CHAPTER 1

INTRODUCTION

In this chapter we present the rationale for the book, define several terms that will be used throughout, and describe various types of systems. The chapter concludes with a description of the particular types of systems to be considered and a summary of the techniques that the reader should be able to apply after finishing the book.

1.1 RATIONALE

The importance of understanding and being able to determine the dynamic response of physical systems has long been recognized. It has been traditional in engineering education to have separate courses in dynamic mechanical systems, circuit theory, chemical-process dynamics, and other areas. Such courses develop techniques of modeling, analysis, and design for the particular physical systems that are relevant to that specific discipline, even though many of the techniques taught in these courses have much in common. This approach tends to reinforce the student's view of such courses as isolated entities with little in common and to foster a reluctance to apply what has been learned in one course to a new situation.

Another justification for considering a wide variety of different types of systems in an introductory book is that the majority of systems that are of practical interest contain components of more than one type. In the design of electronic circuits, for example, attention must also be paid to mechanical structure and to dissipation of the heat generated. Hydraulic motors and pneumatic process controllers are other examples of useful combinations of different types of elements. Furthermore, the techniques in this book can be applied not only to pneumatic, acoustical, and other traditional areas but also to systems that are quite different, such as sociological, physiological, economic, and transportation systems.

Because of the universal need for engineers to understand dynamic systems, and because there is a common methodology applicable to such systems regardless of their physical origin, it makes sense to present them all together. This book considers both the problem of obtaining a mathematical description of a physical system and the various analytical techniques that are widely used.

1.2 ANALYSIS OF DYNAMIC SYSTEMS

Because the most frequently used key word in the text is likely to be *system*, it is appropriate to define it at the outset. A **system** is any collection of interacting elements for which there are cause-and-effect relationships among the variables. This definition is necessarily general, because it must encompass a broad range of systems. The most important feature of the definition is that it tells us we must take interactions among the variables into account in system modeling and analysis, rather than treating individual elements separately.

Our study will be devoted to **dynamic systems**, for which the variables are time-dependent. In nearly all our examples, not only will the excitations and responses vary with time but at any instant the derivatives of one or more variables will depend on the values of the system variables at that instant. The system's response will normally depend on initial conditions, such as stored energy, in addition to any external excitations.

In the process of analyzing a system, two tasks must be performed: modeling the system and solving for the model's response. The combination of these steps is referred to as **system analysis**.

Modeling the System

A **mathematical model**, or **model** for short, is a description of a system in terms of equations. The basis for constructing a model of a system is the physical laws (such as the conservation of energy and Newton's laws) that the system elements and their interconnections are known to obey.

The type of model sought will depend on both the objective of the engineer and the tools for analysis. If a pencil-and-paper analysis with parameters expressed in literal rather than numerical form is to be performed, a relatively simple model will be needed. To achieve this simplicity, the engineer should be prepared to neglect elements that do not play a dominant role in the system.

On the other hand, if a computer is available for carrying out simulations of specific cases with parameters expressed in numerical form, a comprehensive mathematical model that includes descriptions of both primary and secondary effects might be appropriate. In short, a variety of mathematical

models are possible for a system, and the engineer must be prepared to decide what form and complexity are most consistent with the objectives and the available resources.

One example of a dynamic system that is familiar to everyone is the automobile. In order to limit the complexity of any model we wish to make of this system, we must omit some of its features. And, in fact, many of the parameters may be relatively unimportant for the objective of a particular study. Among many possible concerns are ease of handling on the straightaway or while turning a corner; comfort of the driver; the vehicle's fuel efficiency, stopping ability, and crash resistance; and the effect of wind gusts, potholes, and other obstacles.

Suppose that we limit our concern to the forces on the driver when the vehicle is traveling over a rough road. Some of the key characteristics of the system are represented in Figure 1.1(a) by masses, springs, and shock absorbers.[1] The chassis has by far the largest mass, but other masses that may be significant are the front and rear axles and wheels and the driver. Suspension systems between the chassis and the axles are designed to minimize the vertical motion of the chassis when the tires undergo sudden motion because of the road surface. The tires themselves have some elasticity, which is represented by additional springs between the wheels and the road. The driver is somewhat cushioned from the chassis motion by the characteristics of the seat, and there is also some friction between the driver and the seat back.

