simulation that the system is in equilibrium when $x = -1$, $y = 0$, and the input is -3. Then simulate the response to the following conditions. In each case, plot the input and the output y.

a) $x(0) = -0.5$, $y(0) = 0$, and the input is -3.

b) $x(0) = -1$, $y(0) = 0$, and the input is $-3 + 0.2\cos t$.

15.29 Simulate the response of the nonlinear circuit shown in Figure 9.11(a) that was studied in Example 9.9. Use the initial conditions $e_C(0) = 2$ V and $i_L(0) = 1$ A, which correspond to the operating point. Let the amplitude of the incremental input have the values $A = 0.1$ V, 1.0 V, and 10.0 V and plot the inductor current i_L. The responses should agree with the curves that are marked "nonlinear" in Figure 9.12.

15.30 Simulate the response of the nonlinear circuit shown in Figure 9.13 that was analyzed in Example 9.10. Use the initial conditions $e_C(0) = -6.7808$ V and $i_L(0) = -0.7808$ A, which correspond to the steady-state condition when the input voltage is $e_i(t) = -2$ V. For $t > 0$, $e_i(t) = 2 + A\cos 4t$. Let the amplitude of the incremental input have the values $A = 1.0$ V, 5.0 V, and 20.0 V and plot the output current i_o. The responses should agree with the curves that are marked "nonlinear" in Figure 9.14.

15.31 Simulate the zero-input and zero-state responses of the rack and pinion system that was modeled in Example 4.11, for which a block diagram was drawn in Example 13.9. The physical system is shown in Figures 4.27 and 13.20. The following parameter values should be used: $M = 500$ kg, $J = 40$ kg·m², $R = 0.50$ m, $K = 2000$ N/m, $B_1 = 25$ N·m·s/rad, and $B_2 = 25$ N·s/m. The simulation should cover the interval $0 \le t \le 40$ s. Specifically,

a) Simulate the zero-input response when $x(0) = 0.5$ m and $\theta(0) = \omega(0) = v(0) = 0$.

b) Simulate the zero-state response when the applied torque is 50 times the function shown in Figure P15.26.

In each case plot the horizontal displacement of the mass, the contact force on the mass (positive sense to the left), and the applied torque.

15.32 Prepare a simulation of the nonlinear pendulum shown in Figure 4.23(a) that was modeled in Example 4.7. The parameter values to be used are $M = 20$ kg, $L = 2.0$ m, and $B = 50$ N·m·s/rad. The gravitational constant is 9.807 m/s². Select the lengths of the runs so that all transients have died out.

a) Simulate the zero-input response when the initial angle is $\theta(0) = 0.1$, 1.5, 3.0, 3.3, and 6.0 rad and the initial angular velocity is zero. Plot the angle of rotation θ. Comment on the character of the responses and compare them with the behavior of a linearized model.

b) Simulate the zero-state response to a step-applied torque of 275 N·m. Plot the angle of rotation and the applied torque.

APPENDIX A

Units

We use the **International System of Units**, abbreviated as SI for **Système International d'Unités**. In Section A.1, we list the seven basic SI units, of which we use only the first five in this book. A supplementary unit for plane angles is the radian (rad).

In Section A.2, we give those derived SI units that we use. In addition to the physical quantity, the unit, and its symbol, there is a fourth column that expresses the unit in terms of units previously given. For example, 1 N = 1 kg · m/s².

A.1 BASIC UNITS

TABLE A.1 Names and Symbols of the Basic Units

Physical Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electrical current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

A.2 DERIVED UNITS

TABLE A.2 Names, Symbols, and Equivalents of the Derived Units

Physical Quantity	Name	Symbol	In Terms of Other Units
Force	newton	N	$kg \cdot m/s^2$
Energy	joule	J	$kg \cdot m^2/s^2$
Power	watt	W	J/s
Electrical charge	coulomb	C	$A \cdot s$
Voltage	volt	V	W/A
Electrical resistance	ohm	Ω	V/A
Electrical capacitance	farad	F	$A \cdot s/V$
Inductance	henry	H	$V \cdot s/A$
Magnetic flux	weber	Wb	$V \cdot s$

A.3 PREFIXES

The standard prefixes for decimal multiples and submultiples of a unit are given in elementary physics books. The only ones used in this book are kilo (k), milli (m), and micro (μ). The terms in which they appear are as follows: $1\ k\Omega = 10^3\ \Omega$, $1\ mV = 10^{-3}\ V$, $1\ \mu F = 10^{-6}\ F$.

A.4 CONSTANTS

The following three constants are used in the numerical solution of many examples and problems.

Atmospheric pressure at sea level: $p_a = 1.013 \times 10^5\ N/m^2$

Base of natural logarithms: $\epsilon = 2.718$

Gravitational constant at the surface of the earth: $g = 9.807\ m/s^2$

APPENDIX B

Laplace Transforms

In Table B.1, we list the Laplace transforms for common functions of time. When using the table to take inverse transforms, the reader should keep in mind that the time functions are valid only for positive values of t.

Table B.2 contains the most important properties that are needed in the application of the transform method to dynamic models. The restrictions on the use of these properties are not included in the table but may be found in Chapter 7. In that chapter, we discuss the proper interpretation of the initial-condition terms and the conditions on the initial-value and final-value theorems. The functions of time are again valid only for $t > 0$.

TABLE B.1 Transforms of Functions

Time Functions	Transformed Functions
$\delta(t)$	1
$U(t)$	$\frac{1}{s}$
A	$\frac{A}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n for $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
ϵ^{-at}	$\frac{1}{s+a}$
$t\epsilon^{-at}$	$\frac{1}{(s+a)^2}$
$t^2\epsilon^{-at}$	$\frac{2!}{(s+a)^3}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\epsilon^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\epsilon^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\epsilon^{-at}\left[B \cos \omega t + \left(\frac{C - aB}{\omega}\right) \sin \omega t\right]$	$\frac{Bs+C}{(s+a)^2+\omega^2}$
$2K\epsilon^{-at} \cos(\omega t + \phi)$	$\frac{K\epsilon^{j\phi}}{s+a-j\omega} + \frac{K\epsilon^{-j\phi}}{s+a+j\omega}$

TABLE B.2 Transform Properties

Time Functions	Transformed Functions
$f(t)$	$F(s)$
$af(t)$	$aF(s)$
$f(t) + g(t)$	$F(s) + G(s)$
$\epsilon^{-at} f(t)$	$F(s+a)$
$tf(t)$	$-\frac{d}{ds}F(s)$
$f(t/a)$	$aF(as)$
$[f(t-a)]U(t-a)$	$\epsilon^{-sa}F(s)$
$\dot{f}(t)$	$sF(s) - f(0)$
$\ddot{f}(t)$	$s^2F(s) - sf(0) - \dot{f}(0)$
$\dddot{f}(t)$	$s^3F(s) - s^2f(0) - s\dot{f}(0) - \ddot{f}(0)$
$\frac{d^n f}{dt^n}$ for $n = 1, 2, 3, \ldots$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
$\int_0^t f(\lambda)d\lambda$	$\frac{1}{s}F(s)$
$f(0+)$	$\lim_{s\to\infty} sF(s)$
$f(\infty)$	$\lim_{s\to 0} sF(s)$

APPENDIX C

Matrices

This appendix is intended as a refresher for the reader who has had an introductory course in linear algebra and is studying those sections of the book that use matrix methods. It is not a suitable introduction for someone who has not had formal exposure to matrices. Only those aspects of the subject that are used in Sections 3.3, 6.6, and 8.7 are emphasized. References to more complete treatments appear in Appendix D.

C.1 DEFINITIONS

A **matrix** is a rectangular array of elements that are either constants or functions of time. We refer to a matrix having m rows and n columns as being of **order** $m \times n$. The element in the ith row and the jth column of the matrix $\mathbf{A}$ is denoted by a_{ij}, such that

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

A matrix having the same number of rows as columns, in which case $m = n$, is a **square matrix** of order n. A matrix with a single column is a **column vector**. Examples of column vectors are

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \qquad \mathbf{c} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \qquad \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

A matrix with a single row is called a **row vector**.

A matrix that consists of a single element—that is, one that has only one row and one column—is referred to as a **scalar**. A square matrix of n rows and n columns that has unity for each of the elements on its main diagonal and zero for the remaining elements is the **identity matrix** of order n, denoted by $\mathbf{I}$. For example,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

are the identity matrices of order 2 and 3, respectively. A matrix all of whose elements are zero is the **null matrix**, denoted by $\mathbf{0}$.

We obtain the **transpose** of a matrix by interchanging its rows and columns such that the ith row of the original matrix becomes the ith column of the transpose, for all rows. Hence if $\mathbf{A}$ is of order $m \times n$ with elements a_{ij}, then its transpose, $\mathbf{A}^T$, is of order $n \times m$ and has elements $(\mathbf{A}^T)_{ij} = a_{ji}$.

