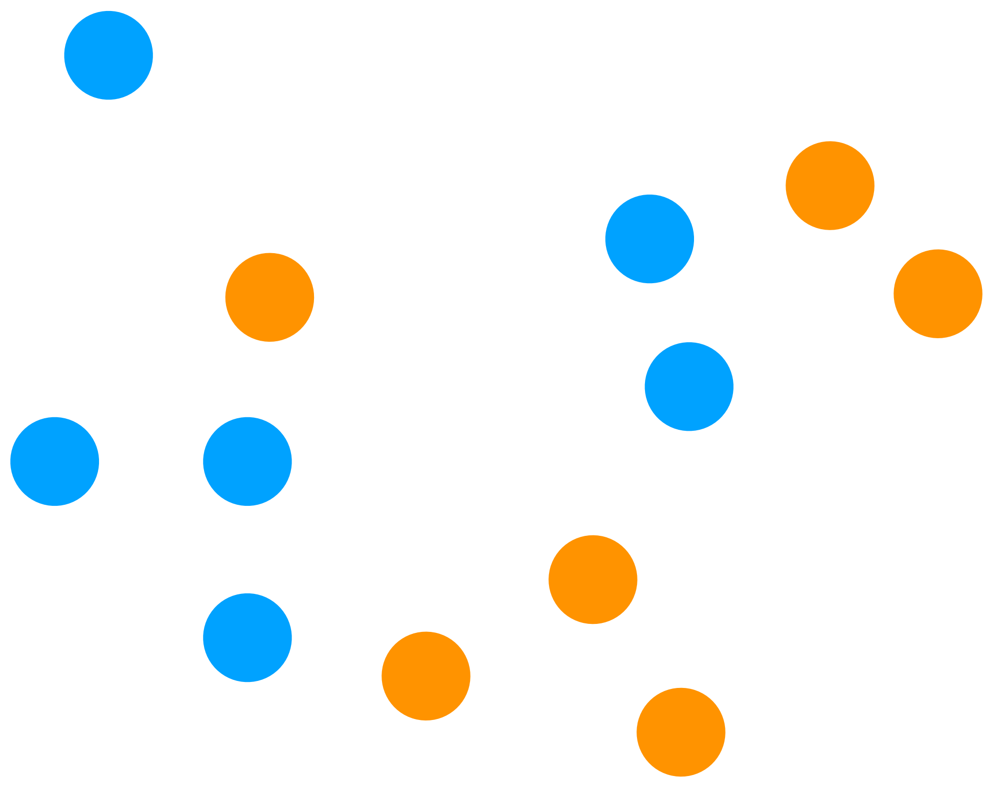
Maximization-Step

Optimizing the deformation field given a correspondence

Goal: maximize the probability that the given correspondence comes from this particular deformation field

Unlikely: after applying the deformation field none of the corresponding points land close to each other

Likely: after applying the complicated deformation field most corresponding points land close to each other



Maximization-Step

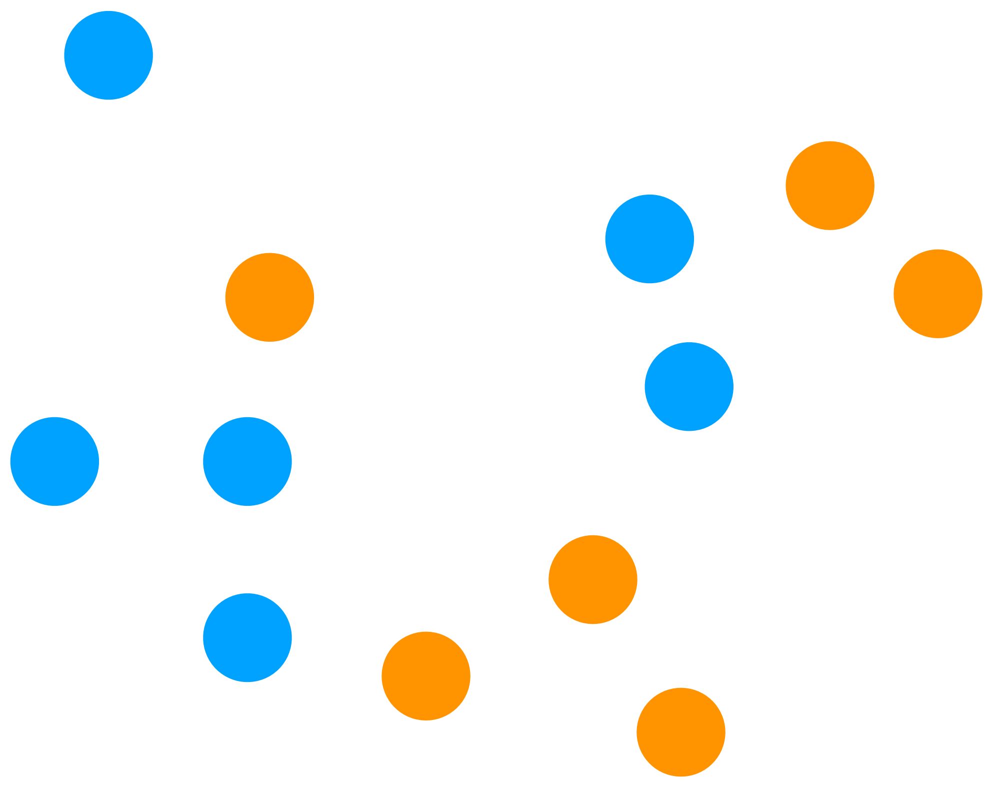
Optimizing the deformation field given a correspondence

