Sinks Tests

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Contents

1 PorousFlow sink boundary conditions

A number of different sink boundary conditions have been implemented in PorousFlow. (To make these into sources instead of sinks, the strength of the flux just needs to be made negative.) All the sinks are implemented using full upwinding. This is to prevent the sink from attempting to remove fluid from a node that actually contains no fluid.

The basic sink uses a Function to specify the flux on the boundary, and also has the option of multiplying by any combination of: the fluid mobility, the relative permeability, or a mass fraction. These latter multiplying factors are all useful in the case of sinks to prevent an unlimited amount of fluid being withdrawn from the porous medium, which can lead to extremely poor nonlinear convergence even if only one node in the entire mesh is "running dry".

Derived from the basic one, is another boundary condition that allows the flux to be modified by a piecewise-linear function of porepressure, which is useful for the case where transfer coefficients are defined across the boundary, or more complicated situations.

Also derived from the basic one are two others, in which the flux is governed by a half Gaussian or half cubic function of porepressure, which are useful for modelling evapotranspiration through a boundary.

1.1 Basic sink

1.1.1 Test 1

A sink flux of strength $6\,\mathrm{kg.m^{-2}.s^{-1}}$ is applied to the left edge ($x = 0$) of a 3D mesh. A single-phase, single-component fluid is used, and the porepressure is initialised to $p = y + 1$ (for $0 \leq y \leq 1$). No fluid flow within the element is used, so the masses of fluid at the finite-element nodes behave independently. The fluid is assumed to have density $\rho = 1.1 \exp(p/1.3)$ [$\mathrm{kg.m^{-3}}$]. The porosity is 0.1.

Under these conditions, assuming $p \geq 0$ so that the porous medium is fully saturated with fluid, the fluid mass at a node should obey

$$m = V\phi\rho = V \times 0.1 \times 1.1 \exp(p/1.3) = m(t = 0) - 6At \ , \tag{1.1}$$

where V is the volume occupied by the node, and A is its area exposed to the flux. MOOSE correctly produces this result, as illustrated in Figure 1.1.

1.1.2 Test 2

An identical setup to Test 1 is used here, but with the sink flux strength being multiplied by the mobility:

$$\text{mobility} = n_i k_{ij} n_j \rho/\nu \ , \tag{1.2}$$

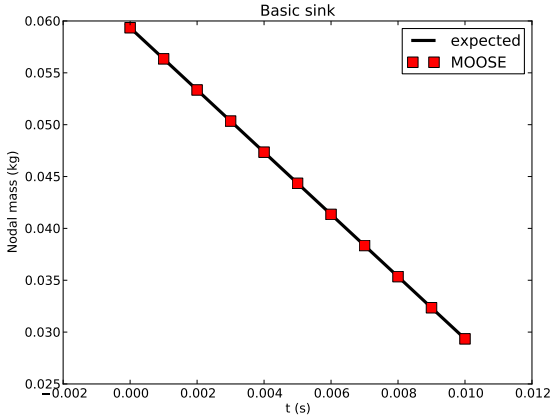

Figure 1.1: Results of Test 1, illustrating that MOOSE correctly applies a constant sink flux to boundary nodes.

where $n_i k_{ij} n_j$ is the permeability tensor k projected onto the normal direction to the boundary n, and the fluid density and viscosity are ρ and ν, respectively. In this example $\nu = 1.1\,\mathrm{Pa.s}$ and $n_i k_{ij} n_j = 0.2\,\mathrm{m}^2$. The other parameters are the same as Test 1, except now the strength of the flux is $6\,\mathrm{Pa.s}^{-1}$.

In this case, the expected result is (for $p > 0$)

$$\frac{\mathrm{d}m}{\mathrm{d}t} = V\phi\frac{\mathrm{d}\rho}{\mathrm{d}t} = -6A\frac{n_i k_{ij} n_j \rho}{\mu} \ . \tag{1.3}$$

MOOSE correctly produces this result, as illustrated in Figure 1.2.