C.2 OPERATIONS

We can add or subtract two matrices that have the same order by adding or subtracting their respective elements. For example, if $\mathbf{A}$ and $\mathbf{B}$ are both $m \times n$, the ijth element of $\mathbf{A} + \mathbf{B}$ is $a_{ij} + b_{ij}$. We can form the product $\mathbf{C} = \mathbf{AB}$ only if the number of rows of $\mathbf{B}$, the right matrix, is equal to the number of columns of $\mathbf{A}$, the left matrix. If this condition is met, we find the $i\ell$th element of $\mathbf{C}$ by summing the products of the elements in the ith row of $\mathbf{A}$ with the corresponding elements in the ℓth column of $\mathbf{B}$. If $\mathbf{A}$ is $m \times n$ and $\mathbf{B}$ is $n \times p$, then $\mathbf{C}$ is $m \times p$ and

$$c_{i\ell} = \sum_{j=1}^{n} a_{ij} b_{j\ell} \qquad \text{for } i = 1, 2, \ldots, m \qquad \text{and} \qquad \ell = 1, 2, \ldots, p \tag{1}$$

For example, if

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ -2 & 1 \end{bmatrix}$$

then

$$\begin{aligned} c_{11} &= (2)(1) + (-1)(2) + (3)(-2) = -6 \\ c_{12} &= (2)(-1) + (-1)(3) + (3)(1) = -2 \\ c_{21} &= (1)(1) + (0)(2) + (4)(-2) = -7 \\ c_{22} &= (1)(-1) + (0)(3) + (4)(1) = 3 \end{aligned}$$

giving

$$\mathbf{C} = \begin{bmatrix} -6 & -2 \\ -7 & 3 \end{bmatrix}$$

We obtain the product of a matrix and a scalar by multiplying each element of the matrix by the scalar. For example, if k is a scalar, the ijth element of $k\mathbf{A}$ is ka_{ij}.

Some properties of matrices are summarized in Table C.1, where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ denote general matrices and where $\mathbf{I}$ and $\mathbf{0}$ are the identity matrix and the null matrix, respectively. Some of the properties that hold for scalars are not always valid for matrices. For example, except in special cases, $\mathbf{AB} \neq \mathbf{BA}$. Also, the equation $\mathbf{AB} = \mathbf{AC}$ does not necessarily imply that $\mathbf{B} = \mathbf{C}$. Furthermore, the equation $\mathbf{AB} = \mathbf{0}$ does not necessarily imply that either $\mathbf{A}$ or $\mathbf{B}$ is zero.

TABLE C.1 Matrix Properties

$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
$\mathbf{AI} = \mathbf{IA} = \mathbf{A}$
$\mathbf{0A} = \mathbf{0}$
$\mathbf{A0} = \mathbf{0}$
$\mathbf{A} + \mathbf{0} = \mathbf{A}$

C.3 THE DETERMINANT AND THE INVERSE MATRIX

Associated with any square matrix $\mathbf{A}$ is a scalar quantity called the **determinant** and denoted by $|\mathbf{A}|$. For the 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

the determinant is defined as

$$|\mathbf{A}| = a_{11}a_{22} - a_{12}a_{21} \tag{2}$$

One way of evaluating the determinant for larger matrices is in terms of the determinants of some of its submatrices, as indicated in the following discussion.

The **minor** of the ijth element of an $n \times n$ matrix $\mathbf{A}$ is the determinant of the $(n-1) \times (n-1)$ submatrix we obtain by deleting the ith row and the jth column of $\mathbf{A}$. It is denoted by M_{ij}. The **cofactor** of the ijth element is denoted by C_{ij} and is

$$C_{ij} = (-1)^{i+j} M_{ij} \tag{3}$$

Thus the ijth cofactor is identical to the ijth minor if $i+j$ is an even integer and is the negative of the ijth minor if $i+j$ is odd. The **cofactor matrix** of an $n \times n$ matrix **A** is another $n \times n$ matrix whose elements are the cofactors of **A**. For the 3×3 matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -2 & 1 \\ -1 & 2 & 1 \end{bmatrix} \tag{4}$$

the minors of the 1, 1 and 2, 1 elements are

$$M_{11} = \begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix} = (-2)(1) - (1)(2) = -4$$

and

$$M_{21} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = (1)(1) - (0)(2) = 1$$

The corresponding cofactors are $C_{11} = -4$ and $C_{21} = -1$. Evaluating the cofactors of the remaining elements of **A**, we find the cofactor matrix to be

$$\mathbf{C} = \begin{bmatrix} -4 & -4 & 4 \\ -1 & 2 & -5 \\ 1 & -2 & -7 \end{bmatrix} \tag{5}$$

We can evaluate the determinant of **A** by selecting any one row or column of **A** and summing the products of the elements a_{ij} and their respective cofactors. Expanding along the ith row gives

$$|\mathbf{A}| = \sum_{j=1}^{n} a_{ij} C_{ij} \qquad \text{for } i = 1, 2, \ldots, n \tag{6}$$

while expanding down the jth column gives

$$|\mathbf{A}| = \sum_{i=1}^{n} a_{ij} C_{ij} \qquad \text{for } j = 1, 2, \ldots, n \tag{7}$$

For example, consider the matrix **A** defined by (4) and its cofactor matrix given by (5). Expanding $|\mathbf{A}|$ along the first row, we have

$$\begin{aligned} |\mathbf{A}| &= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} \\ &= (2)(-4) + (1)(-4) + (0)(4) = -12 \end{aligned}$$

Expanding the determinant along any other row or down any column will give the same result.

For a 4×4 matrix, each cofactor is a determinant of order 3×3, which can in turn be evaluated in terms of three 2×2 determinants. We can always continue this process for any square matrix until only 2×2 arrays are left, and we can then use (2) to evaluate each of these 2×2 determinants.

The **adjoint** of the square matrix **A** is denoted by adj[**A**] and is the transpose of the cofactor matrix; that is,

$$(\text{adj}[\mathbf{A}])_{ij} = C_{ji}$$

For the matrix **A** in (4),

$$\text{adj}[\mathbf{A}] = \begin{bmatrix} -4 & -1 & 1 \\ -4 & 2 & -2 \\ 4 & -5 & -7 \end{bmatrix}$$

If a matrix **B** can be found such that $\mathbf{AB} = \mathbf{BA} = \mathbf{I}$, then **B** is called the **inverse** of **A**, written $\mathbf{A}^{-1}$. Thus

$$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \tag{8}$$

If the inverse exists, it is unique. A necessary but not a sufficient condition for **A** to have an inverse is that **A** must be square. A square matrix **A** of order $n \times n$ has an inverse $\mathbf{A}^{-1}$ if and only if $|\mathbf{A}| \neq 0$. If $|\mathbf{A}|$ is nonzero, it can be shown that

$$\mathbf{A}^{-1} = \frac{\text{adj}[\mathbf{A}]}{|\mathbf{A}|} \tag{9}$$

For the matrix **A** in (4), $|\mathbf{A}| = -12$ and

$$\mathbf{A}^{-1} = -\frac{1}{12}\begin{bmatrix} -4 & -1 & 1 \\ -4 & 2 & -2 \\ 4 & -5 & -7 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/12 & -1/12 \\ 1/3 & -1/6 & 1/6 \\ -1/3 & 5/12 & 7/12 \end{bmatrix} \tag{10}$$

You can readily verify that (4) and (10) satisfy both of the relationships in (8). Although the methods described here for evaluating determinants and inverses are satisfactory for hand calculations with small matrices, other methods exist that are better suited for computer use with large matrices.

C.4 CHARACTERISTIC VALUES

Any square matrix of order n has associated with it a **characteristic equation** which, when written in terms of the variable s, is

$$|s\mathbf{I} - \mathbf{A}| = 0 \tag{11}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix, and where

$$s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & s - a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & s - a_{nn} \end{bmatrix} \tag{12}$$

To evaluate $|s\mathbf{I} - \mathbf{A}|$, we can apply either (6) along any row or (7) down any column of the right side of (12). Regardless of the choice made, $|s\mathbf{I} - \mathbf{A}|$ is a unique polynomial of degree n in the variable s, and it is called the **characteristic polynomial**. Thus

$$|s\mathbf{I} - \mathbf{A}| = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 \tag{13}$$

where the coefficients $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}$ depend on the elements of $\mathbf{A}$.

Because the characteristic polynomial is of degree n, the characteristic equation given by (11) will have at most n distinct solutions, known as the **characteristic values (eigenvalues)** of the matrix $\mathbf{A}$. Should (11) have fewer than n distinct solutions, one or more of its solutions must be a multiple solution such that the total number of solutions, each one counted according to its multiplicity, is exactly n. The characteristic values may be real or complex, but because the coefficients in (13) are real, any complex characteristic values must occur in complex conjugate pairs.

To find the characteristic values of the matrix $\mathbf{A}$ in (4), we form

$$s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s - 2 & -1 & 0 \\ -3 & s + 2 & -1 \\ 1 & -2 & s - 1 \end{bmatrix}$$

and then evaluate its determinant. Expanding along the first row, we find that

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}| &= (s-2)\begin{vmatrix} s+2 & -1 \\ -2 & s-1 \end{vmatrix} - (-1)\begin{vmatrix} -3 & -1 \\ 1 & s-1 \end{vmatrix} \\ &= (s-2)[(s+2)(s-1) - 2] + [(-3)(s-1) + 1] \\ &= s^3 - s^2 - 9s + 12 \end{aligned}$$

Since $|s\mathbf{I} - \mathbf{A}|$ is a cubic function of s, we use a digital computer or a calculator to find that the three characteristic values are $s_1 = 1.432$, $s_2 = 2.687$, and $s_3 = -3.119$.

