Skateboard Optimisation

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ABSTRACT

A skateboard is a popular means of short distance commuting and entertainment. There are several types of Skateboard including electric, longboard, old school and cruiser[1]. This report looks at the minimisation of cost of an old school skateboard whilst ensuring it is safe to use and durable. The two key subsystems analysed are the deck and the wheels.

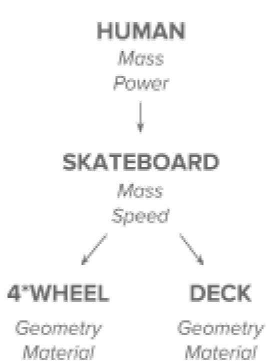
Geometric and material constraints were found through anthropometric data and conducting regression analysis to create metamodels.

Within Matlab fmincon was used to apply a range of optimisation algorithms to the subsystem to find the optimal point. After each subsystem was optimised, the overall system was optimised. The cost was minimised to £3.57. This was an improvement of 77.4%.

1. INTRODUCTION

The objective of this project is to minimise the cost of the skateboard. The system of the skateboard was simplified to include only a deck and four wheels.

Figure 1 | System Diagram with External Influencers



2. SYSTEM LEVEL

The objective function is four lots of the wheel, C_W, and the deck, C_D .The board must be safe to use and so a Safety Factor of 2, *SF*, was applied to the whole subsystem. The stress on the wheel needs to be half of the tensile strength of polyurethane rubber.

It is assumed that the skateboard will be used on tarmac and will not be used for tricks. Therefore the force the skateboard needs to withstand is from a 100 kg mass of a human[2].

So that the skateboard can be carried easily, it must weigh less than 0.8 kg[2]. This will also enable the user move faster.

A trade off must be reached between Von Mises Stresses, displacement and mass. A stiffer board and larger wheels are more safe and stable, however both of these increase the cost.

min **c** $f(C_W, C_D)\ 4C_W + C_D$

where **c** (C_W, C_D)

subject to

h1 (M_H) $M_H - 100 = 0$

h2 $(C_W, Mass_W, C_{pu})$
$$C_W - Mass_W * C_{PU} = 0$$

h3 $(C_D, Mass_D, C_M)$
$$C_D - Mass_D * C_M = 0$$

g1 (SF) $SF - 2 \le 0$

g2 $(Mass_W, Mass_D)$
$$4 * Mass_W + Mass_D - 0.8 \le 0$$

g3$(displacement)$
$$displacement - 2.1 \le 0$$

g4(a, D, W)
$$2 \times tan(a) \times (\tfrac{D}{2} - 11) - W + 15 \le 0$$

g5(A, $Mass_D$, $Mass_W$)
$$\frac{Mass_D + Mass_W}{A} * 9.81 - 20 \le 0$$

3. DECK

3.1 Optimisation formulation

This subsystem consists of the deck of the skateboard; the aim is to minimize cost. The objective function, equation 1.1, is achieved by minimising mass.

$$C_D = Mass_D * C_M \tag{1.1}$$

Design Variables

This problem includes both continuous and discrete variables. The continuous variables are the length, width, thickness and curve width of the deck, shown in *figure 2.i*. Each variable has been given an upper and lower bound based off anthropometric data and existing skateboard data[1][2], shown in *Table 1*.

Table 1 | Lower and Upper bound for variables

Material	Thickness	Width	Curve width	Length
Lower bound (mm)	1	190	60	711
Upper bound (mm)	15	222	170	812

The discrete variable is the material; those used in this subsystem and their parameters are shown in *Table 2*.

Table 2 | Materials used for discrete variables[3]

Material	Plywood	ABS	Aluminium	Carbon Fibre	PET
Cost £/kg	0.4	0.73	1	8.73	157
Yield N/mm²	1.38E+7	3E+7	6.8E+7	8.4E+7	2.3E+7

Displacement, Von Mises Stress and the relationship between length and curve width were used as constraints. In order to ensure there is enough space for the users foot on the board the length and width should follow *equation 1.2*.

$$2x_3 - x_4 \leq 400 \tag{1.2}$$

The stiffness of the board should be high enough such that the board is not unstable. The stiffness of current skateboard decks is 48.2 kg/mm[4]. As the load is fixed, from *equation 1.1* it can be found that the displacement cannot be greater than 2.1mm.

$$k = \frac{W_U}{displacement} \tag{1.3}$$

$$displacement \leq 2.1 \tag{1.4}$$

To give the board a safety factor of 2, the maximum VMS of the board must be less than twice the yield.

