Fundamental Distributed Algorithms

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Outline

Introducing distributed algorithms and basic solutions for common problems

- Classes of algorithms
- Snapshots
- Mutual exclusion
- Working in faulty settings…
- Election

Classes of Algorithms

Basic Alg. vs Control Algorithms

Often a target *distributed* algorithm is supposed to run *at the same time* the overall system is performing a given job.

The algorithm run to perform such a job is named **Basic Algorithm**.

The other, i.e. our target algorithm, is named **Control Algorithm**.

Sometimes it may be convenient regarding a control algorithm as executed by a *controller process* cp_i that pairs to p_i at node i

Centralized vs Decentralized

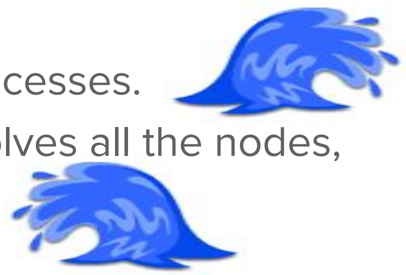
In a given algorithm, a process is said “**initiator**” if it can start performing events *without any input from other processes*.

The first event of an initiator is either an internal one, or a send.

An algorithm is **centralized** if it has one single initiator.

An algorithm is **decentralized** if it admits multiple initiators.

Wave Algorithms



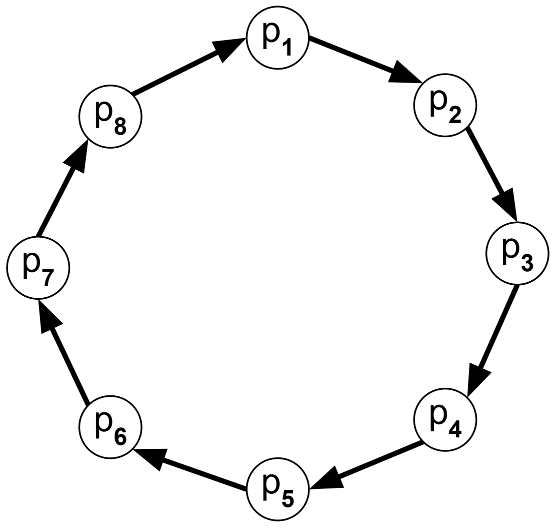
Common need: collection of information spread across processes.
Typically, a “request” originates at one node, and then involves all the nodes, which in turn must provide back the required information.

In a **wave algorithm,** a computation satisfies the following properties:

- It is finite
- It contains one or more *decision* events
- For each decision event d and process p_i , $c \xrightarrow{HB} d$ holds for some c at p_i

Note that, after the definition, each process must take part in the computation

A Trivial Wave Algorithm: Ring



Network topology: directed ring

The initiator (whatever p_x) sends out a token msg:

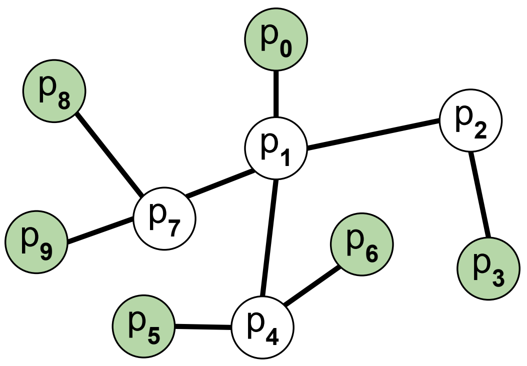
```
send(toNeighbor, OutToken)
receive(fromNeighbor, inToken)
decide()
```

Any process p that receives a token msg performs:

```
receive(fromNeighbor, inToken)
send(toNeighbor, outToken)
```

A More Complex Example: Tree Wave

Undirected Tree Topology.
Each process sends only one msg.
Initial msgs: out of leaves.



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// loop executed n_neigh - 1 times
while len(filter(false, recv)) > 1:
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//toNeighbor: the only proc with recv == false
send(toNeighbor, outToken)
receive(toNeighbor, inToken)
recv[toNeighbor] = true //not necessary…
decide()
//this way, only two processes decide. Possibly:
//foreach n in neighbors\toNeighbor:
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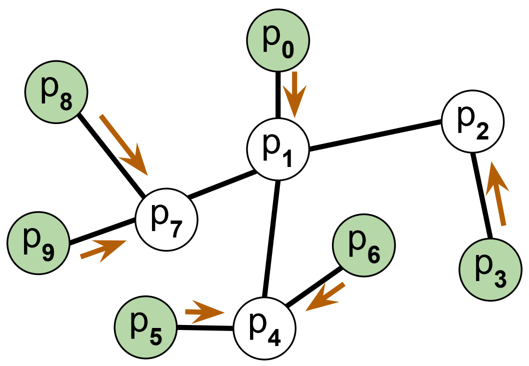

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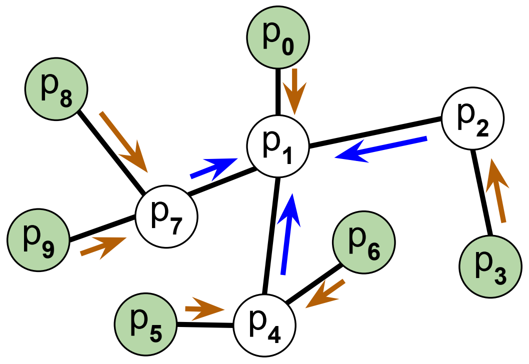

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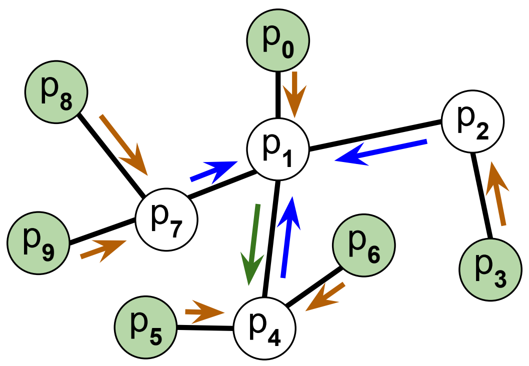

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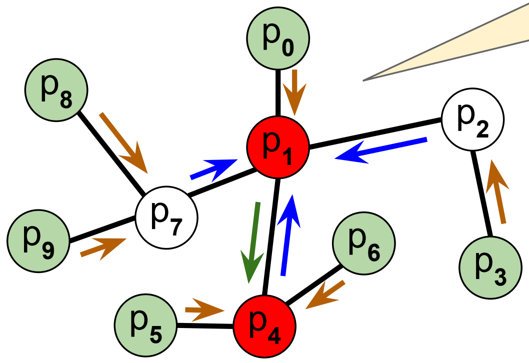

Processes getting
to the decision point
have (potentially) received information
from all the other nodes

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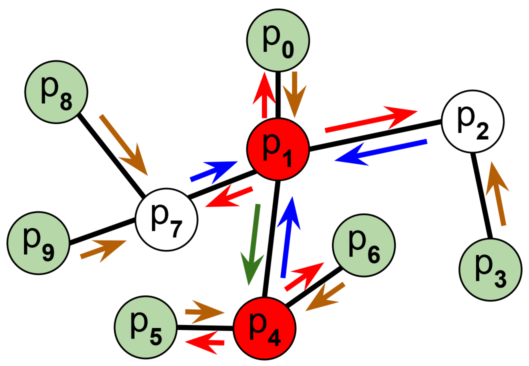

