Online Graph Partitioning

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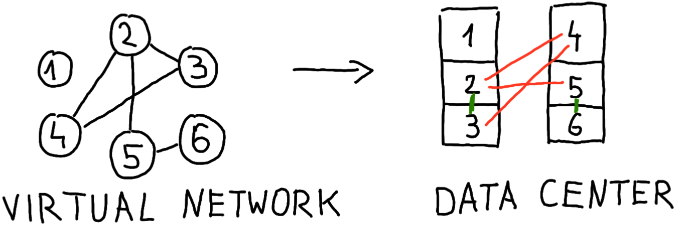
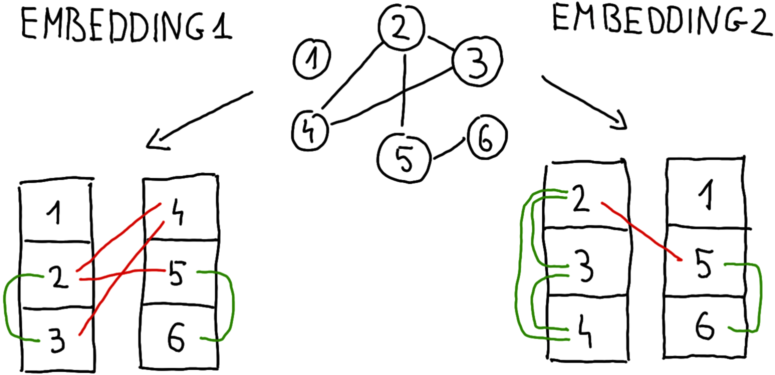
(DISC '16, PODC '20, INFOCOM '20)

MOTIVATION:

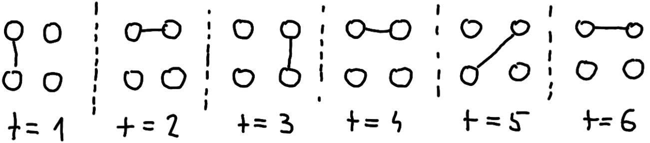
VIRTUAL MACHINES IN DATACENTERS

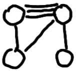



EMBEDDING 1
EMBEDDING 2
1
2
3
4
5
6

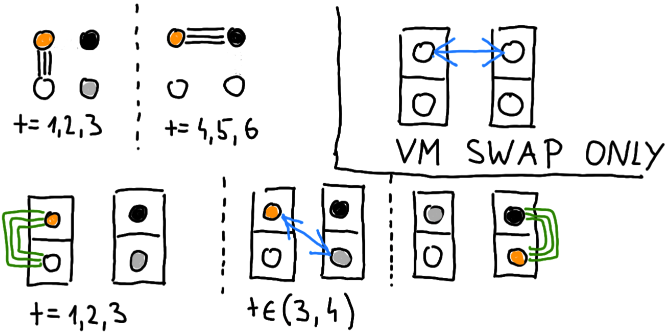
STATIC → DYNAMIC EMBEDDINGS

(ADD TIME DIMENSION)

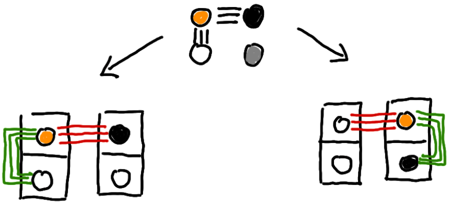


(STATIC VIEW:  ⇒ LOOSES INFORMATION)

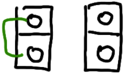
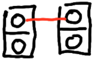
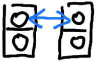
VIRTUAL MACHINE MIGRATION

