MECE-606 Systems Modeling Project
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DC Motor & Inertia Modeling

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ABSTRACT

The DC motor‘s dynamic model behavior is observed through different scenarios such as second order and first order with and without connecting to the shaft/load. The first order case is when armature inductance is assumed zero. When there is no load connected to the motor, the values of motor data sheet are used. When there is a load, the inertia and damping values must be estimated optimizing the cost function for just speed or both speed and current.

INTRODUCTION

The armature controlled DC motor schematic can be seen as below.

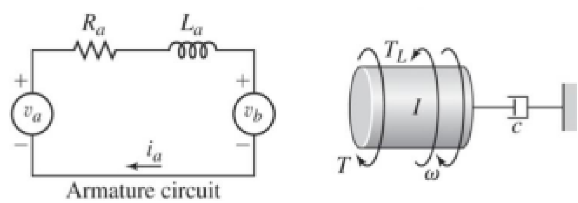


Figure 1. Armature controlled DC motor schematic

From the figure 1, Using Kirchhoff's Law

$$V_a - R_a i_a - L_a \frac{d}{dt}(i_a) - K_B w = 0$$

$$I_a(s) = \frac{1}{L_a s + R_a}(V_a(s) - K_B \Omega(s)) \_\_(1)$$

Using Newton's 2nd Law,

$$J\frac{d}{dt}w = T - Dw - T_L$$

$$= K_T i_a - Dw - T_L$$

where D is the viscous damping

$$\Omega(s) = \frac{1}{J_s + D}(K_t I_a(s) - T_L(s)) \_\_(2)$$

The motor transfer functions are

$$\frac{I_a}{V_a}(s) = \frac{J_s + D}{L_a J_s^2 + (R_a J + DL_a)s + DR_a + K_B K_T} \_\_\_\_\_\_(3)$$

$$\frac{I_a}{T_L}(s) = \frac{K_b}{L_a J_s^2 + (R_a J + DL_a)s + DR_a + K_B K_T} \_\_\_\_\_(4)$$

$$\frac{\Omega}{V_a}(s) = \left(\frac{K_T}{L_a J_s^2 + (R_a J + DL_a)s + DR_a + K_B K_T}\right) \_\_\_\_ (5)$$

$$\frac{\Omega}{T_L}(s) = \left(\frac{L_a s + R_a}{L_a J_s^2 + (R_a J + DL_a)s + DR_a + K_B K_T}\right) \_\_\_\_(6)$$

The equation 1 and 2 can be used to implement the Simulink model. For this lab system, $T_L = 0, J_t = J_{motor} + J_{shaft}$ and $D_t = D_{motor} + D_{bearings}$.

ANALYSIS`

The differential equations are derived below.

$$.\frac{d}{dt}(i_a) = \frac{1}{L_a}(v_a - R_a i_a - K_B w)$$

$$\frac{d}{dt}(w) = \frac{1}{J}(K_a i_a - Dw - T_L),\ i_a \text{and } w \text{ are state variables.}$$

The state space representation as below is used in Simulink and Matlab to find the viscous damping and J, the inertia. $T_L = 0$ in this project.

$$\begin{bmatrix} i_a' \\ w' \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_B}{La} \\ \frac{K_T}{J_t} & -\frac{D}{J_t} \end{bmatrix} \begin{bmatrix} i_a' \\ w' \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J_t} \end{bmatrix} \begin{bmatrix} V_a \\ T_L \end{bmatrix}$$

The Simulink being derived from equation 1 and 2 is implemented to find the cost function since it takes less time than the state space representation to simulate. When using Matlab optimization toolbox 'fminsearch' command, the state space model is used for its convenience.

SIMULATION

The simulation for the second order case optimizing only the speed can be seen in seen in figure 1 and 2