APPENDIX D

Selected Reading

This appendix suggests a number of books that are suitable for undergraduates. Some provide background for specific topics or can be used as collateral reading. Others extend the topics we treat to a more advanced level or offer an introduction to additional areas. The works cited here are included in the list of references that follows. This appendix and the references provide typical starting points for further reading, but these lists are not intended to be comprehensive.

Background Sources

Introductory college physics textbooks describe the basic mechanical, electrical, and electromechanical elements and some other components as well. They also illustrate simple applications of physical laws to such systems. One good reference is *Halliday and Resnick*. Comprehensive lists of units and conversion factors are given in *Wildi*.

A good example of a book on differential equations is *Boyce and DiPrima*. For a background in linear algebra and matrices, such as would be useful for Sections 3.3, 6.6, and 8.7, see *Anton*.

Books at a Comparable Level

The following three introductory systems books discuss a variety of physical components, as well as analysis techniques: *Rosenberg and Karnopp*, *Ogata* (1992), and *Palm*. Included are electrical, mechanical, hydraulic, pneumatic, and thermal systems. The references include several books, easily recognized by their titles, that illustrate the application of the MATLAB and ACSL computer programs.

Most books dealing with modeling and analysis are restricted to a particular discipline. *Kimbrell*, *Meriam and Kraige*, and *Thomson* deal with mechanical components. *Thorpe* and *de Silva* also include hydraulic and mechanical devices. Textbooks on chemical processes include *Seborg, Edgar, and Mellichamp* and *Luyben*.

Two of the standard books on electrical circuits are *Johnson, Johnson, and Hilburn* and *Nilsson*. *Sedra and Smith* covers electronic circuits in detail. Books such as *Smith and Dorf*, *Carlson and Gisser*, and *Del Toro* treat devices and applications in both electrical and electromechanical systems. *Krause and Wasynczuk*, *Guru and Hiziroglu*, and *McPherson* are confined to electromechanical machinery.

Books that Extend the Analytical Techniques

It is natural to follow up your study of this book with more advanced books on feedback control systems, such as *D'Souza*, *Raven*, *Ogata* (1990), and some others in the list of references. Some of these include discrete-time systems in addition to continuous systems. Two books that emphasize discrete-time control systems are *Phillips and Nagle*, and *Franklin, Powell, and Workman*.

A number of books broaden the study of transform methods to include both the Fourier and z transforms, as well as the Laplace transform. Good examples are *Chen*, *Kamen*, and *Sinha*.

APPENDIX E

Answers to Selected Problems

CHAPTER 2

2.1 $M_1\ddot{x}_1 + B_1\dot{x}_1 + (K_1 + K_2)x_1 - K_2x_2 = f_a(t)$,
$-K_2x_1 + M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2x_2 = 0$

2.4 $M_1\ddot{x}_1 + (B_1 + B_3)\dot{x}_1 + (K_1 + K_3)x_1 - B_3\dot{x}_2 - K_3x_2 = f_a(t)$,
$-B_3\dot{x}_1 - K_3x_1 + M_2\ddot{x}_2 + (B_2 + B_3)\dot{x}_2 + (K_2 + K_3)x_2 = 0$

2.7 **(a)** $M_1\ddot{x}_1 + B_1\dot{x}_1 + (K_1 + K_2)x_1 = K_2x_2(t)$
(b) $f_2 = -K_2x_1 + M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2x_2(t)$

2.10 $M_1\ddot{x}_1 + K_1x_1 - K_2x_2 = 0,\quad M_2\ddot{x}_1 + B\dot{x}_1 + M_2\ddot{x}_2 + B\dot{x}_2 + K_2x_2 = f_a(t)$

2.12 $M_1\ddot{x}_1 + K_1x_1 + B\dot{x}_2 = K_1x_3(t)$,
$-M_2\ddot{x}_1 - K_2x_1 + M_2\ddot{x}_2 + B\dot{x}_2 + K_2x_2 = 0$

2.15 **(a)** $M_1\ddot{x}_1 + B\dot{x}_1 + K_1x_1 - B\dot{x}_2 - K_1x_2 = M_1g$,
$-B\dot{x}_1 - K_1x_1 + M_2\ddot{x}_2 + B\dot{x}_2 + (K_1 + K_2)x_2 = M_2g + f_a(t)$
(b) $x_{1_0} = [(M_1 + M_2)/K_2 + (M_1/K_1)]g,\quad x_{2_0} = [(M_1 + M_2)/K_2]g$
(c) $M_1\ddot{z}_1 + B\dot{z}_1 + K_1z_1 - B\dot{z}_2 - K_1z_2 = 0$,
$-B\dot{z}_1 - K_1z_1 + M_2\ddot{z}_2 + B\dot{z}_2 + (K_1 + K_2)z_2 = f_a(t)$

2.18 **(a)** $M_1\ddot{x}_1 + B\dot{x}_1 + 2Kx_1 - B\dot{x}_2 - Kx_2 = M_1g$,
$-B\dot{x}_1 - Kx_1 + M_2\ddot{x}_2 + B\dot{x}_2 + 3Kx_2 - Kx_3 = M_2g$,
$-Kx_2 + M_3\ddot{x}_3 + Kx_3 = M_3g - f_a(t)$
(b) $x_{1_0} = (2M_1 + M_2 + M_3)g/(3K)$,
$x_{2_0} = (M_1 + 2M_2 + 2M_3)g/(3K)$,
$x_{2_0} - x_{1_0} = (-M_1 + M_2 + M_3)g/(3K),\quad x_{3_0} - x_{2_0} = M_3g/K$

2.22 $M_1\ddot{x}_1 + (K_1 + K_2)x_1 - K_2x_2 = M_1g$,
$-K_2x_1 + M_2\ddot{x}_2 + B\dot{x}_2 + (K_2 + K_3)x_2 = -M_2g$

2.25 $\dot{x}_1 = [B_2/(B_1 + B_2)]\dot{x}_2,\quad B_{\text{eq}} = B_1B_2/(B_1 + B_2)$

2.27 $M\ddot{x}_1 + (B_1 + B_2)\dot{x}_1 + (K_1 + K_2)x_1 - B_2\dot{x}_2 - K_2x_2 = 0$,
$-B_2\dot{x}_1 - K_2x_1 + B_2\dot{x}_2 + (K_2 + K_3)x_2 = K_3x_3(t)$

CHAPTER 3

3.3 $\dot y = v,\quad \dot v = -2y - 4v + x,\quad \dot x = -y - x + f_a(t)$

3.5 $\dot x_1 = v_1,\quad \dot v_1 = [-K_1x_1 - (B_1 + B_2 + B_3)v_1 + B_2v_2]/M_1,\quad \dot x_2 = v_2,$
$\dot v_2 = [B_2v_1 - K_2x_2 - B_2v_2 + f_a(t)]/M_2,\quad y_1 = B_2(v_2 - v_1),$
$y_2 = K_2x_2$

3.9 $\dot x_1 = v_1,\quad \dot v_1 = [-K_1x_1 - Bv_1 + K_1x_2 + Bv_2 + M_1g]/M_1,\quad \dot x_2 = v_2,$
$\dot v_2 = [K_1x_1 + Bv_1 - (K_1 + K_2)x_2 - Bv_2 + M_2g + f_a(t)]/M_2,$
$y_1 = x_1 - x_2,\quad a_1 = [-K_1x_1 - Bv_1 + K_1x_2 + Bv_2 + M_1g]/M_1$

3.11 $\dot x_1 = v_1,\quad \dot v_1 = [-3Kx_1 - Bv_1 + Kx_2 + M_1g]/M_1,\quad \dot x_2 = v_2,$
$\dot v_2 = [Kx_1 - Kx_2 + M_2g + f_a(t)]/M_2,\quad y = x_2 - x_1$

3.13 $\dot x_1 = v_1,\quad \dot v_1 = [-K_1x_1 - B_1v_1 - K_3x_2 + K_3x_3(t)]/M,$
$\dot x_2 = [K_2x_1 + B_2v_1 - (K_2 + K_3)x_2 + K_3x_3(t)]/B_2,$
$y = K_2x_1 - (K_2 + K_3)x_2 + K_3x_3(t)$

3.15 $\dot q_1 = -3q_1 + 2q_2 + 3u_1(t) - 6u_2(t),\quad \dot q_2 = 2q_1 + q_2 + u_1(t) + 4u_2(t),$
$y = q_1 - q_2 - u_1(t) + 3u_2(t)$ where $q_1 = x_1 - 2u_2(t)$ and
$q_2 = x_2 - u_1(t)$