We assume that the vehicle is traveling at a constant speed and that the horizontal motion of the chassis does not concern us. We must certainly allow for the vertical motion caused by the uneven road surface. We may also wish to consider the pitching effect when the front tires hit a bump or depression, causing the front of the chassis to move up or down before the rear. This would require us to consider not only the vertical motion of the chassis but also rotation about its center of mass.

The complexity of a system model is sometimes measured by the number of independent energy-storing elements. For Figure 1.1(a), energy can be stored in four different masses and in five different springs. If we ignore the pitching effect, we might simplify the analysis by combining the front and rear axles into a single mass, as shown in part (b) of the figure, which has only three masses and three springs.

In the initial phase of the analysis, we might make other simplifying assumptions. Perhaps some of the elements remaining in Figure 1.1(b) could be omitted. Perhaps we would use a mathematical description of the individual elements that is simpler than that required for the final analysis.

[1] This figure is adapted from a drawing in Chapter 42 of *The Shock and Vibration Handbook*, third edition (1988), edited by Cyril M. Harris. It is used with the permission of the publisher, McGraw-Hill, Inc.

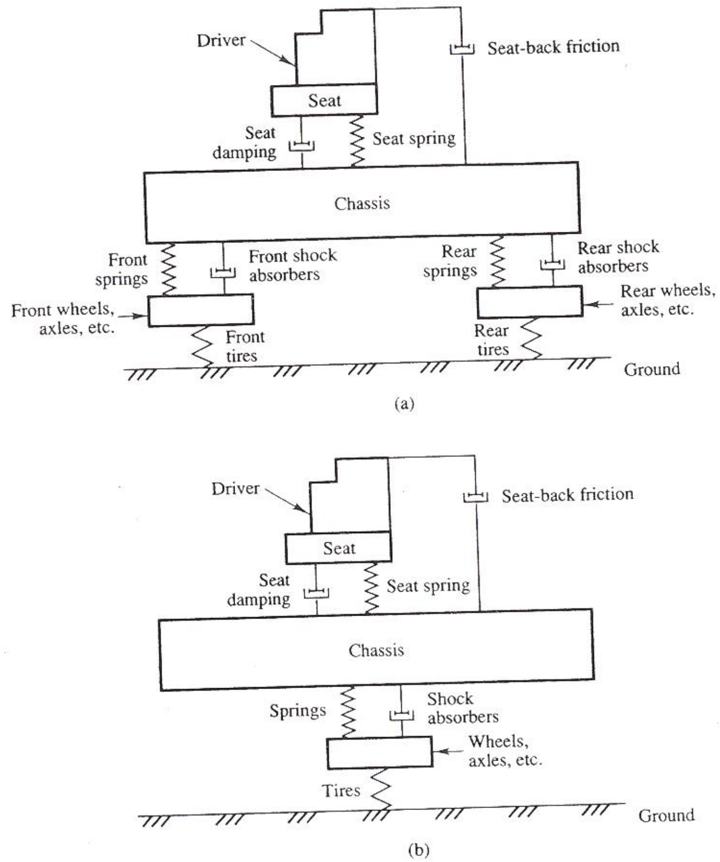

FIGURE 1.1 (a) One representation of an automobile. (b) A simplified representation.

On the other hand, for a more thorough study of the effect of a bumpy road on the driver, it might be necessary to add other characteristics to those represented in Figure 1.1(a). When one of the two wheels on the front axle encounters a bump or depression, the displacement and forces on it are different from those on its mate. Thus we might want to consider each of the four wheels as a separate mass and to allow for side-to-side rotation of the chassis, in addition to the vertical and pitching motions.

When devising models at various stages in the design process, engineers usually give considerable thought to how detailed the representation of the system's characteristics should be. Many of the remarks we have made about the automobile can be extended to airplanes, boats, rockets, motorcycles, and other vehicles. In the next few chapters, we shall show how to describe the important characteristics by sets of equations.

Solving the Model

The process of using the mathematical model to determine certain features of the system's cause-and-effect relationships is referred to as **solving the model**. For example, the responses to specific excitations may be desired for a range of parameter values, as guides in selecting design values for those parameters. As described in the discussion of modeling, this phase may include the analytical solution of simple models and the computer solution of more complex ones.

