MECE-606 Systems Modeling Project
April 19-26, 2016, Rochester, NY, USA

Coffee Cup Thermal Model

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ABSTRACT

The thermal dynamics behavior of two different types of the commercial coffee cups is modeled based on the output data and the size, thermal conductivity, specific heat and density of the water and cups. The convection coefficient value of h is measured estimating the better fit with the output data. The four different models are used in this lab to compare against each other.

INTRODUCTION

The coffee cups are set up as below.

Figure 1. The setup of coffee cups

The fluid and material properties for water, ceramic and Styrofoam can be seen as in table 1.

Parameter	Density (ρ)	Thermal Conductivity (k)	Specific Heat (C_p)
Units	*kg/m^3*	*W/(m^{o}K)*	*J/(kg^{o}K)*
Water	1000	0.609	4187
Ceramic	2400	1.5	1070
Styrofoam	100	0.033	1300

Table 1. The fluid and material properties

The same amount of water quantity is used in both cups. The diameter of ceramic cup is 8.5cm, the thickness 0.4 cm, and the height 12cm while for Styrofoam, the cup diameter is 7.65, average of the top and bottom diameters, the thickness is 0.3cm, and height is 12cm. The thermocouples are placed inside the center of the cup with one-inch-thick Styrofoam plates on the top and bottom to make 1-D radial thermal project. The 3-D case will require much more time to complete considering heat conduction and convection.

ANALYSIS

There are four different cases being taken consideration to model the system. The first case is 1st order with 1 mass. The second case is 2nd order "half-cup" model lumping cup and water. Since thermocouples are placed at the center, the other half of the cup will be ignored for all cases. The third case is 2nd order radial model with 2 states: temperature of the water and the cup. The fourth case is higher order non-radial model multi lumps systems meeting the requirement for Biot number condition. For all cases, the water is treated as one solid mass, and the conduction of water is ignored as well since too much mixing is going on in the water. If conduction of water is taken into account, the modeling result will not get good fit.

Finding the Biot condition is the first thing to do in thermal problems. In this project, Biot number is:

$$Biot = \frac{L_c h}{k} \text{ where } L_c = \frac{V}{A}$$

The volume used is the volume of the hollow cylinder and surface area is just the area in 1-D but not the top and bottom areas.

$$Biot = .3811 * \frac{10}{1.5} = 2.541 * 10^{-2} \quad \text{for}$$

ceramic case.

Since Biot < 0.1, the discretization is not required.

$Biot = 2.882 * 10^{-3} * \frac{10}{.003} = .873$ for the Styrofoam case.

Since Biot >.1, the Biot number should be divided by 9. The discretization is required.

For the first case out of four different cases, $\frac{du}{dt} = \frac{T_f - T}{R_{conv}}$

$$T' = \frac{hA}{pvC_p} * T - \frac{hA}{pvC_p} * T_0$$

The state space representation is as follow.

$$T' = \left[\frac{hA}{pvC_p}\right] T + \left[-\frac{hA}{pvC_p}\right] T_0 \text{\_\_\_\_\_\_(1)}$$

The second case differential equation being used is:

$$M_w C_w T_w' = q_{in} - q_{w_c}$$

$$C_w T_w' = -\frac{T_w - T_c}{R_w} \text{\_\_\_\_\_\_\_\_\_(2)}$$

$$M_c C_c T_c' = q_{Wc} - q_{air}$$

$$C_c T_b' = \frac{T_a - T_b}{R_W} - \frac{T_b - T_0}{R_c} \text{\_\_\_\_\_\_(3)}$$

From the equation 2 and 3, the state space is

$$\begin{bmatrix} T_W' \\ T_c' \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_W R_W} & \frac{1}{C_W R_W} \\ \frac{1}{C_c R_W} & -\frac{1}{C_c}\left(\frac{1}{R_W} + \frac{1}{R_c}\right) \end{bmatrix} \begin{bmatrix} T_W \\ T_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_c C_C} \end{bmatrix} [T_0]$$

The third case uses the second case state space representation structure. The only difference between third and second case is that third one considers the radial model. Logarithmic equation, $R = \frac{\ln\left(\frac{r_0}{r_i}\right)}{2*pi*K_t*L}$ is used.

The fourth case of ceramic is the same as the second case since discretization is not needed for Biot number being less than 0.1. The fourth case of Styrofoam has 9 multi lumps. The matrix used for state space representation has 10 rows and 10 columns. The structure is similar to the case 2 and 3. The equations being used are

$$R_{con} = \frac{1}{hA}$$

$$R_{cond} = \frac{t_{sty}}{BK_{sty}*A}$$

$$T_W' = \frac{1}{C_W}[1/R_{condW}(T_{c9} - T_{cW})]$$

$$T_{c1}' = \frac{1}{C_n}\left[\frac{1}{R_{conv}}(T_0 - T_{C1}) - \frac{1}{R_{conv}}(T_{c1} - T_{c2})\right]$$

The T_{c1}' will go up to T_{c9}' in similar structure.