3.18 $\dot x_1 = q_1 + (B_1/M_1)x_3(t),$
$\dot q_1 = [-(K_1 + K_2)x_1 - B_1q_1 + K_2x_2 + (K_1 - B_1^2/M_1)x_3(t) + f_a(t)]/M_1,$
$\dot x_2 = q_2 + (B_2/M_2)x_3(t),$
$\dot q_2 = [K_2x_1 - K_2x_2 - B_2q_2 - (B_2^2/M_2)x_3(t)]/M_2,$
$v_1 = q_1 + (B_1/M_1)x_3(t),\quad v_2 = q_2 + (B_2/M_2)x_3(t)$

3.21 $\dot x_1 = v_1,\quad \dot v_1 = [-Kx_1 - Bv_1 + Bv_2 - f_a(t)]/M_1,\quad \dot x_2 = v_2,$
$\dot v_2 = [Bv_1 - Bv_2 + M_2g]/M_2,\quad m_1 = M_1v_1,\quad m_2 = M_2v_2,$
$f_c = B(v_2 - v_1)$

3.24 $M_1\ddot x_1 + B\dot x_1 + (K_1 + K_2)x_1 = B\dot x_2 + K_2x_2(t)$

3.26 $x_1^{(iv)} + 2x_1^{(iii)} + 5\ddot x_1 + \dot x_1 + 2x_1 = 4g + \ddot f_a + \dot f_a + 2f_a(t)$

3.29
$$\dot{\mathbf{q}} = \begin{bmatrix} 0 & -1 & 1 \\ K_2/M_1 & -B/M_1 & B/M_1 \\ -K_2/M_2 & B/M_2 & -B/M_2 \end{bmatrix}\mathbf{q} + \begin{bmatrix} 0 \\ 0 \\ 1/M_2 \end{bmatrix} f_a(t),$$
$$\mathbf{y} = \begin{bmatrix} K_2 & 0 & 0 \\ 0 & M_1 & M_2 \end{bmatrix}\mathbf{q} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f_a(t)$$

CHAPTER 4

4.2 $J_1J_2\dddot\omega_2 + BJ_2\ddot\omega_2 + K(J_1 + J_2)\dot\omega_2 + KB\omega_2 = BK\omega_a(t)$

4.6 **(a)** With θ_1, ω_1, θ_2, ω_2 as state variables: $\dot\theta_1 = \omega_1,$
$\dot\omega_1 = [-K_1\theta_1 - B\omega_1 + B\omega_2 + \tau_a(t)]/J_1,\quad \dot\theta_2 = \omega_2,$
$\dot\omega_2 = (B\omega_1 - K_2\theta_2 - B\omega_2)/J_2,\quad \tau_B = B(\omega_2 - \omega_1)$
(b) $\theta_2^{(iv)} + 2\theta_2^{(iii)} + 2\ddot\theta_2 + 2\dot\theta_2 + \theta_2 = \dot\tau_a$

4.7 $J\ddot\theta + K_{eq}\theta = \tau_a(t)$ where $K_{eq} = K_1 + [K_2K_3/(K_2 + K_3)]$

4.10 **(a)** $\dot x_1 = v_1,\quad \dot v_1 = [-(K_1 + K_2)x_1 - B_1v_1 + (K_2L/4)\theta]/M,$
$\dot\theta = (16/9B_2L)[(K_2/4)x_1 - (K_2L/16)\theta - (3/4)f_a(t)]$
(b) $f_r = (4K_2/3)x_1 - (K_2L/3)\theta$

4.13 With x_1, v_1, θ, ω as state variables: $\dot x_1 = v_1,$
$\dot v_1 = [-(K_1 + K_2)x_1 - B_1v_1 + (1/4)K_2L\theta]/M,\quad \dot\theta = \omega,$
$\dot\omega = (3/7ML)[4K_2x_1 - LK_2\theta - 9LB_2\omega - 12f_a(t)]$
$f_{K_2} = -K_2x_1 + (1/4)LK_2\theta$

4.18 **(a)** $J_1\ddot\theta_1 + B\dot\theta_1 - R_1f_c = \tau_a(t),\quad J_2\ddot\theta_2 + K\theta_2 + R_2f_c = 0$
(b) With θ_1, ω_1 as state variables: $\dot\theta_1 = \omega_1,$
$\dot\omega_1 = [-(K/N^2)\theta_1 - B\omega_1 + \tau_a(t)]/[J_1 + (J_2/N^2)]$
$m_T = [J_1 + (J_2/N)]\omega_1,\quad \tau_B = -B\omega_1$
(c) $[J_1 + (J_2/N^2)]\ddot\theta_1 + B\dot\theta_1 + (K/N^2)\theta_1 = \tau_a(t)$

4.19 $\dot\theta_1 = \omega_1,\quad \dot\omega_1 = [-K_1\theta_1 - B\omega_1 + K_1\theta_2 + \tau_a(t)]/J_1,\quad \dot\theta_2 = \omega_2,$
$\dot\omega_2 = [K_1\theta_1 - (K_1 + N^2K_2)\theta_2]/(J_2 + N^2J_3),\quad \theta_3 = -N\theta_2$

4.26 With θ, ω as state variables: $\dot\theta = \omega,$
$\dot\omega = [-KR\theta - (B/R)\omega + Mg + f_a(t)]/[(J/R) + MR]$
$z = R\theta - Mg/K$

4.29 With θ, ω, x_2, v_2 as state variables: $\dot\theta = \omega,$
$\dot\omega = [-(R_1K_1 + K_2)\theta + (K_2/R_2)x_2 + R_1K_1x_1(t)]/J_1,\quad \dot x_2 = v_2,$
$\dot v_2 = [(K_2/R_2)\theta - (K_2/R_2^2 + 2K_3)x_2 - Bv_2 + M_2g]/M_2,$
$y = K_2\theta - K_2R_2x_2$

4.32
$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \\ x \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(K_1 + K_2R^2)/J & -B/J & K_2R/J & 0 \\ 0 & 0 & 0 & 1 \\ K_2R/M & 0 & -K_2/M & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \omega \\ x \\ v \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \end{bmatrix}\begin{bmatrix} Mg \\ f_a(t) \end{bmatrix}$$

CHAPTER 5

1 **5.3** $\dot e_o + e_o = (7/24)\dot e_i + (2/3)e_i(t)$
2 **5.7** $2\ddot e_o + 2\dot e_o + 4e_o = 3e_i(t)$
5.9 $\ddot e_o + \dot e_o + (1/2)e_o = 2di_i/dt$
5.12 $\ddot e_o + 2\dot e_o + 2e_o = \ddot e_i + (1/2)\dot e_i$
5.17 $e_o = 12$ V
5.18 **(a)** With i_L (positive to right), e_C (positive at upper node) as state variables: $\dot e_C = [-e_C - R_1i_L + R_1i_i(t) - 6]/(R_1C),$
$di_L/dt = (e_C - R_2i_L)/L$

(b) $i_o = i_i(t) - (e_C + 6)/R_1$

5.21 With i_L (positive downward), e_o as state variables: $\dot{e}_o = -(1/2)e_o - 2i_L + 2i_i(t)$, $di_L/dt = (1/8)e_o - (1/2)i_L + (1/2)i_i(t)$

3 **5.24** With i_L (positive to right), e_C (positive at upper node) as state variables: $\dot{e}_C = [-e_C + 2i_L + e_i(t)]/3$, $di_L/dt = [-e_C - 4i_L + e_i(t)]/6$, $e_o = (2/3)[-e_C + 2i_L + e_i(t)]$

5.27 $\dot{e}_{C_1} = -i_L + i_i(t)$, $\dot{e}_{C_2} = [-e_{C_2} - 2i_L + 6i_i(t)]/12$, $di_L/dt = (3e_{C_1} + e_{C_2} - 4i_L)/9$, $i_o = (1/6)e_{C_2} + (1/3)i_L$

5.31 $6\dot{e}_o + (10 - \alpha)e_o = [1 + (\alpha/2)]i_i(t)$

5.34 $e_o = -(R_3/R_1)e_1(t) - (R_3/R_2)e_2(t)$, summer with gains determined by R_1, R_2, and R_3.

5.37 $C_2\dot{e}_o + (1/R_2)e_o = -C_1\dot{e}_i - (1/R_1)e_i(t)$

5.41
$$\frac{d}{dt}\begin{bmatrix} e_C \\ i_L \end{bmatrix} = \begin{bmatrix} -0.6 & 0.3 \\ -1.2 & -0.8 \end{bmatrix}\begin{bmatrix} e_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} e_i(t),$$
$$e_o = \begin{bmatrix} -0.6 & -0.2 \end{bmatrix}\begin{bmatrix} e_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.2 \end{bmatrix} e_i(t)$$

CHAPTER 6

Note: Expressions for time functions are valid for $t \geq 0$ unless noted otherwise.

