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**Course: Data Structures and Algorithms**

# Algorithm Analysis and Problem Solving

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# Why do we even need algorithms?


Because even your

favorite app’s

‘**loading**’ screen

has a story to tell...

I blame “the algorithm” behind this

# Algorithm Fundamentals

# Importance of algorithms

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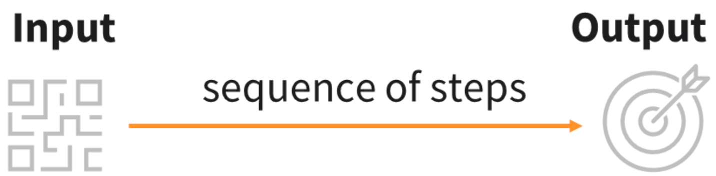
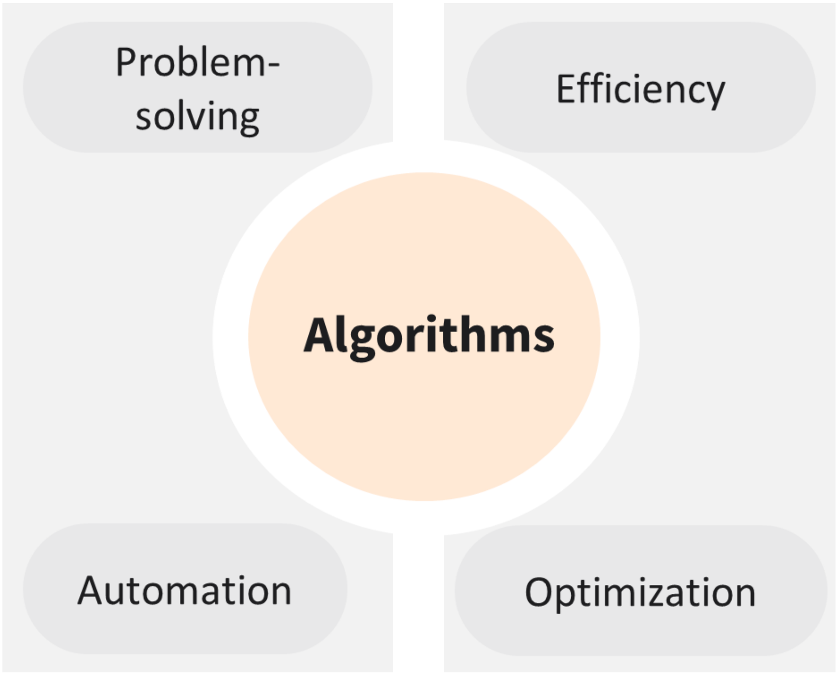
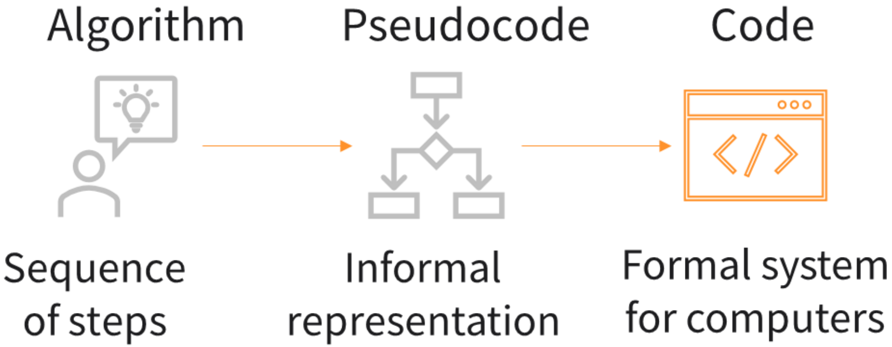



- **Step-by-step instructions to solve tasks efficiently**
- Definite: clear and unambiguous
- Termination: ends in finite steps



