
MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

Second Edition

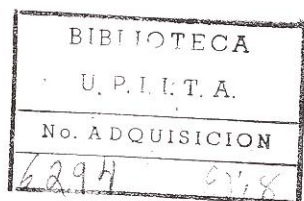
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To my wife Margo
CMC

To my mother
Elizabeth Dean Frederick
and to the memory of my father
Charles Elder Frederick

DKF

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► Preface

The primary purpose of this edition remains the same as in the first edition: to provide an introductory treatment of dynamic systems that is suitable for all engineering students regardless of discipline. We have, however, made major changes as a result of experiences with many thousands of our students, comments from dozens of professors around the country, and the increasing educational use of standard computer packages. We have also arranged the contents to provide more flexibility in the selection and ordering of material.

The book can be adapted to several types of courses. One such use is for students who need a detailed treatment of modeling mechanical and electrical systems and of obtaining time-domain and Laplace-transform solutions before proceeding to more advanced concepts. For example, this text is the basis for a one-semester course at Rensselaer Polytechnic Institute taken by every engineering student in his or her sophomore or junior year. The course serves as a foundation for such subsequent courses as circuits and electronics, vehicular dynamics, chemical process control, linear systems, feedback systems, nuclear reactor control, and bioccontrol systems.

The book also covers such general topics as transfer functions, the linearization of nonlinear models, computer solutions, block diagrams, feedback systems, state variables, and matrix methods. Hence it is very suitable for a general dynamic systems course for students who have completed a disciplinary course such as machine dynamics, electrical circuits, or chemical process dynamics.

This text can also be used for students with significant modeling and analysis experience who wish to emphasize computer techniques and feedback control systems. Topics include computer solutions for both linear and

nonlinear models, as well as root-locus diagrams, Bode plots, block diagrams, and operational amplifiers. We explain some of the practical design criteria for control systems and illustrate the use of analytical and computer methods to meet those criteria. Finally, the book can provide a general introduction to dynamic systems for students in broad-based engineering programs or in programs (such as biomedical and materials engineering) that may have limited time for this subject.

We assume that the reader has had differential and integral calculus and basic college physics, including mechanics and electrical phenomena. A course in differential equations is recommended at least concurrently. We have been careful to present the mathematical results precisely (although without the rigorous proofs required for a mathematics book), so that the concepts learned will remain valid in subsequent courses. For example, the impulse has been treated in a manner that is consistent with distribution theory but is no more difficult to grasp than the usual approach taken in introductory engineering books.

Approach

The book reflects the approach we have used for over two decades in teaching basic courses in dynamic systems. Whether for a particular discipline or for a general engineering course, we have found it valuable to include systems from at least two disciplines in some depth. This illustrates the commonality of the modeling and analysis techniques, encourages students to avoid compartmentalizing their knowledge, and prepares them to work on projects as part of an interdisciplinary team.

Mechanical systems are examined first because of their greater familiarity. However, there are also chapters on electrical, electromechanical, thermal, and hydraulic systems. Each type of system is modeled in terms of its own fundamental laws and nomenclature.

We develop linear models in terms of state-variable equations, input-output differential equations, and transfer functions. We introduce both time-domain solutions and Laplace transforms fairly early. We give considerable emphasis to such concepts as time constants, damping ratios, transfer functions, poles and zeros, mode functions, and frequency-response functions. Although the use of matrices in undergraduate courses is increasing, we recognize that time limitations and possible gaps in the student's background might preclude their use. Accordingly, we have confined the analytical solution of models in matrix form to optional sections at the end of Chapters 3, 6, and 8.

We have always treated both linear and nonlinear models, although the book allows nonlinear systems to be deemphasized if desired. Students should realize that inherent nonlinearities generally cannot be ignored in the formulation of an accurate model. Techniques are introduced for approximating nonlinear systems by linear models. We also show that it is not

difficult to write the equations needed for computer solutions for nonlinear models.

