## Algebra

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# **Chapter 1**

# Namespace Index

## 1.1 Namespace List

Here is a list of all namespaces with brief descriptions:

Algebra

Is Linear Algebra library of required mathematics for, Kinetics, and Kinematics mechanics . . .

2 Namespace Index

# Chapter 2

# File Index

## 2.1 File List

Here is a list of all files with brief descriptions:

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algebra_test.cpp	1
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File Index

## **Chapter 3**

## Namespace Documentation

#### 3.1 Algebra Namespace Reference

Is Linear Algebra library of required mathematics for, Kinetics, and Kinematics mechanics.

#### **Functions**

bool Epsilon (const double val)

Find if the value is negligible, or close to zero.

Eigen::Matrix3d VecToso3 (const Eigen::Vector3d &omg)

Get the skew symmetric matrix representation of a vector 
$$v = \begin{vmatrix} v_1 & v_2 & v_3 \end{vmatrix} skew(v) = \begin{vmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & 1v_1 \\ -v_2 & v_1 & 0 \end{vmatrix}$$
.

Eigen::Vector3d so3ToVec (const Eigen::MatrixXd &so3mat)

Returns vector represented by the skew symmetric matrix.

• Eigen::MatrixXd RpToTrans (const Eigen::Matrix3d &R, const Eigen::Vector3d &p)

Combines a rotation matrix and position vector into a single Special Euclidian Group (SE3) homogeneous transformation matrix.

std::vector< Eigen::MatrixXd > TransToRp (const Eigen::MatrixXd &T)

Separates the rotation matrix and position vector from the transformation matrix representation.

• Eigen::MatrixXd VecTose3 (const Eigen::VectorXd &V)

Translates a spatial velocity vector into a transformation matrix.

• Eigen::VectorXd se3ToVec (const Eigen::MatrixXd &T)

Translates a transformation matrix into a spatial velocity vector.

• Eigen::MatrixXd Normalize (Eigen::MatrixXd V)

Returns a normalized version of the input vector.

- Eigen::Vector4d AxisAng3 (const Eigen::Vector3d &expc3)
- Eigen::Matrix3d MatrixExp3 (const Eigen::Matrix3d &so3mat)

Translates an exponential rotation into a rotation matrix using rodrigues formula.

• Eigen::Matrix3d MatrixLog3 (const Eigen::Matrix3d &R)

Function: Computes the matrix logarithm of a rotation matrix.

Eigen::MatrixXd Adjoint (const Eigen::MatrixXd &T)

Provides the adjoint representation of a transformation matrix Used to change the frame of reference for spatial velocity vectors.

Eigen::MatrixXd MatrixExp6 (const Eigen::MatrixXd &se3mat)

Rotation expanded for screw axis.

Eigen::MatrixXd MatrixLog6 (const Eigen::MatrixXd &T)

Computes the matrix logarithm of a homogeneous transformation matrix.

• Eigen::MatrixXd TransInv (const Eigen::MatrixXd &transform)

Inverts a homogeneous transformation matrix.

Eigen::MatrixXd RotInv (const Eigen::MatrixXd &rotMatrix)

Inverts a rotation matrix.

• Eigen::VectorXd ScrewToAxis (Eigen::Vector3d q, Eigen::Vector3d s, double h)

Takes a parametric description of a screw axis and converts it to a normalized screw axis.

Eigen::VectorXd AxisAng6 (const Eigen::VectorXd &expc6)

Converts a 6-vector of exponential coordinates into screw axis-angle form.

• Eigen::MatrixXd ProjectToSO3 (const Eigen::MatrixXd &M)

Returns a projection of mat into SO(3)

• Eigen::MatrixXd ProjectToSE3 (const Eigen::MatrixXd &M)

Returns a projection of mat into SE(3)

double DistanceToSO3 (const Eigen::Matrix3d &M)

Returns the Frobenius norm to describe the distance of mat from the SO(3) manifold.

double DistanceToSE3 (const Eigen::Matrix4d &T)

Returns the Frobenius norm to describe the distance of mat from the SE(3) manifold.

- bool TestIfSO3 (const Eigen::Matrix3d &M)
- bool TestIfSE3 (const Eigen::Matrix4d &T)
- Eigen::MatrixXd ad (Eigen::VectorXd V)

Calculate the 6x6 matrix of the given 6-vector V.