# Programming the algorithms

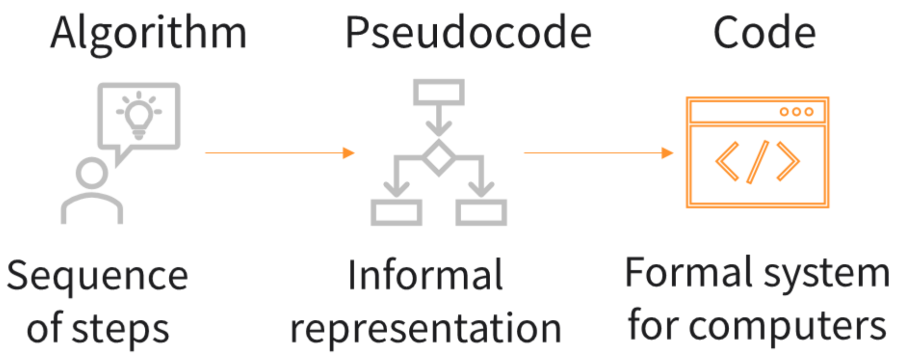



- Programming: convert algorithms to machine-readable instructions
- Language types: machine, assembly, low and high-level languages

**How can we choose the most effective algorithm for a task?**

- Consider time and space complexity
- Time concerns the number of algorithmic steps
- Space efficiency relates to memory usage

# Programming the algorithms

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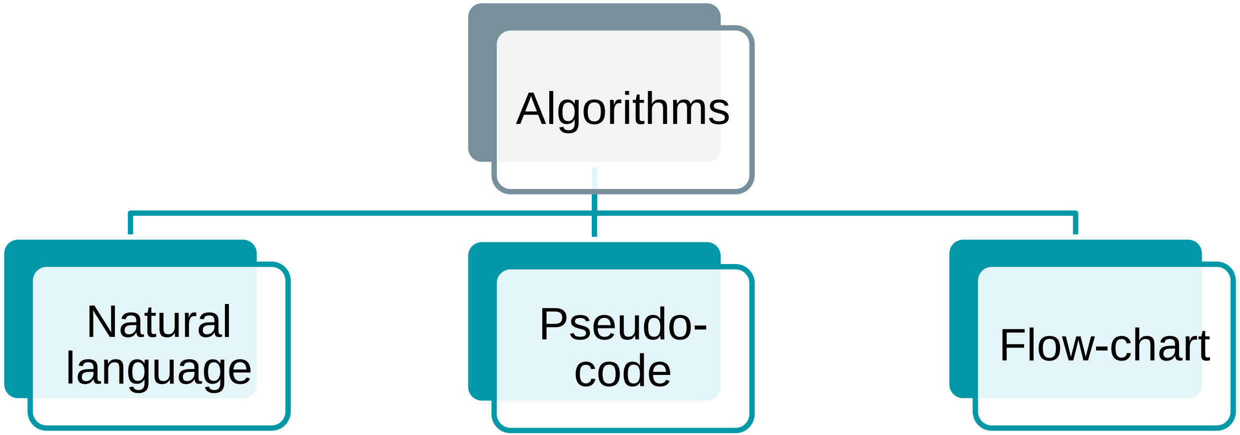
# Detailing and abstraction