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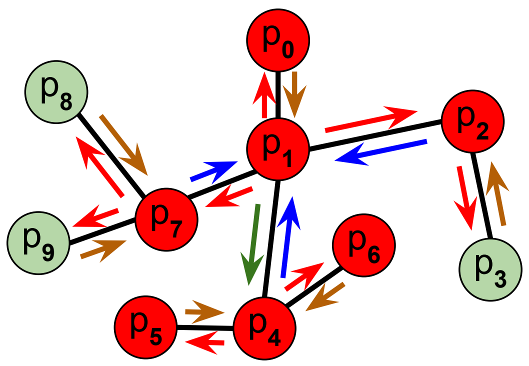

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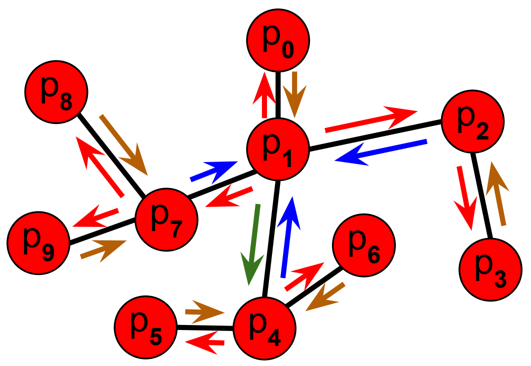

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Traversal Algorithms

A **traversal algorithm** is a *centralized* wave algorithm.
In a computation, the initiator sends out token(s), and:

- All the processes receive the token
- Finally, the token(s) returns back to the initiator, which performs a decision event

Taking Snapshots

What Do We Mean by “Snapshot”?

Snapshot of the execution of a distributed algorithm: configuration consisting of the **local states** of processes, along with **messages in transit** (i.e. channel states).

A snapshot is **consistent** if it is taken just after a **consistent cut** (corresponding to a consistent global state).

Why Snapshots?

Snapshots can be aimed at:

- Check properties
  (in particular, **stable predicates**, those that will keep on being true):
  - Deadlock
  - Termination
  - Etc.
- Checkpointing

Snapshot Algorithm (I)

A Snapshot Algorithm is a **Control Algorithm**
that must obtain a picture of what it is going on
with a basic distributed algorithm at a given progression point.

Actually, it "monitors" executions...

This operation in a distributed system
is not so trivial as it is in a centralized system.

A snapshot algorithm is asked to run "on-the-fly," along with
the basic one, with no need to stop the system to inspect its state.

Snapshot Algorithm (II)

A Snapshot Algorithm is required to take a **consistent snapshot**, i.e. a *possible* configuration of the ongoing execution. The related state-recording operations can occur at different points in time.

In general, a snapshot algorithm can be initiated by any process; each process is in charge of locally recording its own portion of the snapshot. Later, all the portions can be collected (e.g. by a traversal algorithm, etc.).

Chandy-Lamport Algorithm (I)

The most popular snapshot algorithm. System hypotheses:

- FIFO channels
- Strongly connected (di)graph ➔ all the processes can be reached

Idea: local recording actions driven by special “marker” messages.

Any process can start a snapshot.

A channel state (corresponding to *basic* msgs in transit through it) is recorded by the destination process.

Chandy-Lamport Algorithm (II)

The initiator performs:

```
procedure takeSnapshot()
   if not marked then
      marked = true
      foreach c in outchannels send(c, marker)
      <record local state + incoming channels' state>
```

Any process p receiving a marker msg performs:

```
procedure onMarker(channel c)
   takeSnapshot()
   chmark[c] = true
   if chmark[ic] for each incoming channel ic then
      <local snapshot termination>
```

Any process p receiving a basic msg performs:

```
procedure onBasicMsg(channel c, msg m)
   if marked and not chmark[c] then
      chstate[c].append(m) //state for chann. c
```

Chandy-Lamport Algorithm: Complexity

Message complexity:

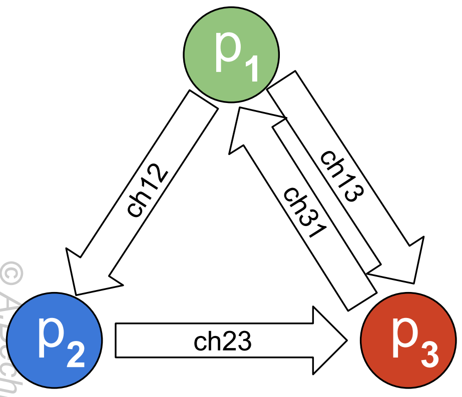
we have just one marker message per FIFO channel, so it is $\Theta(E)$, with E as # of channels.

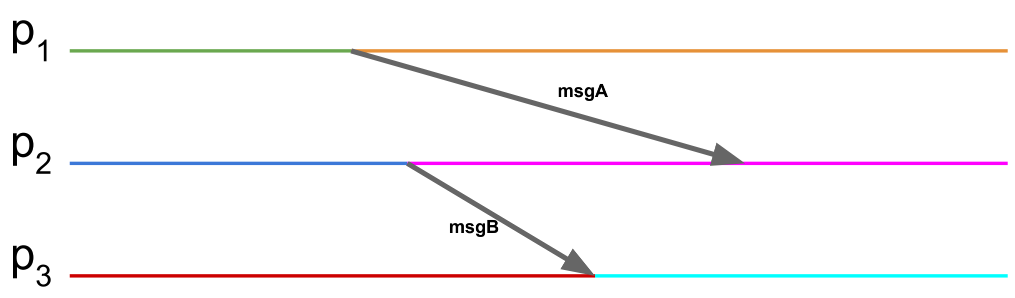
Time complexity:

all the processes have to be reached, thus in the worst case it depends on the longest of the paths (# of hops) between those that "link" two generic processes in the network, i.e. the *network diameter* D. The time complexity is $\boldsymbol{O}(D)$.

Chandy-Lamport Alg. Example (I)

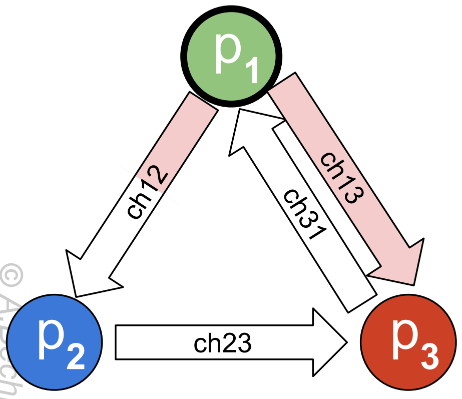
Three processes; in the basic algorithm, p_1 sends msgA to p_2 and p_2 sends msgB to p_3, as shown.