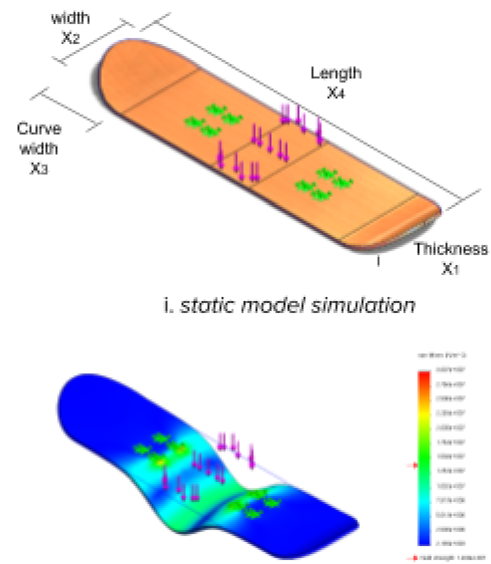
$$VMS \leq 2Y_M \tag{1.5}$$

3.2 Modelling approach

The relationship between the variables and constraints are not directly mathematically modellable. A metamodel was created to obtain data and calculate the relationship between variables and constraints. A CAD model of the deck was created in Solidworks, as shown in *Figure 2.i*; a design study was created to rapidly change the variables.

Latin Hypercube Sampling was used to generate 30 entries which were imported into a design study.The results for mass, Von Mises Stress and displacement for each material were exported from Solidworks and imported into Matlab.

The maximum moment on the skateboard occurs when the weight is concentrated in the centre of the board. As shown in *Figure 2.ii*, the 981 N load was equally distributed across the centre of the board with an assumed foot width of 100 mm[2].



i. *static model simulation*

ii. *Von mises results*

Figure 2: *Static model simulation and example of results from the Solidworks model [5]*

3.3 Explore the problem space

Function Fitting

Linear regression was used to map the inputs to the outputs. It was found that mass was a linear constraint as the R^2 of the model for all materials was above 0.96 and the residuals were randomly distributed. A general equation was derived for mass, with m_i being coefficients for variables.

$$mass = m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5 \tag{1.6}$$

Linear models gave low R^2 for VMS and displacement. The residuals had a curved distribution showing a non-linear model should be used. Through visual inspection it was found the thickness of the material had an inverse relationship both VMS and displacement. An estimation of the model was created using fitnlm; the fit of the real data to the test data can be seen in *Figures 3 and 4.* The non-linear models had a low RSME in comparison to the bounds of the outputs.

$$Displacement = \frac{D_1}{x_2 x_1^2} + D_2 x_4 + D_3 x_3 + D_4 \tag{1.7}$$

$$VMS = \frac{V_1}{x_1 x_2} + V_2 x_4 + V_3 x_3 + V_4 \tag{1.8}$$

Problem Formulation

With all initial constraints taken into account, the optimisation formulation is shown below.

min **x** $\quad f(x_1,x_2,x_3,x_4,m_1,m_2,m_3,m_4,C_M)$
$(m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + m_5) C_M$

where **x** $\quad x_1,x_2,x_3,x_4$

subject to

$g1\,(m_1,m_2,m_3,m_4,m_5,x_1,x_2,x_3,x_4)$
$m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + m_5 - 0.5 < 0$

$g2\,(m_1,m_2,m_3,m_4,m_5,x_1,x_2,x_3,x_4)$
$-m_1 x_1 - m_2 x_2 - m_3 x_3 - m_4 x_4 - m_5 < 0$

