MECE-606 Systems Modeling Project
March 3-10, 2016, Rochester, NY, USA

Single Mass Pendulum

Zeyar Win
Department of Electrical Engineering
Rochester Institute of Technology
Rochester, NY 14623
Email. Zxw6805@rit.edu

Guilherme Mollica
Department of Mechanical Engineering
Rochester Institute of Technology
Rochester, NY 14623
Email. ghg3352@rit.edu

ABSTRACT

The period, natural frequency and characteristics of pendulum motion can be calculated when the mass of the pendulum, initial position, displacement, and damping coefficient values are known. In this lab project, the damping coefficient, period, natural frequency and other characteristics of pendulum are unknown but only output data are provided. System identification is needed to solve the damping coefficient of the pendulum evaluating 4 cases with and without the mass of a rod consideration. 45 degrees are used as training data set whereas 30 and 90 degrees initial conditions are used as validation data set.

INTRODUCTION

The free body diagram of the pendulum can be seen as below.

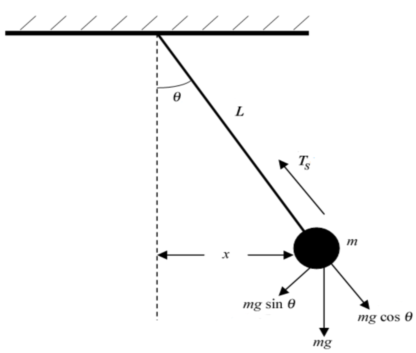

Figure 1. Free body diagram of a pendulum

From free body diagram,

$$-T_d - Lmgsin\theta = mL^2\theta''$$

$$-b\theta' - Lmgsin\theta = mL^2\theta''$$

The equations of motion without and with the mass of rod are as below respectively.

$$\theta'' + \frac{b}{mL^2}\theta' + \frac{g}{L}\theta = 0 \text{\_\_\_\_\_(1)}$$

$$\theta'' + \frac{b}{J_0}\theta' + \frac{mgL + \frac{mrgL_r}{2}}{J_0}\theta = 0 \text{\_\_\_\_\_(2)}$$

where $J_0 = J_{RG} + m_r\left(\frac{L_r}{2}\right)^2 + mL^2$

$$\partial = \frac{1}{n}ln\left(\frac{B_n}{B_{n+1}}\right) \text{\_\_\_\_\_\_\_\_\_\_\_(3)}$$

$$s^2 + 2\zeta w_n s + w_n^2 = 0 \text{\_\_\_\_\_(4)}$$

The equation 3 logarithmic decrement and equation 4 are used to find the natural frequency, damping frequency, period and the length of point mass from a fixed point. After finding period and location of point of mass, damping coefficient value is calculated considering 4 different scenarios.

ANALYSIS

The state representation as below is used in Simulink and Matlab to find the damping coefficient.

$$\theta' = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & -b/mL^{\wedge}2 \end{bmatrix}\theta \text{\_\_\_\_\_\_\_\_\_(5)}$$

With the mass of the rod,

$$\theta' = \begin{bmatrix} 0 & 1 \\ -\frac{mgL + \frac{mrgL_r}{2}}{J_0} & -b/J_0 \end{bmatrix}\theta \text{\_\_\_\_\_(6)}$$

The coefficient b is the viscous friction with small angles assumption linear model in case 1.The value of b is viscous friction with sinθ in case 2. In case 3, the equation Fd damping force is turbulent flow friction, sign$(\theta)b\theta'^2$. In case 4, the Fd is combined Coulomb and viscous friction:

$F_d = sgn(\theta')[K_g|\theta'| + K_0]$. With two unknown values in case 4, the simulation will take more time to estimate the damping factor.

SIMULATION

The 4 cases of massless rod are simulated.

