



函数式编程简介

PARKER LIU

The program is magic words



```
void qsort(int *A, int len) {  
    if (len < 2) return;  
  
    int pivot = A[len / 2];  
  
    int i, j;  
    for (i = 0, j = len - 1; ; i++, j--) {  
        while (A[i] < pivot) i++;  
        while (A[j] > pivot) j--;  
  
        if (i >= j) break;  
  
        int temp = A[i];  
        A[i] = A[j];  
        A[j] = temp;  
    }  
  
    qsort(A, i);  
    qsort(A + i, len - i);  
}
```

```

void qsort(int *A, int len) {
    if (len < 2) return;

    int pivot = A[len / 2];

    int i, j;
    for (i = 0, j = len - 1; ; i++, j--) {
        while (A[i] < pivot) i++;
        while (A[j] > pivot) j--;

        if (i >= j) break;

        int temp = A[i];
        A[i] = A[j];
        A[j] = temp;
    }

    qsort(A, i);
    qsort(A + i, len - i);
}

```

```

qsort [] = []
qsort (x:xs) = qsort left
               ++ (x : qsort right)
               where (left, right) = partition (< x) xs

```



```
char* sieve(int n, int *c)
{
    char* sieve;
    int i, j, m;

    if(n < 2)
        return NULL;

    *c = n-1; /* primes count */
    m = (int) sqrt((double) n);

    /* calloc initializes to zero */
    sieve = calloc(n+1, sizeof(char));
    sieve[0] = 1;
    sieve[1] = 1;
    for(i = 2; i <= m; i++)
        if(!sieve[i])
            for (j = i*i; j <= n; j += i)
                if(!sieve[j]){
                    sieve[j] = 1;
                    --(*c);
                }
    return sieve;
}
```

```

char* sieve(int n, int *c)
{
    char* sieve;
    int i, j, m;

    if(n < 2)
        return NULL;

    *c = n-1; /* primes count */
    m = (int) sqrt((double) n);

    /* calloc initializes to zero */
    sieve = calloc(n+1, sizeof(char));
    sieve[0] = 1;
    sieve[1] = 1;
    for(i = 2; i <= m; i++)
        if(!sieve[i])
            for (j = i*i; j <= n; j += i)
                if(!sieve[j]){
                    sieve[j] = 1;
                    --(*c);
                }
    return sieve;
}

```

```

primes = filterPrime [2..]
where
    filterPrime (p:xs) = p : filterPrime [x | x <- xs, x `mod` p /= 0]

```

```
r = requests.get('http://chrisdone.com/ontime.csv.zip', stream=True)
```

```
with open("flights.csv", 'wb') as f:
```

```
    for chunk in r.iter_content(chunk_size=1024):
```

```
        if chunk:
```

```
            f.write(chunk)
```

```
zf = zipfile.ZipFile("flights.csv.zip") filename = zf.filelist[0].filename fp = zf.extract(filename) df =  
pd.read_csv(fp, parse_dates="FL_DATE").rename(columns=str.lower)
```

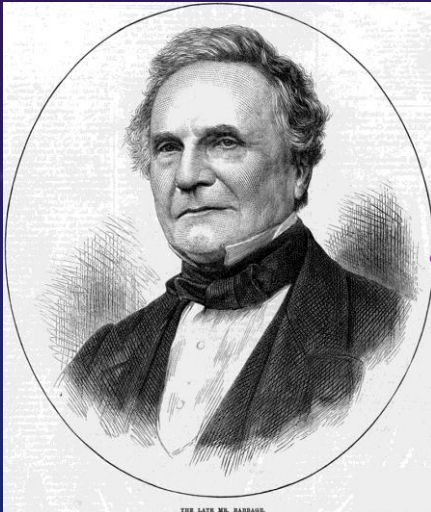
```
r = requests.get('http://chrisdone.com/ontime.csv.zip', stream=True)
with open("flights.csv", 'wb') as f:
    for chunk in r.iter_content(chunk_size=1024):
        if chunk:
            f.write(chunk)

zf = zipfile.ZipFile("flights.csv.zip") filename = zf.filelist[0].filename fp = zf.extract(filename) df =
pd.read_csv(fp, parse_dates="FL_DATE").rename(columns=str.lower)
```