**Clear algorithm design increases programming efficiency**




Plain language

Code-like representation

Visualized process step

# Example: finding greatest common divisor

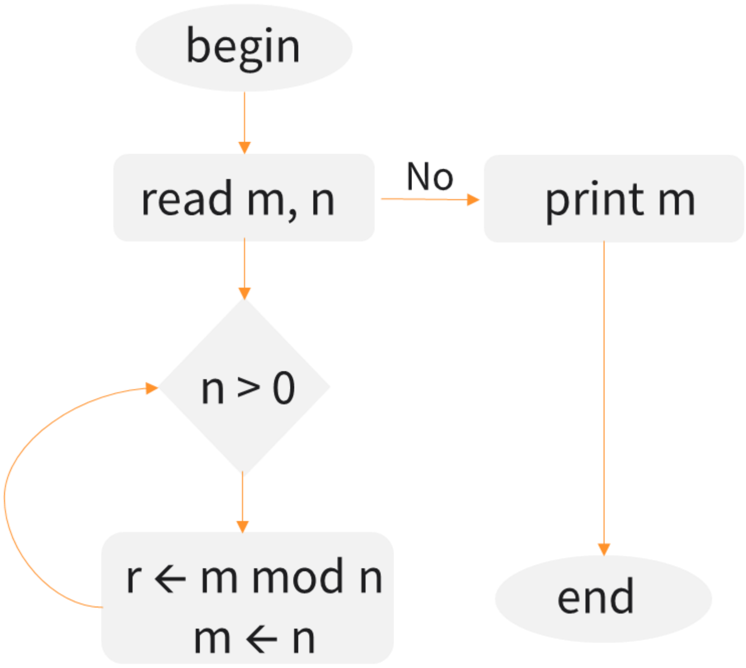
## Plain language

```
1. Input two numbers.
2. Determine the maximum (m) and minimum (n).
3. If n is zero, output m.
4. Otherwise, find the remainder (r) of m divided by n.
4. Set m = n and n = r.
5. Repeat from step 3.
```

## Pseudocode

```
begin
    read a, b
    m ← maximum(a, b)
    n ← minimum(a, b)
    while (n ≠ 0)
        r ← m mod n
        m ← n
        n ← r
    endwhile
    return m
end
```

## Flowchart



# Problem solving and algorithm design

**A clear, structured approach ensures correct and efficient solutions to problems**

1. **Understand the problem** — Clarify the problem, inputs, outputs, constraints, and requirements
2. **Plan the solution** — Break the problem into smaller steps or subproblems
3. **Solve the problem** — Design an algorithm using appropriate techniques
4. **Test and optimize** — Validate the solution and improve efficiency

# Evaluating algorithms

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Algorithm selection is about balancing efficiency, not just optimizing one factor

- Analyze how algorithm runtime grows with input size (Big-O notation)
- Measure memory usage as input size increases (Big-O space complexity)
- Optimizing time may increase space usage, or vice versa

Select based on problem constraints: faster solution or lower memory usage?

# Practical aspects on evaluating algorithms

## Time measurement helps to understand algorithm performance under different conditions

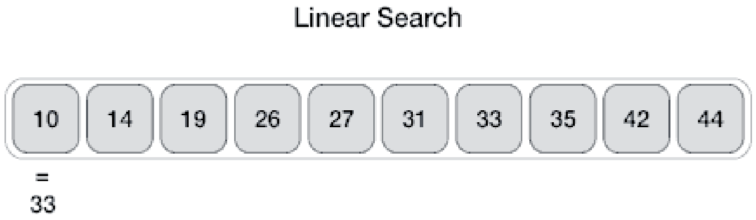
- Measure the time before execution
- **Run the algorithm**
- Measure the time after execution
- Run multiple tests with varying input sizes and conditions to gather meaningful results. Average time over multiple runs for more reliable data

