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1

$$\min\ c^T x \quad Ax \leq b \quad x \geq 0 \longrightarrow \min \begin{bmatrix} 3 & 9 \end{bmatrix} x \quad \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} x \leq \begin{bmatrix} 12 \\ 6 \end{bmatrix} \quad x \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} z_0 - 3x_1 - 9x_2 \geq 0 & (E_0) \\ 2x_1 + 3x_2 \leq 12 & (E_1) \\ x_1 + x_2 \leq 6 & (E_2) \\ x_1 \geq 0 & (E_3) \\ x_2 \geq 0 & (E_4) \end{cases}$$

$$\begin{cases} z_0 - 3x_1 - 9x_2 \geq 0 & (E_0) & [<0] \\ -x_1 - \frac{3}{2}x_2 \geq -6 & (-E_1/2) & [<0] \\ -x_1 - x_2 \geq -6 & (-E_2) & [<0] \\ x_1 \geq 0 & (E_3) & [>0] \\ x_2 \geq 0 & (E_4) & [=0] \end{cases}$$

forma $Ax \geq b$

$$z_0 - 9x_2 \geq 0 \quad (E_0) + (E_3) \quad [<0]$$

$$-\frac{3}{2}x_2 \geq -6 \quad (-E_1/2) + (E_3) \cdot 2/3 \quad [<0]$$

$$-x_2 \geq -6 \quad (-E_2) + (E_3) \quad [<0]$$

$$x_2 \geq 0 \quad (E_4) \quad [>0]$$

$z_0 \geq 0 \quad (E_0) + (E_3) + 9(E_4)$

Tautologia $\begin{cases} 0 \geq -6 & (-E_1/3) + \frac{2}{3}(E_3) + (E_4) \\ 0 \geq -6 & (-E_2) + (E_3) + (E_4) \end{cases}$

$z_0 \geq d_K$

$z_0 = 0 \Rightarrow$

$$\left.\begin{array}{lll} -9x_2 \geq 0 & \to & x_2 \leq 0 \\ -3/2\, x_2 \geq -6 & \to & x_2 \leq 4 \\ -x_2 \geq -6 & \to & x_2 \leq 6 \\ x_2 \geq 0 & \to & x_2 \geq 0 \end{array}\right\} x_2 = 0$$

$z_0 = 0, \quad x_2 = 0$

$$\left.\begin{array}{lll} -3x_1 \geq 0 & \to & x_1 \leq 0 \\ -x_1 \geq -6 & \to & x_1 \leq 6 \\ -x_1 \geq -6 & \to & x_1 \leq 6 \\ x_1 \geq 0 & \to & x_1 \geq 0 \end{array}\right\} x_1 = 0$$

$\therefore \; X^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2) $\tilde{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \in proj_y(P)$?

$P = \{(x,y) \in \mathbb{R}^2 \times \mathbb{R}^2 \mid Ax + By \geq b\}$ //

$$P: \begin{array}{l} x_1 - 2x_2 + y_1 + 6y_2 \geq 6 \\ -2x_1 + 3x_2 + 4y_1 + y_2 \geq 5 \\ 4x_1 - 4x_2 + y_1 + y_2 \geq 10 \\ x_1 + 2x_2 \geq 0 \\ x_2 \geq 0 \\ y_1 \geq 0 \\ y_2 \geq 0 \end{array}$$

Questão 2

Para verificar se $\tilde{y}$ pertence a $\text{proj}_y(P)$ note que se u é um vetor que projeta, então

$$u^T(Ax + By) \geq u^T b$$
$$u^T By - u^T b \geq 0, \ \forall \ y.$$

Basta então encontrar $\min_u u^T(B\tilde{y} - b)$ sujeito às restrições de projeção $A^T u = 0$ e $u \geq 0$. Se o mínimo for maior ou igual a zero, então é claro que não existe u tal que $(u^T By - u^T b) < 0$, logo $\tilde{y}$ pertence à projeção. Isso é confirmado pelo seguinte script, que resolve a PL

$$\begin{aligned} \min & (B\tilde{y} - b)^T u \\ \text{s.t.} \quad & A^T u = 0 \\ & u \geq 0 \end{aligned}$$