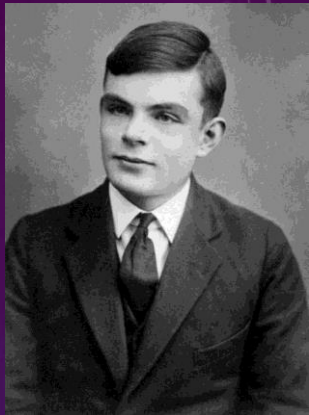
```
main = runResourceT $
  httpSource "http://chrisdone.com/ontime.csv.zip" responseBody
  .| zipEntryConduit "ontime.csv"
  .| fromCsvConduit @("fl_date" := Day, "tail_num" := String) (set #downcase True csv)
  .| dropConduit 10
  .| takeConduit 5
  .> tableSink
```



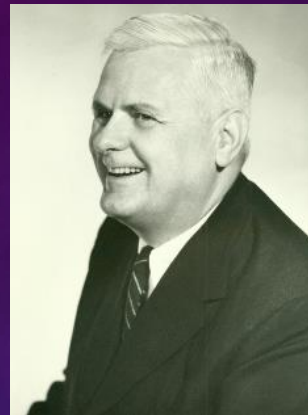

Ada Lovelace



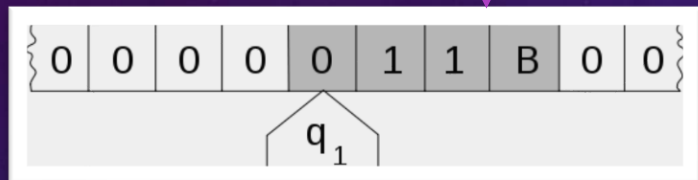
Charles Babbage



Alan Turing

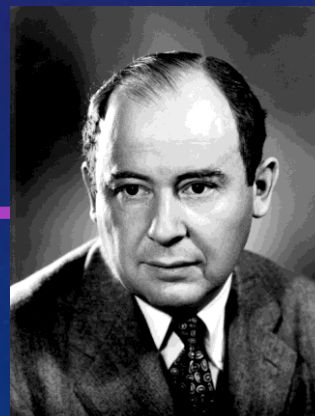
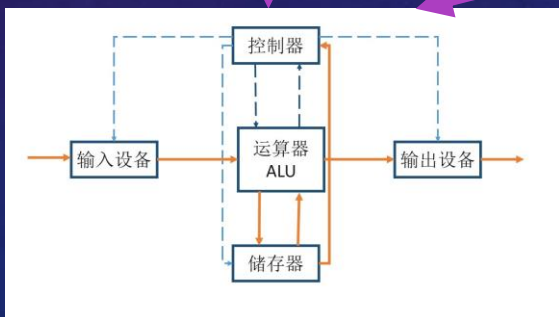