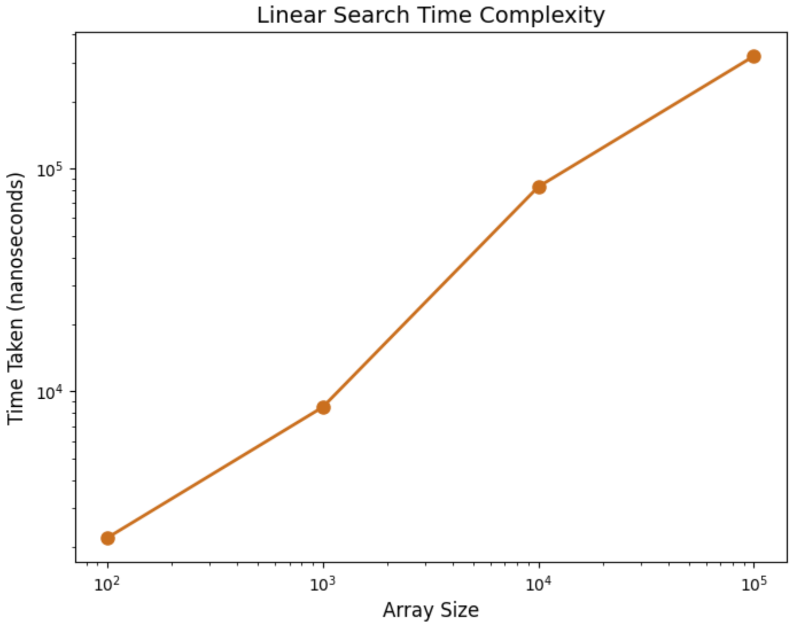
```
START
  startTime = GetCurrentTime()
  result = RunAlgorithm(inputData)
  endTime = GetCurrentTime()
  elapsedTime = endTime - startTime
END
```

# Basic algorithm example: linear search

Search for an element in a list by checking each element sequentially

```java
public static int linearSearch(int[] arr, int target) {
    for (int i = 0; i < arr.length; i++) {
        if (arr[i] == target) {
            return i;
        }
    }
    return -1;
}
```



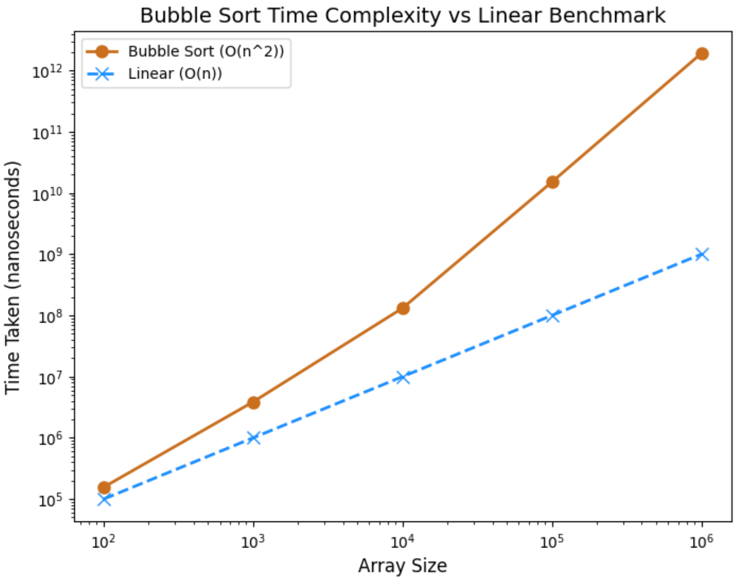



# Basic algorithm example: bubble sort

Bubble Sort repeatedly swaps adjacent elements until the array is sorted

8 5 3 1 4 7 9

```
public static void bubbleSort(int[] arr) {
   for (int i = 0; i < arr.length - 1; i++) {
       for (int j = 0; j < arr.length - i - 1; j++) {
          if (arr[j] > arr[j + 1]) {
             int temp = arr[j];
             arr[j] = arr[j + 1];
             arr[j + 1] = temp;
          }
       }
    }
}
```



# Algorithm Design Techniques

# Fundamental algorithm design techniques



Multiple techniques exist to tackle diverse problem types and ensure optimal solutions

- Algorithm techniques
  - Brute force
  - Divide and conquer
  - Recursion
  - Greedy
  - Dynamic programming

# Brute force approach

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A straightforward problem-solving method that involves trying all possible solutions to find the correct one

## When to apply

- Small problem size
- Solution space is manageable
- Sophisticated algorithms non available or necessary

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- Easy to implement
- Guaranteed solution
- No assumptions