Goal: maximize the probability that the given correspondence comes from this particular deformation field

Unlikely: after applying the deformation field none of the corresponding points land close to each other

Likely: after applying the complicated deformation field most corresponding points land close to each other

Very Likely: after applying the easy smooth deformation field most corresponding points land close to each other



Maximization-Step

Chair for Computer Vision and Artificial Intelligence

Optimizing the deformation field given a correspondence

checking how close the corresponding points are after the deformation

$$E(a) \propto \frac{1}{2} a^\top L^{-1} a + \boxed{\frac{1}{\sigma^2} \sum_{m=1}^{M} \sum_{n=1}^{N} W_{nm} \rho(\|y_m - f_n\|_2)}$$

prefers smooth/easy deformations = weighting of higher frequency coefficients

Huber-loss: robustness against outliers

Notation Reminder

a	deformation coefficients
x_n	source vertex set
y_n	target vertex set
f_n	deformed vertex set, depends on a

Maximization-Step

Optimizing the deformation field given a correspondence

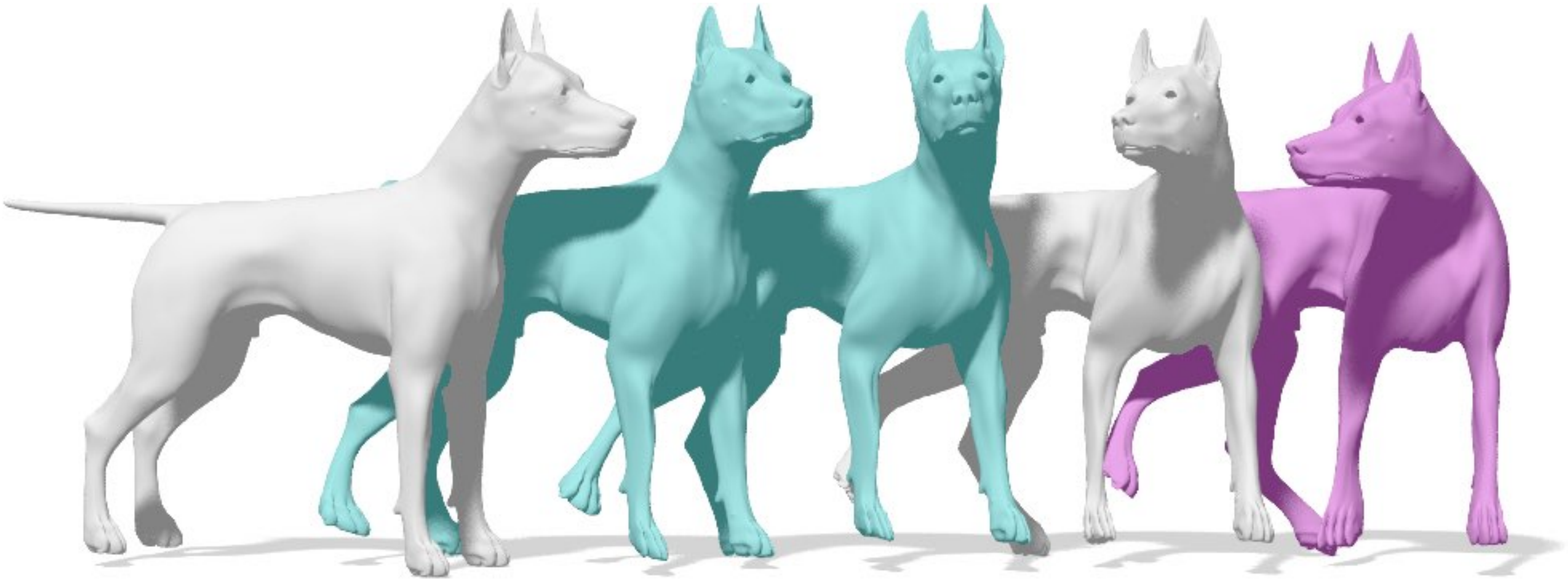
$$E(a) \propto \frac{1}{2} a^\top L^{-1} a + \frac{1}{\sigma^2} \sum_{m=1}^{M} \sum_{n=1}^{N} W_{nm} \rho(\|y_m - f_n\|_2)$$