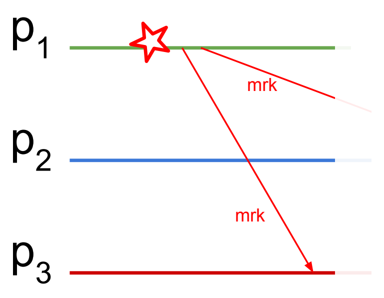




Chandy-Lamport Alg. Example (II)

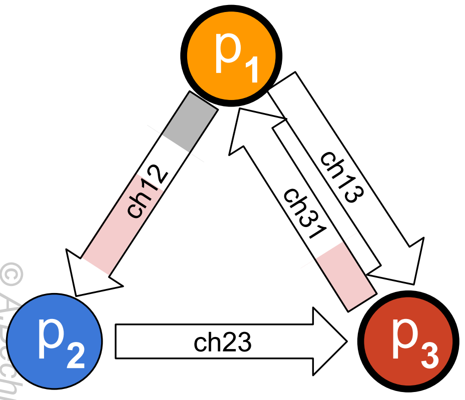
p_1 starts, recording its state (green), and sends mrk to p_2 and p_3. p_3 receives mrk first, so…

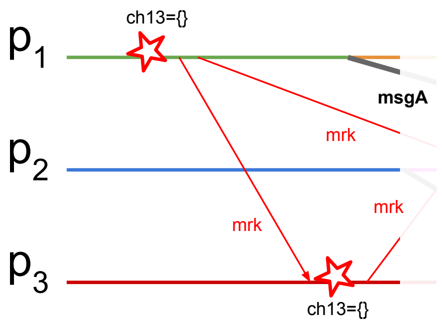




Chandy-Lamport Alg. Example (III)

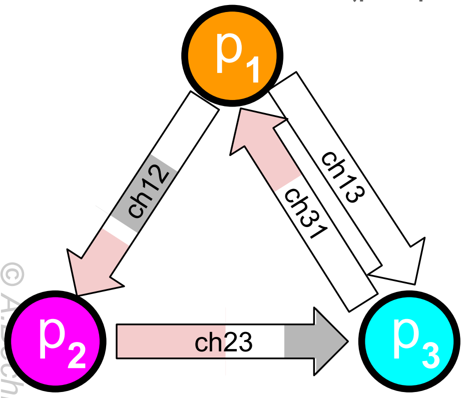
p_3 records its state (red), and sends mrk to p_1. Meanwhile,
p_1 sends basic msgA to p_2
and changes state (➔orange). Then...

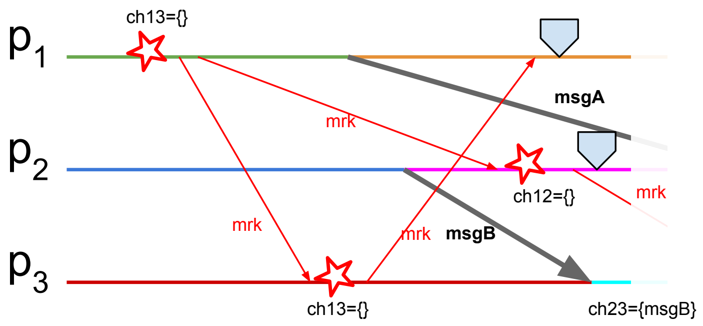




Chandy-Lamport Alg. Example (IV)

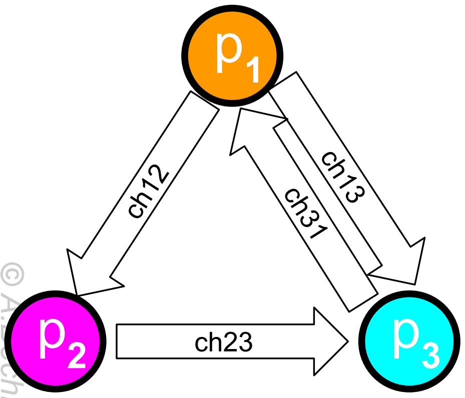
p_2 sends msgB to p_3, changes state (➔purple); then, it receives mrk from p_1,
records its state (purple), sends mrk to p_3, and terminates its local snapshot.
p_1 receives mrk from p_3, and terminates.
p_3 receives msgB from p_2, and changes state (➔cyan).

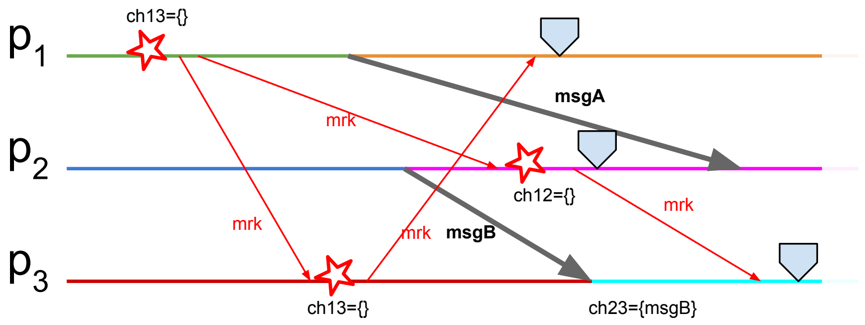




Chandy-Lamport Alg. Example (V)

p_2 receives basic **msgA** from p_1, but its local snapshot is already terminated;
p_3 receives mrk from p_2, and terminates.

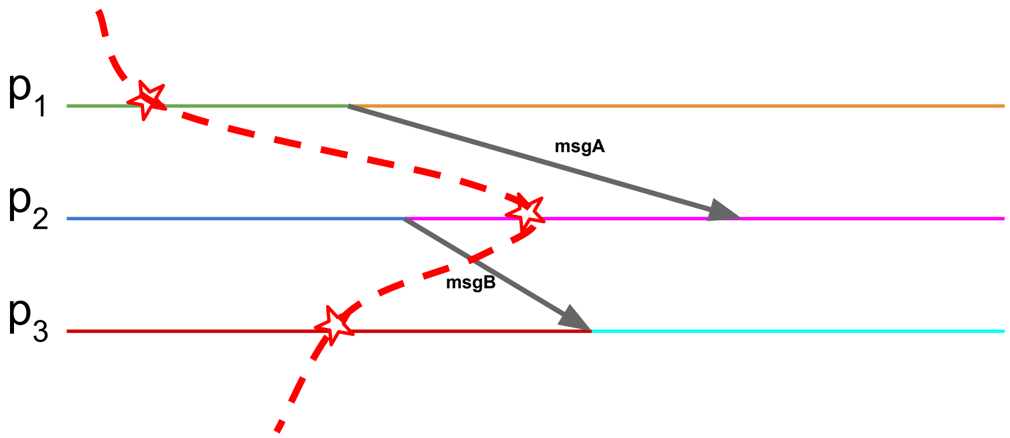




Chandy-Lamport Alg. Example (VI)

The obtained snapshot corresponds to the consistent cut shown below;
anyway, the overall system has not experienced such a state in any specific point in (global) time...

p_1 🟢
p_2 🟣
p_3 🔴
ch_{12}= {}
ch_{13}= {}
ch_{23}= {msgB}
ch_{31}= {}



Chandy-Lamport Alg. Correctness

Correctness means: a snapshot corresponds to a *consistent cut*, i.e.:

For any two events a, b such that $a \rightarrow b$, if b is pre-snapshot, so is a.

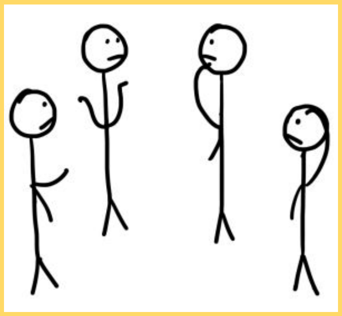
Proof:

The FIFO hypothesis is crucial!

1) a and b on same process: trivial
2) a in p_i, b in p_j, $a = send(m_x)$, $b = receive(m_x)$
   In this case, as b is pre-snapshot, p_j has not received mrk yet at b.
   Because of FIFO channels, p_i has not sent mkr yet at a,
   so a is pre-snapshot as well. □

Pause for Thought

Correctness Depends on Hypotheses !



Removing FIFO Hypothesis…

The FIFO property is crucial for the algorithm correctness.
How to arrange an algorithm to avoid recurring to FIFO?

Avoid that any message *sent after* a local snapshot will be *received before* the local snapshot at the receiving side.