In [1]:
```
import numpy as np
from scipy.optimize import linprog

A = np.array([
    [1, -2],
    [-2, 3],
    [4, -4],
    [1, 2],
])

b = np.array([6,5,10,0]).reshape(-1, 1)
B = np.array([
    [1, 6],
    [4, 1],
    [1, 1],
    [0, 0],
])

y_tilde = np.array([1, 3]).reshape(-1, 1)

res = linprog(c=(B @ y_tilde - b).ravel(), A_eq=A.T, b_eq=np.zeros((2,1)), bounds=(
print('u:', res.x)
print('Custo ótimo:', res.fun)
```

```
u: [-0. -0.  0.  0.]
Custo ótimo: 0.0
```

$Ax + By \geq b$, se u projeta: $u^T A = 0$, $u > 0$

$\underbrace{u^T A x}_{0} + u^T B y \geq u^T b$

$u^T B y \geq u^T b \Rightarrow u^T B y - u^T b \geq 0$

$\min \; u^T(B\tilde{y} - b)$ $\qquad$ $\tilde{y} \in \text{Proj}_y(P)$ se satisfaz

s.t. $A^T u = 0$

$u \geq 0$

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 3 \\ 4 & -4 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 6 \\ 4 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$u^T B\tilde{y} - u^T b = u^T\left(\begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 18 \\ 3 \\ 3 \\ 0 \end{bmatrix}\right) - u^T \begin{bmatrix} 6 \\ 5 \\ 10 \\ 0 \end{bmatrix}$$

$$= u^T\left(\begin{bmatrix} 13 \\ 2 \\ -6 \\ 0 \end{bmatrix}\right) \rightsquigarrow 13u_1 + 2u_2 - 6u_3 + 0u_4$$

$$A^T u = \begin{bmatrix} 1 \\ -2 \end{bmatrix} u_1 + \begin{bmatrix} -2 \\ 3 \end{bmatrix} u_2 + \begin{bmatrix} 4 \\ -4 \end{bmatrix} u_3 + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_4$$

$$\rightarrow \begin{aligned} u_1 - 2u_2 + 4u_3 + u_4 &= 0 \\ -2u_1 + 3u_2 - 4u_3 + 2u_4 &= 0 \end{aligned}$$

$$\hookrightarrow \begin{cases} \min \; 13u_1 + 2u_2 - 6u_3 \\ \text{s.t.} \\ \quad u_1 - 2u_2 + 4u_3 + u_4 = 0 \\ \quad -2u_1 + 3u_2 - 4u_3 + 2u_4 = 0 \\ \quad 0 \leq u_i \leq 1 \end{cases}$$

u_1

$z_0 - 13u_1 - 2u_2 + 6u_3 \geq 0$ E_0 [< 0]

igualdade
$$\begin{cases} u_1 - 2u_2 + 4u_3 + u_4 \geq 0 & E_1 \quad [>0] \\ -2u_1 + 3u_2 - 4u_3 + 2u_4 \geq 0 & E_2 \quad [<0] \\ -u_1 + 2u_2 - 4u_3 - u_4 \geq 0 & E_3 \quad [<0] \\ 2u_1 - 3u_2 + 4u_3 - 2u_4 \geq 0 & E_4 \quad [>0] \end{cases}$$

$u_1 \geq 0$ E_5 [>0]

$u_2 \geq 0$ E_6

$u_3 \geq 0$ E_7

$u_4 \geq 0$ E_8

Proj$_{u_2,u_3,u_4}$

$E_0 + 13E_1$: $z_0 - 28u_2 + 58u_3 + 13u_4 \geq 0$ [<0]

$E_0 + \frac{13}{2}E_4$: $z_0 - \frac{43}{2}u_2 + 32u_3 - 13u_4 \geq 0$ [<0]

$E_0 + 13E_5$: $z_0 - 2u_2 + 6u_3 \geq 0$ [<0]

$E_2 + 2E_1$: $-u_2 + 4u_3 + 4u_4 \geq 0$ [<0]

$E_2 + E_4$: $0 \geq 0$

$E_2 + 2E_5$: $3u_2 - 4u_3 + 2u_4 \geq 0$ [>0]

$E_3 + E_1$: $0 \geq 0$

$E_3 + E_4/2$: $\frac{1}{2}u_2 - 2u_3 - 2u_4 \geq 0$ [>0]

$E_3 + E_5$: $2u_2 - 4u_3 - u_4 \geq 0$ [>0]

E_6: $u_2 \geq 0$ [>0]