1.1.3 Test 3

An identical setup to Test 1 is used here, but with the sink flux strength being multiplied by the relative permeability, which is chosen to be:

$$\kappa_{\mathrm{rel}} = S^2 \ , \tag{1.4}$$

with S being the fluid saturation. A van-Genuchten capillary relationship is used:

$$S = \left(1 + (-\alpha p)^{1/(1-m)}\right)^{-m} \ , \tag{1.5}$$

with $\alpha = 1.1\,\mathrm{Pa}^{-1}$, and $m = 0.5$. The porepressure is initialised to be $p = -y$. The other parameters are identical to Test 1.

In this case, the expected result is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = V\phi\frac{\mathrm{d}\rho S}{\mathrm{d}t} = -6AS^2 \ . \tag{1.6}$$

MOOSE correctly produces this result, as illustrated in Figure 1.3.

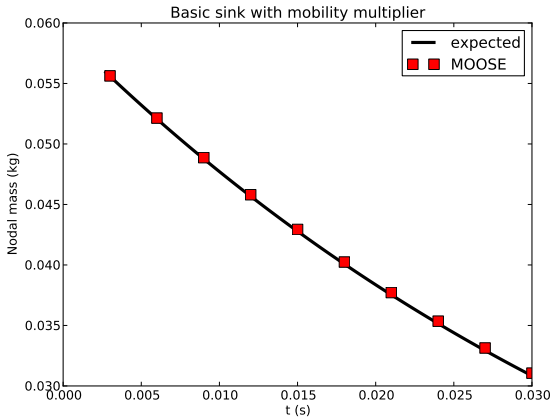

Figure 1.2: Results of Test 2, illustrating that MOOSE correctly applies a constant sink flux modified by the fluid mobility. (A slight drift away from the expected result is due to MOOSE taking large time steps.)

1.1.4 Test 4

A similar setup to Test 1 is used here, but with a 3-component, single-phase fluid, with the sink flux only extracting the second component, with a rate proportional to the mass fraction of that component. This test checks that the flux is correctly implemented (see Figure 1.4) and that the correct fluid component is being withdrawn from the correct nodes. MOOSE produces the expected result.

1.1.5 Test 5

A sink is applied to the left edge ($x = 0$) of a 3D mesh. A 3-component, 2-phase fluid is used. Call the two phases "water" and "gas". The porepressures are initialised to $p_{\rm water} = y$ and $p_{\rm gas} = y + 3$. The mass fractions are initialised to $(0.3, 0.35, 0.35)$ in the water phase, and $(0.1, 0.8, 0.1)$ in the gas phase. The water phase is assumed to have density $\rho_{\rm water} = 1.5 \exp(p_{\rm water}/2.3)$, and the gas phase $\rho_{\rm gas} = 1.1 \exp(p_{\rm gas}/1.3)$. A van-Genuchten capillary relationship is used:

$$S_{\rm water} = \left(1 + (\alpha(p_{\rm gas} - p_{\rm water})^{1/(1-m)}\right)^{-m} \ , \tag{1.7}$$

with $\alpha = 1.1\,{\rm Pa}^{-1}$, and $m = 0.5$. The water relative permeaility is assumed to be Corey type with exponent 1, and the gas phase has exponent 2 (that is $\kappa_{\rm rel,gas} = S_{\rm gas}^2$, with $S_{\rm gas} = 1 - S_{\rm water}$).

The sink flux acts only on the second component. It is multiplied by the relative permeability of the gas phase, and the mass fraction of the second component in the gas phase. This is possibly meaningless physically, but acts as a good test of the PorousFlowSink. In this test the mass fractions remain fixed: there is nothing to induce a

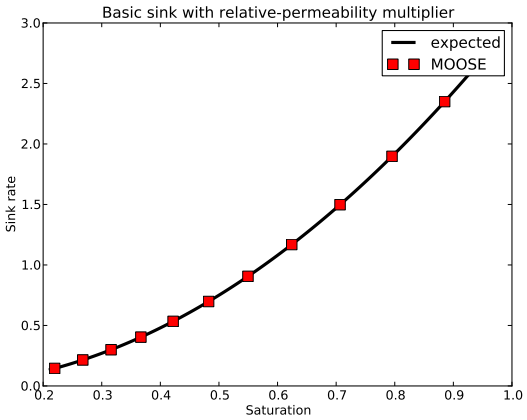

Figure 1.3: Results of Test 3, illustrating that MOOSE correctly applies a constant sink flux modified by the fluid relative permeability.

change of a component from the water phase to the gas phase since the only Kernels used are mass-conservation Kernels that simply demand mass conservation of each fluid component (summed over each phase).