6.2 (a) $y(t) = 0.4\epsilon^{-t/2} + 1.2\sin t - 0.4\cos t$
(b) $y(t) = \epsilon^{-t/2} + t\epsilon^{-t/2}$

6.5 $y(t) = 2\epsilon^{-t} - \epsilon^{-2t} + t - 1$

6.8 $y(t) = (A/2)(1 - \epsilon^{-2t/3})$, $\tau = 3/2$ s, $y_{ss} = A/2$, $y_{\text{transient}} = -(A/2)\epsilon^{-2t/3}$

6.12 (a) $\tau = 3/2$ s
(b) $e_o(t) = 2 + K\epsilon^{-2t/3}$
(c) $e_o(0+) = 2/3$ V, $K = -4/3$
(d) $e_o(t) = 2 - (4/3)\epsilon^{-2t/3}$
(e) $(e_o)_{ss} = 2$ V

6.16 (a) $y_U(t) = 2(1 - \epsilon^{-t/2})$
(b) $y(t) = 4 - 5\epsilon^{-t/2}$
(c) $y(t) = 2(1 - \epsilon^{-t/2})$ for $0 \leq t \leq 2$ and $y(t) = 2(\epsilon - 1)\epsilon^{-t/2}$ for $t \geq 2$
(d) $h(t) = \epsilon^{-t/2}$

6.20 (b) $\zeta = [(B_1 + B_2)/2]/\sqrt{K_2(M_1 + M_2)}$ and $\omega_n = \sqrt{K_2/(M_1 + M_2)}$
(c) $x_{ss} = (1 - M_2 g)/K_2$

6.25 (a) $\zeta = 1/\sqrt{2}$, $\omega_n = \sqrt{2}$
(b) $y_U(t) = -0.5\epsilon^{-t}(\cos t + \sin t) + 0.5$, $h(t) = \epsilon^{-t}\sin t$

6.27 (a) $\zeta = 1/\sqrt{5} = 0.4472$, $\omega_n = \sqrt{5} = 2.236$
(b) $h(t) = 3.414\delta(t) - 4.467\epsilon^{-t}[\sin(2t + 1.249)]U(t)$

6.28 $y(t) = 2 - \epsilon^{-2t} - \cos t + 3\sin t$

6.34 (a) $\phi(t) = \begin{bmatrix} 2\epsilon^{-t} - \epsilon^{-2t} & \epsilon^{-t} - \epsilon^{-2t} \\ -2\epsilon^{-t} + 2\epsilon^{-2t} & -\epsilon^{-t} + 2\epsilon^{-2t} \end{bmatrix}$
(b) $y_{zi}(t) = (2\epsilon^{-t} - 3\epsilon^{-2t})q_1(0) + (\epsilon^{-t} - 3\epsilon^{-2t})q_2(0)$

6.35 $\mathbf{y}(t) = \begin{bmatrix} 12.5\epsilon^{-t} - 20\epsilon^{-2t} + 7.5\epsilon^{-3t} \\ -5.5\epsilon^{-t} + 10\epsilon^{-2t} - 4.5\epsilon^{-3t} \end{bmatrix}$

CHAPTER 7

Note: Expressions for time functions are valid for $t > 0$.

7.3 (a) $F_1(s) = (s^2 + 4s - 5)/[(s^2 + 4s + 13)^2]$
(b) $F_2(s) = (6\omega s^2 - 2\omega^3)/[(s^2 + \omega^2)^3]$
(c) $F_3(s) = 2s/[(s + 1)^3]$
(d) $F_4(s) = 2/[s(s + 1)^3]$

7.7 (a) $f(t) = 2 - (5 + t)\epsilon^{-t} + 4\epsilon^{-2t}$
(b) $f(t) = 2 + 2\sqrt{2}\,\epsilon^{-t}\cos(2t - \pi/4)$
(c) $f(t) = 3\delta(t) + (1/3) - [(37/3) - 20t]\epsilon^{-3t}$
(d) $f(t) = \delta(t) + 3\epsilon^{-t} - 8\epsilon^{-2t}$

7.8 (a) $f(t) = 2\epsilon^{-2t} + 2.692\epsilon^{-t}\cos(2t + 1.190)$
(b) $f(t) = 2\epsilon^{-2t} + \epsilon^{-t}\cos 2t - 2.5\epsilon^{-t}\sin 2t$

7.10 $f(t) = 1.25 - 2\epsilon^{-t} + 0.75\epsilon^{-4t}$

7.14 $y(t) = 2.5t - 3.75 + 6\epsilon^{-t} - 1.25\epsilon^{-2t}$

7.17 $e_o(t) = 2 - (4/3)\epsilon^{-2t/3}$

7.21 (a) $x_0 = -M_2 g/K_2$
(b) $X(s) = 1/s[(M_1 + M_2)s^2 + (B_1 + B_2)s + K_2] - (M_2 g/sK_2)$
(c) $x(t) = 1 - \epsilon^{-0.5t}(\cos 0.5t + \sin 0.5t) - g$

7.23 $\theta(t) = \theta(0)\cos\sqrt{g/L}\,t + \dot{\theta}(0)\sqrt{L/g}\,\sin\sqrt{g/L}\,t$, $\phi(t) = 0.5\left[\phi(0) - \sqrt{L/g}\,\dot{\phi}(0)\right]\epsilon^{-\sqrt{g/L}\,t} + 0.5\left[\phi(0) + \sqrt{L/g}\,\dot{\phi}(0)\right]\epsilon^{+\sqrt{g/L}\,t}$

7.25 $e_o(t) = 1 + (1/3)\epsilon^{-t} + (2/3)\epsilon^{-4t}$

7.28 (b) $E_o(s) = 2se_A(0)/(16s^3 + 19s^2 + 10s + 8)$

7.32 (a) $F_1(s) = A(1 - 2\epsilon^{-s} + \epsilon^{-2s})/s$
(b) $F_2(s) = A(1 - \epsilon^{-s} - \epsilon^{-2s} + \epsilon^{-3s})/s^2$

7.35 (a) $f(0+) = 1$, $f(\infty) = 2$
(b) $f(0+) = 4$, $f(\infty) = 2$
(c) The initial-value theorem is not applicable, $f(\infty) = 1/3$
(d) The initial-value theorem is not applicable, $f(\infty) = 0$

CHAPTER 8

Note: Expressions for time functions are valid for $t > 0$.

8.2 $\theta_1(t) = 0.6552 \sin 0.6622t + 0.0319 \sin 2.136t$

8.5 $e_o(t) =$
$\epsilon^{-2t} \left\{ -e_C(0) \cos \sqrt{12}\, t + (2/\sqrt{12}) \left[e_C(0) - i_L(0) \right] \sin \sqrt{12}\, t \right\}$,
$\epsilon^{-2t} \cos \sqrt{12}\, t$ is absent if $e_C(0) = 0$, $\epsilon^{-2t} \sin \sqrt{12}\, t$ is absent if $i_L(0) = e_C(0)$

8.7 $X_R(s)/F_a(s) = M_1/[M_1 M_2 s^2 + (M_1 + M_2)Bs + (M_1 + M_2)K_2]$

8.10 $X(s)/\tau_a(s) = (K/R)/P(s)$ where $P(s) = s[JMs^3 + (B_1 M + JB_2)s^2 + (KM + B_1 B_2 + JK/R^2)s + KB_2 + (KB_1/R^2)]$

8.12 (a) $E_o(s) = [-(30s + 12)e_C(0) - (10s + 15)i_L(0)]/P(s)$ where $P(s) = 50s^2 + 70s + 42$
(b) $E_o(s)/E_i(s) = (10s^2 + 4s)/P(s)$

8.14 (a) Double pole at $s = -2$, zeros at $s = -1 \pm j1$
(b) $y_{zi}(t) = (K_1 + K_2 t)\epsilon^{-2t}$, $\ddot{y} + 4\dot{y} + 4y = \ddot{u} + 2\dot{u} + 2u(t)$
(c) $y_U(0+) = 1$, $y_U(\infty) = 0.5$
(d) $y_U(t) = 0.5 + (0.5 - t)\epsilon^{-2t}$, $h(t) = \delta(t) - 2(1 - t)\epsilon^{-2t}$

8.17 (a) Poles at $s = -0.5$, $\pm j2$, double zero at $s = 0$
(b) $y_{zi}(t) = K_1 \epsilon^{-0.5t} + K_2 \cos(2t + \phi)$ or
$y_{zi}(t) = K_1 \epsilon^{-0.5t} + K_3 \cos 2t + K_4 \sin 2t$, $2\dddot{y} + \ddot{y} + 8\dot{y} + 4y = \ddot{u}$
(c) $y_U(0+) = 0$, $y_U(\infty) = 0$
(d) $y_U(t) = -0.05882\epsilon^{-0.5t} + 0.2426 \cos(2t - 1.326)$,
$h(t) = 0.02941\epsilon^{-0.5t} + 0.4850 \cos(2t + 0.2450)$

8.20 $h(t) = 10.91\epsilon^{-2t} \sin 4.583t$,
$y_U(t) = 2 - 2.182\epsilon^{-2t} \cos(4.583t - 0.4115)$ or
$y_U(t) = 2 - \epsilon^{-2t}[2 \cos \sqrt{21}\, t + (4/\sqrt{21}) \sin \sqrt{21}\, t]$

8.23 $h(t) = \delta(t) + t\epsilon^{-2t} - 2\epsilon^{-2t}$, $y_U(t) = 0.25 - 0.5t\epsilon^{-2t} + 0.75\epsilon^{-2t}$