Alonzo Church

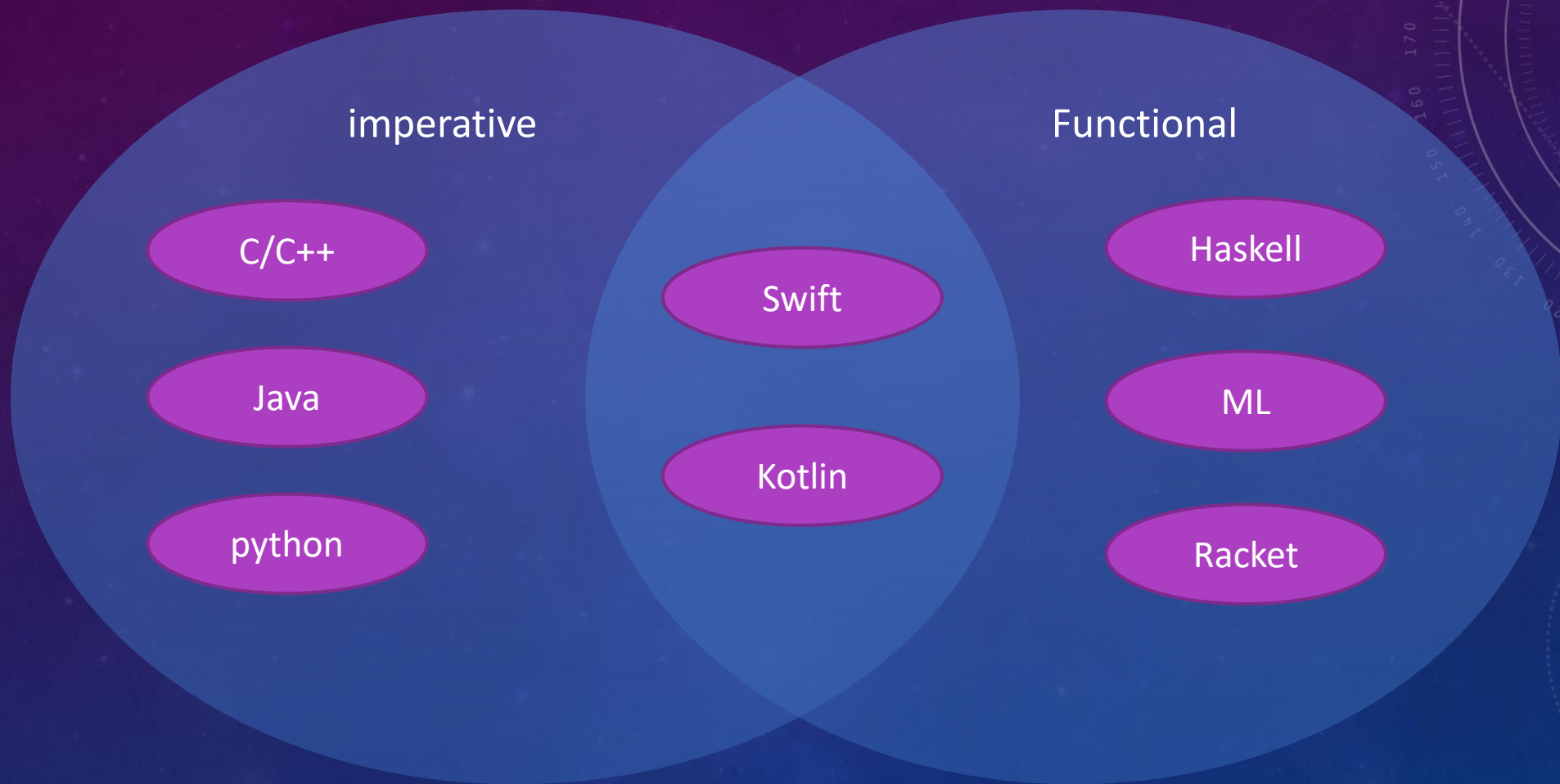


Church-Turing thesis

```
0 := λf.λx.x
1 := λf.λx.f x
2 := λf.λx.f (f x)
3 := λf.λx.f (f (f x))
```



John Von Neumann



WHAT IS FUNCTIONAL PROGRAMMING

- What to do not how to do
- Immutable, no state
- Recursive, no loop
- Function is first class object
- Type system
- Category theory

函数式编程思维：

函数式编程关心类型（代数结构）之间的关系，以及如何将这些关系组合起来，用数学的构造主义来构造程序

THE OLD CONVENTIONAL IDEAS

Imperative Programming

- Fast
- Easy to learn
- No GC
- A lots of industry application

Functional Programming

- Slow
- Difficult to learn – Immutable, recursive, cats
- Have GC
- Less industry application

THE WORLD IS CHANGED

- Multi-core processor
 - GPU and DSP
 - Much more complex task
 - Cloud and distribution
- How to programming with parallel and concurrent
 - How to write safety and stability program
 - How to transfer the program to other remote computer

WHY HASKELL

- The one ring lord of rings
- Type system – System F
- Lazy evaluation
- Category theory
- A lots of packages
- Enough performance for normal application – 2~5 times less than C/C++
- A industry level compiler -- ghc



Haskell Brooks Curry

READY TO START HASKELL

- Install the Haskell-platform from <https://www.haskell.org/downloads>
- Launch ghci
- Or use ghc to compile the Haskell program

```
ghc -make -O2 example.hs
```

VARIABLE AND FUNCTION, OPERATOR

- `2 :: Int, 'a' :: Char, "Hellow, world!" :: String`

- `a :: Int`

`a = 2`

- `add :: Int -> Int -> Int`

`add a b = a + b`

- `mult2 a = a * 2`

`catString s1 s2 = s1 ++ s2`

FUNCTION APPLY

```
> add 2 3
```

```
5
```

```
> mult2 3
```

```
6
```

```
> mult2 $ 3
```

```
6
```

```
> catString "Hello " "world!"
```

```
"Hello world!"
```


