

# MECE-606 Systems Modeling

## Computer Project #2: *Single Mass Pendulum*

**Goal:** Develop a dynamic model of the single pendulum and estimate the moment of inertia, location (X) of the point mass, and the damping coefficient. First, assume a point mass and neglect the rod, and then account for the rod inertia as well as the mass. Apply the developed linear and nonlinear models to the validation data sets.

**Measurement:** Angular position from rotary encoder (2500 CPR quadrature U.S. Digital encoder).

**System:** A 138.9 gram mass is connected to a 92.7-cm steel rod (diameter = 0.635-cm, density  $\rho = 7850\text{-kg/m}^3$ ) an unknown distance “X” from the rotation point. The pendulum rotates on an aluminum shaft through two pillow block ball bearings. The modeling data set initial condition is  $\theta = +45^\circ$  and the validation data sets are  $\theta = +30^\circ$  and  $\theta = +90^\circ$ .

**Considerations:** Model and simulate 4 different scenarios: (i) assume small angles and viscous friction (linear model); (ii) do not assume small angles and use viscous friction; (iii) do not assume small angles and use the turbulent flow friction model; (iv) do not assume small angles and use the combined Coulomb/viscous friction model. For the turbulent flow model use the following functional form for the damping force  $F_d = \text{sgn}(\dot{\theta})b\dot{\theta}^2$  and for the case where there is a combined Coulomb and viscous friction use  $F_d = \text{sgn}(\dot{\theta})[K_g|\dot{\theta}| + K_o]$  where  $K_o$  is the Coulomb friction value and  $K_g$  is the coefficient of viscous friction.

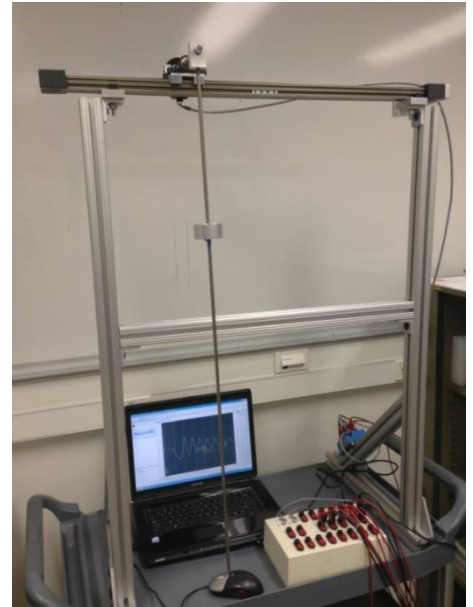


Figure 1 - Single Pendulum System

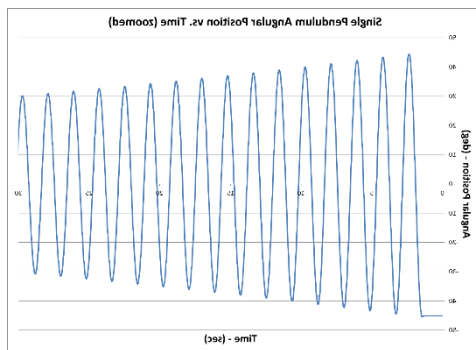
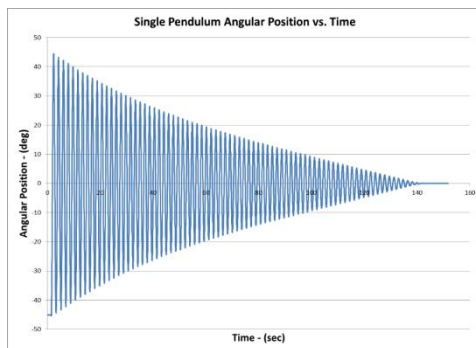


Figure 2 - Angular Position vs. Time

For the linear model case (i) use the concept of logarithmic decrement and the natural frequency to identify the model parameters. From the estimated inertia ( $J$ ) determine the location of the mass ( $l$ ) assuming a point mass, and (2) accounting for the mass of the rod. (Hint:  $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ ) For each of the next three cases (ii-iv) you will need to identify the damping coefficient ( $b$ ). Set up a 1D parameter sweep based on a range of possible damping coefficients. Simulate the model for each and compare to the data by setting up a cost function. Plot the cost function versus damping to identify the optimum. Simulate this optimum versus the data for your final model. For the combined Coulomb/viscous damping case you will have to set up a 2D parameter sweep. For every case, how does the model fit compare versus the validation data sets?

Discuss how close the simulations are. Can inertia of the rod/mass assembly be treated as a point mass? How accurate is the linear damping representation? Are the nonlinear models better? Etc.

**Deliverable:** A concise three page (max.) report on the modeling approach with full derivation of all equations from first principles (i.e.  $F=ma$ ) through final model. A discussion of the final results related to quality of the model fit of the data is critical including a quantification of the error (RMS, etc.). The number of plots should be kept to a minimum but be of high quality and description (legends, captions). A portion of the grade is reserved for the quality of the written report. Please submit all of your Matlab/Simulink code separately and consolidate it to as few pages as possible.

**Due Date:** Two weeks after the assigned date.