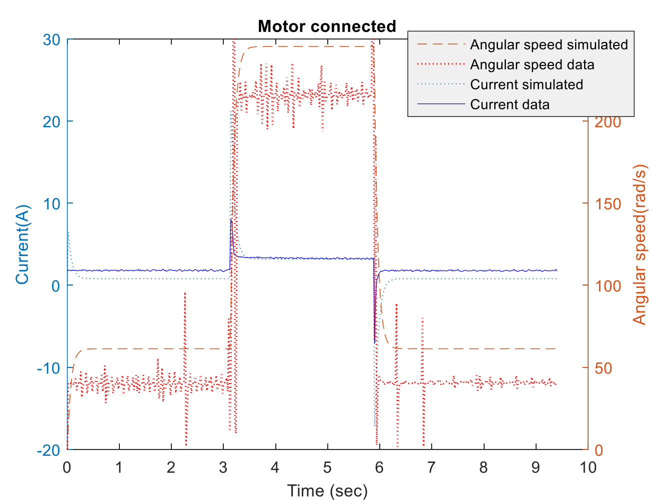


Figure 2. The simulation of the second order case

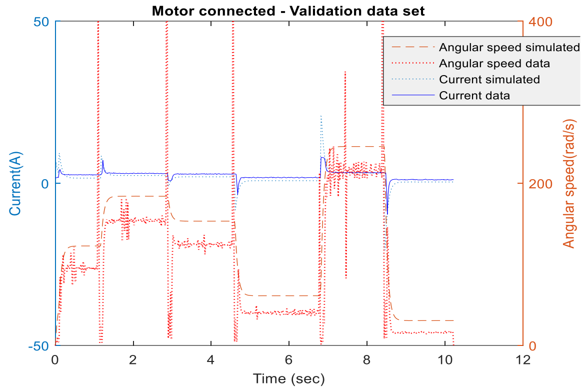


Figure 3. The simulation of the 2nd order validate case

Optimizing both speed and current is as below.

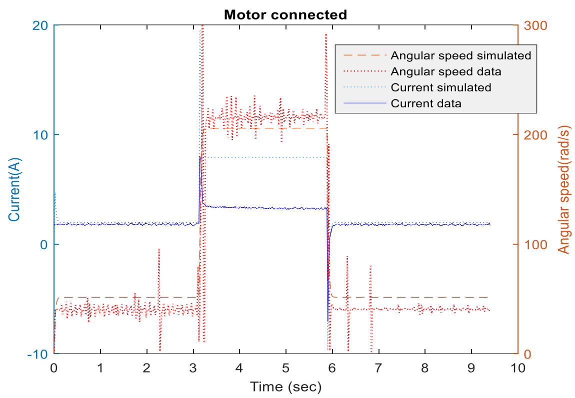


Figure 4. The simulation of the second order case

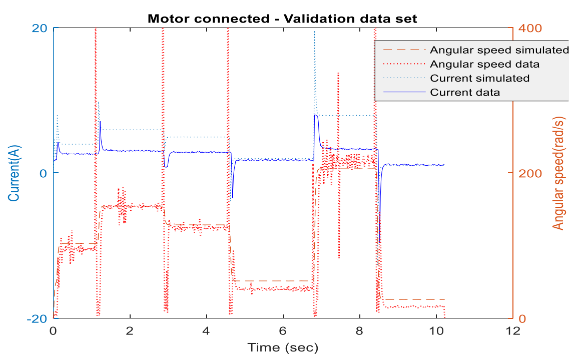


Figure 5. The simulation of the 2nd order validate case

The simulation can have better fit by changing the Ra parameter since Ra value seems to be a bit off for the lab. 2D parameter sweep is used to find the cost function for inertia and viscous damping.

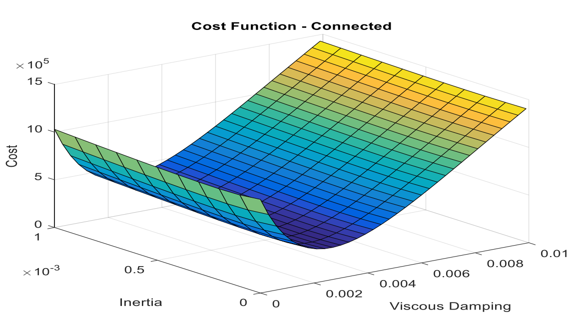


Figure 6. The cost function of optimizing speed only

The graphs of the other cost functions optimizing both speed and current, and functions using fminsearch for optimizing both cases are similar as the above figures.