The type of equation involved in the model has a strong influence on the extent to which analytical methods can be used. For example, nonlinear differential equations can seldom be solved in closed form, and the solution of partial differential equations is far more laborious than that of ordinary differential equations. Computers can be used to generate the responses to specific numerical cases for complex models. However, using a computer to solve a complex model has its limitations. Models used for computer studies should be chosen with the approximations encountered in numerical integration in mind and should be relatively insensitive to system parameters whose values are uncertain or subject to change. Furthermore, it may be difficult to generalize results based only on computer solutions that must be run for *specific* parameter values, excitations, and initial conditions.

The engineer must not forget that the model being analyzed is only an approximate mathematical description of the system, not the physical system itself. Conclusions based on equations that required a variety of assumptions and simplifications in their development may or may not apply to the actual system. Unfortunately, the more faithful a model is in describing the actual system, the more difficult it is to obtain general results.

One procedure is to use a simple model for analytical results and design and then to use a different model to verify the design by means of computer simulation. In very complex systems, it may be feasible to incorporate actual hardware components into the simulation as they become available, thereby eliminating the corresponding parts of the mathematical model.

1.3 CLASSIFICATION OF VARIABLES

A system is often represented by a box (traditionally called a black box), as shown in Figure 1.2. The system may have several **inputs**, or **excitations**, each of which is a function of time. Typical inputs include a force applied to a mass, a voltage source applied to an electrical circuit, and a heat source applied to a vessel filled with a liquid. For the model of an automobile in Figure 1.1(a), the inputs might be the vertical displacements of the bottoms

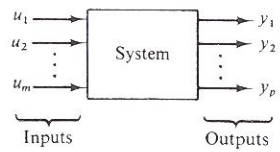

FIGURE 1.2 Black-box representation of a system.

of the springs representing the tires, as the tires move over the bumps in the road. In general discussions that are not related to specific systems, we shall use the symbols $u_1(t)$, $u_2(t)$, ... , $u_m(t)$ to denote the m inputs, shown by the arrows directed into the box.

Outputs are variables that are to be calculated or measured. Typical outputs include the velocity of a mass, the voltage across a resistor, and the rate at which a liquid flows through a pipe. For the model in Figure 1.1(a), one of the outputs might be the vertical acceleration of the driver. The p outputs are represented in Figure 1.2 by the arrows pointing away from the box representing the system. They are denoted by the symbols $y_1(t)$, $y_2(t)$, ... , $y_p(t)$. There is a cause-and-effect relationship between the outputs and inputs. To calculate any one of the outputs for all $t \geq t_0$, we must know the inputs for $t \geq t_0$ and also the accumulated effect of any previous inputs. One approach to constructing a mathematical model is to find equations that relate the outputs directly to the inputs by eliminating all the other variables that are internal to the system. If we are interested only in the input-output relationships, eliminating extraneous variables may seem appealing. However, by deleting information from the model, we may lose potentially important aspects of the system's behavior.

Another modeling technique is to introduce a set of **state variables**, which generally differs from the set of outputs but may include one or more of them. The state variables must be chosen so that a knowledge of their values at any reference time t_0 and a knowledge of the inputs for all $t \geq t_0$ is sufficient to determine the outputs and state variables for all $t \geq t_0$. An additional requirement is that the state variables must be independent; that is, it must not be possible to express one state variable as an algebraic function of the others. This approach is particularly convenient for working with multi-input, multi-output systems and for obtaining computer solutions. In Figure 1.3, the representation of the system has been modified to include the state variables denoted by the symbols $q_1(t)$, $q_2(t)$, ... , $q_n(t)$ within the box. The state variables can account for all important aspects of the system's internal behavior, regardless of the choice of output variables. Equations for the outputs are then written as algebraic functions of the state variables, the inputs, and time.

Whenever it is appropriate to indicate units for the variables and parameters, we shall use the International System of Units (abbreviated SI, from

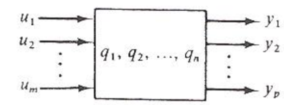

FIGURE 1.3 General system representation showing inputs, state variables, and outputs.

the French *Système International d'Unités*). A list of the units used in this book appears in Appendix A.

1.4 CLASSIFICATION OF SYSTEMS

Systems are grouped according to the types of equations that are used in their mathematical models. Examples include partial differential equations with time-varying coefficients, ordinary differential equations with constant coefficients, and difference equations. In this section we define and briefly discuss ways of classifying the models, and in the next section we indicate those categories that will be treated in this book. The classifications that we use are listed in Table 1.1.