TYPES AND TYPE VARIABLE

- Primary types: **Int, Integer, Float, Double, Char, BOOL, Word**
- Container types: **[], Map, IntMap, Tree, Vector, Array**
- **type String = [Char]**
- A type variable is **a** in type **[a], Maybe a**

head :: [a] -> a, length :: [a] -> Int

fst :: (a, b) -> a, snd :: (a, b) -> b

TYPE CONSTRUCTION

A rich type is constructed by other types

```
data Maybe a = Nothing
              | Just a
              deriving(Eq, Show)

data [a] = [] | a : [a]

data Gendar = Male | Female deriving(Eq, Ord, Show)
data Person = Person {
    name :: String
  , age  :: Int
  , gendar :: Gendar
}
```

TYPE CLASS

A type constraint

```
class Eq a where
  (==) :: a -> a -> Bool
  a == b = not (a /= b)

  (/=) :: a -> a -> Bool
  a /= b = not (a == b)

class Ord a where
  compare :: a -> a -> Ordering
  (<) :: a -> a -> Bool
  (>) :: a -> a -> Bool
  max :: a -> a -> a
  min :: a -> a -> a
```

TYPE AND NEWTYPE

Type alias and newtype for wrapper type that real same as original type

```
type Name = String
type Age  = Int

newtype Fd = Fd CInt
newtype State s a = State { runState :: s -> (a, s) }

newtype Any = Any { getAny :: Bool } deriving(Show)
instance Monoid Any where
    mempty  = Any False
    mappend = (||)

newtype All = All { getAll :: Bool } deriving(Show)
Instance Monoid All where
    mempty  = All True
    mappend = (&&)
```

FUNCTION USAGE – PATTERN MATCHING & GUARDS

Pattern matching

```
sayMe :: (Integral a) => a -> String
sayMe 1 = "One!"
sayMe 2 = "Two!"
sayMe 3 = "Three!"
sayMe x = "Not between 1 and 3"
```

```
addVectors (Num a) => (a, a)
                        -> (a, a)
                        -> (a, a)
addVectors (x1, y1) (x2, y2) =
    (x1 + x2, y1 + y2)
```

Guards

```
myCompare :: (Ord a) => a -> a -> Ordering
myCompare a b
    | a > b      = GT
    | a == b    = EQ
    | otherwise = LT

isOdd (Integral a) => a -> Bool
isOdd a
    | a `mod` 2 /= 0 = True
    | otherwise      = False
```


FUNCTION USAGE – LET, WHERE AND CASE

let and where

```
cylinder :: (RealFloat a) => a -> a -> a
cylinder r h =
    let sideArea = 2 * pi * r * h
        topArea  = pi * r ^ 2
    in  sideArea + 2 * topArea
```

```
qsort []      = []
qsort (x:xs) = qsort left
               ++ (x : qsort right)
  where
    (left, right) = partition (< x) xs
```

case expression

```
describeList :: [a] -> String
describeList xs = "The list is "
                ++ case xs of
                    []   -> "empty."
                    [x]  -> "a singleton list."
                    xs   -> "a longer list."
```

```
describeList :: [a] -> String
describeList xs = "The list is " ++ what xs
  where what []   = "empty."
        what [x] = "a singleton list."
        what xs  = "a longer list."
```

HIGH ORDER FUNCTION

A function that takes other functions as arguments or returns a function as result.

```
applyTwice :: (a -> a) -> a -> a  
applyTwice f x = f (f x)
```

```
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]  
zipWith _ [] _ = []  
zipWith _ _ [] = []  
zipWith f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

CURRY AND UNCURRY

All multi-parameters function is can be treated as single parameter function

```
curry :: ((a, b) -> c) -> a -> b -> c  
curry f a b = f (a, b)
```

```
uncurry :: (a -> b -> c) -> (a, b) -> c  
uncurry f (a, b) = f a b
```

FUNCTION COMPOSITION

The large function can be constructed by small functions

```
(.) :: (b -> c) -> (a -> b) -> a -> c  
f . g = \x -> f (g x)
```

```
id :: a -> a  
id x = x
```

```
oddSquareSum :: Integer  
oddSquareSum =  
    sum . takeWhile (<10000)  
    . filter odd . map (^2) $ [1..]
```

Laws

- $id . f = f$
- $f . id = f$
- $(f . g) . h = f . (g . h)$

RECURSIVE

Who are you like?