The optimization can be done with a subsampled version of the inputs. (~3000 vertices in our experiments)
The final deformation field can still be applied to any resolution in the end.

Results

source shape interpolated target shape

Results



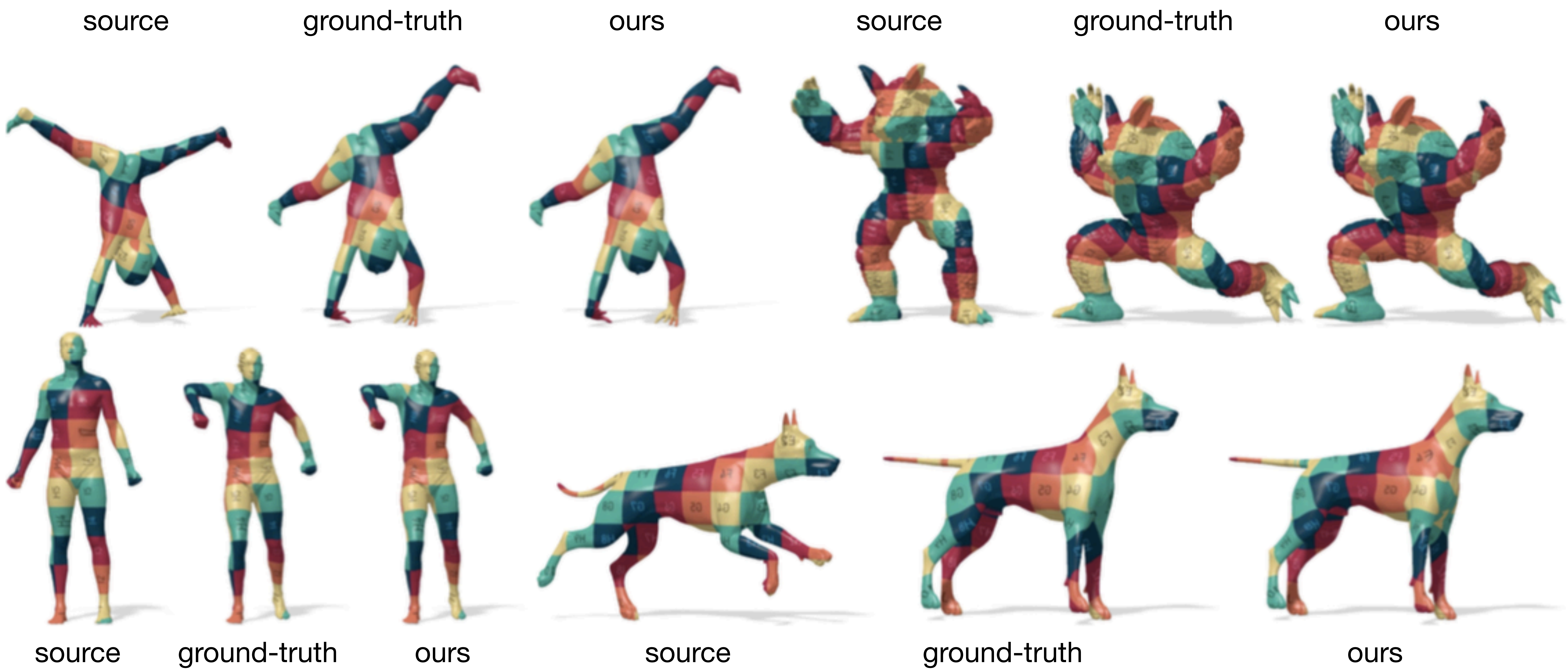
source shape interpolated target shape extrapolated