TABLE 1.1 Criteria for Classifying Systems

Criterion	Classification
Spatial characteristics	Lumped Distributed
Continuity of the time variable	Continuous Discrete-time Hybrid
Quantization of the dependent variable	Nonquantized Quantized
Parameter variation	Fixed Time-varying
Superposition property	Linear Nonlinear

Spatial Characteristics

A **distributed system** does not have a finite number of points at which state variables can be defined. In contrast, a **lumped system** can be described by a finite number of state variables.

To illustrate these two types of systems, consider the flexible shaft shown in Figure 1.4(a) with one end embedded in a wall and with a torque applied to the other end. The angle through which a point on the surface of the shaft is twisted depends on both its distance from the wall and the applied

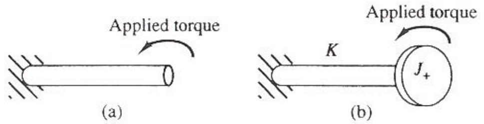

FIGURE 1.4 (a) A torsional shaft. (b) Its lumped approximation.

torque. Hence the shaft is inherently distributed and would be modeled by a partial differential equation. However, if we are interested only in the angle of twist at the right end of the shaft, we may account for the flexibility of the shaft by a rotational spring constant K and represent the effect of the distributed mass by the single moment of inertia J. Making these approximations results in the lumped system shown in Figure 1.4(b), which has the important property that its model is an ordinary differential equation. Because ordinary differential equations are far easier to solve than partial differential equations, converting from a distributed system to a lumped approximation is often essential if the resulting model is to be solved with the resources available.

Another example of a distributed system is an inductor that consists of a wire wound around a core, as shown in Figure 1.5(a). If an electrical excitation is applied across the terminals of the coil, then different values of voltage exist at all points along the coil, characteristic of a distributed system. To develop a lumped circuit whose behavior as calculated at the terminals closely approximates that of the distributed device, we might account for the resistance of the wire by a lumped resistance R and for the inductive effect related to the magnetic field by a single inductance L. The resulting lumped circuit is shown in Figure 1.5(b). Note that in these two examples (though not in all cases), the two elements in the lumped model do not correspond to separate physical parts of the actual system. The stiffness and moment of inertia of the flexible shaft cannot be separated into two physical pieces, nor can the resistance and inductance of the coil.

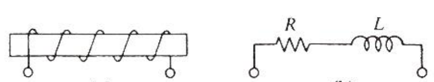

FIGURE 1.5 (a) An inductor. (b) Its lumped approximation.

Continuity of the Time Variable

A second basis for classifying dynamic systems is the independent-variable time. A **continuous system** is one for which the inputs, state variables, and outputs are defined over some continuous range of time (although the signals may have discontinuities in their waveshapes and not be continuous

functions in the mathematical sense). A **discrete-time system** has variables that are determined at distinct instants of time and that are either not defined or not of interest between those instants. Continuous systems are described by differential equations, discrete-time systems by difference equations.

Examples of the variables associated with continuous and discrete-time systems are shown in Figure 1.6. In fact, the discrete-time variable $f_2(kT)$ shown in Figure 1.6(b) is the sequence of numbers obtained by taking the values of the continuous variable $f_1(t)$ at instants separated by T units of time. Hence $f_2(kT) = f_1(t)|_{t=kT}$, where k takes on integer values. In practice, a discrete-time variable may be composed of pulses of very short duration (much less than T) or numbers that reside in digital circuitry. In either case, the variable is assumed to be represented by a sequence of numbers, as indicated by the dots in Figure 1.6(b). There is no requirement that their spacing with respect to time be uniform, although this is often the case.

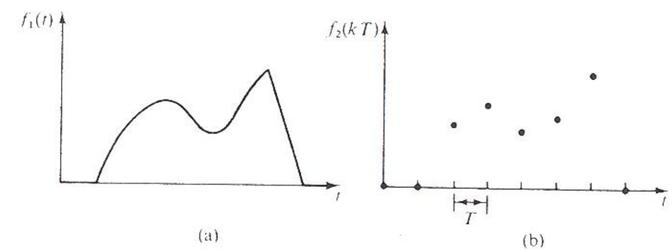

FIGURE 1.6 Sample variables. (a) Continuous. (b) Discrete-time.

A system that contains both discrete-time and continuous subsystems is referred to as a **hybrid system**. Many modern control and communication systems contain a digital computer as a subsystem. In such cases, those variables that are associated with the computer are discrete in time, whereas variables elsewhere in the system are continuous. In such systems, sampling equipment is used to form discrete-time versions of continuous variables, and signal reconstruction equipment is used to generate continuous variables from discrete-time ones.