$g3\,(x_3,x_4)\;\; 2x_3 - x_4 - 400 \le 0$

$g4\,(D_1,D_2,x_1,x_2,x_4)$
$\frac{D_1}{x_2 x_1^2} + D_2 x_4 + D_3 x_3 + D_4 - 2.1 \le 0$

$g5\,(v_1,v_2,x_1,x_2,x_4,Yield)$
$\frac{V_1}{x_1 x_2} + V_2 x_4 + V_3 x_3 + V_4 - 2Y \le 0$

$g6(x_1)\;\; 2 - x_1 \le 0$

$g7(x_1)\;\; x_1 - 15 \le 0$

$g8(x_2)\;\; 190 - x_2 \le 0$

$g9(x_2)\;\; x_2 - 222 \le 0$

$g10(x_3)\;\; 60 - x_3 \le 0$

$g11(x_3)\;\; x_3 - 170 \le 0$

$g12(x_4)\;\; 711 - x_4 \le 0$

$g13(x_4)\;\; x_4 - 812 \le 0$

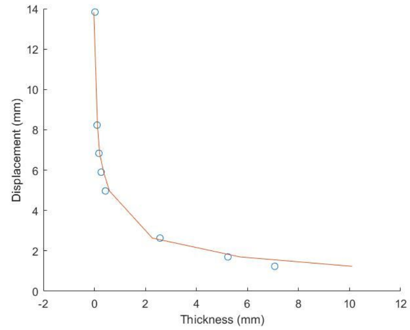


Figure 3 | Fit of displacement model for plywood

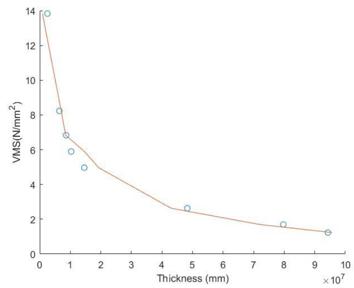


Figure 4 | Fit of VMS model for plywood

As seen in *Table 3*, all variables are well bounded. Some constraints can be relaxed and removed. As the model is minimising cost, which in turn minimises mass constraint *g1* is not active and so was removed from the model. Due to lower bounds being set for all constraints, g2 is also not active and is removed.

g6:g13 were input as upper and lower bounds into the meta model. The constraints that are non active are; g7, g9, g10 and g13 so could be removed from the formulation.

By relaxing g4 and g5 it can be seen that their activity is dependent on material. For ABS, Aluminium, PET removing g5 is not active as the g4 constraint bounds the problem as is active with respect to x_1. For Plywood and Carbon fibre g4 is not active whilst g5 is active with respect to x_1.

Table 3 | Monotonicity table

	X_1	X_2	X_3	X_4		X_1	X_2	X_3	X_4
f	+	+	-	+	g_7	+			
g_1	+	+	-	+	g_8		-		
g_2	-	-	+	-	g_9		+		
g_3			+	-	g_{10}			-	
g_4					g_{11}			+	
g_5					g_{12}				-
g_6	-				g_{13}				+

3.4 Optimise

Fmincon is a non-gradient framework that was applied to this problem. Within this, the interior points and SQP algorithm was used. Interior points is an algorithm suitable for both linear and non-linear models which reaches the solution by traversing the interior of the feasible region. It is a computationally expensive algorithm. [6]The results of applying it to this model are found in Table 4. A random point of [12.59 216 149 793] was used as the starting point.

Table 4 | Optimal results for interior points algorithm

Material	**Plywood**	**ABS**	**Aluminium**	**Carbon Fibre**	**PET**
x_1(mm)	4.047	4.3	2.0	2.0	4.1
x_2(mm)	190	190	190	190	190
x_3(mm)	155.5	60	155.5	155.5	60
x_4(mm)	711	711	711	711	711
Mass(kg)	0.549	0.106	0.795	0.524	0.137
Cost(£)	2.20	7.70	7.95	45.79	2146

SQP is an iterative method suitable for non-linear optimization, *Table 5*. It was selected as it is one of the most successful methods for finding a numerical solution to a problem [6].