```
length :: [a] -> Int
length [] = 0
length (x:xs) = 1 + length xs
```

```
sum :: (Num a) => [a] -> a
sum [] = 0
sum (x:xs) = x + sum xs
```

```
maximum :: (Ord a) => [a] -> a
maximum [] = error "maximum of empty list"
maximum [x] = x
maximum (x:xs) = x `max` (maximum xs)
```


FUNCTION ABSTRACTION

A general function that can accept a function parameter

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z []      = z
foldr f z (x:xs) = x `f` foldr f z xs
```

```
length :: [a] -> Int
-- length = foldr (\a len -> 1 + len) 0
length = foldr ((+) . const 1) 0
```

```
sum :: (Num a) => [a] -> a
sum = foldr (+) 0
```

```
product :: (Num a) => [a] -> a
product = foldr (*) 1
```

FUNCTOR AND MAP

A normal function can lift to a container transformation function

```
allAddBy2 :: (Num a) => [a] -> [a]
allAddBy2 [] = []
allAddBy2 (x:xs) = (x + 2) : allAddBy2 xs

allCheckOdd :: (Integral a) => [a] -> [Bool]
allCheckOdd [] = []
allCheckOdd (x:xs) = isOdd x : allCheckOdd xs

map :: (a -> b) -> [a] -> [b]
map f = foldr ((:) . f) []
-- map f [] = []
-- map f (x:xs) = f x : map f xs
```

FUNCTOR AND MAP

A normal function can lift to a container transformation function

```
allAddBy2 :: (Num a) => [a] -> [a]
allAddBy2 [] = []
allAddBy2 (x:xs) = (x + 2) : allAddBy2 xs

allCheckOdd :: (Integral a) => [a] -> [Bool]
allCheckOdd [] = []
allCheckOdd (x:xs) = isOdd x : allCheckOdd xs

map :: (a -> b) -> [a] -> [b]
map f = foldr ((:) . f) []
-- map f [] = []
-- map f (x:xs) = f x : map f xs

class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
allAddBy2 = map (+2)
```

```
allCheckOdd = map isOdd
```

Laws

- $\text{fmap id} = \text{id}$
- $\text{fmap } (f \cdot g) = (\text{fmap } f) \cdot (\text{fmap } g)$

APPLICATIVE

A context computation function can applied by context value

```
class Functor f => Applicative f where
  pure  :: a -> f a

  (<*>) :: f (a -> b) -> f a -> f b

  (<$>) :: (a -> b) -> f a -> f b
  (<$>) = fmap
```

Laws

- $\text{pure id } \langle * \rangle v = v$
- $\text{pure } f \langle * \rangle \text{pure } x = \text{pure } (f x)$
- $f \langle * \rangle \text{pure } y = \text{pure } (\$ y) \langle * \rangle f$
- $\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = u \langle * \rangle (v \langle * \rangle w)$

APPLICATIVE EXAMPLES

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```

```
Instance Applicative [] where
  pure a = [a]
  fs <*> xs = [f x | f <- fs, x <- xs]
```

```
sequenceA :: Applicative f => [f a] -> f [a]
sequenceA [] = pure []
sequenceA (x:xs) = (:) <$> x <*> sequenceA xs
```


MONAD

A context value can apply to a monadic function

```
class Applicative m => Monad m where
  return :: a -> m a

  (>>=) :: m a -> (a -> m b) -> m b
  m >>= f = join (fmap f m)

  join :: m (m a) -> m a
  join m = m >>= id
```

Laws

- $\text{join} \cdot \text{return} = \text{id}$
- $\text{join} \cdot \text{fmap return} = \text{id}$
- $\text{join} \cdot \text{join} = \text{join} \cdot \text{fmap join}$

MONAD EXAMPLES

```
instance Monad Maybe where  
  return = Just  
  Nothing >>= f = Nothing  
  (Just a) >>= f = f a
```

```
Instance Monad [] where  
  return a = [a]  
  as >>= f = concatMap f as
```