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- Slow and expensive
- Impractical for large input size
- Inefficient for complex problems

# Brute force approach example: linear search

```
function linearSearch(arr, target):
    for i = 0 to length(arr) - 1:
        if arr[i] == target:
            return i
    return -1
```

- Best case: $O(1)$
- Worst and average case: $O(n)$

- **Exhaustive search**: checks every element
- **Simple & direct**: no optimizations or shortcuts
- **Guaranteed solution**: finds the target if it exists
- **Inefficient**: slow for large datasets

# Divide and conquer strategy



Break problem into subproblems, solve independently, combine results


**Divide**

Break the problem into smaller subproblems

**Conquer**

Solve each subproblem recursively

**Combine**

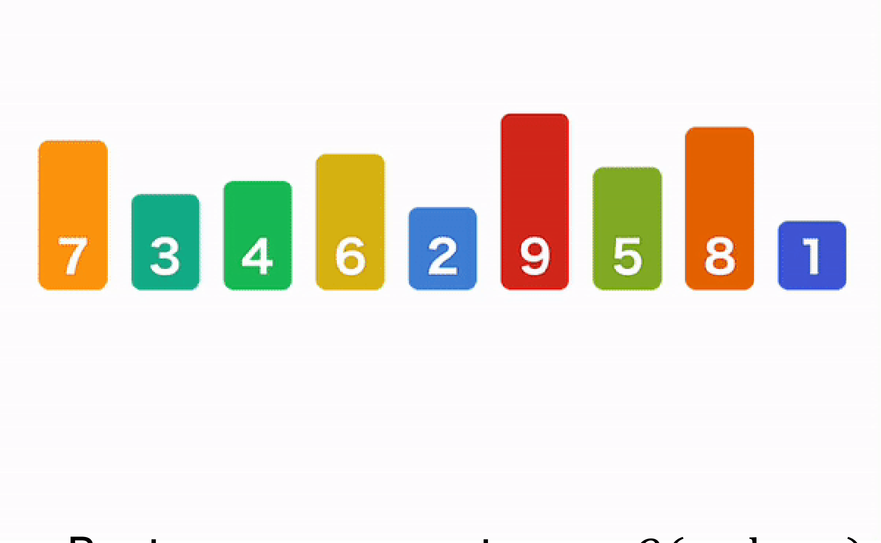
Combine the solutions of the subproblems to form the final solution

# Divide and conquer example: merge sort



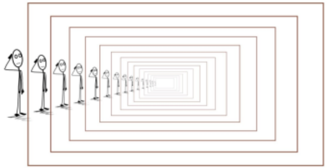
- **Divide:** split the array into two halves
- **Conquer**: recursively sort each half
- **Combine**: merge the sorted halves into a single sorted array

Recursively divide the array, sort each half, then merge them




Best, average, worst case: $O(n \cdot \log n)$

# Recursion



Technique where a function calls itself to solve smaller instances of the same problem

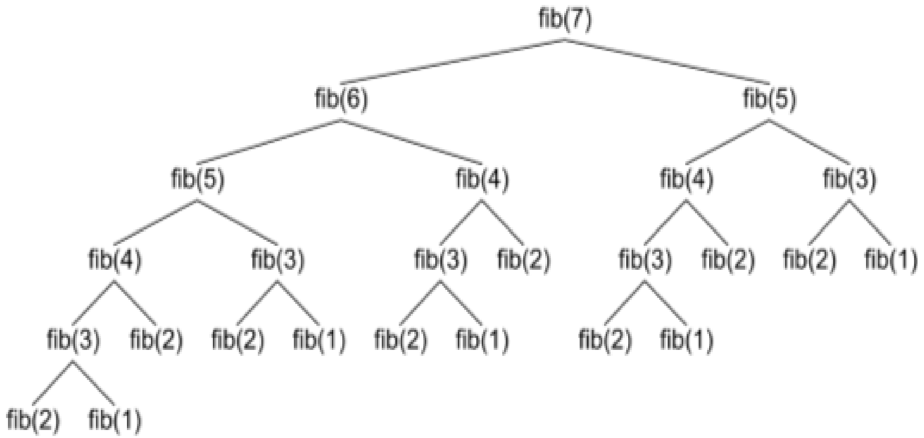
- **Base case:** simple, direct solution for smallest problem
- **Recursive step**: break problem into smaller subproblems
- Self-call: function calls itself with smaller input