Table 5 | Results for SQP

Material	**Plywood**	**ABS**	**Aluminium**	**Carbon Fibre**	**PET**
x_1 (mm)	3.65	4.32	2.0	1	4.1
x_2(mm)	201.9	190	190	190	190
x_3(mm)	152.43	60	155.5	155.5	60
x_4(mm)	787.76	711	711	711	711
Mass (kg)	0.693	0.106	0.795	0.524	0.1367
Cost(£)	2.77	7.70	7.95	45.78	2146

Within a solution, there can be local minima which are found by the solver. To ensure the global minima is found the GlobalSearch algorithm was implemented within fmincon. This algorithm explores different starting points, then rejects the points which will not improve the current local minima[7]. The GlobalSearch(interior points) improved all material solutions, whilst GlobalSearch(SQP) generally achieved the same solution.

Global Search validates the solutions from SQP as global optimum. Parametric optimisation was conducted to select the best material. As Table 6 shows, Plywood was selected as the material.

Table 6 | Results for Global Search(SQP) and Global Search(interior points)

x1 (mm)	**x2 (mm)**	**x3 (mm)**	**x4 (mm)**	**Mass (kg)**	**Cost (£)**
4.05	190	155.5	752.27	0.711	2.20

3.5 Analysis

The optimal model was input into Solidworks to find the estimated mass. The mass within Solidworks was 680g, giving a deviation of 4.5%. Although for

3.6 Discussion

Within this subsystem the use of fmincon and Global Search were investigated. Global Search yielded the lowest results for most materials as it found the global minima. Parametric selection was used to determine that Plywood is the ideal material with dimensions x = [4.05, 190, 752.27, 711] with a mass of 0.711kg and cost of £2.20.

The construction of the models that inform this optimisation have a large impact on the outcome. As discussed, the results for mass are 4.5% different from the Solidworks model. Although this is an acceptable error, the non-linear models created did not fit some materials as well as others. In particular, plastics did not fit the model as well as shown in *Figure 5*. This could be due to plastic deformation. Therefore, for the future a neural network could be fitted based on each material. Although this would be computationally expensive, it would give more accurate models for all materials.

For future works, multi objective optimisation could be implemented in order to minimise Von Mises Stresses to increase the durability of the skateboard. A weighting would have to be given to both Von Mises Stress and cost. A genetic algorithm could also be used to choose between the materials to improve on parametric optimisation.

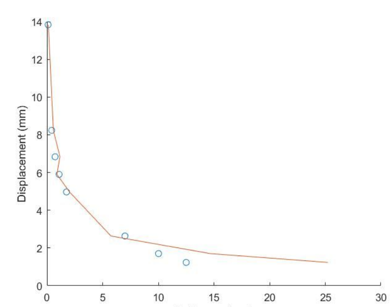


Figure 5 | Goodness of fit of displacement model for PET

4 WHEELS

To minimise the cost of the skateboard, the cost of the wheels will also need to be minimised. Existing skateboard wheels were reviewed to find out if there is any industry standards for the design. The material used is usually made of one single material, polyurethane, and the standard bearings used in the wheels are of size 608. The materials and size of the wheel are used to form the constraints in the optimisation process.

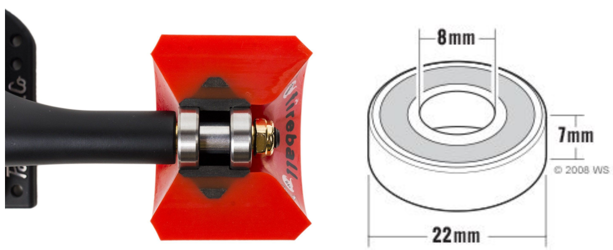


Figure 6 | Skateboard Wheel and 608 Bearing

4.1 Optimisation formulation

The cross-sectional view of the skateboard wheel is shown in Fig.4.1.1. The variables considered in the study are all dimensional, including the width of the wheel (*W*), the diameter of the wheel (*D*), the indented angle (*a*) and the edge fillet size (r). There is also a dependent variable *x*. The cost of manufacturing a wheel is calculated by

$$Cw = Mass_w * C_{PU}$$

where C_w is the total cost, $Mass_w$ is the mass of the wheel and C_{PU} is the cost per kg. As the material is decided to follow the industry standard (using polyurethane rubber), the problem can be written as to minimise the mass, which is also equivalent to minimise the volume.

