Poroelasticitiy Tests

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Contents

1 Introduction **3**

2 Simple tests **4**

2.1 Volumetric expansion due to increasing porepressure 4
2.2 Undrained oedometer test 4
2.3 Porepressure generation of a confined sample 4
2.4 Porepressure generation of an unconfined sample 5

3 Terzaghi consolidation **6**

4 Mandel's consolidation of a drained medium **7**

1 Introduction

The PorousFlow module includes the ability to couple fluid flow to solid mechanics, and thus includes poroelasticity, which is the theory of a fully-saturated single-phase fluid with constant bulk density and constant viscosity coupled to small-strain isotropic elasticity.

There is one important difference between the theories, however. The time-derivative terms of poroelasticity are

$$\frac{1}{M}\dot{P} + \alpha\dot{\epsilon}_{ii} \ , \tag{1.1}$$

where M is the Biot modulus:

$$\frac{1}{M} = \frac{(1-\alpha)(\alpha-\phi)}{K} + \frac{\phi}{K_{\mathrm{f}}} \ , \tag{1.2}$$

P is the fluid porepressure, α is the Biot coefficient, ϵ_{ii} is the volumetric strain, ϕ is the porosity, K is the solid (drained) bulk modulus, and K_{f} is the fluid bulk modulus. Evidently from Eqn (1.2), the Biot modulus, M, should evolve with time as the porosity evolves. Indeed, the terms in Eqn (1.1) are derived from the continuity equation $\partial(\phi\rho)/\partial t + \phi\rho\dot{\epsilon}_{ii}$ using the evolution of ϕ (and that $\rho = \rho_0 \exp(P/K_{\mathrm{f}})$). However, in the standard analytical solutions of poroelasticity theory, the Biot modulus, M is considered fixed.

The PorousFlow module allows porosity to vary with fluid porepressure and volumetric strain, so usually the Biot modulus would vary too, causing differences with the analytical solutions of poroelasticity. Therefore, PorousFlow offers a porosity relationship that evolves porosity in such a way as to keep M fixed. This is called `PorousFlowPorosityHMBiotModulus`.

PorousFlow is also built with finite strains in mind, whereas poroelasticity is not. Therefore, in comparisons with solutions from poroelasticity theory, either the strain should be kept small, or the various finite-strain switches in PorousFlow should be turned off (they are all on by default).

2 Simple tests

2.1 Volumetric expansion due to increasing porepressure

The porepressure within a fully-saturated sample is increased:

$$P_{\mathrm{f}} = t \ . \tag{2.1}$$

Zero mechanical pressure is applied to the sample's exterior, so that no Neumann BCs are needed on the sample. No fluid flow occurs since the porepressure is increased uniformly throughout the sample

The effective stresses should then evolve as $\sigma_{ij}^{\mathrm{eff}} = \alpha t \delta_{ij}$, and the volumetric strain $\epsilon_{00} + \epsilon_{11} + \epsilon_{22} = \alpha t/K$. MOOSE produces this result correctly.

2.2 Undrained oedometer test

A cubic single-element fully-saturated sample has roller BCs applied to its sides and bottom. All the sample's boundaries are impermeable. A downwards (normal) displacement, u_z, is applied to its top, and the rise in porepressure and effective stress is observed. (Here z denotes the direction normal to the top face.) There is no fluid flow in the single element.

Under these conditions, assuming constant porosity, and denoting the height (z length) of the sample by L:

$$\begin{aligned}
P_{\mathrm{f}} &= -K_f \log(1 - u_z) \ , \\
\sigma_{xx}^{\mathrm{eff}} &= (K - \tfrac{2}{3}G)u_z/L \ , \\
\sigma_{zz}^{\mathrm{eff}} &= (K + \tfrac{4}{3}G)u_z/L \ .
\end{aligned} \tag{2.2}$$

2.3 Porepressure generation of a confined sample

A single-element fully-saturated sample is constrained on all sides and its boundaries are impermeable. Fluid is pumped into the sample via a source s (kg.s^{-1}.m^{-3}) and the rise in porepressure is observed.

Denoting the strength of the source by s (units are s^{-1}), the expected result is

$$\begin{aligned}
\text{fluid mass} &= \text{fluid mass}_0 + st \ , \\
\sigma_{ij}^{\mathrm{eff}} &= 0 \ , \\
P_{\mathrm{f}} &= K_{\mathrm{f}} \log(\rho\phi/\rho_0) \ , \\
\rho &= \rho_0 \exp(P_{\mathrm{f}}/K_f) \ , \\
\phi &= \alpha + (\phi_0 - \alpha) \exp\left(P_{\mathrm{f}}(\alpha - 1)/K\right) \ . \qquad (2.3) \\
& \qquad (2.4)
\end{aligned}$$

Here K is the solid bulk modulus.