Possible solution: *piggybacking*, letting a message carry information on the state of the sending process. The receiver *will possibly delay the actual receive*, just performing the snapshot before it.

Lai-Yang Algorithm (I)

This algorithm adopts such a trick to get consistent snapshots, no matter the ordering in message delivery.

Yet another issue: how to obtain channel states?
It is sufficient to keep the log of outgoing/ingoing messages for any channel at any process;
then, the state for channel c_{ij} from p_i to p_j can be computed as

$$state(c_{ij}) = sent\_msgs_i[j] \setminus received\_msg_j[i]$$

Lai-Yang Algorithm (II)

To perform the local snapshot:

```
procedure takeSnapshot()
   marked = true
   <record local_state considering also all logged
    outgoing and incoming messages >
```

Any process p sending out **any** msg m from ch. c:

```
procedure wrappedSend(channel c, msg m)
   m.color = green if not marked else red
   send(c,m)
   log(c,m) // keep record of outgoing msgs
```

Any process p receiving **any** msg m on ch. c:

```
procedure wrappedReceive(channel c, msg m)
   if msg.color == red then
       takeSnapshot()
   receive(c,m)
   log(c,m) // keep record of incoming msgs
```

Lai-Yang Algorithm (III)

The global snapshot will be computed upon the collection of all the local snapshots.

Further complication: in case of no outgoing basic msg after the first snapshot, we should wait all the other processes take their own when they please...
Possible solution: introduction of **a "trigger" control msg** to be sent through all outgoing channels, just after taking a local snapshot. This will "urge" the completion of the algorithm.

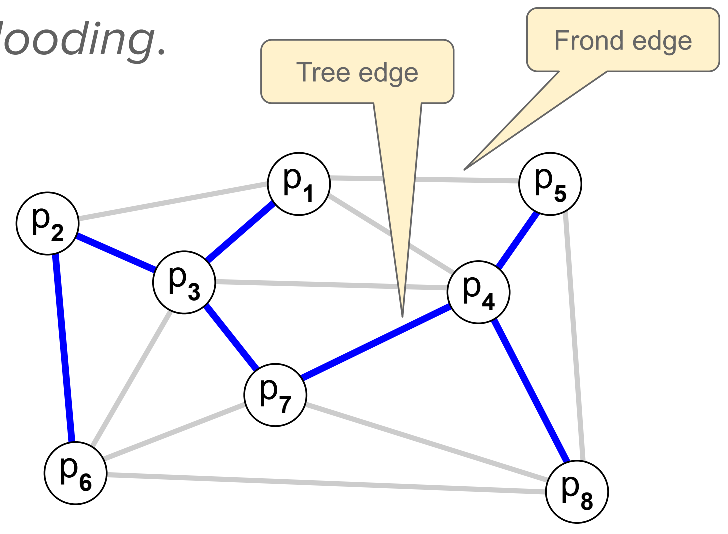
Performing Broadcast/Convergecast

Building a Spanning Tree

A trivial approach to broadcasting is *flooding*.
But what if the initiator wants back
some information from each process?

Let's build up
a **spanning tree**,
to be used later
for broadcast/convergecast.

We assume here to work
with undirected, FIFO channels (and connected network).



On Spanning Trees

- Every connected (and undirected) graph *G* with *n* nodes and *E* edges has *at least* one spanning tree.
- All the possible spanning trees for *G* have *n* vertices and *n-1* edges.
- A spanning tree is *minimally connected*, i.e. the removal of an edge makes it disconnected.
- A spanning tree is *maximally acyclic*, i.e. adding one edge always creates a loop.

“Distributed” Spanning Tree Info

The structure of the spanning tree can be described keeping at each process information about:

1. The process that is the parent (variable **`parent`**)
2. A list with the children (list **`children`**)

The structure can be built in a distributed fashion with a *traversal* algorithm:
“forward” msgs are propagated ahead to explore the network
- the parent of a process is the one the first forward msg is received from;
“backward” msgs keep collected info and state parent-child relationships.

The initiator (the tree root) has just to know its own children, nothing else.

A Simple Spanning Tree Algorithm

```
procedure startSpanning()
   parent=pid //pid: local proc. index
   children=empty; expected = n_neighbors
   foreach p in neighbors send(p, fwd)
```

First action for the initiator:

Any process *p* receiving a msg **<fwd>**:

```
procedure onForward(proc sender)
   if parent == ⊥ then  //first fwd received
      parent=sender; children=empty; expected=n_neighbors-1
      if expected == 0 then //reached a "dead end"
         send(parent, bck(list( (pid, myval) ))
      else
            foreach p in neighbors\sender send(p, fwd)
   else send(sender, bck(empty)) //ie "I'm not your child"
```

Any process *p* receiving a msg **<bck>**:

```
procedure onBackward(proc sender, msg m)
   expected -= 1; val_list.append(m.val_list)
   if m.val_list not empty then children.append(sender)
   if expected == 0 then //no more bck from children
      val_list.append( (pid,myval) )
      if parent != pid then send(parent, bck(val_list))
      else <termination; use val_list>
```

What about Complexity?

Message complexity:

# of *tree* edges: $n\text{-}1$; 1 fwd + 1 bck per tree edge ➔ $2(n\text{-}1)$
# of fronds: $E\text{-}(n\text{-}1)$; 2 fwd +2 bck per frond ➔ $4E - 4(n\text{-}1)$
In total, $4E - 2(n\text{-}1)$ so msg complexity is $\boldsymbol{O}(E)$ with E = # edges.
But E in a connected graph is in the interval $[(n\text{-}1), n(n\text{-}1)/2]$,
so in the worst case the msg complexity is also $\boldsymbol{O}(n^2)$

Time complexity:

all the processes are reached, and a message comes back to the initiator,
so $2D$ hops, with D = *network diameter.*
So the time complexity is $\boldsymbol{O}(D)$.

Pause for Thought

Nondeterministic Behaviors

ALICE SENDS A MESSAGE TO BOB SAYING TO MEET HER SOMEWHERE.
UH HUH.
BUT EVE SEES IT, TOO, AND GOES TO THE PLACE.
WITH YOU SO FAR.
BOB IS DELAYED, AND ALICE AND EVE MEET.
YEAH?
I'VE DISCOVERED A WAY TO GET COMPUTER SCIENTISTS TO LISTEN TO ANY BORING STORY.

Mutual Exclusion: The Distributed Way

Problem Statement

Informally: Only one process *at a time* can “access a resource.”
What does it means “at a time” ?

Actual access to the resource: through a set of operations said *critical section*;
a CS is ideally delimited by the operations *req_access()* and *rel_access()*

Properties:

ME1 - safety - at most one process may execute in the critical section

ME2 - liveness - requests to enter/exit the critical section *eventually* succeed

Process States wrt Critical Sections

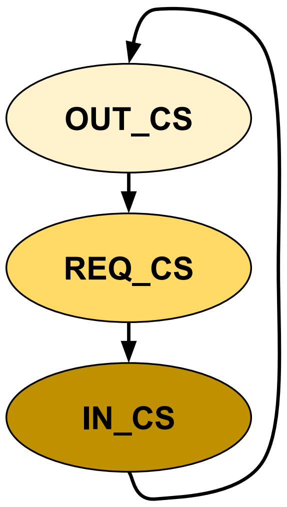
As regards the relationship with a CS,
a process can be in three different states:

OUT_CS - not interested in executing CS

REQ_CS - started *req_access()*,
not yet executing CS

IN_CS - executing CS

A variable in each process can be dedicated to encode such a state.