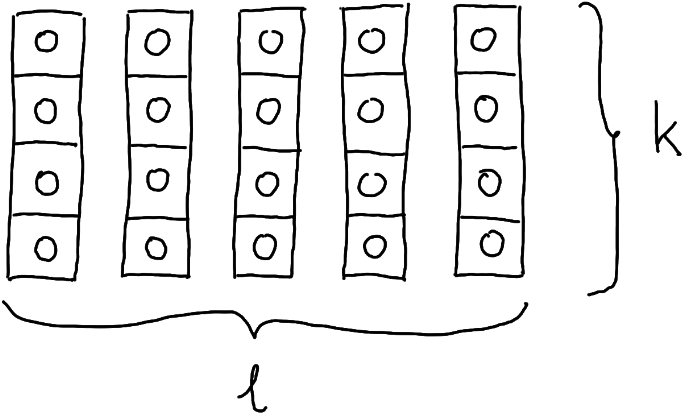


STATIC EMBEDDING IS INHERENTLY INEFFICIENT



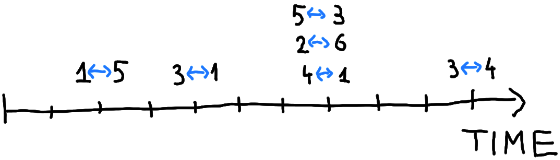
MODEL: ONLINE GRAPH PARTITIONING

- INPUT: EDGES APPEAR OVER TIME
-  COSTS 0,  COSTS 1
- MIGRATION  COSTS $\alpha \geq 1$ (A PARAMETER)
- GOAL: MINIMIZE TOTAL COST


k
ℓ

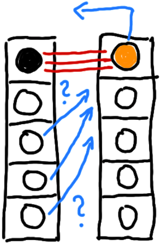
ALGORITHMIC PROBLEM:

DECIDE WHICH VM TO MIGRATE AND WHEN



ALGORITHMIC CHALLENGES:

- FUTURE REQUESTS UNKNOWN
- WHAT TO EVICT?
- WHERE TO COLLOCATE?
- SERVE REMOTELY OR MIGRATE?

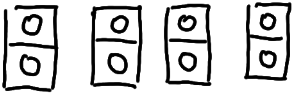


COMPETITIVE ANALYSIS

$$\min \frac{ALG}{OPT}$$

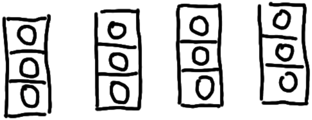
(OPT KNOWS THE FUTURE)

THIS TALK:



k = 2

IS FUNDAMENTALLY DIFFERENT THAN



k = 3

LOWER BOUND FOR $K \geq 2$

THM. DETERMINISTIC ONLINE ALGORITHMS ARE NO BETTER THAN 3-COMPETITIVE

CHOOSE 3 NODES.

INPUT: IF (A,C) SPLIT, REQUEST (A,C)
IF (A,B) SPLIT, REQUEST (A,B)

FIX A SEQUENCE OF R REQUESTS.

ALG PAYS M FOR MIGRATIONS

$$ALG = R + M$$

GOAL: $\frac{ALG}{OPT} \geqslant 3$

3 OFFLINE ALGORITHMS

OFF_{AB} : $\boxed{\begin{matrix}A\\B\end{matrix}}$ $\boxed{\begin{matrix}\ \\ \ \end{matrix}}$

OFF_{AC} : $\boxed{\begin{matrix}A\\C\end{matrix}}$ $\boxed{\begin{matrix}\ \\ \ \end{matrix}}$

OFF_D: DO OPPOSITE OF ALG

ALG $\boxed{\begin{matrix}A\\B\end{matrix}}$ $\Rightarrow$ OFF_D $\boxed{\begin{matrix}A\\C\end{matrix}}$, ALG $\boxed{\begin{matrix}A\\C\end{matrix}}$ $\Rightarrow$ OFF_D $\boxed{\begin{matrix}A\\B\end{matrix}}$

$$ALG = \underset{\text{(MIGRATION)}}{M} + \underset{\text{(REQUESTS)}}{R}$$

$$OFF_{AC} + OFF_{AB} = \emptyset + R + O(1)$$

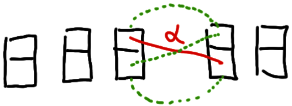
$$OFF_{D} = M + \emptyset + O(1)$$

$$OPT \leqslant \min_i OPT_i \leqslant \frac{1}{3} \sum_i OPT_i = \frac{1}{3}(M+R) + O(1)$$