Quantization of the Dependent Variable

In addition to possible restrictions on the values of the independent-variable time, the system variables may be restricted to certain distinct values. If within some finite range a variable may take on only a finite number of different values, it is said to be **quantized**. A variable that may have any value within some continuous range is **nonquantized**. Quantized variables may arise naturally, or they may be created by rounding or truncating the values of a nonquantized variable to the nearest quantization level.

The variables shown in Figure 1.7(a) and Figure 1.7(b) are both nonquantized, whereas those in the remaining parts of the figure are quantized. Although the variable in Figure 1.7(b) is restricted to the interval $-1 \leq f_b \leq 1$, it is nonquantized because it can take on a continuous set of values within that interval. The variable f_c shown in Figure 1.7(c) is restricted to the two values 0 and 1 and is representative of the signals found in devices that perform logical operations. The variable f_d is restricted to integer values and thus is quantized, although there need not be any other restriction on the magnitude of its values. The variable f_e is a discrete-time variable that is also quantized.

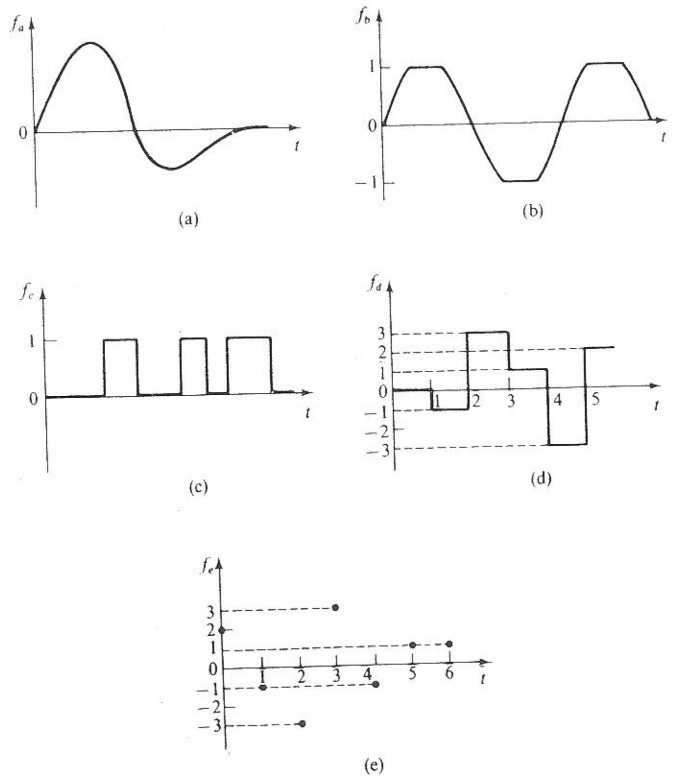

FIGURE 1.7 Sample variables. (a), (b) Nonquantized. (c), (d), (e) Quantized.

Because variables that are both discrete in time and quantized in amplitude, such as f_e, occur within a digital computer, they are referred to as **digital variables**. In contrast, the variable f_a in Figure 1.7(a) is both continuous and nonquantized and is representative of a signal within an analog computer. Hence continuous, nonquantized variables are often referred to as **analog variables**.

Parameter Variation

Systems may be classified according to properties of their parameters as well as of their variables. **Time-varying systems** are systems whose characteristics (such as the value of a mass or a resistance) change with time. Element values may change because of environmental factors such as temperature and radiation. Other examples of time-varying elements include the mass of a rocket, which decreases as fuel is burned, and the inductance of a coil, which increases as an iron slug is inserted into the core. In the differential equations describing time-varying systems, some of the coefficients are functions of time. Delaying the input to a time-varying system affects the size and shape of the response.

For **fixed** or **time-invariant systems**, whose characteristics do not change with time, the system model that describes the relationships between the inputs, state variables, and outputs is independent of time. If such a system is initially at rest, delaying the input by t_d units of time just delays the output by t_d units, without any change in its size or waveshape.