Types of Distributed Mutex Algorithms

They are necessarily **decentralized** algorithms: anyone can be the initiator.

- **Individual Permissions**: Any single process grants a permission just on its own behalf.
- **Token-based** - a unique token is shared across all processes, and CS is executed only when the token is held.
- **Arbiter-Permissions**: there exists at least one process that resolves conflicts between any couple of requesting processes; permissions are returned at CS exit.
  (general case: "Quorum-based approach")

Ricart-Agrawala Algorithm - Basics

Classically, the execution of CS requires

- An enter protocol - `req_access()`
- An exit protocol - `rel_access()`

Ideas for dealing with "individual permissions":

- To enter CS, a process must request the permit to each other process, and wait for all the relative "OKs."
- Possible request conflicts are resolved using totally-ordered timestamps: *the earliest request gets the priority.*
  Remember: $(ts_a, i) < (ts_b, j)$ *iff* $(ts_a < ts_b) \vee ((ts_a = ts_b) \wedge (i < j))$

Lamport timestamps...

Ricart-Agrawala Algorithm

```
procedure req_access() // pid: local pid
   mystate = REQ_CS
   pending_oks = N-1 //all the others
   myreq_lt = myclock+1 // l.timest. for my req
   req.ts = (myreq_lt, pid) //set msg timest.
   foreach p in req_set send(p, req)
   wait_until (pending_oks == 0 )
   mystate = IN_CS // then, CS
```

```
procedure rel_access()
   mystate = OUT_CS
   foreach p in req_delayed send(p, ok)
   req_delayed = empty
```

Enter protocol   Exit protocol

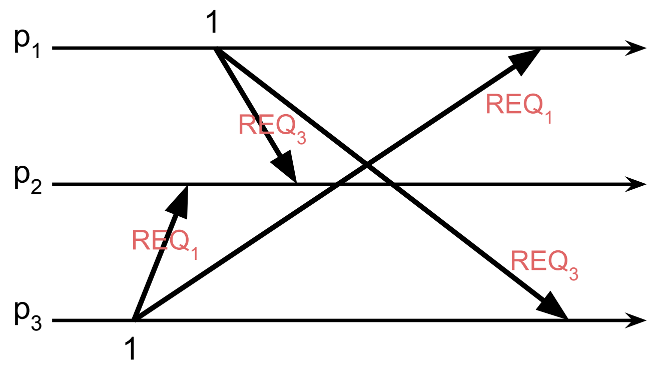
Deal with: OK,
REQ

```
procedure onReq(proc sender, msg m)
   myclock = max(myclock, m.ts)
   my_prior =(mystate != OUT_CS) and (myreq_lt,pid)<m.ts
   if not my_prior then send(sender, ok)
   else req_delayed.append(sender)
```

```
procedure onOk(proc sender, msg m)
   pending_oks = pending_oks-1
```

Ricart-Agrawala Algorithm - Example

In the example shown here, both p_1 and p_3 try to enter the CS "at the same time"



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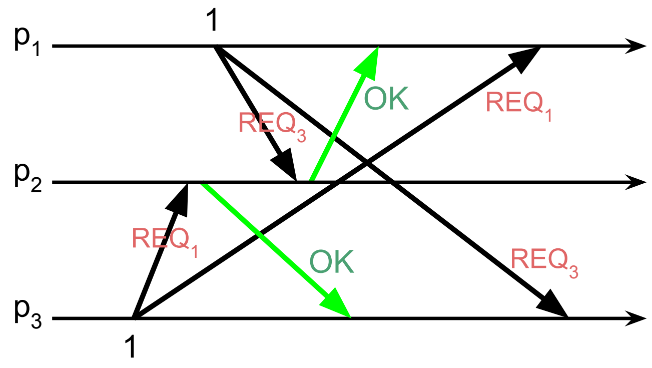

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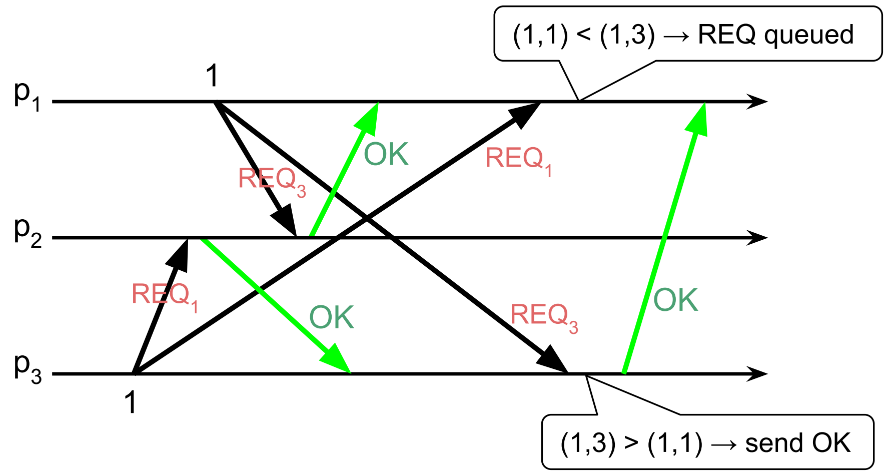

$(1,1) < (1,3) \rightarrow$ REQ queued
p_1
p_2
p_3
1
1
REQ_3
OK
REQ_1
REQ_1
OK
REQ_3
OK
$(1,3) > (1,1) \rightarrow$ send OK

Ricart-Agrawala Algorithm - Example

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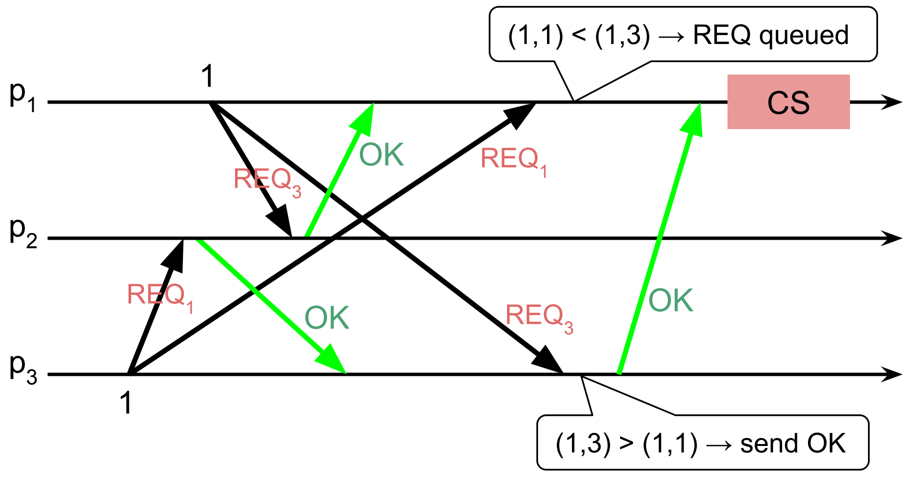