8.27 (a) Pole at $s = -1$, zero at $s = +1$, $M(\omega) = 1$ for all ω,
$\theta(\omega) = \pi - 2 \tan^{-1} \omega$
(b) Poles at $s = 0, -5$, double zero at $s = -1$, $M(\omega) = (1 + \omega^2)/(\omega \sqrt{25 + \omega^2})$,
$\theta(\omega) = 2 \tan^{-1} \omega - \pi/2 - \tan^{-1}(\omega/5)$
(c) Double poles at $s = -0.1 \pm j10$, zero at $s = 0$, $M(\omega) = \omega/[(100 - \omega^2)^2 + 0.04\omega^2]$,
$\theta(\omega) = \pi/2 - 2 \tan^{-1}[0.2\omega/(100 - \omega^2)]$

8.29 $A = 0.0667$, $B = 5.0$, $C = 0.120$, $A/B = 0.01333$, $C/B = 0.0240$

8.31 (a) $H(s) = Cs/(LCs^2 + RCs + 1)$
(b) $\omega = 1/\sqrt{LC}$

8.35 (b) $E_o(s)/E_i(s) = (s^2 + 2s + 1)/(s^2 + 4s + 4)$

8.38 (b) $E_o(s)/E_i(s) = (s^2 + 0.5s)/(s^2 + 2s + 2)$

(c) $\ddot{e}_o + 2\dot{e}_o + 2e_o = \ddot{e}_i + 0.5\dot{e}_i$

8.42 (a) $\Phi(s) = \begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\ \dfrac{-2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix}$
(b) $\phi(t) = \begin{bmatrix} 2\epsilon^{-t} - \epsilon^{-2t} & \epsilon^{-t} - \epsilon^{-2t} \\ -2\epsilon^{-t} + 2\epsilon^{-2t} & -\epsilon^{-t} + 2\epsilon^{-2t} \end{bmatrix}$
(c) $q_1(t) = (2\epsilon^{-t} - \epsilon^{-2t})q_1(0) + (\epsilon^{-t} - \epsilon^{-2t})q_2(0)$,
$q_2(t) = (-2\epsilon^{-t} + 2\epsilon^{-2t})q_1(0) + (-\epsilon^{-t} + 2\epsilon^{-2t})q_2(0)$

8.46 (a) 2 state variables, 2 inputs, 1 output
(b) $\mathbf{H}(s) = \begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \end{bmatrix}$
(c) $\mathbf{h}(t) = \begin{bmatrix} 2\epsilon^{-t} - \epsilon^{-2t} & \epsilon^{-t} - \epsilon^{-2t} \end{bmatrix}$

CHAPTER 9

9.2 For $\bar{x} = -1$, $f(x) \simeq A(0.6321 + 0.3679\hat{x})$; for $\bar{x} = 0$, $f(x) \simeq A\hat{x}$; for $\bar{x} = +1$, $f(x) \simeq A(-0.6321 + 0.3679\hat{x})$

9.5 $f(y) \simeq 2 - 4\hat{y}$

9.8 (b) $\bar{x} = 0.39$ m
(c) $1.5\ddot{\hat{x}} + 0.5\dot{\hat{x}} + 55\hat{x} = 0$
(d) Satisfactory for at least $0.27 < x < 0.60$

9.10 (a) $\bar{x} = 4$, $\ddot{\hat{x}} + 2\dot{\hat{x}} + \hat{x} = B \sin 3t$
(b) For $A = 4$, $\bar{x} = 1$, $k = 2$; for $A = -4$, $\bar{x} = -1$, $k = 2$
(c) $\hat{x}(0) = 0.5$, $\dot{\hat{x}}(0) = 0.5$

9.12 (a) $\bar{y} = -1$, $\ddot{\hat{y}} + 2\dot{\hat{y}} + 4\hat{y} = B \cos t$
(b) $\bar{y} = 3$, $\ddot{\hat{y}} + 2\dot{\hat{y}} + 8\hat{y} = B \cos t$

9.18 (a) $\bar{\theta} = 2$
(b) $\ddot{\hat{\theta}} + 2\dot{\hat{\theta}} + 12\hat{\theta} = \hat{\tau}_a(t)$
(c) $\hat{\theta}(0) = -1.5$ rad, $\dot{\hat{\theta}}(0) = -0.5$ rad/s

9.21 (b) $\bar{x} = \sqrt[3]{Mg}$, $M\ddot{\hat{x}} + B\dot{\hat{x}} + 3(Mg)^{(2/3)}\,\hat{x} = \hat{f}_a(t)$
(c) $\bar{x} = 1.260\sqrt[3]{Mg}$, $M\ddot{\hat{x}} + B\dot{\hat{x}} + 4.762(Mg)^{(2/3)}\,\hat{x} = \hat{f}_a(t)$

9.23 (a) $\bar{x} = -4$, $\bar{y} = -2$
(b) $\dot{\hat{x}} = -2\hat{x} + 12\hat{y}$, $\dot{\hat{y}} = \hat{x} + \cos t$
(c) $\ddot{\hat{x}} + 2\dot{\hat{x}} - 12\hat{x} = 12 \cos t$

9.27 (b) $\bar{i}_L = 0.3622$ A, $\bar{e}_o = 0.3936$ V,
$d\hat{i}_L/dt = -2.515\hat{i}_L + 0.05714 \cos t$, $\hat{e}_o = 2.173\hat{i}_L$
(c) $\tau = 0.3976$ s

9.28 (b) $\bar{e}_o = 4$ V, $\bar{i}_o = 16$ A
(c) $\ddot{\hat{e}}_o + 7\dot{\hat{e}}_o + \hat{e}_o = 2\dot{\hat{e}}_i + \hat{e}_i(t)$

CHAPTER 10

10.2 (a) $L_T\dot{e}_o + R_T e_o = L_2(x)\dot{e}_i + R_2(x)e_i(t)$
(b) $L_T\dot{e}_o + R_T e_o = [x(t)/x_{\max}][L_T\dot{e}_i + R_T e_i(t)]$

10.7 (b) $X(s)/E_i(s) = \alpha/P(s)$ where
$P(s) = MLs^3 + (BL + MR)s^2 + (KL + BR + \alpha^2)s + KR$,
$ML\dddot{x} + (BL + MR)\ddot{x} + (KL + BR + \alpha^2)\dot{x} + KRx = \alpha e_i(t)$
(c) $MR\ddot{x} + (BR + \alpha^2)\dot{x} + KRx = \alpha e_i(t)$,
$\zeta = (\alpha^2 + BR)/(2R\sqrt{MK})$, $\omega_n = \sqrt{K/M}$

10.9 $E_o(s)/F_a(s) = d\mathcal{B}R_o/P(s)$ where
$P(s) = MLs^2 + (BL + MR + MR_o)s + B(R + R_o) + (d\mathcal{B})^2$

10.11 (a) $di/dt = [-Ri + \mathcal{B}dv + e_i(t)]/L$, $\dot{v} = (-\mathcal{B}di + Mg)/M$
(b) $\bar{e}_i = RMg/\mathcal{B}d$
(c) $\bar{v} = RMg/(\mathcal{B}d)^2$

10.14 (a) $(J_R + J_L)\dot{\omega} + (B_R + B_L)\omega = \gamma k_\phi i_F(t)i_A(t)$
(b) $\bar{\omega} = [\gamma k_\phi/(B_R + B_L)]\bar{i}_F\bar{i}_A$
(c) $(J_R + J_L)\dot{\hat{\omega}} + (B_R + B_L)\hat{\omega} = \gamma k_\phi[\bar{i}_F\hat{i}_A(t) + \bar{i}_A\hat{i}_F(t)]$

10.17 (a) $di_A/dt = (-R_A i_A - \gamma k_\phi i_F\omega + E_A)/L_A$, $di_F/dt =$
$[-R_F i_F + e_F(t)]/L_F$, $\dot{\omega} = [\gamma k_\phi i_F i_A - B\omega - \tau_L(t)]/J$
(b) $\bar{\omega} = \gamma k_\phi\bar{i}_F\bar{i}_A/B$ where $\bar{i}_F = \bar{e}_F/R_F$ and
$\bar{i}_A = BR_F^2E_A/[BR_AR_F^2 + (\gamma k_\phi\bar{e}_F)^2]$
(c) $d\hat{i}_A/dt = (-R_A\hat{i}_A - \gamma k_\phi\bar{\omega}\hat{i}_F - \gamma k_\phi\bar{i}_F\hat{\omega})/L_A$, $d\hat{i}_F/dt =$
$[-R_F\hat{i}_F + \hat{e}_F(t)]/L_F$, $\dot{\hat{\omega}} = [\gamma k_\phi\bar{i}_F\hat{i}_A + \gamma k_\phi\bar{i}_A\hat{i}_F - B\hat{\omega} - \hat{\tau}_L(t)]/J$

10.21 $\omega_{ss} = 37.5$ rad/s

CHAPTER 11

11.2 $R_{eq} = R_aR_b/(2R_a + 4R_b)$, $q = [(4/R_a) + (2/R_b)](\theta_1 - \theta_a)$

11.5 (a) $\dot{\theta}_1 = [-(1/R_{eq})\theta_1 + (1/R_1)\theta_a + (1/R_2)\theta_2(t) + q_i(t)]/C$ where
$1/R_{eq} = (1/R_1) + (1/R_2)$
(b) $H_1(s) = (1/R_2)/[Cs + (1/R_{eq})]$, $H_2(s) = 1/[Cs + (1/R_{eq})]$