Superposition Property

A system can also be classified in terms of whether it obeys the **superposition property**, which requires that the following two tests be satisfied when the system is initially at rest with zero energy. (1) Multiplying the inputs by any constant α must multiply the outputs by α. (2) The response to several inputs applied simultaneously must be the sum of the individual responses to each input applied separately. **Linear systems** are those that satisfy the conditions for superposition; **nonlinear systems** are those for which superposition does not hold. For a linear system, the coefficients in the differential equations that make up the system model do not depend on the size of the excitation, whereas for nonlinear systems, at least some of the coefficients do. For a linear system initially at rest, multiplying all the inputs by a constant multiplies the output by the same constant. Likewise, replacing all the original inputs by their derivatives (or integrals) gives outputs that are the derivatives (or integrals) of the original outputs.

Nearly all systems are inherently nonlinear if no restrictions at all are placed on the allowable values of the inputs. If the values of the inputs are confined to a sufficiently small range, the originally nonlinear model of a system may often be replaced by a linear model whose response closely approximates that of the nonlinear model. This type of approximation is desirable because analytical solutions to linear models are more easily obtained.

In other applications, the nonlinear nature of an element may be an essential feature of the system and should not be avoided in the model. Examples include mechanical valves or electrical diodes designed to give completely different types of responses for positive and negative inputs. Devices used to produce constant-amplitude oscillations generally have the amplitude of the response determined by nonlinear elements in the system.

To illustrate the difference between linear and nonlinear models, consider a system with the single input $u(t)$ and the single output $y(t)$. If the input and output are related by the differential equation

$$a_1 \frac{dy}{dt} + a_0 y(t) = b_0 u(t)$$

where a_0, a_1, and b_0 may be functions of time but do not depend on $u(t)$ or $y(t)$ in any way, then the system is linear. However, if one or more of the coefficients is a function of the input or output, as in

$$\frac{dy}{dt} + u(t)y(t) = u(t)$$

or

$$\frac{dy}{dt} + |y(t)|y(t) = u(t)$$

then the system is nonlinear.

Analogous Systems

Different systems that are described by equations that are identical except for the use of different symbols are called **analogs**. Consider the four simple systems depicted in Figure 1.8. In the later chapters, we shall examine mechanical, electrical, and hydraulic systems in detail. For the systems in this figure, we shall give at this time only a very brief explanation of the variables, the symbols used, and the equations describing the systems.

Each type of system has two basic variables, both of which are functions of time t. For the translational mechanical system, the variables used in the figure are force $f(t)$ and velocity $v(t)$; for the rotational mechanical system, torque $\tau(t)$ and angular velocity $\omega(t)$; for the electrical system, voltage $e(t)$ and current $i(t)$; and for the hydraulic system, flow rate $q(t)$ and pressure difference $p(t)$. Instead of velocity, angular velocity, and current, the alternative variables of displacement, angular displacement, and charge could be used just as well. For simplicity, we assume for Figure 1.8 that all the elements are linear and that no energy is stored within the system before the input is applied.

In the translational system shown in part (a) of the figure, an external force $f(t)$ is applied to a mass M, whose motion is restrained by a spring K and a friction element B. For the rotational system in part (b), a torque $\tau(t)$ is exerted on a disk whose moment of inertia is J, which is restrained by the

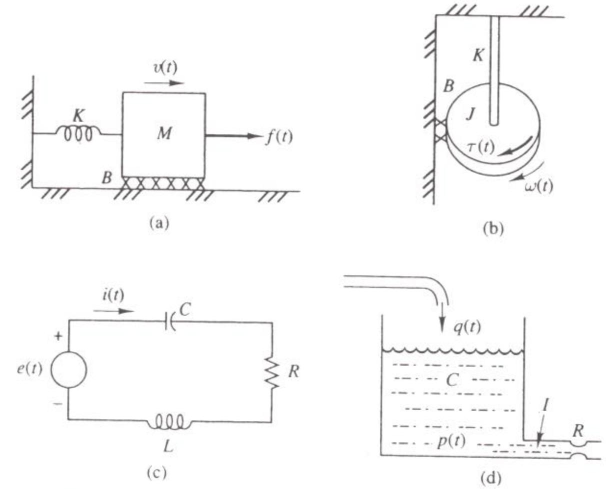

FIGURE 1.8 Analogous systems. (a) Translational mechanical. (b) Rotational mechanical. (c) Electrical. (d) Hydraulic.

torsional bar K and the friction B. The electrical system in part (c) consists of an inductor L, a resistor R, and a capacitor C excited by a voltage source $e(t)$. In part (d), fluid with a known flow rate $q(t)$ enters a vessel that has hydraulic capacitance C. The orifice in the outlet pipe is approximated by the hydraulic resistance R. The inertia effect of the fluid mass can usually be neglected, but in order to complete the analogy, we represent it here by the inertance I.