Ricart-Agrawala Algorithm - Example

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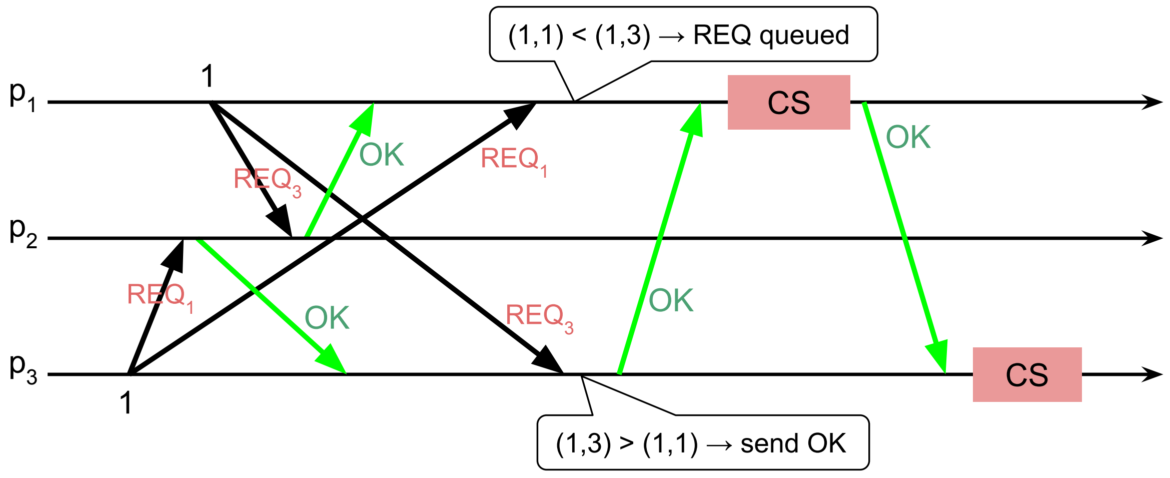


Ricart-Agrawala Algorithm: Complexity

Message complexity:

The use of a CS for one process involves the exchange of n-1 REQs and n-1 OKs, so $2(n$-$1)$ in total.

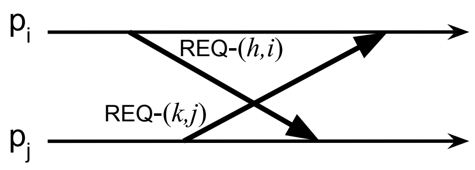
Time complexity:

all the processes have to be reached, thus in the worst case it depends on the longest possible path (# of hops) between two generic processes in the network, i.e. the *network diameter* D. The time complexity is $\boldsymbol{O}(D)$.

Ricart-Agrawala Alg. Correctness (I)

The alg. satisfies ME1, i.e. at most one process may execute in CS.

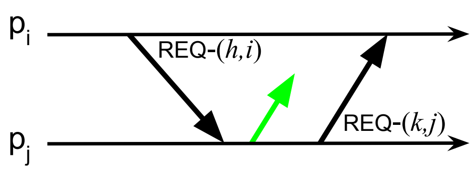
Proof: by contradiction; let suppose both p_i and p_j are in state IN_CS. It follows that each of them has sent REQ to the other (with ts (h,i) and (k,j), respectively), and got OK back. Two scenarios are possible:



1) each process has sent its REQ before receiving the other's REQ.
   Let's assume e.g. $(h,i) < (k,j)$, thus p_j was in the condition to send OK to p_i; on the other hand, under the same condition, p_i didn't send OK to p_j, thus p_j cannot be in CS $\Rightarrow$ contradiction
2) ... *(cont.)*

Ricart-Agrawala Alg. Correctness (II)

1) …
2) One process, say p_j, has sent the other (say p_i) its OK before sending its REQ.
   When p_j receives REQ, it performs $myclock_{(j)}=max(myclock_{(j)},h)$, yielding $myclock_{(j)} \geq h$.
   When subsequently p_j send REQ, the relative ts is set to $k=myclock_{(j)}+1 > h$.
   As this REQ is received by p_i, its state is not OUT_CS, and $(h,i) < (k,j)$ and thus p_i is not in the condition to send back OK to p_j, which thus cannot enter CS $\Longrightarrow$ contradiction. □

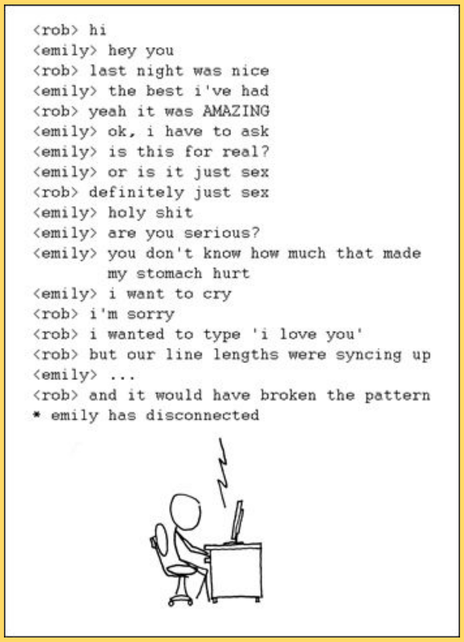


ME2 (liveness) can be proven by showing absence of deadlock and starvation.

Pause for Thought

Adhere to Patterns

(enter/exit protocols)


```
<rob> hi
<emily> hey you
<rob> last night was nice
<emily> the best i've had
<rob> yeah it was AMAZING
<emily> ok, i have to ask
<emily> is this for real?
<emily> or is it just sex
<rob> definitely just sex
<emily> holy shit
<emily> are you serious?
<emily> you don't know how much that made
        my stomach hurt
<emily> i want to cry
<rob> i'm sorry
<rob> i wanted to type 'i love you'
<rob> but our line lengths were syncing up
<emily> ...
<rob> and it would have broken the pattern
* emily has disconnected
```

Token Ring Mutex

W.r.t. the algorithm's purposes, processes must be organized in a directed ring topology - an "overlay network."

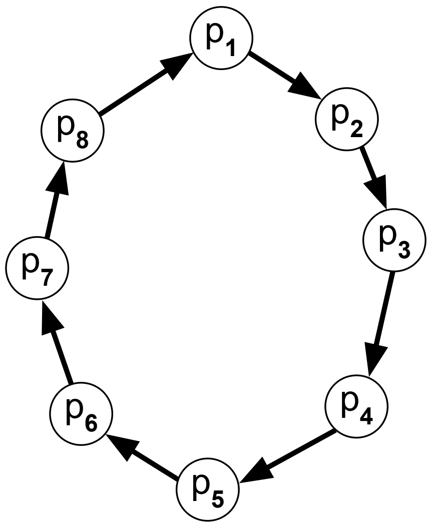
The permit to execute CS is a **token msg** that circulates the ring.

`req_access()` means to set the state to REQ_CS

`onToken()`: if mystate = REQ_CS, then execute CS, otherwise send the token to next process.

`rel_access()` means to set my state to OUT_CS, and then send the token to the next process.

Here, state IN_CS is practically meaningless.



Token Ring Mutex: Correctness

Informally:

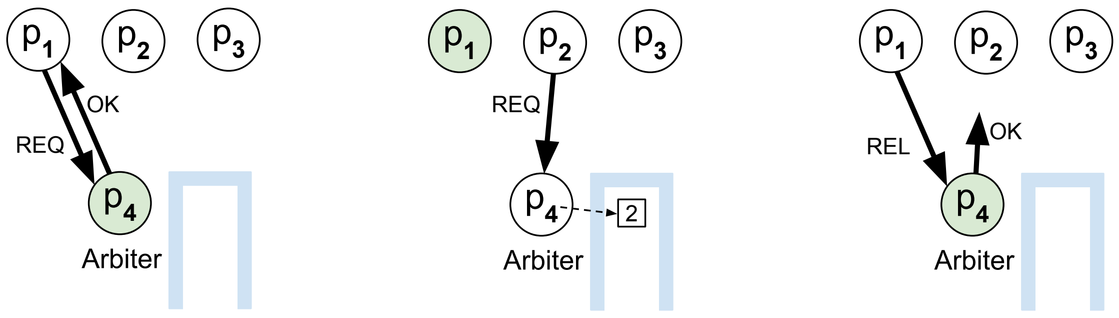
ME1 follows from the unicity of the token.

