Liquidation Target Calculations in Euler

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January 2023

1 Target health

The liquidation module calculates the maximum repay and yield amounts by targeting a health score the liquidated account will result in, currently set at 1.25. It is not always possible to reach the target health score, because the account is no longer sufficiently over-collateralized, or because the liquidated collateral and liability pair balances are not sufficient, given the overall composition of the account. Given that reaching the target health is possible, the calculations depend on the composition of the deposits and debts of the account.

2 General approach

The health score h_s is defined as total risk adjusted collateral value of deposits c_v over total risk adjusted liabilities value l_v

$$h_s = \frac{c_v}{l_v} \tag{1}$$

The liquidation will attempt to calculate the repay r and yield y amounts such that new collateral value c'_v and new liabilities value l'_v match a target health score $T = 1.25$

$$h'_s = \frac{c'_v}{l'_v} = T \tag{2}$$

For collateral factor of the taken token c_f, its price p_c and yield amount y the new collateral value is:

$$c'_v = c_v - \Delta c_v = c_v - y p_c c_f \tag{3}$$

The amount of yield value the liquidator is entitled to depends on the value of repay r and the liquidator's discount d which can be expressed as a conversion rate c_r, where p_l is the liability price

$$c_r = \frac{p_l}{p_c(1-d)} \tag{4}$$

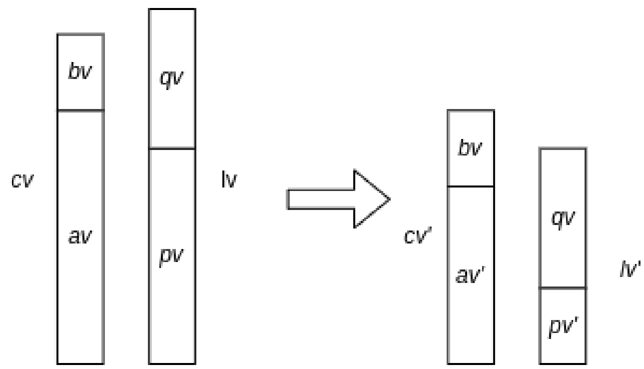
$$y = r c_r \tag{5}$$

The new liability value depends on the resulting composition of the account, because some part of the debt can be supported by override collateral, where the borrow factor applied is $b_{fo} = 1$ while the rest of the debt is supported by a regular collateral for which a regular borrow factor b_f is applied. In general:

$$l'_v = l_v - \Delta l_v \tag{6}$$

3 Account composition

3.1 Regular collateral and liabilities



In this scenario there is one or more regular collateral and one or more liabilities. The liquidated collateral is token A with collateral value a_v. Sum of other collateral value is b_v. The liquidated debt is token P with liability value p_v. Sum of other liabilities value is q_v. In the resulting position there are also no overrides, so the borrow factor applied to the whole liability is b_f

$$l'_v = l_v - \frac{rp_l}{b_f} \tag{7}$$

Solving (2) for r:

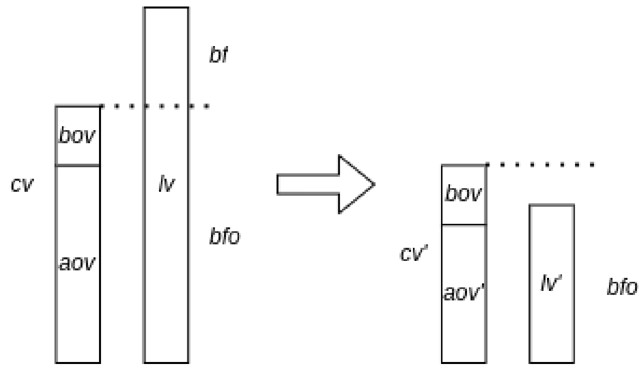
$$r = \frac{Tl_v - c_v}{\frac{T}{b_f} - \frac{c_f}{1-d}} \frac{1}{p_l} \tag{8}$$

The numerator is guaranteed to be positive, since the account is in violation, so $h_s < 1$ and $l_v > c_v$, while $T > 1$. The denominator could turn negative, depending mostly on the discount, but while approaching zero, the repay approaches infinity. Therefore, if the denominator is negative the repay is set to the whole debt amount $r_{max} = l$, and yield limited to the available collateral token balance a.