2.4 Porepressure generation of an unconfined sample

A single-element fully-saturated sample is constrained on all sides, except its top. All its boundaries are impermeable. Fluid is pumped into the sample via a source s ($\mathrm{kg.s}^{-1}.\mathrm{m}^{-3}$)) and the rise in the top surface, the porepressure, and the stress are observed.

Regardless of the evolution of porosity, the following ratios result

$$\begin{aligned}
\sigma_{xx}/\epsilon_{zz} &= K - 2G/3 \ , \\
\sigma_{zz}/\epsilon_{zz} &= K + 4G/3 \ , \\
P/\epsilon_{zz} &= (K + 3G/3 + \alpha^2 M)/\alpha - \alpha M \ . \qquad (2.5)
\end{aligned}$$

where K is the undrained bulk modulus, G the shear modulus, α the Biot coefficient, and M is the initial Biot modulus. MOOSE produces these results when using the `PorousFlowPorosityHM` material.

However, if the Biot modulus, M, is held fixed as the porosity evolves, and the source is

$$s = S\rho_0 \exp(P/K_{\mathrm{f}}) \ , \qquad (2.6)$$

with S being a *constant* volumetric source ($\mathrm{m}^3.\mathrm{s}^{-1}.\mathrm{m}^{-3}$) then

$$\begin{aligned}
\epsilon_{zz} &= \frac{\alpha M st}{K + 4G/3 + \alpha^2 M} \ , \\
P &= M(st - \alpha\epsilon_{zz}) \ , \\
\sigma_{xx} &= (K - 2G/3)\epsilon_{zz} \ , \\
\sigma_{zz} &= (K + 4G/3)\epsilon_{zz} \ . \qquad (2.7)
\end{aligned}$$

MOOSE produces these results when using the `PorousFlowPorosityHMBiotModulus` material.

3 Terzaghi consolidation

This is documented at `http://mooseframework.org/wiki/PhysicsModules/TensorMechanics/TensorMe`
The PorousFlow Material `PorousFlowPorosityHMBiotModulus` must be used.

4 Mandel's consolidation of a drained medium

A sample's dimensions are $-a \leq x \leq a$ and $-b \leq y \leq b$, and it is in plane strain (no z displacement). It is squashed with constant normal force by impermeable, frictionless plattens on its top and bottom surfaces (at $y = \pm b$). Fluid is allowed to leak out from its sides (at $x = \pm a$), but all other surfaces are impermeable. This is called Mandel's problem and it is shown graphically in Fig 4.1

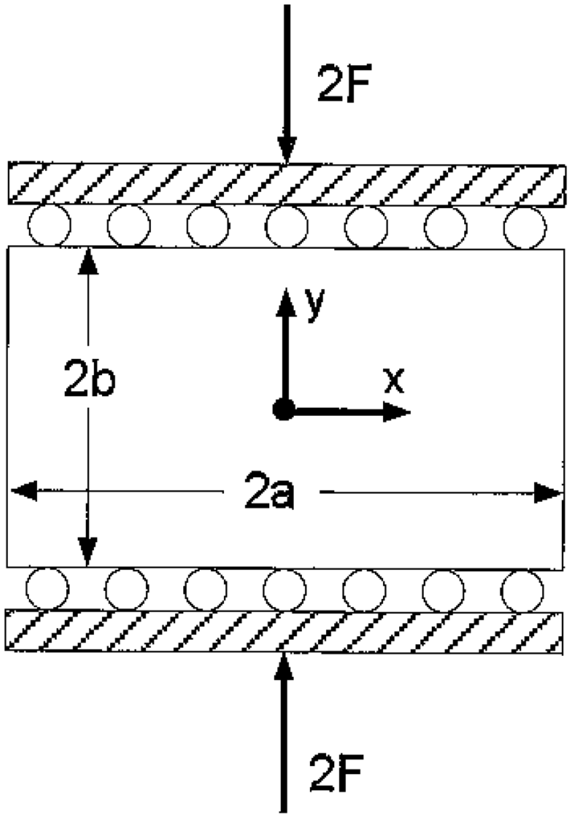

Figure 4.1: The setup of the Mandel experiment: a force F squashes a porous material with impermeable plattens. This causes fluid to seep from the material.

The interesting feature of this problem (apart from that it can be solved analytically) is that the porepressure in the sample's center actually increases for a short time after application of the force. This is because the leakage of the fluid from the sample's sides causes an apparent softening of the material near those sides. This means stress concentrates towards the sample's center which causes an increase in porepressure. Of course, eventually the fluid totally drains from the sample, and the porepressure is zero. As the fluid drains from the sample's sides the plattens move slowly towards each other.