SIMULATION

The four cases of ceramic are simulated as seen in figure 2.

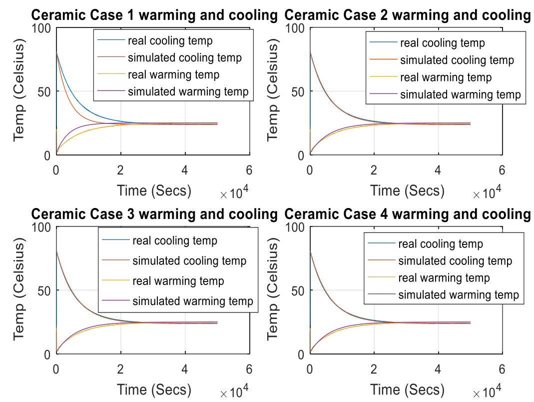

Figure 2. The simulation of Ceramic in four cases

It can be clearly seen from figure 2 that case 1 simulation is a bit off from the real output temperature data. The case 4 should be better than any other cases since multi lumps are considered.

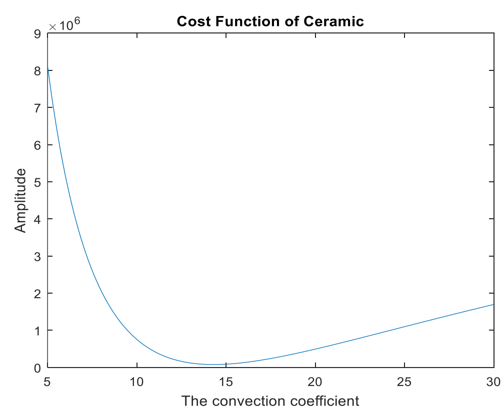

Figure 3. The cost function of ceramic case 4

The convection coefficient value is estimated using the 1D sweep with 5 to 30 range as in figure 3. The minimum cost value is chosen as the convection coefficient value.

The four cases of the Styrofoam are simulated as in figure 4.

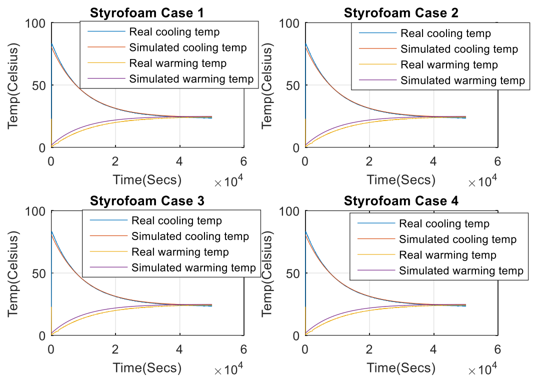

Figure 4. The simulation of Styrofoam in four cases

The H values in table 2 are obtained using the cost function as in figure 3. The ceramic h values as bigger than Styrofoam h values in general. The rate of change of heat in Ceramic is faster than that of Styrofoam. That's the reason why a person will feel the cup's made of Ceramic the heat hotter than that of Styrofoam assuming the heat inside both of them is the same. The case 4 Ceramic is forced to be 2 multi lumps in the simulation to see if the model is different from others.

J/kgK	Case 1	Case 2	Case 3	Case 4
H for Styrofoam	8.12	11.36	10.28	18.58
H for Ceramic	14.84	15.52	14.04	14.28

Table 2. H values

RMS values are obtained as in table 3 to compare which models fit better. The case 4 ceramic case 4 RMS values should be the same case 2. However, in this lab, the case 4 ceramic Biot is divided by 2 to see the better changes. With simulation RMS data from table 3, the case 2 and 4 fit is not much different.

	Case 1	Case 2	Case 3	Case 4
Styrofoam - Cool	1.8232	1.8185	1.8186	1.8192
Styrofoam - Warm	1.8543	1.722	1.8487	1.8453
Ceramic- Cool	4.3621	1.2607	1.2605	1.2622
Ceramic - Warm	2.5315	1.105	1.1036	1.1048

Table 3. RMS values

From RMS value, it can be claimed that as the simulation improves, the values of RMS decrease. This is because for cases 2,3 and 4, the model is better than for case 1. We also can see that for the validation data set of the Styrofoam the RMS values are in general greater than for the cooling data. But, for ceramics, the opposite occurs.

The radial in both cups have fit similar to case 2 and 4. The H value in ceramic is in general greater than that in Styrofoam. The higher order model doesn't necessary mean that it is better fit.

CONCLUSION

The lab objective was achieved using the thermal differential equations, heat conduction, heat convection, finding the approximate value of heat convection coefficient through trial and error, and plotting those in the warming data set as validation data set. The validation data sets are similar to the real output data as seen in figure 2 and 4 for both types of cup. To conclude, the lab is successfully completed utilizing Matlab state space modeling and Simulink.