## 3.1.1 Detailed Description

Is Linear Algebra library of required mathematics for, Kinetics, and Kinematics mechanics.

Build on top of Kevin M. Lynch's, and Frank C. Park 'Modern Robotics' Book http://hades.mech. 
northwestern.edu/index.php/Modern\_Robotics, and courses from Princeton.

#### 3.1.2 Function Documentation

## 3.1.2.1 ad()

Calculate the 6x6 matrix of the given 6-vector V.

#### **Parameters**

V Eigen::VectorXd (6x1), V=[omg, v]

#### Returns

Eigen::MatrixXd (6x6) Note: Can be used to calculate the Lie bracket [V1, V2] = [adV1]V2

## 3.1.2.2 Adjoint()

Provides the adjoint representation of a transformation matrix Used to change the frame of reference for spatial velocity vectors.

#### **Parameters**

```
T 4x4 Transformation matrix SE(3) \begin{vmatrix} R & p \\ 0 & 1 \end{vmatrix}.
```

#### Returns

```
6x6 Adjoint Representation of the matrix T adj(T) = \begin{vmatrix} R & 0 \\ [p]R & R \end{vmatrix}
```

## 3.1.2.3 AxisAng3()

### 3.1.2.4 AxisAng6()

Converts a 6-vector of exponential coordinates into screw axis-angle form.

### **Parameters**

expc6 A 6-vector of exponential coordinates for rigid-body motion S\*theta

### Returns

S The corresponding normalized screw axis theta The distance traveled along/about S

#### 3.1.2.5 DistanceToSE3()

Returns the Frobenius norm to describe the distance of mat from the SE(3) manifold.

#### **Parameters**

```
mat A 4x4 matrix
```

#### Returns

A quantity describing the distance of mat from the SE(3) manifold Computes the distance from mat to the SE(3) manifold using the following method: Compute the determinant of matR, the top 3x3 submatrix of mat. If det(matR) <= 0, return a large number. If det(matR) > 0, replace the top 3x3 submatrix of mat with  $mat \leftarrow R^T.matR$ , and set the first three entries of the fourth column of mat to zero. Then return norm(mat - I).

### 3.1.2.6 DistanceToSO3()

Returns the Frobenius norm to describe the distance of mat from the SO(3) manifold.

### Parameters

```
M A 3x3 matrix
```

#### Returns

A quantity describing the distance of mat from the SO(3) manifold Computes the distance from mat to the SO(3) manifold using the following method: If  $det(mat) \le 0$ , return a large number. If det(mat) > 0, return  $norm(mat^T.mat - I)$ .

## 3.1.2.7 Epsilon()

Find if the value is negligible, or close to zero.

#### **Parameters**

val double value to be checked.

#### Returns

Boolean of true-ignore or false-can't ignore.

## 3.1.2.8 MatrixExp3()

Translates an exponential rotation into a rotation matrix using rodrigues formula.

#### **Parameters**

	so3mat	exponenential representation of a rotation in so3 $[w]\theta$ .	
--	--------	-----------------------------------------------------------------	--

#### Returns

Rotation matrix  $e^{[w]\theta}$ .

## 3.1.2.9 MatrixExp6()

Rotation expanded for screw axis.

#### **Parameters**

se3mat	se3 matrix representation of exponential coordinates $[\boldsymbol{s}]\boldsymbol{\theta}$
--------	--------------------------------------------------------------------------------------------

#### Returns

6x6 Matrix exponential Transformation  $T=e^{[s]\theta}$ 

#### 3.1.2.10 MatrixLog3()

Function: Computes the matrix logarithm of a rotation matrix.

#### **Parameters**

```
R Rotation matrix [w]\theta.
```

#### Returns

matrix logarithm of a rotation  $e^{[w]\theta}$ .

## 3.1.2.11 MatrixLog6()

Computes the matrix logarithm of a homogeneous transformation matrix.

#### **Parameters**

```
T A matrix in SE3.
```

#### Returns

The matrix logarithm of  $e^{[s]\theta}$ .

## 3.1.2.12 Normalize()

Returns a normalized version of the input vector.

