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William F. Morris, Ruth A. Hufbauer, Anurag A. Agrawal, James D. Bever, Victoria A. Borowicz, Gregory S. Gilbert, John L. Maron, Charles E. Mitchell, Ingrid M. Parker, Alison G. Power, Mark E. Torchin, and Diego P. Vázquez. 2007. Direct and interactive effects of enemies and mutualists on plant performance: a meta-analysis. *Ecology* 88:1021–1029.

Appendix B. Effect sizes and their sampling variances.

In this appendix, we outline the measures we used for individual, overall, and interaction effects, based on both Hedges' *d* and the log response ratio, *L*, and we give their sampling variances. For *d*, we made slight modifications to the effect size measures given by Gurevitch et al. (2000) so that negative effects always indicate poorer plant performance and positive effects always indicate improved performance when the agent is at a higher abundance. We also extend to the log response ratio the approach of Gurevitch et al. (2000) for conducting a meta-analysis of factorial experiments. Finally, we outline the procedure we used to test for a difference in the magnitudes of the effects of enemies and mutualists using the log response ratio. Matlab code to compute all of the effect sizes given in this appendix and their sampling variances, and to compute mean effect sizes and compare them using homogeneity tests, is given in Supplement 2.

We first label the two agents (which may be enemies or mutualists of the plant) in a factorial removal or addition experiment as agent *A* and agent *B*. We use $\overline{X}_A$, $\overline{X}_B$, $\overline{X}_{AB}$, and $\overline{X}_C$ to represent the mean plant performances in the four treatment combinations: only agent *A* at high levels, only agent *B* at high levels, both agents *A* and *B* at high levels, and the "control" (both agents *A* and *B* at low levels), respectively. The standard deviation and the number of plants in treatment *Z* (where *Z* is *A*, *B*, *AB*, or *C*) are denoted by s_Z and N_Z, respectively.

Effect sizes for a meta-analysis of factorial experiments using Hedges' d

The individual effect of agent *Z*, d_Z (where *Z* is *A* or *B*, and the individual effect size is subscripted with a lower-case letter to distinguish it from the overall effects below), was calculated as

$$d_z = \frac{\left(\overline{X}_Z - \overline{X}_C\right)}{s} J(m) \qquad \text{(B.1)}$$

where *s* is the pooled standard deviation computed as

$$s = \sqrt{\frac{(N_A - 1)s_A^2 + (N_B - 1)s_B^2 + (N_{AB} - 1)s_{AB}^2 + (N_C - 1)s_C^2}{N_A + N_B + N_{AB} + N_C - 4}} \qquad \text{(B.2)}$$

and $J(m) = 1 - 3/(4m - 1)$ is a correction for small-sample bias (Hedges and Olkin 1985). In (B.1), the degrees of freedom *m* equals $N_Z + N_C - 2$.

The overall effect of agent *A* (comparable to the main effect in an ANOVA) was calculated as the difference in mean performance between the two treatments with *A* at high levels and the two treatments with *A* at low levels:

$$d_A = \frac{\left(\overline{X}_A + \overline{X}_{AB}\right) - \left(\overline{X}_B + \overline{X}_C\right)}{2s} J(m) \qquad \text{(B.3)}$$

where $m = N_A + N_B + N_{AB} + N_C - 4$. The overall effect of agent *B*, d_B, is obtained by switching $\overline{X}_A$ and $\overline{X}_B$ in (B.3).

Note that positive values of these individual and overall effect sizes indicate improved plant performance relative to the control, and show that the agent acts as a mutualist of the plant, whereas negative values indicate reduced plant performance relative to the control, and show that the agent acts as an enemy.

We calculated the effect size of the interaction between agents *A* and *B* as (cf. Gurevitch et al. 2000)

$$d_I = \frac{\left(\overline{X}_{AB} - \overline{X}_B\right) - \left(\overline{X}_A - \overline{X}_C\right)}{s} J(m) \qquad \text{(B.4)}$$

where $m = N_A + N_B + N_{AB} + N_C - 4$. A positive interaction effect indicates that plant performance is greater when both agents are at high levels than would be predicted by the sum of the two individual effects, and a negative interaction effect indicates that, jointly, the two agents have a more detrimental effect on plant performance than the sum of their individual effects would predict. This interpretation holds whether the agents are two enemies, two mutualists, or an enemy and a mutualist.

To combine results across studies, weighted means of the effect sizes are calculated, where the weights are the inverses of the sampling variances of the effect sizes. For individual effects d_z (where *z* is *a* or *b* and *Z* is *A* or *B*), the sampling variance is (Gurevitch et al. 2000)

$$\hat{s}^2(d_z) = \frac{N_Z + N_C}{N_Z N_C} + \frac{d_z^2}{2(N_Z + N_C)} \qquad \text{(B.5)}$$

and for an overall or interaction effect d_Z (where *Z* is *A*, *B*, or *I*), the sampling variance is

$$\hat{s}^2(d_Z) = \frac{1}{N_A} + \frac{1}{N_B} + \frac{1}{N_{AB}} + \frac{1}{N_C} + \frac{d_Z^2}{2(N_A + N_B + N_{AB} + N_C)} \qquad \text{(B.6)}$$

Effect sizes for a meta-analysis of factorial experiments using the log response ratio, L