MONAD EXAMPLES

```
type Birds = Int
type Pole = (Birds, Birds)

leftLand :: Birds -> Pole -> Maybe Pole
leftLand n (l, r)
  | abs ((l + n) - r) < 4 = Just ((l + n), r)
  | otherwise              = Nothing

rightLand :: Birds -> Pole -> Maybe Pole
rightLand n (l, r)
  | abs (l - (r + n)) < 4 = Just (l, (r + n))
  | otherwise              = Nothing
```

MONAD EXAMPLES

```
routine :: Maybe Pole
routine = case landLeft 1 (0,0) of
  Nothing -> Nothing
  Just pole1 -> case landRight 4 pole1 of
    Nothing -> Nothing
    Just pole2 -> case landLeft 2 pole2 of
      Nothing -> Nothing
      Just pole3 -> landLeft 1 pole3
```

```
routine :: Maybe Pole
routine = landLeft 1 (0, 0)
  >>= (\p1 -> (landRight 4 p1
  >>= (\p2 -> (landLeft 2 p2
  >>= (\p3 -> landLeft 1 p3))))))
```


MONAD DO NOTATION

```
routine :: Maybe Pole
routine = landLeft 1 (0, 0)
      >>= (\p1 -> (landRight 4 p1
      >>= (\p2 -> (landLeft 2 p2
      >>= (\p3 -> landLeft 1 p3))))))
```

```
routine :: Maybe Pole
routine = do
  p1 <- landLeft 1 (0, 0)
  p2 <- landRight 4 p1
  p3 <- landLeft 2 p2
  landLeft 1 p3
```

IO MONAD

We can interactive with real world through IO Monad

```
main :: IO ()
main = do
  putStrLn "Hello, World!"
  putStrLn "Please input youre name:"
  name <- getLine
  putStrLn "Good morning, " ++ name
```

COMONAD

The comonadic functions can be applied context value

```
class Functor w => Comonad w where
  extract      :: w a -> a

  duplicate    :: w a -> w (w a)
  duplicate = extend id

  extend      :: (w a -> b) -> w a -> w b
  extend f = fmap f . duplicate
```

Laws

- $\text{extract} \circ \text{duplicate} = \text{id}$
- $\text{fmap extract} \circ \text{duplicate} = \text{id}$
- $\text{duplicate} \circ \text{duplicate} = \text{fmap duplicate} \circ \text{duplicate}$

COMONAD EXAMPLES

```
data Stream a = a :> Stream a

tail :: Stream a -> Stream a
tail (_ :> as) = as

tails :: Stream a -> Stream (Stream a)
tails w = w :> tails (tail w)

instance Comonad Stream where
  extract = head
  duplicate = tails
  extend f w = f w :> extend f (tail w)
```


无用之用 -- IDENTITY

```
newtype Identity a = Identity { getIdentity :: a }
```

```
Instance Functor Identity where  
  fmap f (Identity a) = Identity (f a)
```

```
instance Applicative Identity where  
  pure = Identity  
  (Identity f) <*> (Identity a) = Identity (f a)
```

```
Instance Monad Identity where  
  return = Identity  
  (Identity a) >>= f = f a
```

```
Instance Comonad Identity where  
  extract    = getIdentity  
  duplicate  = Identity  
  extend f   = fmap f . Identity
```



May the fore be with you

Lightness

$$\mathit{Lan}_k F a = \int^b D(k b, a) \times F b$$

```
data Lan k f a = forall b.  
    Lan (k b -> a) (f b)
```

Darkness

$$\mathit{Ran}_k F a = \int_b \mathit{Set}(D(a, k b), F b)$$

```
newtype Ran k f a =  
    Ran { getRan :: forall b. (a -> k b) -> f b }
```

REFERENCE

- Haskell趣学指南 -- 作者: Miran Lipovaca 译者: Fleurer, 李亚舟, 宋方睿
- Haskell函数式编程入门（第二版） -- 作者: 张淞, 刘长生
- Category Theory for Programmers – 作者: Bartosz Milewski
链接: <https://bartoszmilewski.com/2014/10/28/category-theory-for-programmers-the-preface/>
- Generic Programming with Adjunctions – 作者: Ralf Hinze