The test checks whether MOOSE is correctly applying the sink flux, and that the fluid-component masses at the nodes respond correctly to the flux. Figure 1.5 demonstrates that MOOSE produces the expected result.

1.2 Piecewise-linear sink

A sink flux of strength

$$f = \begin{cases} 8 & \text{if } p > 0.8 \\ 8(p + 0.2) & \text{if } 0.3 \leq p \leq 0.8 \\ 4 & \text{if } p < 0.3 \ , \end{cases} \tag{1.8}$$

(measured in $\mathrm{kg.m^{-2}.s^{-1}}$) is applied to the right side ($x = 1$) of a 3D mesh. A single-phase, single-component fluid is used, and the porepressure is initialised to $p = y + 1$ (for $0 \leq y \leq 1$). No fluid flow within the element is used, so the masses of fluid at the finite-element nodes behave independently. The fluid is assumed to have density $\rho = 1.1 \exp(p/1.3)$ [$\mathrm{kg.m^{-3}}$]. The porosity is 0.1.

Under these conditions, the expected result for the fluid mass at a node on the right side of the mesh is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = V\phi \frac{\mathrm{d}\rho S}{\mathrm{d}t} = -fA \ . \tag{1.9}$$

The notation is the same as in previous sections.

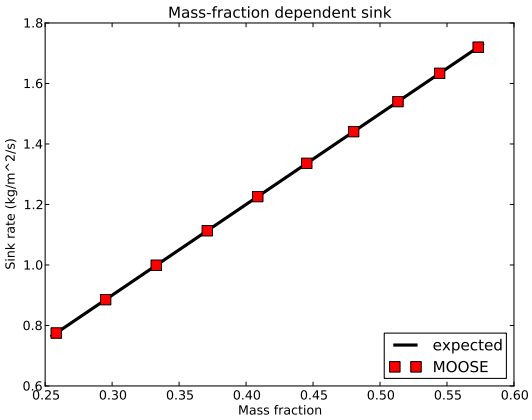

Figure 1.4: Results of Test 4, illustrating that MOOSE correctly applies a sink flux of a particular fluid component proportional to the component's mass fraction.

The test checks that the mass evolves according to this equation, and that the flux is applied correctly. Figure 1.6 demonstrates agreement with the expected flux and the MOOSE implementation.

1.3 Half-Gaussian sink

A sink flux of strength

$$f = \begin{cases} 6 & \text{if } p \geq 0.9 \\ 6\exp\left(-\frac{1}{2}\left(\frac{p-0.9}{0.5}\right)^2\right) & \text{if } p < 0.9 \ . \end{cases} \tag{1.10}$$

(measured in $\mathrm{kg.m^{-2}.s^{-1}}$) is applied to the right side ($x = 1$) of a 3D mesh. This is a half-Gaussian sink with center 0.9 Pa, standard deviation 0.5 Pa and maximum 6. A single-phase, single-component fluid is used, and the porepressure is initialised to $p = y + 1.4$ (for $0 \leq y \leq 1$). No fluid flow within the element is used, so the masses of fluid at the finite-element nodes behave independently. The fluid is assumed to have density $\rho = 1.1\exp(p/1.3)$ [$\mathrm{kg.m^{-3}}$]. The porosity is 0.1. A van-Genuchten capillary relationship is used:

$$S = \left(1 + (-\alpha p)^{1/(1-m)}\right)^{-m} \ , \tag{1.11}$$

with $\alpha = 1.1\,\mathrm{Pa^{-1}}$, and $m = 0.5$.

Under these conditions, the expected result for the fluid mass at a node on the right side of the mesh is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = V\phi\frac{\mathrm{d}\rho S}{\mathrm{d}t} = -fA \ . \tag{1.12}$$

The notation is the same as in previous sections.