For the individual effect of agent *Z* (*Z* = *A* or *B*), the log response ratio is (Hedges et al. 1999)

$$L_z = \log\left(\frac{\overline{X}_Z}{\overline{X}_C}\right) = \log \overline{X}_Z - \log \overline{X}_C \qquad \text{(B.7)}$$

with approximate sampling variance

$$\hat{s}^2(L_z) = \frac{s_Z^2}{\overline{X}_Z^2 N_Z} + \frac{s_C^2}{\overline{X}_C^2 N_C}. \qquad \text{(B.8)}$$

We computed the overall effect of agent *A* as the log of the ratio of the average plant performance in the two treatments with agent *A* at high levels to the average performance in the two treatments with *A* at low levels:

$$L_A = \log\left(\frac{\frac{1}{2}\left(\overline{X}_A + \overline{X}_{AB}\right)}{\frac{1}{2}\left(\overline{X}_C + \overline{X}_B\right)}\right) = \log\left(\overline{X}_A + \overline{X}_{AB}\right) - \log\left(\overline{X}_C + \overline{X}_B\right), \qquad \text{(B.9)}$$

which has an approximate sampling variance

$$\hat{s}^2(L_A) = \left(\frac{1}{\overline{X}_A + \overline{X}_{AB}}\right)^2 \left(\frac{s_A^2}{N_A} + \frac{s_{AB}^2}{N_{AB}}\right) + \left(\frac{1}{\overline{X}_C + \overline{X}_B}\right)^2 \left(\frac{s_C^2}{N_C} + \frac{s_B^2}{N_B}\right). \qquad \text{(B.10)}$$

The overall effect of agent *B* and its sampling variance are obtained by switching $\overline{X}_A$ and $\overline{X}_B$, N_A and N_B, and s_A and s_B in (B.9) and (B.10).

We computed the interaction effect as the difference in log response ratios for agent *A* when agent *B* is at high vs. low levels (and vice versa):

$$L_I = \log\left(\frac{\overline{X}_{AB}}{\overline{X}_B}\right) - \log\left(\frac{\overline{X}_A}{\overline{X}_C}\right) = \log\left(\frac{\overline{X}_{AB}}{\overline{X}_A}\right) - \log\left(\frac{\overline{X}_B}{\overline{X}_C}\right) = \log\overline{X}_{AB} - \log\overline{X}_A - \log\overline{X}_B + \log\overline{X}_C \qquad \text{(B.11)}$$

which has an approximate sampling variance

$$\hat{s}^2(L_I) = \left(\frac{s_A^2}{\overline{X}_A^2 N_A}\right) + \left(\frac{s_B^2}{\overline{X}_B^2 N_B}\right) + \left(\frac{s_{AB}^2}{\overline{X}_{AB}^2 N_{AB}}\right) + \left(\frac{s_C^2}{\overline{X}_C^2 N_C}\right). \qquad \text{(B.12)}$$

As for d_I, L_I is positive if the joint effect of the two agents results in higher plant performance than the sum of the individual effects would predict, and negative if the joint effect reduces plant performance relative to the sum of the individual effects.

In computing confidence limits for weighted mean effect sizes using log response ratios, if the number of studies was less than 50, we corrected the standard error of the mean for small-sample bias using Eq. 7 in Hedges et al. (1999).

Comparing the magnitudes of enemy and mutualist effects using the log response ratio

Using the log response ratio, it is difficult to compare directly the magnitudes of the effects of two types of agents when one type has a negative effect on plant performance and the other type has a positive effect. The reason is that a given proportional increase in performance on the untransformed scale will be of lesser magnitude than the same proportional decrease in performance when both are expressed on the log scale. For example, if P_E is the proportional decrease in performance due to an enemy and P_M is the proportional increase in performance due to a mutualist, then the log response ratios for the enemy and mutualist are

$$L_E = \log\left(\frac{(1-P_E)\overline{X}_C}{\overline{X}_C}\right) = \log(1-P_E) \quad \text{and} \quad L_M = \log\left(\frac{(1+P_M)\overline{X}_C}{\overline{X}_C}\right) = \log(1+P_M),$$

respectively. If $P_E = P_M$, $|L_E| > |L_M|$.

Therefore, to compare the magnitudes of the effects of enemies and mutualists using the log response ratio, we used the following procedure. First, we sampled with replacement from the set of log response ratio effect sizes of enemies, computed the sampling-variance weighted average effect size $\hat{L}_E$, and then estimated the average proportional decrease in performance as $\hat{P}_E = 1 - \exp(\hat{L}_E)$. Next we sampled with replacement from the set of log response ratio effect sizes of mutualists, computed the sampling-variance weighted average effect size $\hat{L}_M$, and then estimated the average proportional increase in performance as $\hat{P}_M = \exp(\hat{L}_M) - 1$. Next, we computed and stored the difference in average proportional effects, $\Delta = \hat{P}_E - \hat{P}_M$. We repeated this procedure 5000 times and examined the distribution of Δ. If <5% of the distribution lies below zero, we conclude that proportional effects of enemies are greater than those of mutualists, while if <5% of the distribution lies above zero, we conclude that mutualists have greater proportional effects than enemies.

For individual effects, only 2.7% of the distribution of Δ fell below zero, indicating that, on average, the magnitude of the enemy effects was stronger than that of mutualists across the studies included in our meta-analysis (cf. Fig. 1). However, for overall effects, 39.6% of the distribution of Δ fell below zero, indicating that the magnitudes of enemy and mutualist effects are comparable when averaged across levels of the other agent in a factorial experiment.

[Back to E088-063]