Proj$_{u_3,u_4}$

$E_0 + 13E_1 + 28/3E_2 + 56/3E_5$: $z_0 + \frac{62}{3}u_3 + \frac{95}{3}u_4 \geq 0$ [>0]

$E_0 + 13E_1 + 56E_3 + 28E_4$: $z_0 - 54u_3 - 99u_4 \geq 0$ [<0] ★

$E_0 + 13E_1 + 14E_3 + 14E_5$: $z_0 + 2u_3 - u_4 \geq 0$ [>0] Ü

$E_0+13E_1+28E_6$: $z_0+58u_3+13u_4 \geq 0$ [>0]

$E_0+\frac{13}{2}E_4+\frac{43}{6}E_2+\frac{43}{3}E_3$: $z_0+\frac{10}{3}u_3+\frac{4}{3}u_4 \geq 0$ [>0]

$E_0+\frac{13}{2}E_4+43E_3+\frac{43}{2}E_4$: $z_0-54u_3-99u_4 \geq 0$ [<0]

$E_0+\frac{13}{2}E_4+\frac{43}{4}E_3+\frac{43}{4}E_5$: $z_0-11u_3-\frac{95}{4}u_4 \geq 0$ [<0]

$E_0+\frac{13}{4}E_4+\frac{43}{2}E_6$: $z_0+32u_3-13u_4 \geq 0$ [>0]

$E_0+13E_5+\frac{2}{3}E_2+\frac{4}{3}E_5$: $z_0+\frac{10}{3}u_3+\frac{4}{3}u_4 \geq 0$ [>0]

$E_0+13E_5+4E_3+2E_4$: $z_0-2u_3-8u_4 \geq 0$ [<0]

$E_0+13E_5+E_3+E_5$: $z_0+2u_3-u_4 \geq 0$ [>0] ü

$E_0+13E_5+2E_6$: $z_0+6u_3 \geq 0$ [>0]

$E_2+2E_1+\frac{1}{3}E_2+\frac{2}{3}E_5$: $\frac{8}{3}u_3+\frac{14}{3}u_4 \geq 0$ [>0]

$E_2+2E_1+2E_3+E_4$: $0 \geq 0$

$E_2+2E_1+\frac{1}{2}E_3+\frac{1}{2}E_5$: $2u_3+\frac{7}{2}u_4 \geq 0$ [>0]

$E_2+2E_1+E_6$ $\quad 4u_3+4u_4 \geq 0$ [>0]

E_7: $u_3 \geq 0$ [>0]

proj u_4:

$E_0+13E_1+56E_3+28E_4+\frac{3\cdot 54}{62}\left(E_0+13E_1+\frac{28}{3}E_2+56/3\,E_5\right)$

$$2z_0-\frac{504}{31}u_4 \geq 0 \quad [<0]$$

$E_0+13E_1+56E_3+28E_4+27\left(E_0+13E_1+14E_3+14E_5\right)$

$$2z_0-126u_4 \geq 0 \quad [<0]$$

$E_0+13E_1+56E_3+28E_4+\frac{54}{58}\left(E_0+13E_1+28E_6\right)$

$$2z_0-\frac{2520}{29}u_4 \geq 0 \quad [<0]$$

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{3\cdot 54}{10}\left(E_0 + \frac{13}{2}E_4 + \frac{43}{6}E_2 + \frac{43}{3}E_3\right)$$

$$2z_0 - \frac{387}{5}u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{54}{32}\left(E_0 + \frac{13}{4}E_4 + \frac{43}{2}E_6\right)$$

$$2z_0 - \frac{1935}{16}u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{3\cdot 54}{10}\left(E_0 + 13E_5 + \frac{2}{3}E_2 + \frac{4}{3}E_5\right)$$

$$2z_0 - \frac{387}{5}u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + 27\left(E_0 + 13E_5 + E_3 + E_5\right)$$

$$2z_0 - 126u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{54}{6}\left(E_0 + 13E_5 + 2E_6\right)$$

$$2z_0 - 99u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{3\cdot 54}{8}\left(E_2 + 2E_1 + \frac{1}{3}E_2 + \frac{2}{3}E_5\right)$$

$$z_0 - 9/2\, u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + 27\left(E_2 + 2E_1 + \frac{1}{2}E_3 + \frac{1}{2}E_5\right)$$

$$z_0 - \frac{191}{2}u_4 \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + \frac{54}{4}\left(E_2 + 2E_1 + E_6\right)$$

$$z_0 - \frac{95}{4} \geq 0$$ [<0]

$$E_0 + 13E_1 + 56E_3 + 28E_4 + 54\left(E_7\right)$$

$$z_0 - 99u_4 \geq 0$$ [<0]