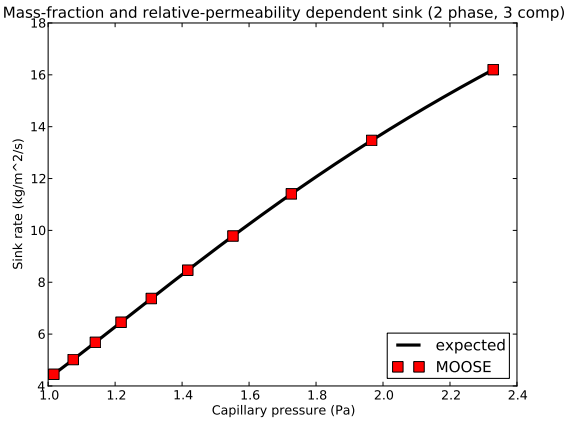

Figure 1.5: Results of Test 5, illustrating that in a 2-phase system MOOSE correctly applies a sink flux of a particular fluid component proportional to the component's mass fraction and the relative permeaility of the gas phase.

The test checks that the mass evolves according to this equation, and that the flux is applied correctly. Figure 1.7 demonstrates agreement with the expected flux and the MOOSE implementation.

1.4 Half-cubic sink

A sink flux of strength

$$f = \begin{cases} 3 & \text{if } p \geq 0.9 \\ \frac{3}{(-0.8)^3}(2(p-0.9)+0.8)(p-0.9+0.8)^2 & \text{if } 0.1 < p < 0.9 \\ 0 & \text{if } p \leq 0.1 \end{cases} \tag{1.13}$$

(measured in $\mathrm{kg.m^{-2}.s^{-1}}$) is applied to the right side ($x = 1$) of a 3D mesh. This is a half-cubic sink with center 0.9 Pa, cutoff -0.8 Pa, and maximum $3\,\mathrm{kg.m^{-2}.s^{-1}}$. A single-phase, single-component fluid is used, and the porepressure is initialised to $p = x(y+1)$ (for $0 \leq y \leq 1$ and $0 \leq x \leq 1$). No fluid flow within the element is used, so the masses of fluid at the finite-element nodes behave independently. The fluid is assumed to have density $\rho = 1.1\exp(p/1.3)$ $[\mathrm{kg.m^{-3}}]$. The porosity is 0.1.

Under these conditions, the expected result for the fluid mass at a node on the right side of the mesh is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = V\phi\frac{\mathrm{d}\rho S}{\mathrm{d}t} = -fA\ . \tag{1.14}$$

The notation is the same as in previous sections.

The test checks that the mass evolves according to this equation, and that the flux is applied correctly. Figure 1.8 demonstrates agreement with the expected flux and the MOOSE implementation.

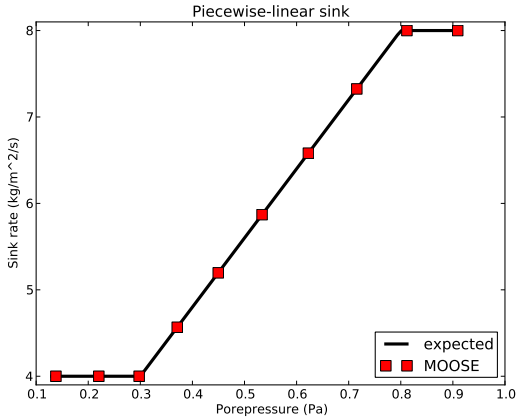

Figure 1.6: A piecewise-linear sink flux is correctly modelled by MOOSE

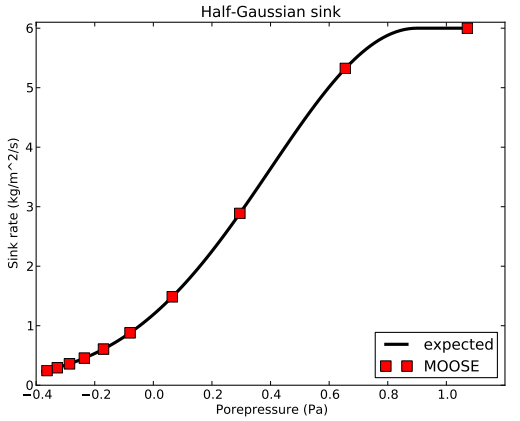

Figure 1.7: A half-Gaussian sink flux with center 0.9 Pa and standard deviation 0.5 Pa is correctly modelled by MOOSE