11.7 (a) $\dot{\theta}_1 = [-(1/R_1 + 1/R_2)\theta_1 + (1/R_2)\theta_2 + (1/R_1)\theta_a + q_1(t)]/C_1$,
$\dot{\theta}_2 = [(1/R_2)\theta_1 - (1/R_2 + 1/R_3)\theta_2 + (1/R_3)\theta_3(t) + q_2(t)]/C_2$
(b) $\dot{\hat{\theta}}_1 = [-(1/R_1 + 1/R_2)\hat{\theta}_1 + (1/R_2)\hat{\theta}_2 + q_1(t)]/C_1$, $\dot{\hat{\theta}}_2 =$
$[(1/R_2)\hat{\theta}_1 - (1/R_2 + 1/R_3)\hat{\theta}_2 + (1/R_3)\hat{\theta}_3(t) + q_2(t)]/C_2$, $H_1(s) =$
$R_1/P(s)$ and $H_2(s) = (R_1R_2C_1s + R_1 + R_2)/P(s)$ where $P(s) =$
$R_1R_2R_3C_1C_2s^2 + [R_1C_1(R_2 + R_3) + R_3C_2(R_1 + R_2)]s + R_1 + R_2 + R_3$

11.9 (b) $\theta_m = \{C_w\theta_w(0) + C_m\theta_m(0) + C_w[\theta_m(0) - \theta_w(0)]\epsilon^{-t/\tau}\}/(C_m +$
$C_w)$, $\theta_w =$
$\{C_w\theta_w(0) + C_m\theta_m(0) - C_m[\theta_m(0) - \theta_w(0)]\epsilon^{-t/\tau}\}/(C_m + C_w)$
for $t \geq 0$ where $\tau = RC_mC_w/(C_m + C_w)$

(c) $\dot{\theta}_m$ is unchanged,
$\dot{\theta}_w = [(1/R)\theta_m - (1/R + 1/R_a)\theta_w + (1/R_a)\theta_a]/C_w$

11.11 $\dot{\hat{\theta}}_1 = (9/RC)[-2\hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_i(t)]$, $\dot{\hat{\theta}}_2 =$
$(9/RC)(\hat{\theta}_1 - 2\hat{\theta}_2 + \hat{\theta}_3)$, $\dot{\hat{\theta}}_3 = (9/RC)(\hat{\theta}_2 - \hat{\theta}_3)$, $P(s) =$
$s^3 + (45/RC)s^2 + [486/(RC)^2]s + [729/(RC)^3]$

CHAPTER 12

12.1 $C = \pi h^2/\rho g$, $p^* = \rho g\sqrt[3]{3v/\pi}$

12.5 $\dot{\hat{p}}_1 = [-(1/K)\hat{p}_1 + (1/K)\hat{p}_2 + \hat{w}_i(t)]/C_1$,
$\dot{\hat{p}}_2 = [(1/K)\hat{p}_1 - (1/K + 1/R)\hat{p}_2]/C_2$

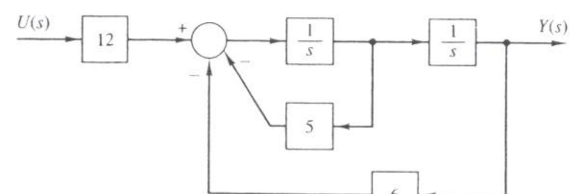
12.7 (b) $\hat{W}_o(s)/\hat{P}_i(s) =$
$[(R_3C_1s + 1)/R_2 + (1/R_1)]/[(R_4C_2s + 1)(R_3C_1s + 1)]$
(c) $(\hat{w}_o)_{ss} = (1/R_1) + (1/R_2)$

12.10 (b) $\tau = \beta A/\alpha\rho g$, $h_{ss} = \beta/\rho g$
(c) $h(t) = (\beta/\rho g)(1 - \epsilon^{-t/\tau})$ for $t \geq 0$

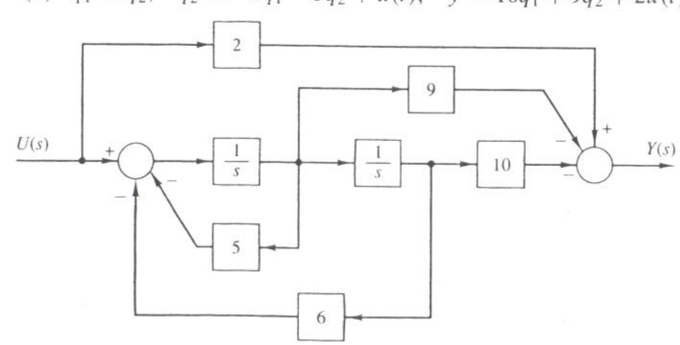
CHAPTER 13

13.3 $T_1(s) = (3s - 6)/(s^3 + 7s^2 + 14s + 8)$, $T_2(s) =$
$(2s^4 + 14s^3 + 28s^2 + 13s + 6)/(s^4 + 7s^3 + 14s^2 + 8s)$

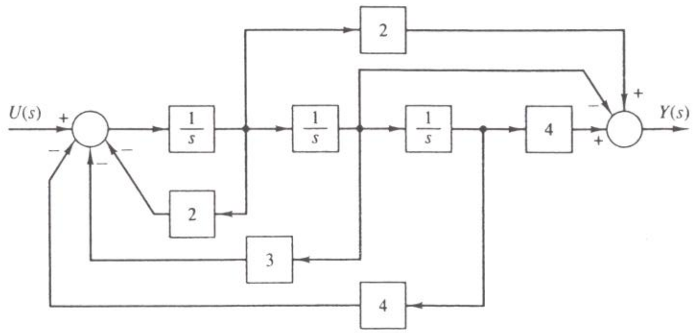
13.7 (a) $\dot{q}_1 = q_2$, $\dot{q}_2 = -6q_1 - 5q_2 + 12u(t)$, $y = q_1$



(b) $\dot{q}_1 = q_2$, $\dot{q}_2 = -6q_1 - 5q_2 + u(t)$, $y = 10q_1 + 9q_2 + 2u(t)$



(c) $\dot{q}_1 = q_2, \quad \dot{q}_2 = q_3, \quad \dot{q}_3 = -4q_1 - 3q_2 - 2q_3 + u(t)$
$y = 4q_1 - q_2 + 2q_3$



13.24 $Y(s)/U(s) = 2/(s^2 + 8s + 9)$

13.26 $X(s)/\tau_a(s) = (K/R)/P(s)$ where $P(s) = JMs^4 + (MB_1 + JB_2)s^3 + [MK + B_1B_2 + (JK/R^2)]s^2 + [B_2K + (B_1K/R^2)]s$

13.30 (a) $T(s) = 2/(s^2 + 9s + 2K)$
(b) $y_{ss} = 1/K$
(c) $\zeta = 9/(2\sqrt{2K}), \quad \omega_n = \sqrt{2K}, \quad K = 20.25, \quad \omega_n = 6.364$ rad/s

13.32 (a) $T_1(s) = 4/(s^2 + 4Ks + 4)$
(b) $T_2(s) = s/(s^2 + 4Ks + 4)$
(c) $\omega_n = 2$ rad/s, $\zeta = K$, so use $K = 1$.
(d) For $u(t) = U(t)$, $y_{ss} = 1$; for $v(t) = U(t)$, $y_{ss} = 0$.

13.34 (a) $T(s) = K_1/[s^2 + (K_2 + 1)s + K_3]$
(b) $\zeta = (K_2 + 1)/(2\sqrt{K_3}), \quad \omega_n = \sqrt{K_3}$
(c) $K_2 > -1$
(d) $-1 < K_2 < 2\sqrt{K_3} - 1$
(e) $K_2 = 1, \quad K_3 = 4$

13.36 $T_1(s) = (AB + BDE + D)/(1 + BE + AC)$,
$T_2(s) = A/(1 + BE + AC)$

CHAPTER 14

14.2 (a) $K_A = 0.0218, \quad T(s) = 1.563/(s^2 + 2.50s + 1.563)$
(b) $K_A = 1.396, \quad K_T = 0.0836$ V·s/rad, $T(s) = 100/(s^2 + 20s + 100)$

14.5 (a) $T(s) = 1/(s^2 + 2Ks + 4)$
(b) Poles at $s = -K \pm \sqrt{K^2 - 4}$; for $K = 0$, $s = \pm j2$; for $K = 1$, $s = -1 \pm j\sqrt{3}$; for $K = 2$, $s = -2, -2$; for $K = 3$, $s = -0.764, -5.236$

14.9 $\sigma_0 = -2, \quad K^* = 75.4$

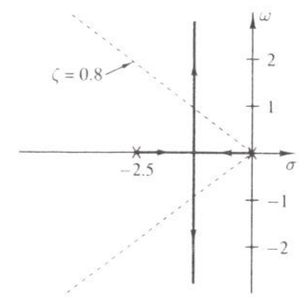
14.14 $\sigma_0 = -5.5, \quad K^* = 1825$