Results

Chair for Computer Vision and Artificial Intelligence

Texture Transfer

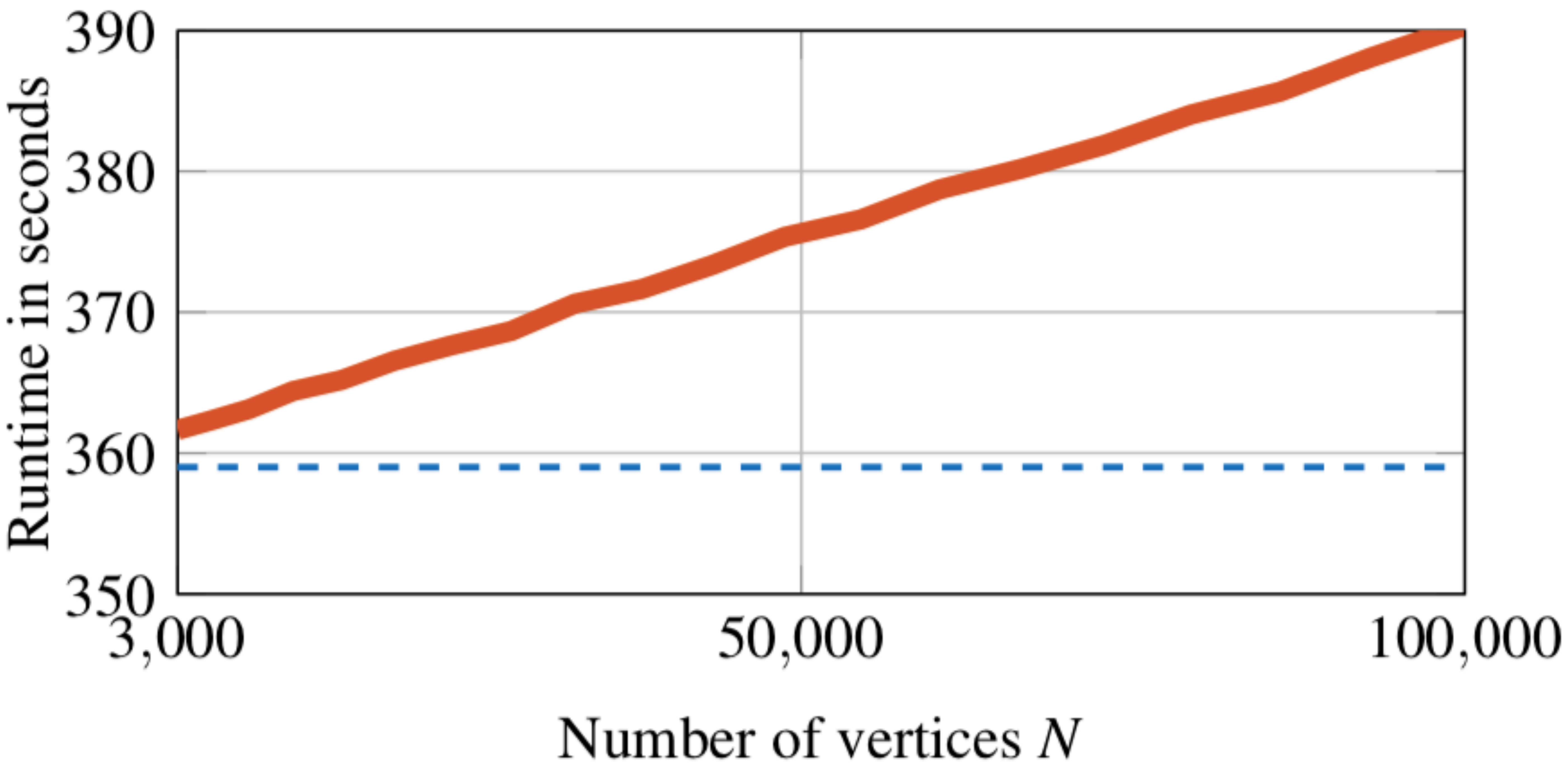
Chair for Computer Vision and Artificial Intelligence