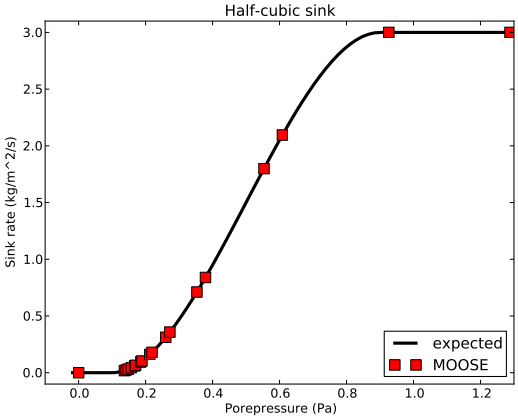

Figure 1.8: A half-cubic sink with center 0.9 Pa, cutoff −0.8 Pa, and maximum 3 $\mathrm{kg.m^{-2}.s^{-1}}$ is correctly modelled by MOOSE.

2 Advection of a fluid component

The porepressure at a boundary may be maintained at a fixed value by applying a sufficiently strong piecewise-linear sink:

$$f = Cp \ , \tag{2.1}$$

(measured in kg.m^{-2}.s^{-1}) for large[1] conductance C. In the multi-component case, the flux of of fluid component κ should be made proportional to the component mass fraction, χ^κ:

$$f^\kappa = C\chi^\kappa p \ . \tag{2.2}$$

This is a "natural" boundary condition, in that fluid exits or enters the porous material at a rate dictated by the mass-fraction within the porous material. This means, for instance, that if fluid is exiting ($p > 0$ in this case) then only components that exist at the boundary system will exit, and MOOSE will not attempt to extract fluid components that have zero mass-fraction.

This example concerns a 1D porous material occupying the space $0 \leq x \leq 1$. It contains a single phase fluid with two fluid components. The porous material initially only contains fluid component 1, and there is a pressure gradient:

$$p(t = 0) = 1 - x \quad \text{and} \quad \chi^0(t = 0) = 0 \ . \tag{2.3}$$

For $t > 0$, fluid component 0 is introduced on the material's left side ($x = 0$), by applying the fixed boundary conditions:

$$p(x = 0) = 1 \quad \text{and} \quad \chi^0(x = 0) = 1 \ . \tag{2.4}$$

The right-hand side, at $x = 1$, is subjected to the flux of Eqn (2.2). The fluid-component 0 flows from the left side to the right side via the pressure gradient. To simplify the following analysis, the fluid bulk modulus is taken to be very large.

Because the fluid bulk modulus is very large, $\partial P/\partial x = -1$ is a solution for all time. This means that the governing equation reduces to

$$\phi \frac{\partial \chi}{\partial t} = (k_{ij}/\mu)\nabla_j \chi \nabla_i P \ . \tag{2.5}$$

In this equation ϕ is the porosity, k_{ij} is the permeability tensor, and μ is the fluid viscosity. This is just the advection equation for the mass fraction χ, with velocity

$$\text{velocity}_j = \nabla_i P \frac{k_{ij}}{\mu\phi} \ . \tag{2.6}$$

[1] If C is too large then it will dominate the numerics and MOOSE will not converge.

In this test, the parameters are chosen such that velocity$= 1\,\mathrm{m.s}^{-1}$.

The sharp front (described by the advection equation with the initial and boundary conditions) is *not* maintained by MOOSE. This is due to numerical diffusion, which is particularly strong in the upwinding scheme implemented in the PorousFlow module. Nevertheless, MOOSE advects the smooth front with the correct velocity, as shown in Figure 2.1.

The sharp front is *not* maintained by MOOSE even when no upwinding is used. In the case at hand, which uses a fully-saturated single-phase fluid, the `FullySaturated` versions of the Kernels may be used in order to compare with the standard fully-upwinded Kernels. The `FullySaturated` Kernels do not employ any upwinding whatsoever, so less numerical diffusion is expected. This is demonstrated in Figure 2.1. Two additional points may also be nocied: (1) the lack of upwinding has produced a "bump" in the mass-fraction profile near the concentrated side; (2) the lack of upwinding means the temperature profile moves slightly slower than it should. These two affects reduce as the mesh density is increased, however.