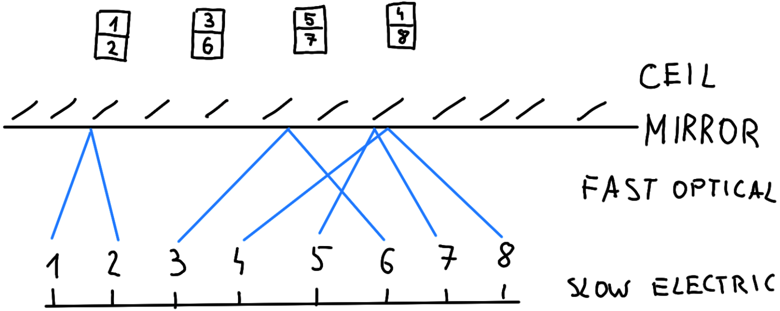
$$\frac{ALG}{OPT} \xrightarrow[R \to \infty]{} 3$$

ALGORITHM FOR K=2

- COUNT REQUESTS BETWEEN EACH PAIR OF VMs
- COUNTER REACHES α
  $\Rightarrow$ SWAP AND RESET 4 COUNTERS



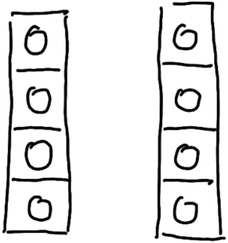
INTERPRETATION OF K=2



TALK OUTLINE

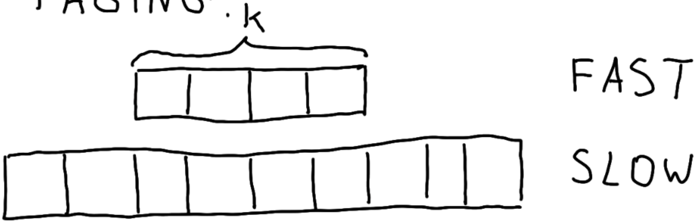
k	l	LOWER BOUND	UPPER BOUND
2	any	3	6
any	2	NEXT	
3	any	NEXT	
any	any		

LOWER BOUND $k \geq 3$, $l = 2$



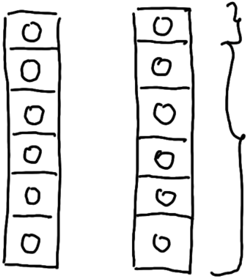
REDUCTION FROM PAGING

PAGING:



NO DETERMINISTIC ONLINE ALGORITHM IS BETTER THAN k-COMPTETITIVE

REDUCTION



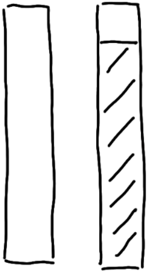
CACHE

1 PAGE OUT OF CACHE

k-1 HELPER NODES

SET $\mathcal{D} = 1$
(MIGRATION COST)

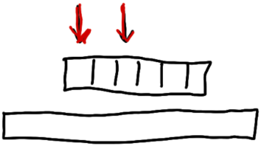
REDUCTION

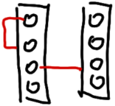


- OFF NEVER SPLITS HELPER NODES
- IF ALG EVER SPLITS HELPER NODES, WE REQUEST THEM

REDUCTION

MAP REQUESTS TO PAGES
TO PAIRS OF NODES





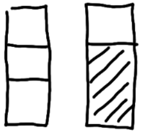
P_1, P_2, P_3 .....

HOW? $(v_1, v_2), (v_2, v_5)$ .....