ME2 follows from the ring topology and the token passing rules, which assure that one token will *eventually* reach each node.

Plain Centralized Mutex

A simple solution makes use of one arbiter process, which handles one single permit that, after each use, has to be sent back, or “released.”

The arbiter usually keeps a queue for processes waiting to access CS.



Failures Right Around The Corner

Classifying Failures

Class of Failure	Affects	Description
Fail-stop	process	Process halts and remains halted. Others may detect this state.
Crash	process	Ditto, but others may not be able to detect this state.
Omission	channel	Message got lost in the way
Send-omission	process	Message “sent”, but not inserted in out-buffer
Receive-omission	process	Message arrived to in-buffer, but not received
Byzantine	proc/channel	Any arbitrary behavior!

Towards Synchronous Models

A *synchronous model* of computation accounts for a sequence
of fixed time-slots according to the global time,
and each operation has to be performed at each process *within a timeslot*.
This assumption let us deal with a “discrete” notion of time,
developing algorithms in a different way.

This “bare” model is clearly unrealistic; anyway, in dealing with faulty settings,
we can imagine to count on *upper bounds on replying times*.
In this way, we can employ **timeouts** to check for possible faults.

Often, the system model letting us legally use timeouts is said *synchronous*.

Leader Election Algorithms

Leader Election: Problem Statement

Informally: All the processes choose one of them (or out a set of candidates) to play a special role in the system (coordination/control).
Leader election is a form of ***symmetry-breaking*** in distributed systems.

Each process p_i is associated with a **unique** "UID$_i$"; the process to be elected is the one (in the set of candidates) with an *extremal* (max, min) UID.

Possible states for a process during an election run: participant/nonparticipant.

Required local state vars:
- bool (or UID type) $leader_i$, to indicate "I'm the leader" (or the UID of the leader)
- (possibly) $done_i$, to indicate that a leader has been elected.

Leader Election: Properties

LE1 - safety -

at most one (non-crashed) process is elected;
elected$_i$ and *done*$_i$ are stable;
termination implies that one process has been elected.

LE2 - liveness - (termination)

a process is eventually elected;
the election is eventually known by all the processes (non-crashed ones).

Note: it has been proven that in an “Anonymous” system, with no personal ids and a regular network topology, the leader election problem has no solution.

Chang and Robert’s Election (I)

An *unidirectional ring topology* is assumed (not necessarily FIFO channels).

Two phases:

1. **Leader identification**
2. **Leader announcement**

Possibly, multiple concurrent initiators

Each process sends its own UID on the ring,
and stops any incoming UID lower than its own.
The only UID that can pass all the way around the ring is the leader’s one.

Two types of exchanged messages: ELECTION and LEADER.

Chang and Robert's Election (II)

For a process to start election:

```
procedure startElection()
   mystate = participant
   election.UID = myUID
   send(election)
```

What to do upon receiving ELECTION msg:

```
procedure onElection(msg m)
   if   myUID < m.UID then
      mystate = participant
      send(m)
   elif myUID > m.UID then
      if mystate == nonparticipant
         mystate = participant
         election.UID = myUID
         send(election)
   elif myUID == m.UID then
      leader.UID = myUID
      send(leader); elected = true
```

To spread the word about the elected process via LEADER msg:

```
procedure onLeader(msg m)
   leader = m.UID;   done = true
   if myUID != m.UID then
      elected = false
      send(m)
```

C&R's Election: Complexity

Message cost: in any case, n LEADER msgs are used.

Best case: one single initiator, with highest UID $\longrightarrow n$ ELECTION msgs
Worst case: all initiate at the same time, and processes on the ring are ordered with decreasing UIDs. The process with the highest UID yields n ELECTION msgs, the one with the second highest UID yields n-1, and so on; in total, $n(n+1)/2$ msgs: $\longrightarrow \boldsymbol{O}(n^2)$
Average case (not proven here): $\boldsymbol{O}(n \log n)$

Time cost: Best case: one initiator, with highest UID, as before, $\boldsymbol{O}(n)$.
Worst case: the only initiator is the process with the second highest UID, and follows the one with the highest; $(n$-$1)+n$ ELECTION msgs are used.

Election with Possible Crashes

Assumptions: Reliable delivery, synchronous system, i.e. an *upper bound* exists *on the msg latency*.

Processes can crash even during election turns, but process failures can be detected with *timeouts*.

Premises for the next algorithm:

- Each process knows what processes have other UIDs, and is also able to communicate with them.
- The leader has to be the process with the current highest UID: but this depends on what processes are currently up/down!

The Bully Algorithm (I)

Types of msgs:

- ELECTION (to announce an election),
- ANSWER (to response to an election),
- LEADER (to announce the elected leader).

The election is typically triggered by any process that, by using timeouts, notices that *the current leader is no more alive*.

The process that knows to hold the highest UID in the group can just send LEADER msgs to the others to inform them that it is the leader.

The Bully Algorithm (II)

One process, to start the election, must send an ELECTION msg to those processes with higher UID.

Then, it awaits for their ANSWER msgs back.

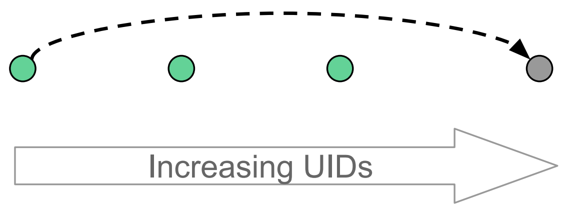
If no answer arrives within time T, it takes itself as the leader, and sends LEADER msgs to the processes with lower UIDs; Otherwise, it waits for an additional T' for a LEADER msg to come from the new coordinator; in case the msg does not arrive, the process starts a new election round.

The Bully Algorithm (III)

As a process p_i receives a LEADER msg from a process with higher UID, it sets its variable *leader* to that identifier.

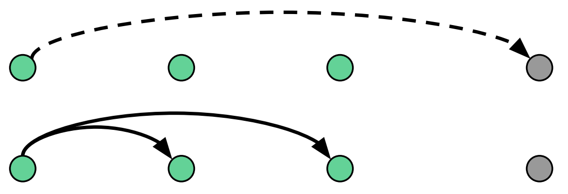
As a process p_i receives an ELECTION msg, it sends back an ANSWER and begins another election (unless it has not done this yet).

The Bully Algorithm: Example



A process finds the current leader is down

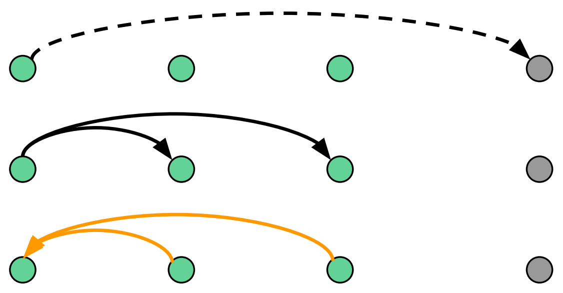
The Bully Algorithm: Example



A process finds the current leader is down

Msg ELECTION - “who is the leader?”