The following equations, which describe the four systems in Figure 1.8, are identical except for the symbols used. The inputs are on the right sides of the equations, the output variables on the left.

$$M\frac{dv}{dt} + Bv(t) + K\int_0^t v(\lambda)d\lambda = f(t)$$

$$J\frac{d\omega}{dt} + B\omega(t) + K\int_0^t \omega(\lambda)d\lambda = \tau(t)$$

$$L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\lambda)d\lambda = e(t)$$

$$C\frac{dp}{dt} + \frac{1}{R}p(t) + \frac{1}{I}\int_0^t p(\lambda)d\lambda = q(t)$$

If the inputs are identical, then the respective responses will have the same form. The expressions for the power supplied by each of the inputs in the four parts of the figure are $f(t)v(t)$, $\tau(t)\omega(t)$, $e(t)i(t)$, and $p(t)q(t)$, respectively.

Other analogous components could have been included in our discussion. Levers, gears, transformers, and double-headed pistons constitute one set of analogous components. Brakes, diodes, and overflow valves are another. Our treatment of analogies could also be extended to other types of systems, such as thermal, pneumatic, and acoustical systems. We shall not overemphasize analogies, because it is generally better to treat each type of system separately. However, it is important to realize that the modeling and analysis tools that we shall develop are applicable to a very wide range of physical systems.

1.5 SCOPE AND OBJECTIVES

This book is restricted to lumped, continuous, nonquantized systems that can be described by sets of ordinary differential equations. Because well-developed analytical techniques are available for solving linear ordinary differential equations with constant coefficients, we shall emphasize such methods. The majority of our examples will involve systems that are both fixed and linear. A method for approximating a nonlinear system by a fixed linear model will be developed. For time-varying or nonlinear systems that cannot be approximated by a fixed linear model, one can resort to computer solutions.

We list as **objectives** the following things that the reader should be able to do after finishing this book. These objectives are grouped in the two general categories of modeling and solving for the response.

After finishing this book, the reader should be able to do the following for dynamic systems composed of mechanical, electrical, thermal, and hydraulic components:

1. Given a description of the system, construct a simplified version using idealized elements and define a suitable set of variables.
2. Use the appropriate element and interconnection laws to obtain a mathematical model generally consisting of ordinary differential equations.
3. If the model is nonlinear, determine the equilibrium conditions and, where appropriate, obtain a linearized model in terms of incremental variables.
4. Arrange the equations that make up the model in a form suitable for solution. Construct and simplify block diagrams.

When a linear mathematical model has been determined or is given, the reader should be able to do the following:

1. For a first- or second-order system, solve directly for the time-domain response without transforming the functions of time into functions of other variables.

2. For a model of moderate order (of order four or less), use the Laplace transform to
   a. Find the complete time response.
   b. Determine the transfer function and its poles and zeros.
   c. Analyze stability and, where appropriate, evaluate time constants, damping ratios, and undamped natural frequencies.
3. Find from the transformed expression the steady-state response to a constant or sinusoidal input without requiring a general solution.
4. Use block diagrams, root-locus plots, and Bode diagrams as aids in analyzing and designing feedback systems.

In addition to using these analytical methods for obtaining the model's response, the reader should be able to use a computer to obtain the time response of a linear or nonlinear model in numerical form.

This book investigates all the foregoing procedures in detail. We illustrate the basic modeling approaches and introduce state variables first in the context of mechanical systems and then for electrical systems. There are several chapters about the analytical solution of mathematical models. In Chapter 6, we review and apply the classical procedures usually introduced in basic mathematics courses for solving the equations describing linear systems. Then we develop and use analytical techniques based on transforming functions of time into functions of a different variable.

The material in Chapter 9 and in much of Chapter 15 can be presented any time after Chapter 5, depending on the organization of a particular course of study. The first of these chapters gives procedures for approximating a nonlinear model by a linear one. The second considers the computer solution of both linear and nonlinear models, with an emphasis on the widely used MATLAB and ACSL programs.

Chapters 10 through 12 extend the procedures of the earlier chapters to electromechanical, thermal, and hydraulic systems. Chapters 13 and 14 present block and simulation diagrams and then introduce the modeling and design of feedback systems.