We believe that obtaining and interpreting computer solutions of both linear and nonlinear models constitute an important part of any course on dynamic systems. We employ MATLAB and ACSL, two computer packages that are widely used in both educational and industrial settings, although other equivalent packages can be used. Because individual instructors introduce numerical solutions at different points in a course, we have put the explanation of these programs in Chapter 15. However, some of the sections can be used as early as Chapter 3.

Organization

The majority of the material can be covered in a one-semester course, but the book can also be used as the basis for a two-quarter or year-long course. A number of chapters (including Chapters 10, 11, 12, and 14) and a number of individual sections (including Sections 3.3, 5.7, 6.5, 6.6, 8.6, and 8.7) can be omitted or abbreviated without any loss of continuity. As we have pointed out, the computer packages described in Chapter 15 can be used throughout the book. The remaining 14 chapters can be grouped into the following four blocks.

1. Modeling of mechanical and electrical systems: Chapters 1 through 5
2. Analytical solutions for linear systems and linearizing nonlinear models: Chapters 6 through 9
3. Modeling other types of systems: Chapters 10 through 12
4. Block diagrams, feedback systems, and design tools: Chapters 13 and 14

A core sequence for students without previous courses in dynamic systems might include Chapters 1 through 9 plus Chapter 13. Readers with prior modeling experience might find parts of the first five chapters to be review, although they should be sure to understand the state-variable and input-output models in Chapter 3. As an illustration of how to extend the modeling and analysis techniques to other types of systems, we suggest including one or more of Chapters 10, 11, and 12.

Chapters 13 and 14 can be used as an introduction to the modeling, analysis, and design of feedback control systems. In Chapter 14, we present root-locus and Bode diagrams. Rather than provide lengthy descriptions of how to draw such diagrams by hand, we emphasize interpreting computer-generated plots and using them in an iterative process to meet some practical design criteria.

The following table of prerequisites should be helpful in selecting material from the later chapters.

Topic	Chapter	Prerequisite Material
Linearization	9	Chapter 4 ¹
Electromechanical Systems	10	Chapters 4–8
Thermal Systems	11	Chapters 8, 9
Hydraulic Systems	12	Chapters 8, 9
Block Diagrams	13	Chapter 8
Feedback Modeling and Design	14	Chapter 13
Computer Analysis	15.1, 15.2	Chapter 8 ²
	15.3, 15.4	Chapter 3 ³

¹ Section 9.3 also requires Chapter 5.

² Parts of Sections 15.1 and 15.2 require Chapter 14.

³ Parts of Sections 15.3 and 15.4 require Chapter 4.

New to this Edition

Many of the major changes in the content and organization of this edition resulted from a comprehensive series of surveys and in-depth reviews. These changes include the following.

1. Electrical systems and Laplace transform techniques are introduced earlier. Electrical systems now appear in the initial block of modeling chapters. Transform methods are treated immediately after the discussion of time-domain solutions.
2. Material on individual transistor and diode circuits has been replaced by optional sections on operational amplifiers and impedances.
3. We have merged the treatment of simulation and block diagrams and have extended the coverage of feedback systems and frequency-response functions.
4. We have also expanded the treatment of matrix methods and have regrouped it into several optional sections.
5. Instead of including the more abstract discussion of simulation and numerical methods contained in the first edition, we have shown how to apply specific computer packages to linear and nonlinear models.

In addition, we have provided increased motivation and guidance for the reader.

1. Summaries have been added to each chapter.
2. The majority of the examples have been modified or replaced, and the number of examples has been increased to nearly 200. Explicit statements about the points to be illustrated have been added before the examples. Where appropriate, comments about the significance of the results follow the examples.

3. The end-of-chapter problems have been heavily revised, and a large number of new problems have been added. There are now well over 400 problems.
4. The answers to selected problems are contained in an appendix at the back of the book.

Solutions Manual

A solutions manual containing detailed solutions to all the problems in the text can be obtained by contacting Houghton Mifflin Company. At the beginning of each chapter in the manual, a table lists the following information for each of the major topics: the section of the text in which it is covered, the examples in which it is illustrated, and the problems that relate to it.

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C.M.C.
D.K.F.