The solution for porepressure and displacements is given in: AHD Cheng and E Detournay "A direct boundary element method for plane strain poroelasticity" International Journal of Numerical and Analytical Methods in Geomechanics 12 (1988) 551-572. The solution involves rather lengthy infinite series, so I will not write it here.

As is common in the literature, this is simulated by considering the quarter-sample, $0 \leq x \leq a$ and $0 \leq y \leq b$, with impermeable, roller BCs at $x = 0$ and $y = 0$ and $y = b$. Porepressure is fixed at zero on $x = a$. Porepressure and displacement are initialised to zero. Then the top ($y = b$) is moved downwards with prescribed velocity, so that the total force that is induced by this downwards velocity is fixed. The velocity is worked out by solving Mandel's problem analytically, and the total force is monitored in the simulation to check that it indeed remains constant.

The simulations in the PorousFlow test suite use 10 elements in the x direction and 1 in the y direction. Four types of simulation are run:

1. HM. This uses standard PorousFlow Materials and Kernels, in particular it uses the "HM" porosity relationship. This is not expected to agree perfectly with the analytical solutions because: the solutions assume constant Biot modulus.

2. constM. This is identical to the HM case, save that it uses a porosity evolution law that keeps the Biot modulus fixed. It is therefore expected to agree with the analytical solutions.

3. FullSat. This uses the FullySaturated versions of the fluid mass time derivative and the fluid flux. In this case the Biot modulus is kept fixed, so it is expected to agree with the analytical solutions.

4. FullSatVol. This uses the FullySaturated versions of the fluid mass time derivative and the fluid flux, and does not multiply by the fluid density. Therefore this version is identical to what is usually implemented in poro-elastic codes. It is linear and therefore converges in only one iteration. In this case the Biot modulus is kept fixed, so it is expected to agree with the analytical solutions.

Of course there are minor discrepancies between the last three and the analytical solution that are brought about through spatial and temporal discretisation errors. The figures below present the results.

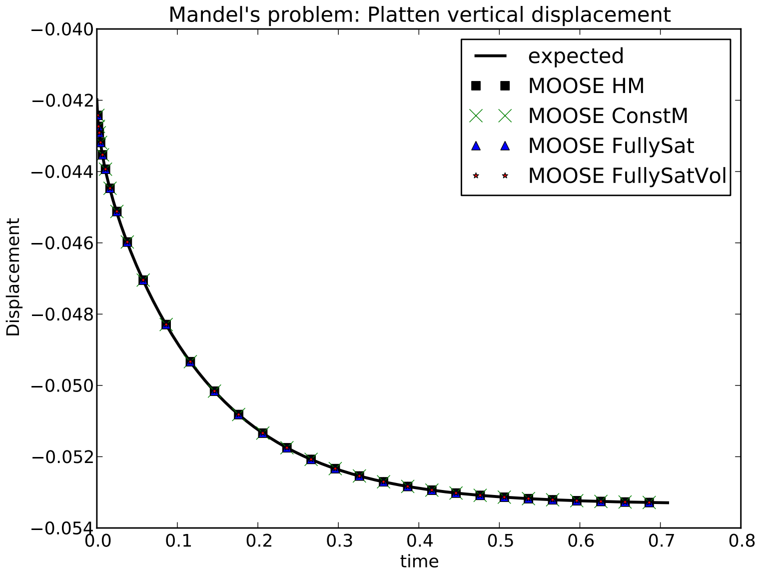

Figure 4.2: The vertical displacement of the platten as a function of time.

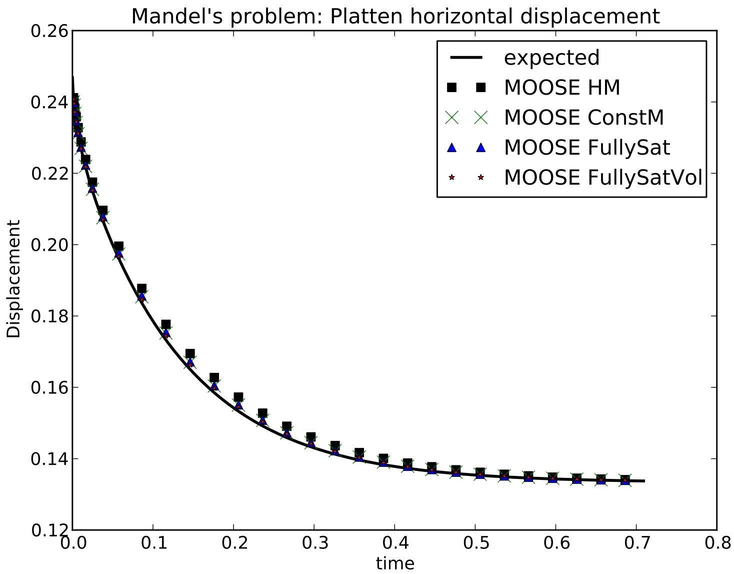

Figure 4.3: The horizontal displacement of the material at $(x, y) = (a, b)$ as a function of time.