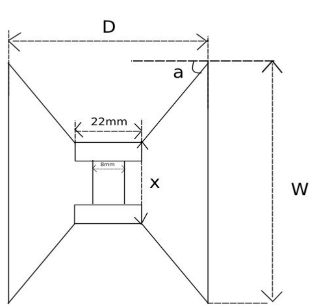


Figure 7 | Cross-Sectional View of the Wheel

4.2 Modelling Approach

Constraints

Existing skateboard wheel dimension were used as lower bounds and upper bounds for the wheel dimensions. There are other constraints caused by the geometry of the wheel, as it needs to hold two 608-size bearings.

The factor of safety used is decided to be 2. As rubber does not have a yield strength, the tensile strength of PUR was used as the reference.

Objective function

To minimise the cost, the objective function is equivalent to minimise the volume of the wheel when the material is defined (PUR rubber). However, because the form of the wheel is not a standard geometry, the volume was decided to be found through the mass property in SolidWorks model. The objective function was then calculated by function fitting in MATLAB.

Latin Hypercube sampling was used after constraints for each variable were established, and 30 sets of values were used to get output von mises stress, displacement and volume from simulations.

While simulating, it is assumed that the force exerted on the model is at the centre of the wheel, and is in contact with lower half of the inner surface.

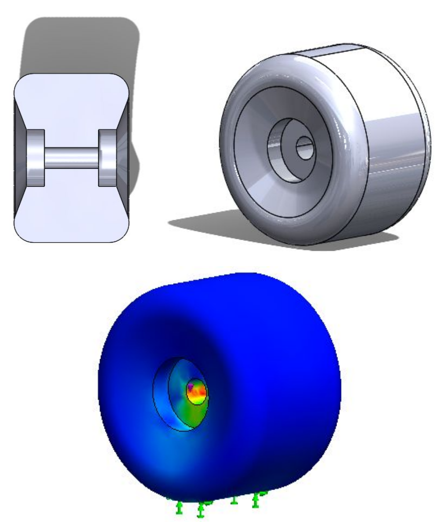


Figure 8 | Skateboard Wheel Model and Simulation Settings

4.3 Explore the problem space

Constraints

One of the design considerations is that, two standard bearings need to be fitted into each skateboard wheel. Therefore, the constraint is

$$2 \times 7 - x \le 0 \rightarrow 14 - x \le 0$$

The problem was further simplified to optimise the wheels in a static situation, rather than rolling. The tensile strength of polyurethane rubber is 40MPa[8] (40N/mm^2). For safety reasons, the design was done under a safety factor of 2, which means that the maximum stress in the wheel is 20MPa. When considering the person weighs 100kg and the deck weighs 500g, each wheel will have an exerted force of

$$(100 + 0.5) \times 9.81 \div 4 = 246.5N$$

As the force is exerted at the centred of the wheel, the smallest contact area exerted by the force will be

$$A = \frac{8\pi}{2}(x - 7 \times 2) = 4\pi(x - 14)$$

Therefore, the stress constraint will be

$$\frac{246.5}{4\pi(x-14)} \le 20 \rightarrow 15 - x \le 0$$

which means that the first inequality is now inactive.

x can be formulated as

$$x = W - 2 \times tan(a) \times (\tfrac{D}{2} - 11),$$

$$2 \times tan(a) \times (\tfrac{D}{2} - 11) - W + 15 \le 0$$

Therefore, the maximum value of a can be found as 24.23 degrees.

Objective function

Model Fitting 1 - Linear Regression

Initially, a multivariate linear regression was done after the corresponding mass of each wheel design was found.

However, the R squared value is 0.96, suggesting that the model is not a very good fit. Apart from that, the residual plot is in a U-shape, which means that the correlation is nonlinear.