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{33}{62}\left(E_0 + 13E_1 + 28/3\,E_2 + 56/3\,E_5\right)$$

$$2z_0 - \frac{855}{124}u_4 \geq 0$$

$[<0]$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{11}{2}\left(E_0 + 13E_1 + 14E_3 + 14E_5\right)$$

$$2z_0 - \frac{117}{4}u_4 \geq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{11}{58}\left(E_0 + 13E_1 + 28E_6\right)$$

$$2z_0 - \frac{2469}{116}u_4 \geq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{33}{10}\left(E_0 + \frac{13}{2}E_4 + \frac{43}{6}E_2 + \frac{43}{3}E_3\right)$$

$$2z_0 - \frac{387}{20}u_4 \leq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{11}{32}\left(E_0 + \frac{13}{4}E_4 + \frac{43}{2}E_6\right)$$

$$2z_0 - \frac{903}{32}u_4 \leq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{33}{10}\left(E_0 + 13E_5 + \frac{2}{3}E_2 + \frac{4}{3}E_5\right)$$

$$2z_0 - \frac{387}{20}u_4 \leq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{11}{6}\left(E_0 + 13E_5 + 2E_6\right)$$

$$2z_0 - \frac{95}{4} \geq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{33}{8}\left(E_2 + 2E_1 + \frac{1}{3}E_2 + \frac{2}{3}E_5\right)$$

$$z_0 - \frac{9}{2}u_4 \geq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + \frac{11}{4}\left(E_2 + 2E_1 + E_6\right)$$

$$z_0 - \frac{51}{4} \geq 0$$

$$E_0 + \frac{13}{2}E_4 + \frac{43}{4}E_3 + \frac{43}{4}E_5 + 54\left(E_7\right)$$

$$z_0 - \frac{95}{4}u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + \frac{6}{62}\left(E_0 + 13E_1 + 28/3\,E_2 + 56/3\,E_5\right)$$

$$2z_0 - \frac{153}{31}u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 2\left(E_0 + 13E_1 + 14E_3 + 19E_5\right)$$

$$2z_0 - 9u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + \frac{2}{58}\left(E_0 + 13E_1 + 28E_6\right)$$

$$2z_0 - \frac{219}{29}u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 6/10\left(E_0 + \frac{13}{2}E_4 + \frac{43}{6}E_2 + \frac{43}{3}E_3\right)$$

$$2z_0 - 36/5\,u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 2/32\left(E_0 + \frac{13}{4}E_4 + \frac{43}{2}E_6\right)$$

$$2z_0 - \frac{191}{16} \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 2/6\left(E_0 + 13E_5 + 2E_6\right)$$

$$2z_0 - 8u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 6/8\left(E_2 + 2E_1 + \frac{1}{3}E_2 + \frac{2}{3}E_5\right)$$

$$z_0 - 9/2\,u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 1/2\left(E_2 + 2E_1 + E_6\right)$$

$$z_0 - 6u_4 \geq 0$$

$$E_0 + 13E_5 + 4E_3 + 2E_4 + 2\left(E_7\right)$$

$$z_0 - 8u_4 \geq 0$$

E8: $u_4 \geq 0$

$$\Rightarrow z_0 \geq d_k \, , \quad d_k = 0 \ \forall k$$

$$z_0^* = \max\{0, \cdots, 0\} = 0$$

Tome $z_0 = 0$:

As restrições agregadas de $proj_{u_4}(P)$ são todas da forma

$$-\alpha u_4 \geq 0 \, , \quad \mathbb{R} \ni \alpha > 0$$

$$\therefore u_4 = 0$$

Restrições de $proj_{u_3, u_4}$: $(z_0 = 0 = u_4)$

$+\frac{62}{3}u_3$	≥ 0	$+\frac{10u_3}{3}$	≥ 0
$-54u_3$	≥ 0	$-2u_3$	≥ 0
$+2u_3$	≥ 0	$+2u_3$	≥ 0
$+58u_3$	≥ 0	$+6u_3$	≥ 0
$\frac{10}{3}u_3$	≥ 0	$\frac{8}{3}u_3 +$	≥ 0
$-54u_3$	≥ 0	$2u_3$	≥ 0
$-11u_3$	≥ 0	$4u_3$	≥ 0
$+32u_3$	≥ 0	u_3	≥ 0