Recursion is a key technique used in Divide and Conquer to solve subproblems recursively

# Recursion example: Fibonacci numbers



```
function fibonacci(n):
    if n <= 1:
        return n
    return fibonacci(n-1) + fibonacci(n-2)
```




- **Fibonacci series:** each number is the sum of the two preceding ones

- Time Complexity: $O(2^n)$ due to redundant recursive calls

Recursive functions can be elegant but may lead to high time complexity and inefficiency

# Greedy algorithms

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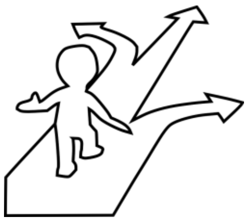

Approach where locally optimal choices lead to a globally optimal solution

- **Greedy property:** making the best choice at each step ensures an optimal solution

- A problem can be broken into subproblems, and <u>optimal solutions to subproblems lead to an overall optimal solution</u>

Use for optimization problems where local choices lead to global solutions, like scheduling and networking

# Greedy algorithms typical use cases

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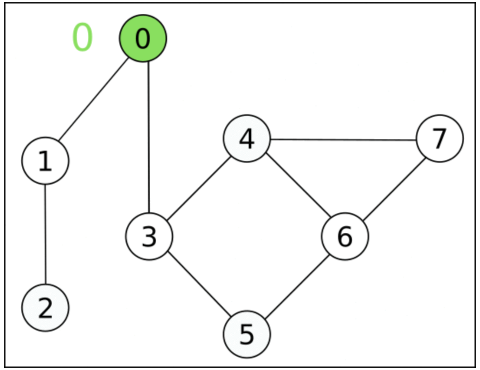
## Graph problems

Finds the shortest path from a single source to all other nodes



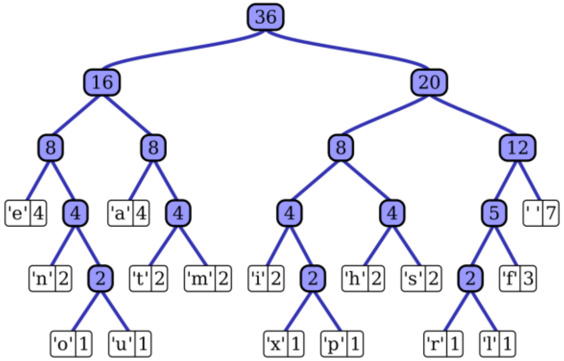

## Scheduling and optimization

Choose the maximum number of non-overlapping activities

| ACTIVITY | START TIME | END TIME |
|---|---|---|
| A1 | 1 | 3 |
| A2 | 2 | 5 |
| A3 | 3 | 4 |
| A4 | 4 | 7 |
| A5 | 7 | 10 |
| A6 | 8 | 9 |

## Resource allocation

**Huffman coding:** creates optimal prefix-free encoding for data compression



# Greedy algorithm example: activity selection

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Select the maximum number of non-overlapping activities given their start and end times

- **Greedy Choice:** always pick the activity that finishes the earliest
- Time Complexity: $O(n \log n)$ due to sorting, followed by $O(n)$ selection
- Real-World Applications: scheduling tasks, meeting room allocation, interview scheduling

```
FUNCTION MaxActivities(activities):
    SORT activities by end time in ascending order
    count ← 1
    lastEnd ← activities[0].end

    FOR i FROM 1 TO length(activities) - 1:
        IF activities[i].start ≥ lastEnd:
            count ← count + 1
            lastEnd ← activities[i].end
    RETURN count
```