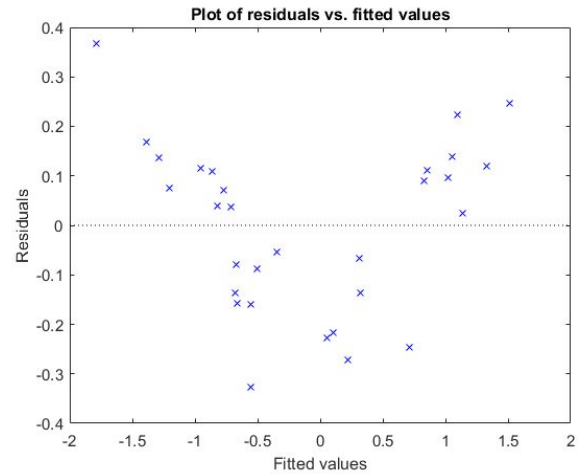


Figure 9 | Linear Regression Residual Plot

Model Fitting 2 - Polynomial Fit

polyfitn and related functions were used to fit the data into a polynomial function. By experimenting with different number of dimensions, the R squared value in the final model is 0.9996, with dimensions of 2 (the highest dimension can be reached for this model). Several points from the input were used to predict the results from the model, and the results were compared with the actual mass (shown in Table 7).

Table 7 | Polynomial Model Testing

Point	Actual Mass	Predicted Mass	% difference
1	0.205	0.2087	1.8%
2	0.338	0.3393	0.3%
3	0.238	0.2377	0.12%
4	0.125	0.1246	0.32%

The model obtained from the polyfitn is used as the objective function for the study.

Therefore, the optimisation problem can be simplified and written as

Min **x** f (W, D, a, r, c_w) ($k_1W-k_2D+k_3a+k_4r+k_5WD+k_6Wa-k_7Wr-k_8Da-k_9Dr-k_{10}ar+k_{11}W^2+k_{12}D^2-k_{13}a^2-k_{14}r^2+k_{15}$) $\times\, c_w$

Where **x** $= (W, D, a, r)$

s.t. $g1(W)\ 30 - W \le 0$

$g2(W)\ W - 70 \le 0$

$g3(D)\ D - 100 \le 0$

$g4(D)\ 47 - D \leq 0$

$g5(a)\ a - 24.23 \leq 0$

$g6(a, D, W)\quad 2 \times tan(a) \times (\frac{D}{2} - 11) - W + 15 \leq 0$

$g7(r)\ r - 8 \leq 0$

The boundaries for the dimension of W and D are based on existing skateboard wheel design.[1]

4.4 Optimise

fmincon and sqp

The nonlinear inequality constraint *g6* was included in the setup to make sure the outcome will satisfy this condition. The starting points were selected as so that it can cover as much as the possible range.

Table 8 | Results from fmincon and SQP

Starting Point	Optimal Point	Mass (kg)	Cost (£)
33.61, 99.25, 9.48,6.32	30,47,0,8	0.0453	0.272
42.68, 78.24, 14.3, 1.74	30,47,0,8	0.0453	0.272
55.25, 78.95, 11.0, 3.01	30,47,20.3,8	0.0492	0.295
42.68, 78.24, 14.3, 1.74	30,52.2,24.2,8	0.0484	0.290

Genetic Algorithm

Genetic algorithm was used as the second algorithm to check if the optimum found is global. Another set of points were used as the input. The same lower bounds, upper bounds and inequality constraint were used. The results showed that the minimal point is the same as the one found in the previous method.

The default density of the material in Solidworks is 1g/mm^3. As the density of PUR is 1.26g/mm^3 [5], the mass of each wheel will be 45.3g, and the cost of one wheel will be 0.342 pounds. The cost of materials for four wheels will be 1.37 pounds.

Table 9 | Optimised Result for Each Wheel

Cw(£)	Mass(kg)	D(mm)	W(mm)	a(degree)	r(mm)
0.342	0.0453	30	47	0	8

4.5 Discussion

The optimal point in the study of wheel occurs when width is 30mm, diameter is 47mm, angle a is 0 degree and the radius of the fillet is 8mm. It can be seen that each value is either at its maximum or minimum value. This can be a result as the objective is just to minimise the mass (i.e. volume). If other useful constraints can be identified (e.g. the maximum displacement which is considered as safe), the study could be more meaningful in terms of achieving other requirements.