$$\Rightarrow u_3 = 0$$

Restrições de $proj_{u_2, u_3, u_4}$ $(z_0 = u_4 = u_3 = 0)$

$-28u_2 +$	≥ 0	$3u_2$	≥ 0
$-\frac{43}{2}u_2$	≥ 0	$\frac{1}{2}u_2$	≥ 0
$-2u_2$	≥ 0	$2u_2$	≥ 0
$-u_2 +$	≥ 0	u_2	≥ 0

$\Rightarrow u_2 = 0$

Restrições de P:

$$\begin{array}{llll} -13u_1 \geq 0 & & -u_1 & \geq 0 \\ u_1 \geq 0 & & 2u_1 & \geq 0 \\ -2u_1 \geq 0 & & u_1 & \geq 0 \end{array}$$

$\Rightarrow u_1 = 0$

$\therefore \; u = [0 \quad 0 \quad 0 \quad 0]$

$\min \; u^T(B\tilde{y} - b) = 0$

$\therefore \; \nexists \; u > 0 \;$ t.q. $\; u^T(B\tilde{y} - b) < 0$

$\Rightarrow \tilde{y} \in \mathrm{proj}_y(P)$

3 $f(x) = x_1 + x_2 + 3y_1 + 3y_2$

$$= [1 \quad 1 \quad 3 \quad 3] \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} = c^T x$$

(1) $\min \; c^T x$

s.t. $Ax \geq b$, $A = \begin{bmatrix} 1 & -2 & 1 & 6 \\ -2 & 3 & 4 & 1 \\ 4 & -4 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 5 \\ 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Primal

PL (1) é dado por: $\min \; z_0$

s.t. $z_0 - c^T x \geq 0$

$Ax \geq b$

Suponha (u_0, u) um vetor que projeta x:

$$\begin{bmatrix} -c^T \\ A \end{bmatrix} x \geq \begin{bmatrix} -z_0 \\ b \end{bmatrix} \to [u_0 \; u^T] \begin{bmatrix} -c^T \\ A \end{bmatrix} x \geq [u_0 \; u^T] \begin{bmatrix} -z_0 \\ b \end{bmatrix}$$

$$\to [-u_0 c^T \;\; u^T A] x \geq -u_0 z_0 + u^T b$$

$$-u_0 c^T x + u^T A x + u_0 z_0 \geq u^T b$$

$$(u^T A - u_0 c^T) x + u_0 z_0 \geq u^T b$$

Se (u_0, u) projeta, então $(u^T A - u_0 c^T) = 0$

$\Rightarrow u_0 z_0 \geq u^T b$

- Se $u_0 = 0 \Rightarrow 0 \geq u^T b$ sistema ilimitado, se for viável

$\Rightarrow (u^T A - 0) = 0 \Rightarrow u^T A = 0, u \geq 0$

- Se $u_0 \neq 0$, tome $u_0 = 1$ sem perda de generalidade (basta tomar $u' = u/u_0$ t.q. $u_0' = 1$)

$\Rightarrow z_0 \geq u^T b$ *

Note que a projeção irá resultar em:

$$z_0 \geq d_k, \quad k = 1, \cdots, p$$
$$0 \geq d_i, \quad i = p+1, \cdots, q$$

A restrição * então é obtida por projeção: $u^T b = d_k$.
Então, basta encontrar o máximo de $u^T b$ t.q. $u^T A - u_0 c^T = 0$

$\to u'^T A = c^T, \; u' \geq 0$

$\Rightarrow$ Problema dual:

$$\begin{aligned} \max \;& b^T u \\ \text{s.t. } & A^T u = c \\ & u \geq 0 \end{aligned}$$

$$\longrightarrow \max \begin{bmatrix} 6 & 5 & 10 & 0 & 0 & 0 & 0 \end{bmatrix} U$$

$$\text{s.t.} \begin{bmatrix} 1 & -2 & 4 & 1 & 0 & 0 & 0 \\ -2 & 3 & -4 & 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 & 1 & 0 \\ 6 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} U = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix}$$

$$U \geq 0$$