Results

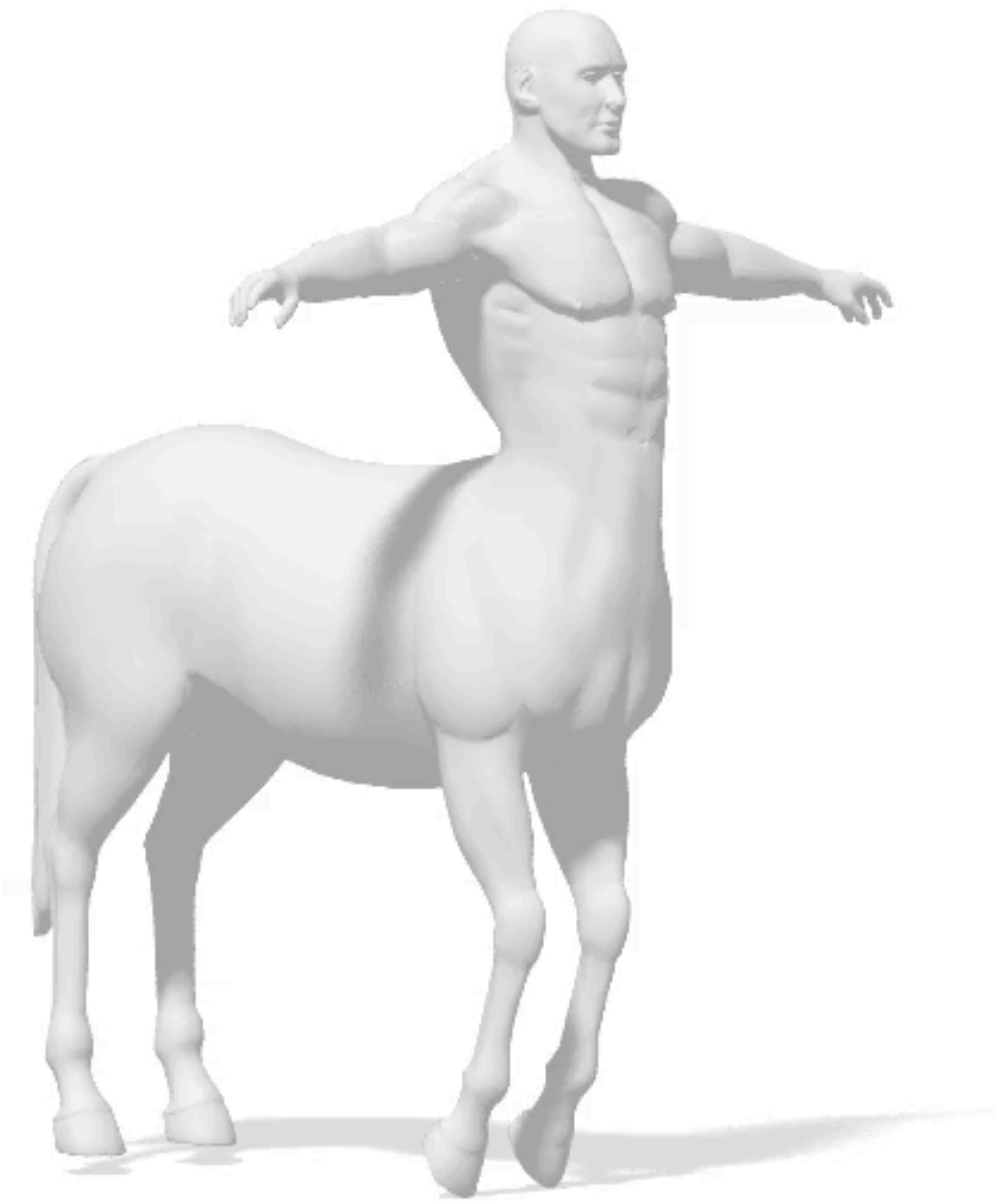
over 200k vertices, complete optimization and interpolation in ~10 mins