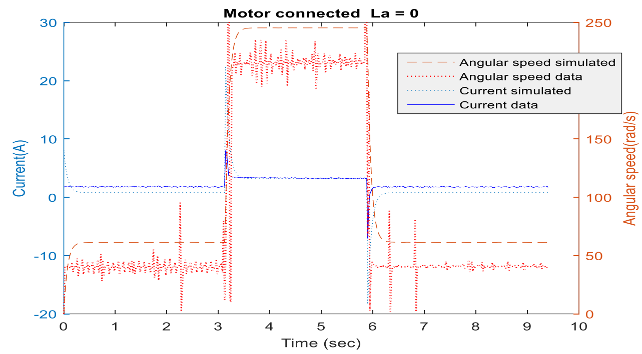


Figure 7. Simulation of motor connected La=0

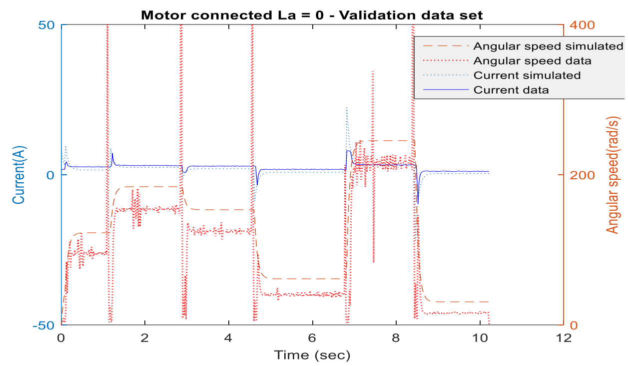


Figure 8.Simulation of first order motor validation data set

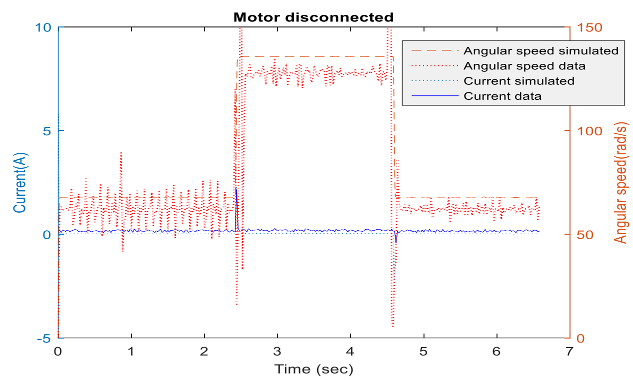


Figure 9. Simulation of motor disconnected

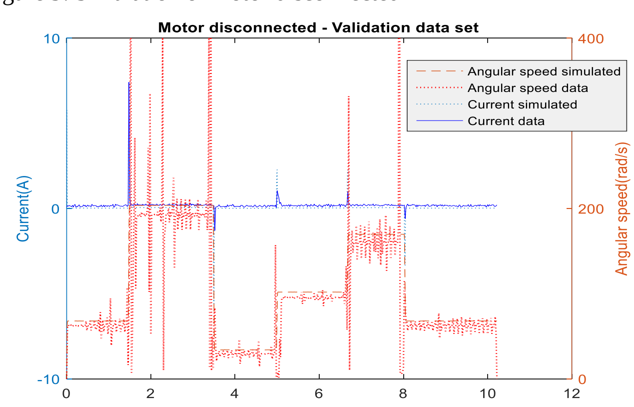


Figure 10. Simulation of motor disconnected validation

From figures 6 to 9, it can be seen that the validation data fit along with the output of angular speed and current. From the real output data and simulated output data, root mean square RMS can be evaluated to determine which model or which optimization fits better.