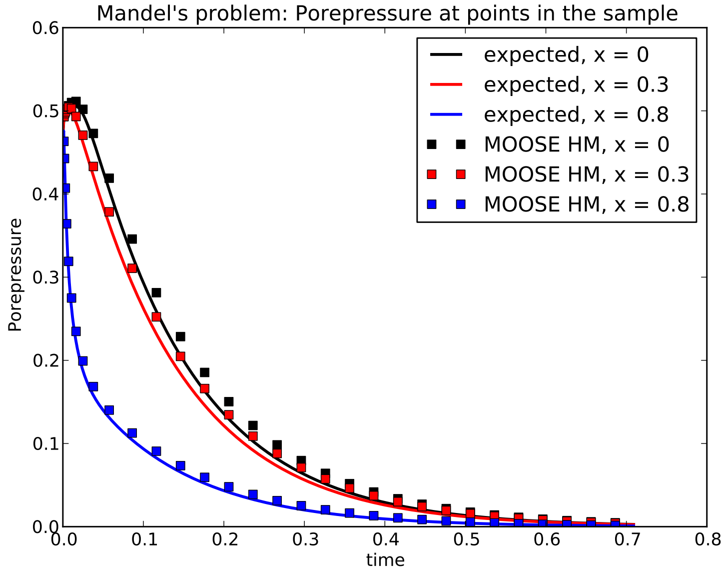

Figure 4.4: The porepressure at various points in the sample in the HM model with $a = 1$.

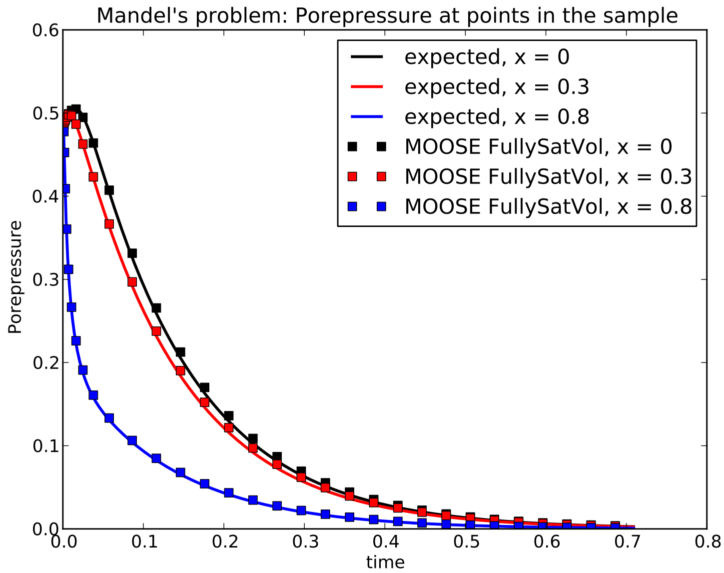

Figure 4.5: The porepressure at various points in the sample in the FullSatVol model with $a = 1$.

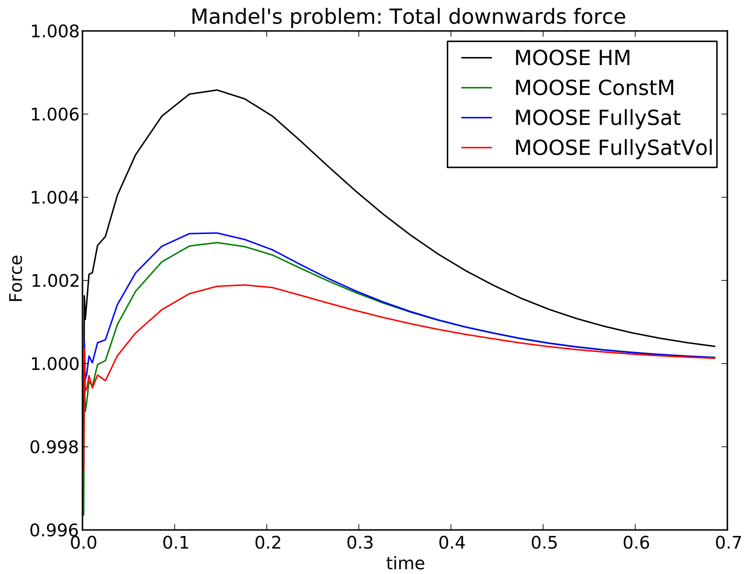

Figure 4.6: The total downwards force on the platten as a function of time. This should be unity.