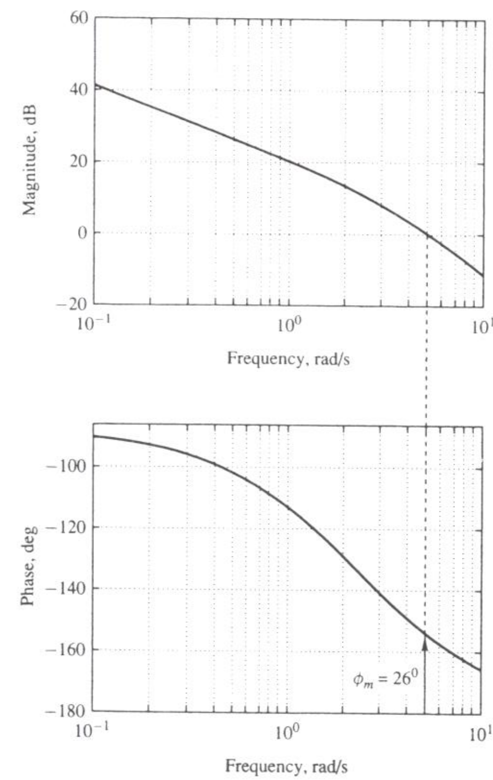
14.16 $\sigma_0 = -2, \quad K^* = -64/9 = -7.11$

14.24 (a) $k_m = 11.1$ dB, $\phi_m = 28.9°$
(b) $K = 45.1$
(c) $K^* = 143$

14.26 (a) $H(s) = -(R_2Cs + 1)/(R_1Cs)$
(b) Pole at $s = 0$, zero at $s = -1/(R_2C)$
(c) PI compensator

14.28 (c) (i) $\theta_{ss} = 0.400$ rad, (ii) $\theta_{ss} = -0.384$ rad
(d) $K_A = 0.0341$ V/V, $\omega_n = 1.563$ rad/s
(e) $\phi_m = 26°$, phase is always $\geq -180°$.





14.32 (a) The block diagram is based on the equation
$\hat{\Theta}(s) = \tau\hat{Q}_h(s)/[C(\tau s + 1)] + \hat{Q}_i(s)/(\tau s + 1)$.

(b) $\hat{\Theta}(s)/\hat{\Theta}_d(s) = \tau G_c(s)/P(s)$, $\hat{\Theta}(s)/\hat{\Theta}_i(s) = C/P(s)$ where $P(s) = \tau Cs + C + \tau G_c(s)$

(c) For unit step in $\hat{\theta}_d(t)$, $\hat{\theta}_{ss} = \tau K_c/(C + \tau K_c)$; for unit step in $\hat{\theta}_i(t)$, $\hat{\theta}_{ss} = C/(C + \tau K_c)$

(d) $\hat{\Theta}(s)/\hat{\Theta}_d(s) = \tau K_c(s + K_I)/P(s)$ and $\hat{\Theta}(s)/\hat{\Theta}_i(s) = Cs/P(s)$ where $P(s) = \tau Cs^2 + (C + \tau K_c)s + \tau K_c K_I$

CHAPTER 15

15.3 (a) $T(s) = (6s^3 + 9s^2 + 49s + 79)/(s^4 + 7s^3 + 18s^2 + 45s + 41)$

(b) Zeros are $s = 0.0429 \pm j2.881,\ -1.586$; poles are $s = -0.4218 \pm j2.502,\ -1.316,\ -4.840$; gain = 6

15.5 (a) $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2 & -4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$\mathbf{C} = \begin{bmatrix} -3 & -7 & -8 \end{bmatrix}$, $\mathbf{D} = 4$

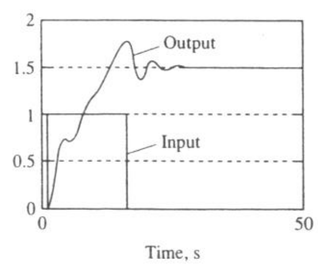
(b) $T(s) = (4s^3 - 6s + 2)/(s^3 + 4s^2 + 2s + 2)$

15.7 Zeros are $s = -2,\ -5$; poles are $s = 0,\ -1.500 \pm j2.398,\ -12.00$; gain = 2; $T(s) = (2s^2 + 14s + 20)/(s^4 + 15s^3 + 44s^2 + 96s)$

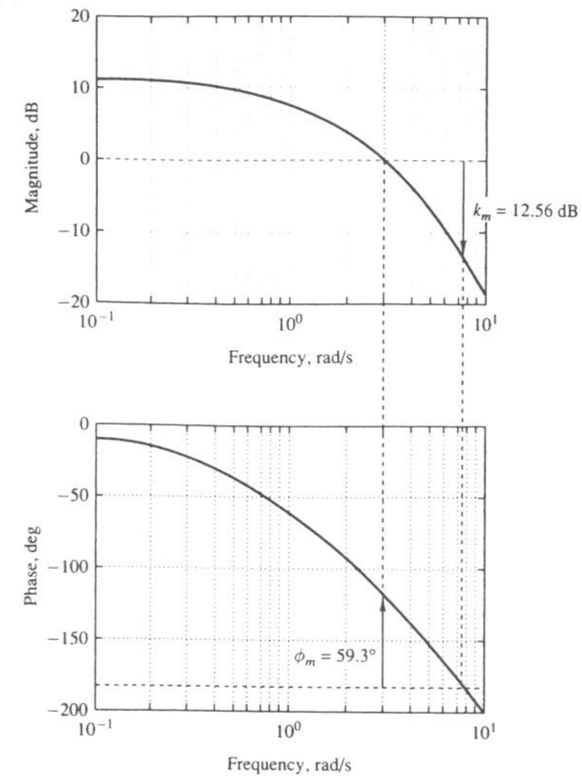
15.12 Zeros are $s = -3,\ -5$; poles are $s = -1.0956,\ -6.952 \pm j2.536$, $T(s) = (4s^2 + 32s + 60)/(s^3 + 15s^2 + 70s + 60)$,

$\mathbf{A} = \begin{bmatrix} -15 & -70 & -60 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 4 & 32 & 60 \end{bmatrix}$, $\mathbf{D} = 0$

15.15



15.19 (a)

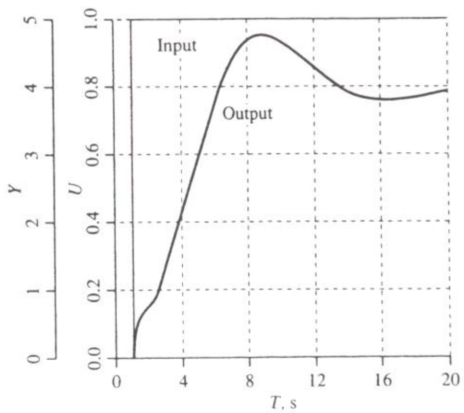


(b) $k_m = 12.56$ dB, $\omega_{gm} = 7.21$ rad/s, $\phi_m = 59.3°$, $\omega_{pm} = 3.00$ rad/s

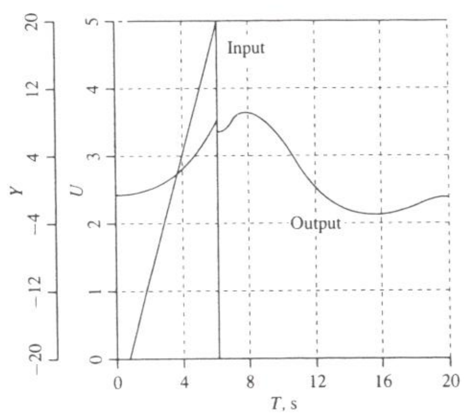
(c) $K = 636$

15.24 (b) $K = 39.7$, $s = -0.574 \pm j2.00,\ -2.50,\ -7.36$

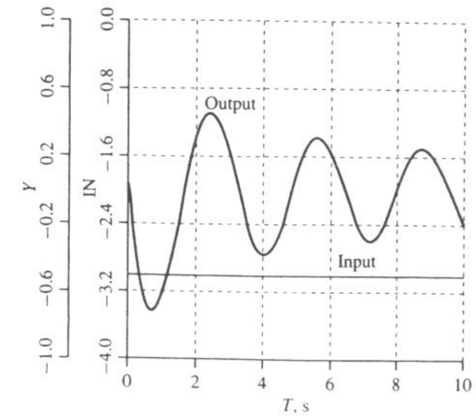
15.25 (a) Unit step function starting at $t = 1$ s.



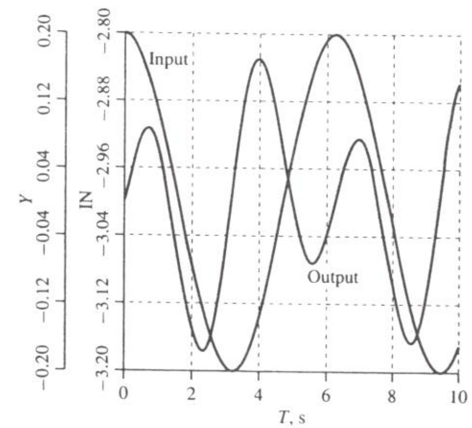
(b) Triangular signal in Figure P15.16.



15.28 (a)



(b)



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