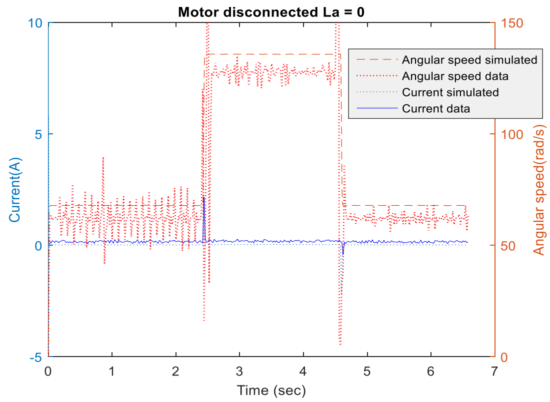


Figure 11. Simulation of first order motor disconnected

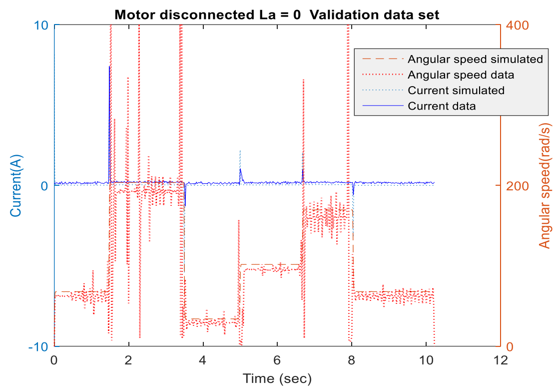
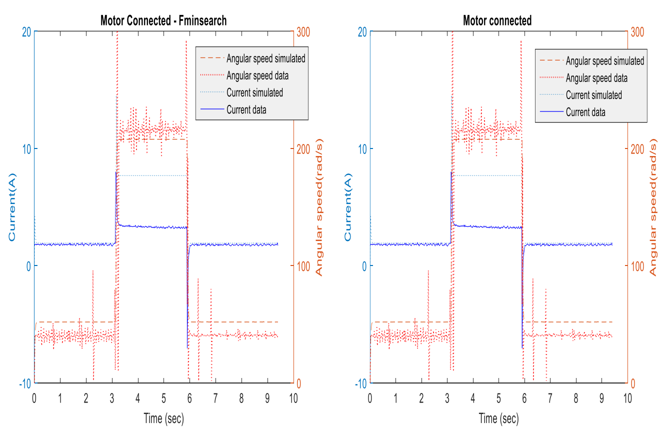


Figure 12. Simulation of motor disconnected validation



Figures 13. Simulation of motor connected using fminsearch for optimizing both speed and current, and speed only.

	J Inertia	D viscous Damping
Connected: speed only	0.000647	9.17E-04
Connected: both speed and current	2.47E-04	0.002717
disconnected, speed only	4.70E-05	1.70E-05
fminsearch, speed only	0.002593	1.52E-04
fminsearch both speed and current	0.0025931	1.52E-04

Table 1. J and D values for connected, disconnected and fminsearch cases

	current	current_val	speed	speed_val
connected, speed only	1.724	2.064	34.773	64.156
speed and current	2.564	2.651	34.737	64.1375
connected,La=0	1.7513	2.1068	26.1038	60.638
disconnected,La=0	0.2015	2.064	26.9186	64.1562
disconnected	0.2016	0.3595	26.918	58.9392

Table 2. RMS values for current and speed with validation sets

fminsearch	current	speed
speed and current	2.4942	26.2779
speed only	2.4942	26.2779

Table 3. RMS values for fminsearch

FOPDT	speed	speed_val
load	26.4709	60.6882
no load	27.8811	61.2259

Table 4. RMS values for FOPDT with load and no load

The RMS of speed being greater than that of the current can be clearly observed in table 2. From table 2 and 3 comparison, the user defined cost function performs better than the Matlab toolbox. The first order case fits better than second order in general. Optimizing speed only is better than optimizing both speed and current due to current spikes.

For First Order Plus Dead Time (FOPDT), the system gain K is obtained diving the steady state output by the steady state input. The time constant T is the time it takes to reach from step response to 63% of its final value. The no load case K is 13.54 and the time constant is .15 secs. The load case K is 5.5128 and time constant is .03 secs.

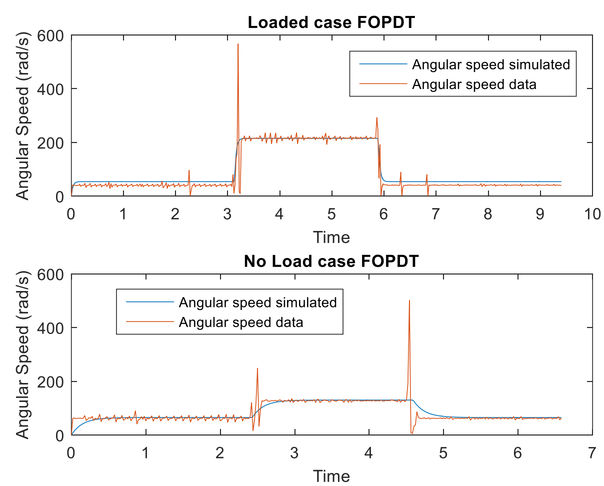


Figure 14. The loaded case of FOPDT simulation

CONCLUSION

The lab objective was accomplished applying Kirchhoff's law, Newton's 2nd law, using differential equations, finding the approximate values of viscous damping, and inertia through trial and error, and confirm the result with the validation data set simulation. To conclude, the lab is successfully completed utilizing Matlab state space modeling and Simulink.