#### **Parameters**

```
V Eigen::MatrixXd
```

#### Returns

Eigen::MatrixXd normalized V Note: MatrixXd is used instead of VectorXd for the case of row vectors Requires a copy Useful because of the MatrixXd casting

### 3.1.2.13 ProjectToSE3()

Returns a projection of mat into SE(3)

#### **Parameters**

M A 4x4 matrix to project to SE(3)

#### Returns

The closest matrix to T that is in SE(3) Projects a matrix mat to the closest matrix in SE(3) using singular-value decomposition (see <a href="http://hades.mech.northwestern.edu/index.php/Modern\_cobotics\_Linear\_Algebra\_Review">http://hades.mech.northwestern.edu/index.php/Modern\_cobotics\_Linear\_Algebra\_Review</a>). This function is only appropriate for matrices close to SE(3).

### 3.1.2.14 ProjectToSO3()

Returns a projection of mat into SO(3)

#### **Parameters**

M A matrix near SO(3) to project to SO(3)

#### Returns

The closest matrix to R that is in SO(3) Projects a matrix mat to the closest matrix in SO(3) using singular-value decomposition (see <a href="http://hades.mech.northwestern.edu/index.php/Modern\_condition-linear\_Algebra\_Review">http://hades.mech.northwestern.edu/index.php/Modern\_condition-linear\_Algebra\_Review</a>). This function is only appropriate for matrices close to SO(3).

#### 3.1.2.15 Rotlnv()

Inverts a rotation matrix.

#### **Parameters**

R A rotation matrix

#### Returns

: The inverse of R

## 3.1.2.16 RpToTrans()

Combines a rotation matrix and position vector into a single Special Euclidian Group (SE3) homogeneous transformation matrix.

#### **Parameters**

R	Rotation Matrix
р	Position Vector

#### See also

TransToRp

#### Returns

```
Transformation Matrix \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}.
```

## 3.1.2.17 ScrewToAxis()

Takes a parametric description of a screw axis and converts it to a normalized screw axis.

#### **Parameters**

(	7	A point lying on the screw axis
5	s	A unit vector in the direction of the screw axis
ŀ	h	The pitch of the screw axis

#### Returns

A normalized screw axis described by the inputs

## 3.1.2.18 se3ToVec()

Translates a transformation matrix into a spatial velocity vector.

#### **Parameters**

```
T Transformation matrix \begin{vmatrix} [w] & v \\ 0 & 0 \end{vmatrix}
```

See also

VecTose3

Returns

```
Spatial velocity vector \begin{bmatrix} w \\ v \end{bmatrix}
```

## 3.1.2.19 so3ToVec()

Returns vector represented by the skew symmetric matrix.

$$v = \begin{vmatrix} v_1 & v_2 & v_3 \end{vmatrix} skew(v) = \begin{vmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & 1v_1 \\ -v_2 & v_1 & 0 \end{vmatrix}$$

## **Parameters**

See also

VecToso3

Returns

v\_ret Eigen::Vector3d 3x1 vector

### 3.1.2.20 TestIfSE3()

## 3.1.2.21 TestIfSO3()

### 3.1.2.22 TransInv()

Inverts a homogeneous transformation matrix.

#### **Parameters**

T A homogeneous transformation matrix

#### Returns

The inverse of T Uses the structure of transformation matrices to avoid taking a matrix inverse, for efficiency.

## 3.1.2.23 TransToRp()

Separates the rotation matrix and position vector from the transfomation matrix representation.

#### **Parameters**



### See also

**RpToTrans** 

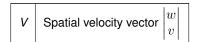
#### Returns

Rp\_ret std::vector of rotation matrix (R), and position vector (p).

#### 3.1.2.24 VecTose3()

Translates a spatial velocity vector into a transformation matrix.

#### **Parameters**



See also

se3ToVec

#### Returns

linear representation of Twist in special Euclidian group se3  $egin{bmatrix} [w] & v \\ 0 & 0 \end{bmatrix}$ 

#### 3.1.2.25 VecToso3()

 $\text{Get the skew symmetric matrix representation of a vector } v = \begin{vmatrix} v_1 & v_2 & v_3 \end{vmatrix} skew(v) = \begin{vmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & 1v_1 \\ -v_2 & v_1 & 0 \end{vmatrix}.$ 