Apart from that, if a rolling wheel can be simulated, more constraints could be applied to make the study more realistic.

5. SYSTEM LEVEL OPTIMISATION

As both subsystems were minimised for the same objective, cost, the outputs of the subsystem can be directly input into the system objective function. There are four wheels and only one deck and hence this is

$$4C_W + C_D \tag{1.9}$$

Table 10 | Results for system level objective function

C_D(£)	C_W (£)	C(£)
2.20	0.342	3.57

As shown in table 9, the optimised cost for the Skateboard is £3.57.

6. CONCLUSION

As the two subsystems are quite independent, each subsystem was optimised individually, and the minimised cost of both subsystems were combined to calculate the total cost. The Deck has reduced the cost by 77.1%, and the cost of wheels is reduced by 78% comparing with the starting point.

Table 11 | Cost Comparison After Optimisation

Subsystem	Starting Cost(£)	Minimised Cost(£)
Deck	9.62	2.20
4 Wheel	6.20	1.37

Therefore, the cost of the skateboard in total is minimised by 77.4%.

Due to being largely mechanical, there were few interdependencies between the system. The system could be more complicated for further improvement. For example, the battery or energy transmission of an electric skateboard could be optimised. The optimisation also could have taken into account more environmental factors, such as the frictional coefficient of the tarmac and force applied by the user. This would increase the possible acceleration achievable by the skateboard. Multi-objective optimisation could be carried out to achieve both optimisation goals.

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APPENDIX. NOMENCLATURE

Nomenclature	Meaning	Units
SYSTEM		
C_M	Cost of deck material	GBP/kg
C_{PU}	Cost of wheel material (polyurethane)	GBP/kg
C_W	Cost of wheel	GBP
C_D	Cost of deck	GBP
M_H	Mass human	kg
$Mass_W$	Mass of wheel	kg
$Mass_D$	Mass of Deck	kg
Displacement	Displacement	mm
SF	Safety Factor	
A	Area on the wheel where normal force exerts	mm^2
D	Diameter of the wheel	mm
α	Angle of indent on the wheel	degree
W	Width of the wheel	mm
SUBSYSTEM 1		
D_1	Displacement coefficient for thickness	
D_2	Displacement coefficient for width	
D_3	Displacement coefficient for curve width	
D_4	Displacement coefficient for length	
k	Stiffness	N/mm^2
Mass		Kg
m_1	Mass coefficient for width	
m_2	Mass coefficient for width	
m_3	Mass coefficient for curve width	
m_4	Mass coefficient for length	
m_5	Mass constant	mm
x_1	Deck thickness	mm
x_2	Deck width	mm
x_3	Deck curve width	mm
x_4	Deck length	mm
Y_m	Yield strength of material	N/mm^2
VMS	Von Mises Stress	N/mm^2
v_1	VMS coefficient for thickness	

v_2	VMS coefficient for width	
v_3	VMS coefficient for curve width	
v_4	VMS coefficient for length	
SUBSYSTEM 2		
W	Width of the wheel	mm
D	Diameter of the wheel	mm
a	Angle of incident	degree
r	Radius of fillet	mm
k_1	Coefficient for width	
k_2	Coefficient for diameter	
k_3	Coefficient for angle a	
k_4	Coefficient for fillet radius	
k_5	Coefficient for width times diameter	
k_6	Coefficient for width times angle a	
k_7	Coefficient for width times fillet radius	
k_8	Coefficient for diameter times angle a	
k_9	Coefficient for diameter times fillet radius	
k_{10}	Coefficient for angle a times fillet radius	
k_{11}	Coefficient for the square of width	
k_{12}	Coefficient for the square of diameter	
k_{13}	Coefficient for the square of angle a	
k_{14}	Coefficient for the square of fillet radius	
k_{15}	Constant	
x	Distance between two bearings	mm