# Dynamic programming

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Solve problems by breaking them into subproblems and reusing results

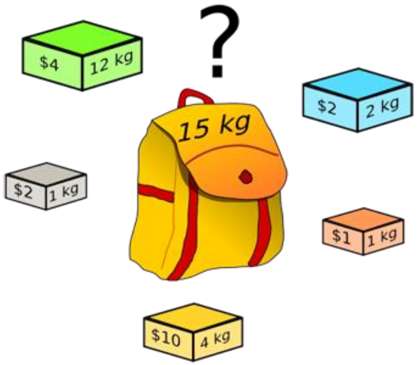
- Optimal solution is built from optimal subproblem solutions
- Overlapping subproblems: **subproblems are solved multiple times, so store results.**

Dynamic programming is used for optimization, decision-making, resource allocation, problem decomposition

# Dynamic programming major use cases

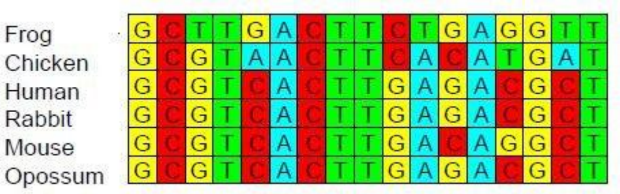
## Optimization

Knapsack problem by maximizing profit within constraints




## Sequence alignment

Finding the best match between two sequences (e.g., DNA, text)




## Shortest path

Computing the shortest path in weighted graphs (e.g., Dijkstra’s algorithm)

# Backtracking

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A general algorithm for finding solutions to problems incrementally, by exploring all possibilities

- Build solutions step by step, abandoning partial solutions that fail constraints (backtrack)

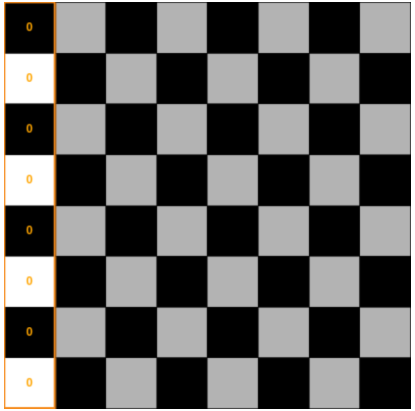
- Use cases: solving combinatorial problems like puzzles, pathfinding, and constraint satisfaction

Backtracking tries all possibilities step by step, undoing choices when they don't work

# Backtracking major use cases

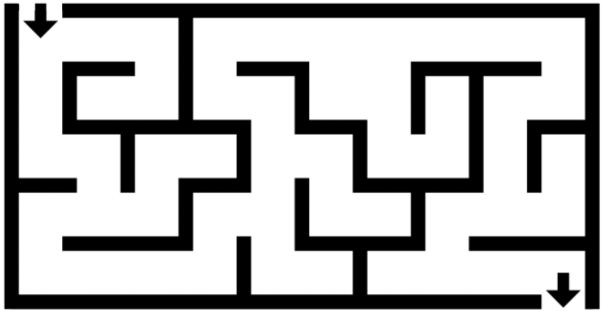
## Solving puzzles

Exploring all possible moves in puzzles like Sudoku and N-Queens to find a solution



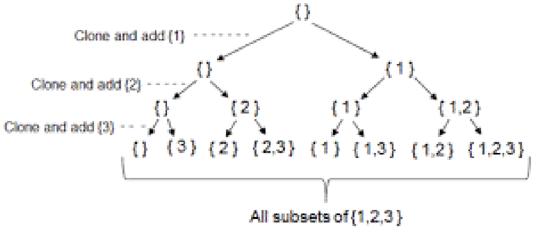

## Pathfinding

Finding possible paths in mazes or graphs by trying different routes and backtracking