$$y = max(r_{max}c_r, a) \tag{9}$$

$$r = \frac{y}{c_r} \tag{10}$$

3.2 Only override collateral and a single liability



All collateral assets have overrides set with the liability. By definition overrides are only active if there is just a single liability. Before the liquidation the liability value is greater than total collateral value c_v, therefore the part that is supported is calculated with $b_{fo} = 1$ while the excess at a regular b_f. After the liquidation, all of the liability is covered by override collateral, so only b_{fo} applies. Given the original debt amount l:

$$l'_v = (l - r)p_l \tag{11}$$

Solving (2) for r:

$$r = \frac{Tlp_l - c_v}{T - \frac{c_f}{1-d}} \frac{1}{p_l} \tag{12}$$

Note that the result is the same as (8) when the override borrow factor is applied $b_f = b_{fo} = 1$. The numerator is still guaranteed to be positive, since all of the collateral was initially in override, so $c_v = l_o p_l$, where l_o is the collateralized debt amount, $l > l_o$ because $l_v > c_v$, therefore $c_v < lp_l$

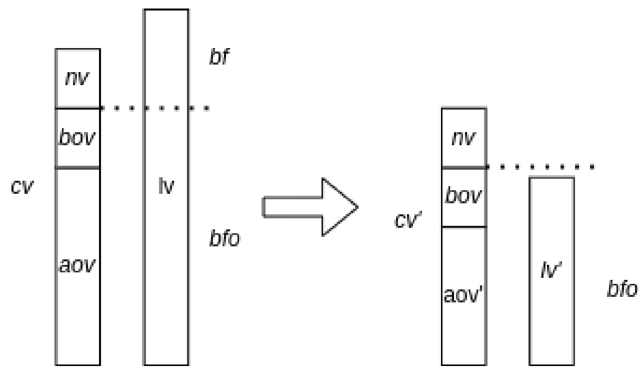
3.3 Regular and override collateral, liquidating override collateral

In this scenario, the collateral value is a sum of override collateral values a_{ov} and b_{ov}, and a sum of the regular collateral values n_v. Part of the liability value is counted with override b_{fo}, while the rest, including excess, is counted with regular b_f. The collateral liquidated is token A.

There are 2 possible outcomes of the liquidation. The first is that the resulting liability is fully covered by the override collateral. Second is that the resulting liability is still in part covered by override and in part by regular collateral. The repay calculation will be different for each case.

It is possible to determine the type computationally, but in practice the calculation is similar in size to the repay amount calculation itself and includes its own edge-cases. We can opt instead for another approach, which on average should have similar costs: without knowing the type in advance, we can assume the first and carry out the calculations accordingly. If the choice was correct, the result will comply with the assumed composition, which can be verified. If it doesn't, then the other type must be true, and the second calculation correct.

Assuming the result is of the first type:



New liability value is again:

$$l'_v = (l - r)p_l \tag{13}$$

And repay is the same as in 3.2.

$$r = \frac{Tlp_l - c_v}{T - \frac{c_f}{1-d}} \frac{1}{p_l} \tag{14}$$

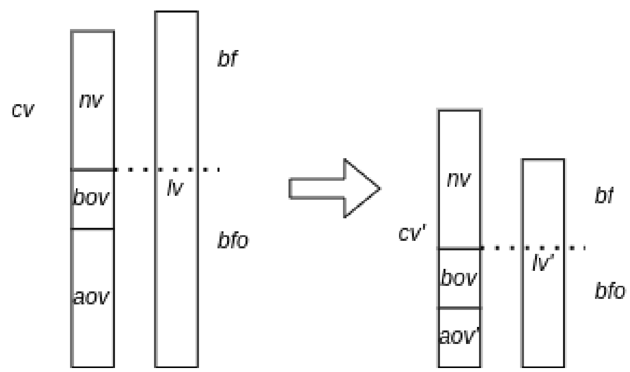
Again note that the result is the same as (8) when the override borrow factor is applied $b_f = b_{fo} = 1$. The numerator though is no longer guaranteed to be positive. Since some of the liability is supported by regular collateral and $Tlp_l < l_v$. If the amount of the regular collateral in proportion to the override collateral is high enough, the numerator will be negative. In this case we can deduce that the resulting composition will be of the second type.