#### **Parameters**

omg | Eigen::Vector3d 3x1 angular velocity vector

See also

so3ToVec

Returns

Eigen::MatrixXd 3x3 skew symmetric matrix in so3

## **Chapter 4**

## **File Documentation**

## 4.1 algebra.hpp File Reference

```
#include <Eigen/Dense>
#include <cmath>
#include <vector>
#include <iostream>
Include dependency graph for algebra.hpp:
```

## 4.2 algebra\_test.cpp File Reference

```
#include <iostream>
#include <Eigen/Dense>
#include "gmock/gmock.h"
#include "algebra.hpp"
Include dependency graph for algebra_test.cpp:
```

#### **Functions**

- TEST (ALGEBRA, Epsilon)
- TEST (ALGEBRA, So3ToVecTest)
- TEST (ALGEBRA, VecToSO3Test)
- TEST (ALGEBRA, RpToTrans)
- TEST (ALGEBRA, TransToRp)
- TEST (ALGEBRA, VecToso3)
- TEST (ALGEBRA, so3ToVec)
- TEST (ALGEBRA, AxisAng3)
- TEST (ALGEBRA, Adjoint)
- TEST (ALGEBRA, Adjoint2)
- TEST (ALGEBRA, MatrixLog3)
- TEST (ALGEBRA, TransInv)
- TEST (ALGEBRA, RotInv)
- TEST (ALGEBRA, ScrewToAxis)
- TEST (ALGEBRA, AxisAng6)
- TEST (ALGEBRA, MatrixLog6)
- TEST (ALGEBRA, DistanceToSO3Test)
- TEST (ALGEBRA, DistanceToSE3Test)
- TEST (ALGEBRA, TestIfSO3Test)
- TEST (ALGEBRA, TestIfSE3Test)
- TEST (ALGEBRA, adTest)
- int main (int argc, char \*\*argv)

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## 4.2.1 Function Documentation

## 4.2.1.1 main()

```
int main ( \label{eq:int_argc} \text{int } \textit{argc,} \label{eq:char_argv} \text{char ** argv })
```

## 4.2.1.2 TEST() [1/21]

```
TEST (

ALGEBRA ,

Adjoint )
```

## 4.2.1.3 TEST() [2/21]

## 4.2.1.4 TEST() [3/21]

```
TEST ( ALGEBRA , adTest )
```

## 4.2.1.5 TEST() [4/21]

```
TEST (
ALGEBRA ,
AxisAng3 )
```

```
4.2.1.6 TEST() [5/21]
```

```
TEST (
ALGEBRA ,
AxisAng6 )
```

## 4.2.1.7 TEST() [6/21]

```
TEST (
          ALGEBRA ,
          DistanceToSE3Test )
```

## 4.2.1.8 TEST() [7/21]

```
TEST (
          ALGEBRA ,
          DistanceToSO3Test )
```

## 4.2.1.9 TEST() [8/21]

```
TEST (

ALGEBRA ,

Epsilon )
```

## 4.2.1.10 TEST() [9/21]

```
TEST (

ALGEBRA ,

MatrixLog3 )
```

## 4.2.1.11 TEST() [10/21]

```
TEST (

ALGEBRA ,

MatrixLog6 )
```

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```
4.2.1.12 TEST() [11/21]
```

```
TEST ( ALGEBRA , RotInv )
```

## 4.2.1.13 TEST() [12/21]

```
TEST ( ALGEBRA , RpToTrans )
```

## 4.2.1.14 TEST() [13/21]

```
TEST (

ALGEBRA ,

ScrewToAxis )
```

## 4.2.1.15 TEST() [14/21]

```
TEST (
ALGEBRA , so3ToVec )
```

## 4.2.1.16 TEST() [15/21]

```
TEST (

ALGEBRA ,

So3ToVecTest )
```

## 4.2.1.17 TEST() [16/21]

```
TEST (
          ALGEBRA ,
          TestIfSE3Test )
```

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TEST (
            ALGEBRA ,
            VecToso3 )
4.2.1.22 TEST() [21/21]
TEST (
           ALGEBRA ,
            VecToSO3Test )
```

## 4.3 c1-c3.cpp File Reference

```
#include <iostream>
#include "algebra.hpp"
Include dependency graph for c1-c3.cpp:
```

## **Functions**

int main (int argc, char \*\*args)

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## 4.3.1 Function Documentation

## 4.3.1.1 main()

```
int main (
          int argc,
          char ** args )
```

## 4.4 c1w3.cpp File Reference

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