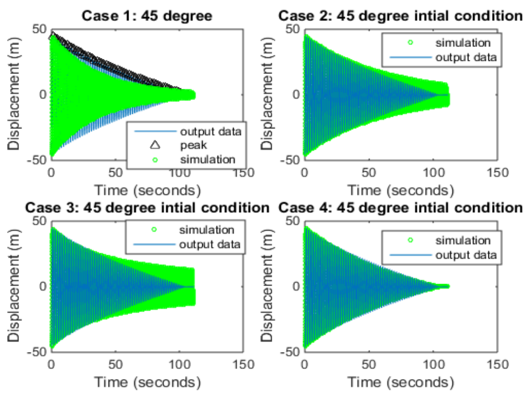

Fig 2.The 45 degree initial condition in 4 cases

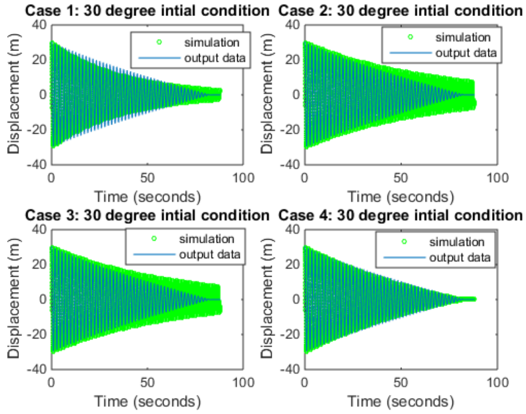

Fig 3 .The 30 degree initial condition in 4 cases

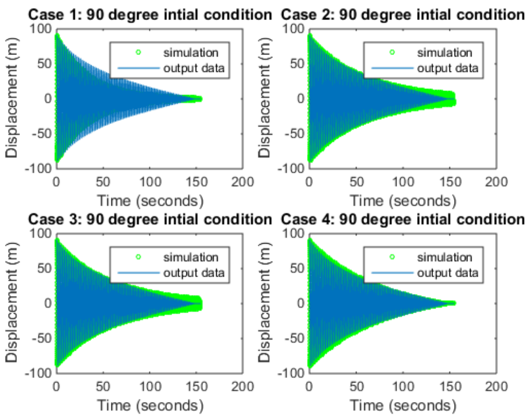

Fig 4.The 90 degree initial condition in 4 cases

It is intriguing to see that case 4 in 90 degree is worse than any other degree cases. Case 2 30 degree has bigger error value when time goes on compared to 90 degree. Small angle approximation is better than sin theta assumption. The one with mass cases fit less than with massless since location of point mass increases by 20 cm due to inertia of the rod.

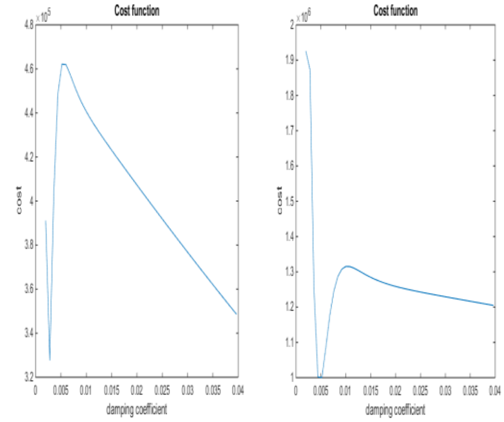

Fig 5.The cost function of case 2 without and with the mass of the rod

The damping constant value is determined using cost function in the Matlab loop. The minimum value is chosen for damping constant. The same principle is applied to 2D arrays as in figure 6.

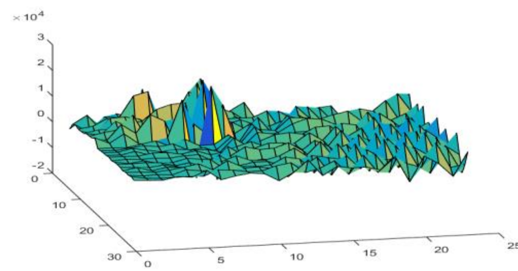

Fig 6. The cost function of number 4 case

The coefficient damping value obtained from 4 different cases in massless rod condition can be seen in the table 1.

case1	case2	case3	case4(ko/kg)
0.0049	0.0028	0.0042	.0019,.002

Table 1. Massless rod damping value

The case with rod mass can be observed as in table 2.

	case1	case2	case3	case4(ko/kg)
damping	0.0105	0.0042	0.0015	.0018 , .0026
Jo	0.1915	0.1915	0.1915	0.1915

Table 2. Mass rod damping value

	case1	case2	case3	case4
RMS45	21.1172	16.6646	16.4339	14.7788
RMS30	3.3352	11.4099	11.4099	11.8229
RMS90	35.0775	21.1813	21.1813	35.1459

Table 3: The RMS value for massless rod cases

	case1	case2	case3	case4
RMS45	20.1193	17.7439	32.2855	23.3087
RMS30	20.0094	14.2035	20.9054	16.9188
RMS90	35.5728	36.3516	32.0618	46.9596

Table 4: The RMS value for mass of the rod cases

The RMS Matlab function is used to find the root mean square error of the 24 different situations. Those data can be seen in table 3 and 4.

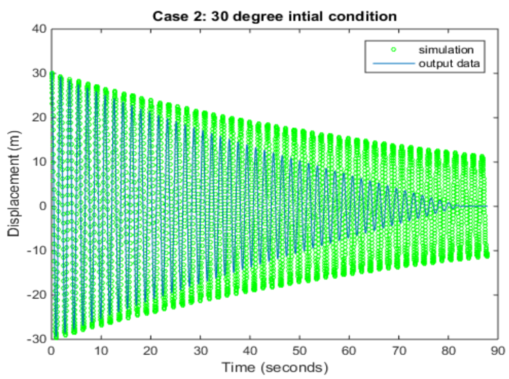

Figure 7. The simulation of pendulum considering mass of the rod with 30’ in case 2

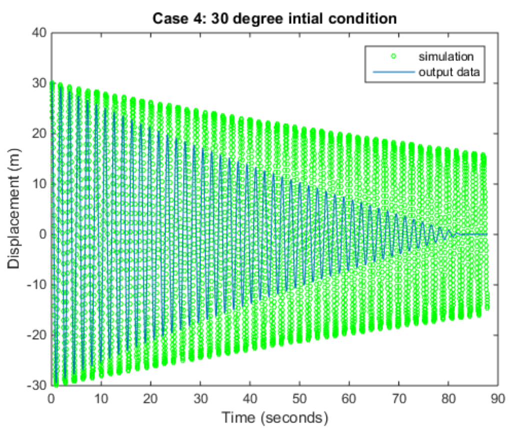

Figure 8. The simulation of pendulum considering mass of the rod with 30’in case 4

CONCLUSION

The lab objective was achieved using the equations of motion, finding the approximate value of the damping constant through trial and error, and plotting those using the validation data set. The validation graphs are similar to the original output data samples. It is useful to know that Simulink simulate faster than only execution of Matlab ODE45 function. The ODE45 function was taking hours to simulate. Overall, Matlab is a powerful software to analyze, design and model the systems.