Having the calculation result, we can now test if the resulting composition matches the assumption, which is that the override collateral value is greater than override liability value:

$$(l - r)p_l < a_{ov} + b_{ov} - yc_f p_c \tag{15}$$

If the denominator in the initial calculation is negative, which means full debt amount, then there is no point in carrying out the secondary calculation. By assuming first that resulting liability will be fully covered by overrides, we can expect that if the assumption doesn't hold, then there was too little debt liquidated, because the assumption requires that at least some part of the liquidated amount would be calculated at b_{fo}, which in token amount would be less than in the second scenario, where all of the liquidated amount is calculated at b_f, and $b_f < b_{fo}$. In summary, the secondary calculation always yields greater r than the first, so if the first calculation gives max yield, then we can be sure that the whole collateral must be liquidated. Note that with the current settings of maximum discount (20%) and target health (1.25), the denominator is always positive.

If the assumption doesn't hold, then the correct result will be in the second type:



Now the resulting liability is partially covered by regular collateral. With the initial token amount of liquidated collateral a and resulting debt amount l':

$$l'_v = b_{ov} + a'_{ov} + (l' - \frac{b_{ov} + a'_v}{p_l})p_l/b_f \tag{16}$$

$$a'_v = (a - y)c_f p_c = a_v - y c_f p_c \tag{17}$$

$$l' = l - r \tag{18}$$

Solving (2) for r:

$$r = \frac{T(lp_l/b_f - (a_{ov} + b_{ov})k) - c_v}{\frac{T}{b_f} - \frac{c_f(Tk+1)}{1-d}} \frac{1}{p_l} \tag{19}$$

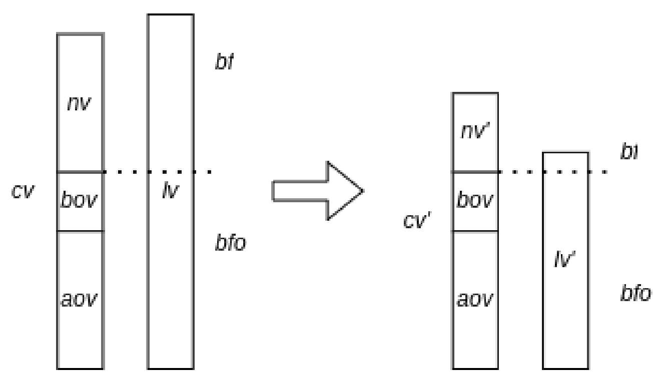
$$k = \frac{1}{b_f} - 1 \tag{20}$$

If the nominator or denominator are negative, the whole collateral is seized.

3.4 Regular and override collateral, liquidating regular collateral

Similar to 3.3 the liquidation could result in two types of composition - debt fully or partially supported by override collateral. The liquidation calculations are also carried out in 2 steps, but in reverse order - first testing for partial coverage result and then full coverage.

While liquidating the regular collateral, liability value is always counted with regular b_f. As the r increases, the account will first be partially collateralized by overrides and only after possibly starts being fully collateralized by overrides. If assuming partial scenario the calculation gives max yield already, there is no need to check the other scenario.



The resulting liability value is given as:

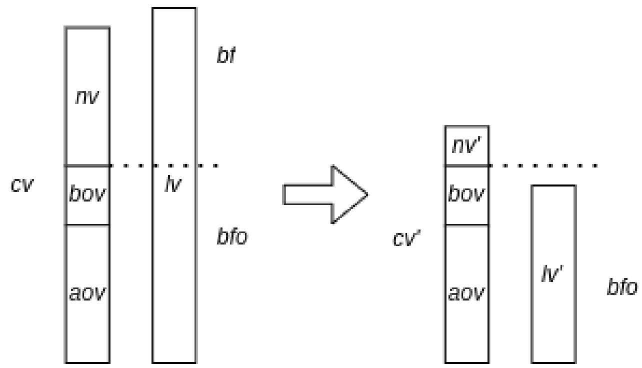
$$l'_v = l_v - rp_l/b_f \tag{21}$$

and it is the same equation as in case of a regular liquidation without any overrides in 3.1 and (8) can be applied.

If the assumption of the resulting composition was correct, the override collateral is not sufficient to cover the new liability:

$$(l - r)p_l > a_{ov} + b_{ov} \tag{22}$$

If the condition is not true, we can expect the final scenario:



$$l'_v = (l - r)p_l \tag{23}$$

Which again matches 3.2 and can be applied to (8) with $bf = b_{fo}$