The Bully Algorithm: Example

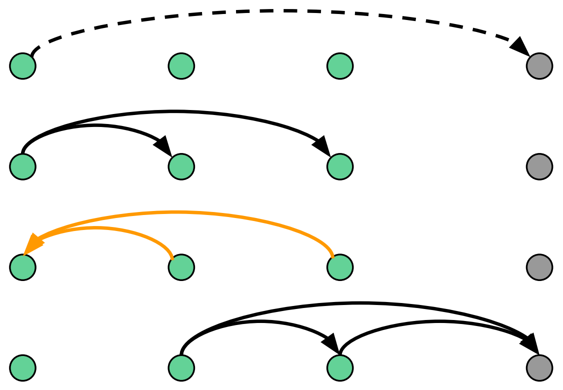


A process finds the current leader is down

Msg ELECTION - “who is the leader?”

Msg ANSWER - “for sure not you!”

The Bully Algorithm: Example



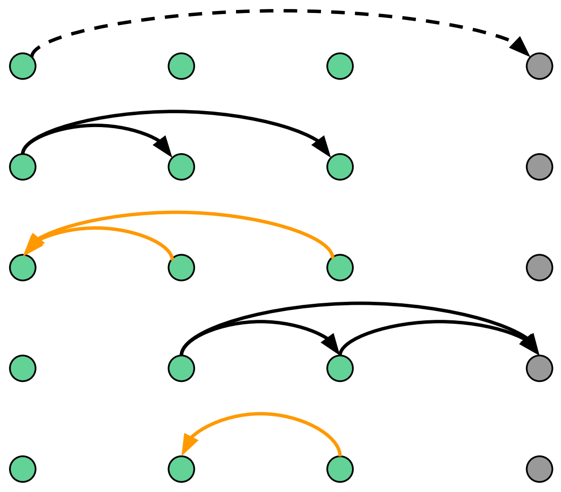
A process finds the current leader is down

Msg ELECTION - “who is the leader?”

Msg ANSWER - “for sure not you!”

Msg ELECTION - “who is the leader?”

The Bully Algorithm: Example



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Msg ELECTION - “who is the leader?”

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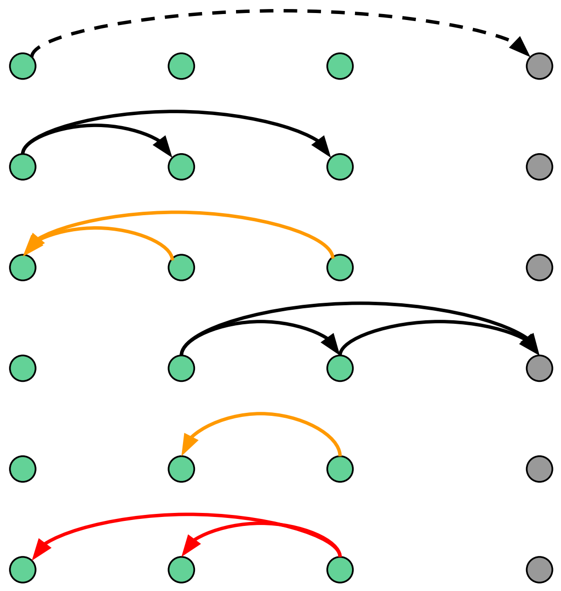
The Bully Algorithm: Example



A process finds the current leader is down

Msg ELECTION - “who is the leader?”

Msg ANSWER - “for sure not you!”

Msg ELECTION - “who is the leader?”

Msg ANSWER - “for sure not you!”

Msg LEADER - “I’m the leader!”

But… What about Bullism?

When a process is started to replace a crashed process, it initiates a new election round.

If it holds the highest possible UID, it just decides it is the leader, and announces this to the others: no matter if the current leader is still up!

Such an “impolite” behavior inspired the name for the algorithm.

The Bully Algorithm: Properties

About LE1 (safety): at most one (non-crashed) process is elected;
In fact, it is not possible to have two leaders,
because the process with lower UID would discover the other,
and would defer leadership to it.

About LE2 (liveness): guaranteed by the assumed reliable delivery.

Message Complexity:
worst case - initiator with the *lowest* UID ➔ $\boldsymbol{O}(n^2)$;
best case - initiator with the *highest* UID ➔ n-1 LEADER msgs, $\boldsymbol{O}(n)$.

Multicast: What Multicast?

Group Communication

Group up processes, and *address groups instead of single processes*.

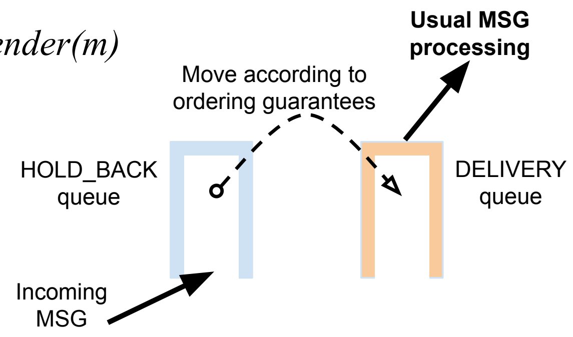
Relative primitives:

Xmulticast(group g, msg m) - **X** indicates the specific type, according to different possible semantics about ordering guarantees

Xdeliver(msg m) - getting *m*, it must be possible to obtain *sender(m)*

Why "deliver" and not "receive"?
Possible reorderings of msgs at destination require an **hold-back queue**.

Similar trick already seen in Lai-Yang alg.



Basic Multicast

For the time being, let us suppose processes may fail *only by crashing*.

The most straightforward implementation for a multicast:

$Bmulticast(g,m)$ ➔ for each process p in g, $send(p,m)$

On $receive(m)$ at p: $Bdeliver(m)$ at p.

"Basic" multicast guarantees that a correct process
will *eventually* deliver the message,
as long as the multicaster does not crash.

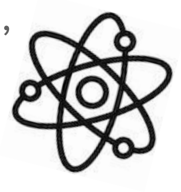
Reliable Multicast

A reliable multicast ($Rmulticast$) satisfies the following properties:

Integrity - A correct process delivers a message m at most once

Validity - If a correct process multicasts msg m, it will eventually deliver it
(this is self-validity! It's meaningful only along with the next property).

Agreement (a.k.a. *"Atomicity"*) - If a correct process delivers msg m,
then all the other correct processes in $group(m)$
will eventually deliver it.



Trivial Implementation of Rmulticast

Rmulticast can be simply implemented on top of *Bmulticast* in the following way:

- *Rmulticast* corresponds exactly to *Bmulticast*
- *Rdeliver* corresponds to: *Bdeliver* + (*only once*) *Bmulticast* of the same msg to the group; it is assumed that only one copy of the msg is actually delivered, and the (possible) others are discarded.

Agreement comes from observing that, after the *Bdeliver*, each processes issues a *Bmulticast*: if a correct process does not *Rdeliver m*, this may happen only because no other process *Bdeliver*ed it either (in fact, there should be an HB relation from any other *Bdeliver* and the local *Rdeliver*)

Tradeoff between
stronger guarantee and msg complexity

Other Multicast Semantics

FIFO: if on one correct process *Fmulticast*(g,m_1) and then *Fmulticast*(g,m_2) occur,
at any destination in g, m_1 is *Fdeliver*-ed before m_2

Causal: if, on correct processes, *COmulticast*(g,m_1) H.B. *COmulticast*(g,m_2), at any destination in g, m_1 is *COdeliver*-ed before m_2. Causal implies FIFO.

Total: if a correct process *Tdeliver* m_1 first and m_2 later,
the same order of delivering will be experienced by any process.

COmulticast is implemented using vector timestamps; messages are kept at destination inside the hold-back queue until the checks on their precedence relation with the local vector clock would allow the actual deliver operation.