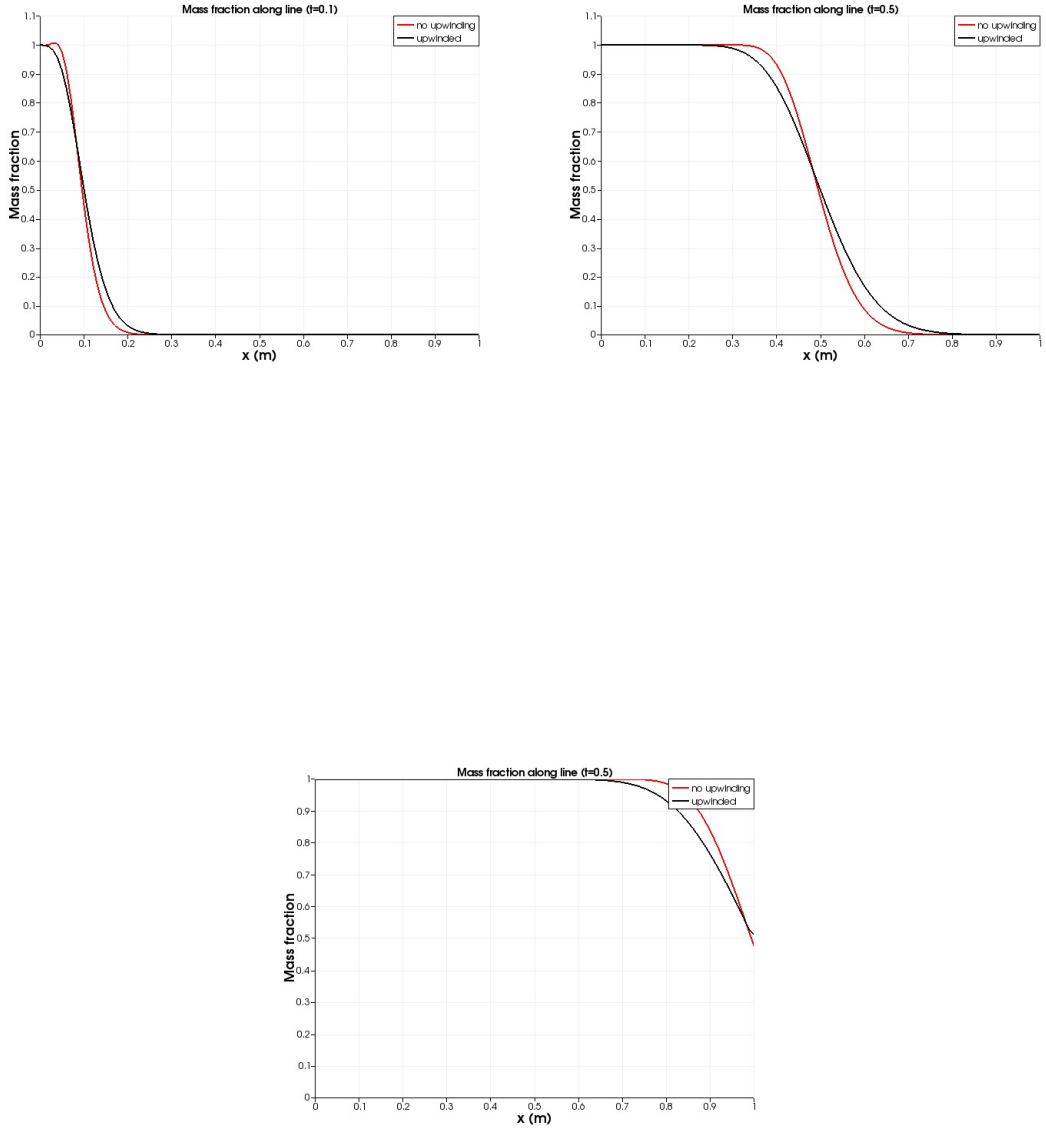

Figure 2.1: Results of the advection of a fluid component test, illustrating that the numerical implementation of porous flow within MOOSE diffuses sharp fronts, but advects them at the correct velocity ($v = 1\,\mathrm{m.s}^{-1}$ in this case, and notice the centre of the front is at the correct position in each picture). Less diffusion is experienced when upwinding is not used. Top left: mass fraction at $t = 0.1\,\mathrm{s}$. Top right: mass fraction at $t = 0.5\,\mathrm{s}$. Bottom: mass fraction at $t = 1.0\,\mathrm{s}$.