## Subset generation

Generating all possible subsets or combinations, like in the subset sum problem



# Summary of algorithm design techniques

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## Optimization and problem solving

- **Greedy:** fast solutions by making local optimal choices
- **Dynamic programming:** optimizes by breaking problems into overlapping subproblems

## Divide and conquer

- **Divide and conquer:** divides problems into smaller subproblems, solving them independently
- **Recursion:** break problems into smaller, similar subproblems

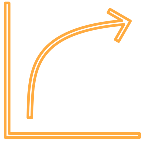
## Exhaustive search

- **Brute force:** tries all solutions without optimization, best for small problems
- **Backtracking:** explore all solutions, backtrack when constraints are violated

# Algorithm Complexity Classes

# What are complexity classes





Categories of problems based on
how their time/space requirements grow

- P: Problems solvable in polynomial time (efficient)
- NP: Problems whose solutions can be verified in polynomial time
- NP-Complete: Hardest problems in NP, no known fast solution
- NP-Hard: Problems at least as hard as NP problems, may not be in NP

Helps determine if a problem can be solved efficiently

# Class P

Problems that can be solved in time proportional to a polynomial function of input size

- Sorting: arranging data in a specific order (e.g., Merge Sort, Quick Sort)
- Searching: finding an element in a list (e.g., Binary Search, Linear Search)
- Shortest Path: finding the shortest path in a graph (e.g., Dijkstra’s Algorithm)

Easy to solve.

Feasible even with large inputs

crucial for real-world applications and scalability

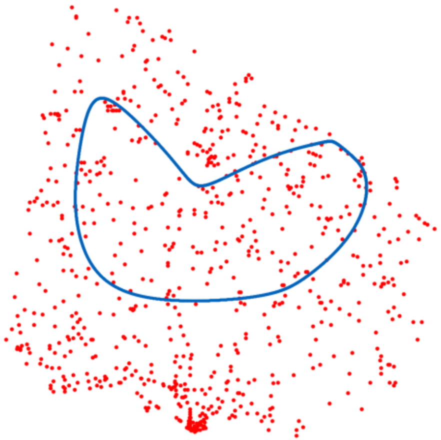
# Class NP



Problems where, if given a solution, we can verify its correctness in polynomial time

- Significance: solutions may be hard to find, but easy to check

- Example: **knapsack problem** (finding the most valuable combination of items without exceeding weight limit)



**Traveling Salesman Problem**: finding the shortest possible route that visits each city once

# The NP-hard and NP-complete problems

## NP – hard

- Problems at least as hard as NP but may not be in NP
- No known polynomial-time solution; may not even have verifiable solutions in polynomial time

## NP – complete

- Problems that are both in NP and NP-Hard
- *<u>If one NP-Complete problem is solved in polynomial time, all NP problems can be solved in polynomial time</u>*

$$P \subseteq NP \subseteq NP\ \mathrm{hard} \subseteq NP\ complete$$

Understanding these classes helps in determining problem feasibility

Many real-world optimization problems fall into these categories

# Key Takeaways

- **Algorithm design strategies**: Brute Force, Divide & Conquer, Recursion, Greedy, Dynamic Programming, and Backtracking solve different types of problems

- **Efficiency matters**: Polynomial-time (P) problems are feasible; NP and NP-Hard problems are much harder to solve

- **NP vs. P**: P problems can be solved efficiently; NP problems can be verified quickly but may not be solvable efficiently

- Computational Limits: NP-Complete problems connect NP and NP-Hard; solving one efficiently could solve all NP problems

# Helpful Resources on Algorithms & Complexity

- 📝 GeekForGeeks Algorithm design techniques
  Concise explanations and examples
- 📖 P versus NP in simple plain English
- 📝 Khan's academy comprehensive list of important algorithms
- 🎥 Example of divide and conquer strategy

# Quote of the Week

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The right algorithm can
save hours of computing…
or years of waiting