Runtime

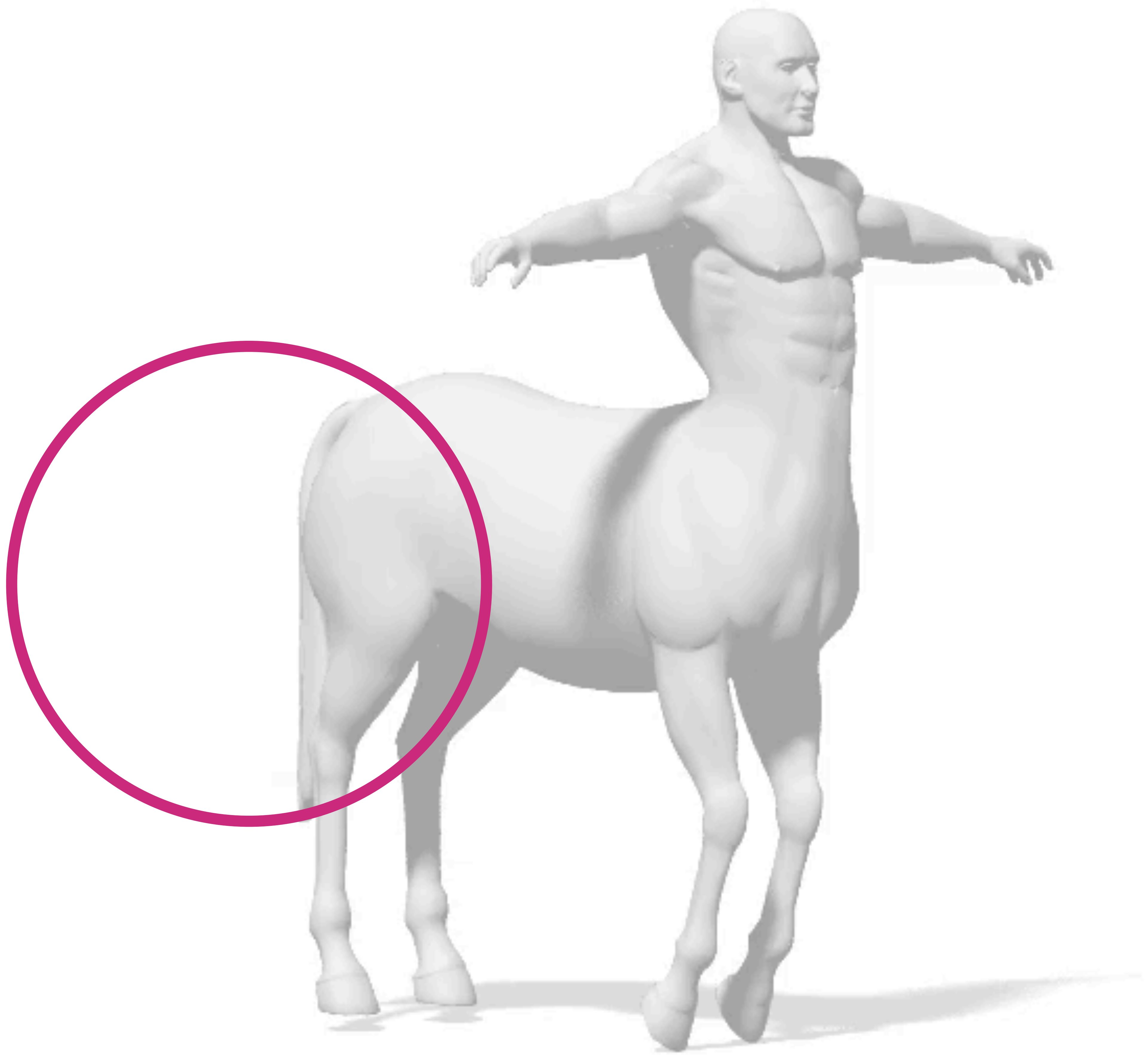


390
380
370
360
350
Runtime in seconds
3,000
50,000
100,000
Number of vertices N

Results



Results



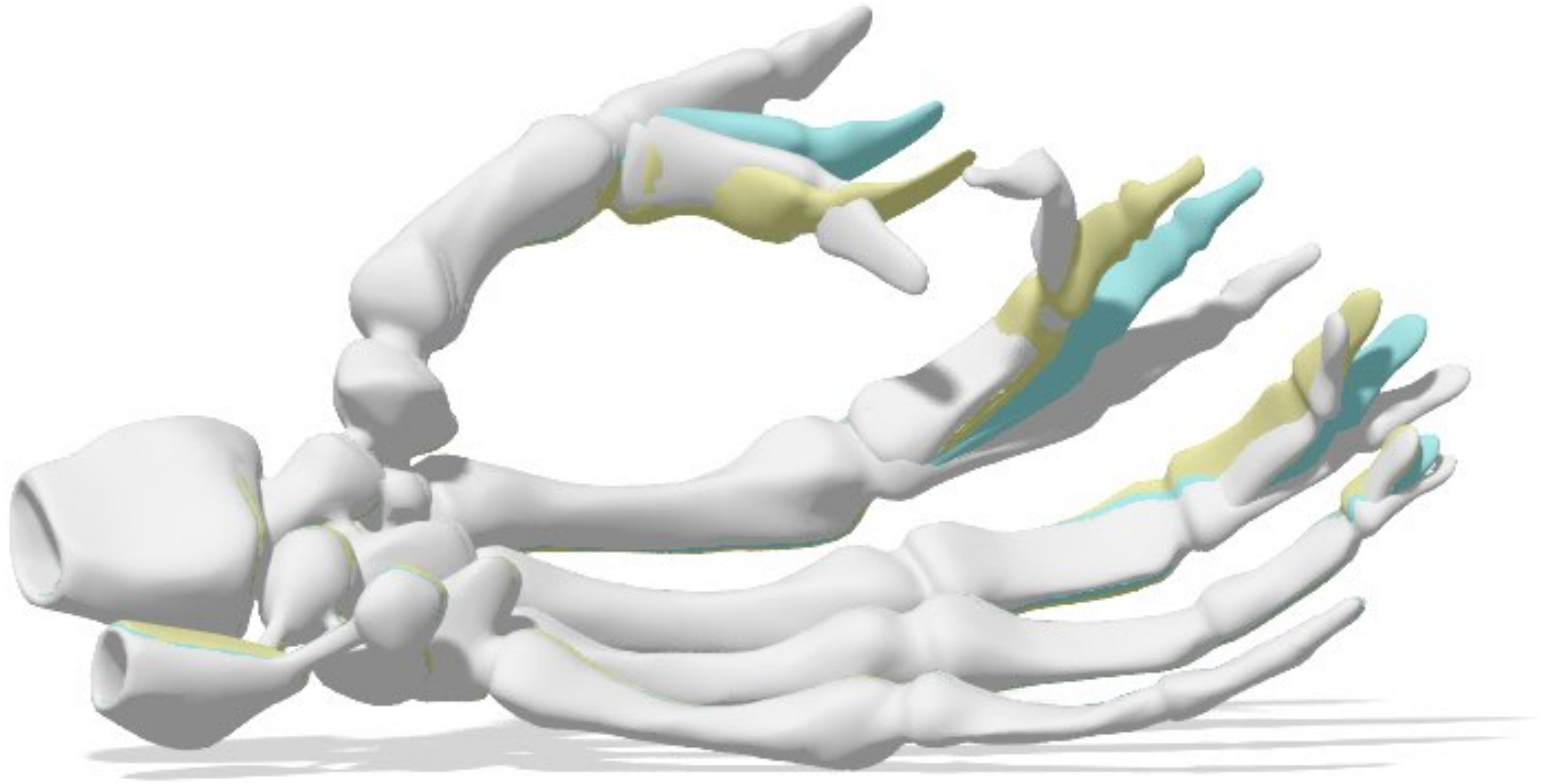
Volume preserving and
as-smooth-as-possible
is not the same thing as
as-rigid-as-possible

Hard Cases


avg=0.67cm max=7.65cm
avg=0.66cm max=10.12cm
avg=0.63cm max=14.45cm
avg=0.65cm max=7.43cm
avg=1.21cm max=15.23cm
avg=0.42cm max=3.65cm
5
0

Failure Case

Chair for Computer Vision and Artificial Intelligence



white	source and target shapes
blue	interpolated shape at t=0.5
yellow	final shape at t=1 (supposed to be the same as the target)

Conclusion

Chair for Computer Vision and Artificial Intelligence

- We can produce good correspondences and interpolations independent of shape resolution and meshing, and will never end up with degenerated or self-intersecting shapes
- But problems are still
  - (semi) topological changes
  - the volume preservation can not be relaxed
  - in some cases the interpolations are not as-rigid-as-possible
  - the deformation field cannot change over time

Thank you for your attention!