MAPPING PAGES TO PAIRS WITH HELP FROM OPT

ALL WE NEED IS A SECOND NODE THAT IS IN THE CACHE

THE REQUESTED PAGE →



FIX OPT FOR PAGING SEQUENCE δ

MAP δ_i to (δ_i, p_i^{OPT}) WHERE p_i^{OPT} IS IN OPT'S CACHE AT TIME i

IF ALG NEVER SPLITS HELPER NODES,

$$\text{cost}_{\text{PAGING}}(\text{ALG}) = \text{cost}_{\text{G.P}}(\text{ALG})$$

$$\frac{\text{ALG}}{\text{OPT}} \geqslant \frac{\text{cost}_{\text{PAGING}}(\text{ALG})}{\text{cost}_{\text{PAGING}}(\text{OPT})} \geqslant k$$

THE LAST PROOF

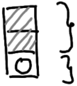
$k=3$

LOWER BOUND $\Omega(l)$

(IN GENERAL $\Omega(k \cdot l)$)

TWO IDEAS

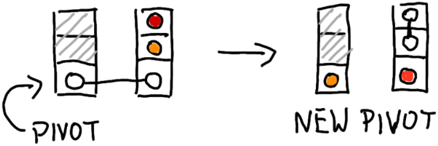
1. SETS OF UNSPLITTABLE NODES
2. PIVOT NODES



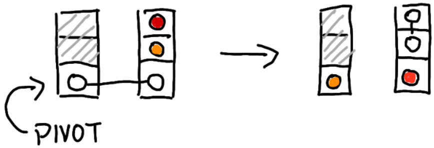
UNSPLITTABLE
PIVOT

CONSTRUCTION

1. FIRST REQUEST IS SPECIAL
2. NEXT REQUESTS
   DEPEND ON ALG
3. REPEAT REQUESTS IF
   ALG SPLITS



REQUEST THE CURRENT PIVOT,
AND NEW PIVOT REPLACES IT



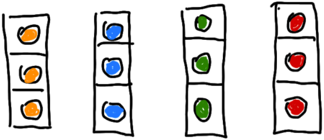
REPEAT $\Omega(\ell)$ TIMES

BUT:

- (PIVOT, ?)
- WHEN TO STOP SO OPT CHEAP?

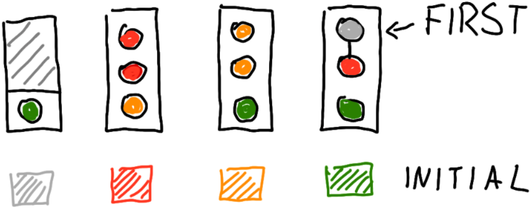
PIVOT AND X?

LOOK AT INITIAL CONFIGURATION



FIRST REQUEST HAS 2 COLORS

NEXT REQUESTS UNICOLOR

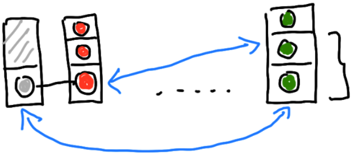


2 GREEN NODES EVICTED FROM GREEN SERVER

REPEAT AS LONG AS WE CAN?

NO.

OPT: ONLY MOVE 2 NODES



DO NOT REQUEST

GREEN = THE LAST SERVER OF ALG

REPEAT WHILE ALG HAS UNTOUCHED SERVER

$$\left.\begin{array}{l} \text{ALG} \geqslant L-2 \\ \\ \text{OPT} \leqslant 1 \end{array}\right\} \text{\# OF SWAPS}$$

k	ℓ	LB	UB
2	any	3	6
any	2	k	
3	any	$\Omega(\ell)$	$O(\ell)$
any	any	$\Omega(\ell \cdot k)$	$O(2^{o(k)} \cdot \ell)$ $O(k \cdot \ell \cdot \log k)$ $(O(k^2 \cdot \ell^2))$

OUR

WAOA '21

ESA '22

ALSO: SIGMETRICS '16, SODA '21 } ON A MODEL VARIANT