Heat Advection Tests

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Contents

1 One-dimensional heat advection via a single-phase fluid

Consider the case of a single-phase fluid in 1 dimension, $0 \leq x \leq 1$, with the porepressure fixed at the boundaries:

$$P(x=0,t) = 1 \quad \text{and} \quad P(x=1,t) = 0 \ . \tag{1.1}$$

With zero gravity, and high fluid bulk modulus, the Darcy equation implies that the solution is $P(x,t) = 1 - x$, with

$$v = k/\mu \tag{1.2}$$

being the constant[1] "Darcy velocity" from $x = 0$ to $x = 1$. Here k is the porous medium's permeability, and μ is the fluid dynamic viscosity.

Suppose that the fluid internal energy is given by CT, where C is the specific heat capacity and T is its temperature. Assuming that $P/\rho \ll CT$, then the fluid's enthalpy is also CT. In this case, the energy equation reads

$$((1-\phi)\rho_R C_R + \phi\rho C)\frac{\partial T}{\partial t} + C\rho v\frac{\partial T}{\partial x} = 0 \ . \tag{1.3}$$

This is the wave equation with velocity

$$v_T = \frac{C\rho v}{(1-\phi)\rho_R C_R + \phi\rho C} \ . \tag{1.4}$$

Recall that the "Darcy velocity" is $v = k/\mu$.

Let the initial condition for T be $T(x, t = 0) = 200$. Apply the boundary conditions

$$T(x=0,t) = 300 \quad \text{and} \quad T(x=1,t) = 200 \ . \tag{1.5}$$

At $t = 0$ this creates a front at $x = 0$. Choose the parameters $C = 2$, $C_R = 1$, $\rho = 1000$, $\rho_R = 125$, $\phi = 0.2$, $k = 1.1$, $\mu = 4.4$ (all in consistent units), so that $v_T = 1$ is the front's velocity.

The sharp front is *not* maintained by MOOSE. This is due to numerical diffusion, which is particularly strong in the upwinding scheme implemented in the PorousFlow module. Nevertheless, MOOSE advects the smooth front with the correct velocity, as shown in Figure 1.1.

The sharp front is *not* maintained by MOOSE even when no upwinding is used. In the case at hand, which uses a fully-saturated single-phase fluid, the `FullySaturated`

[1] To get the velocity of the individual fluid particles, this should be divided by the porosity.

versions of the Kernels may be used in order to compare with the standard fully-upwinded Kernels. The `FullySaturated` Kernels do not employ any upwinding whatsoever, so less numerical diffusion is expected. This is demonstrated in Figure 1.1. Two additional points may also be nocied: (1) the lack of upwinding has produced a “bump” in the temperature profile near the hotter side; (2) the lack of upwinding means the temperature profile moves slightly slower than it should. These two affects reduce as the mesh density is increased, however.

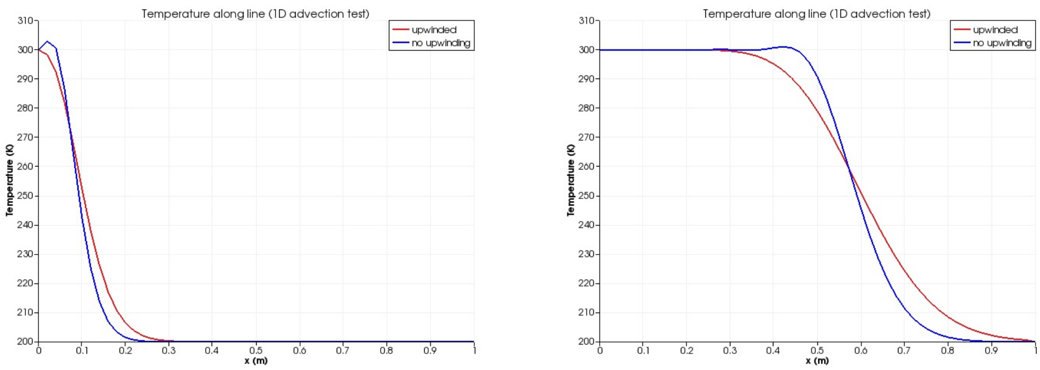

Figure 1.1: Results of heat advection via a fluid in 1D. The fluid flows with constant Darcy velocity of $0.25\,\mathrm{m.s}^{-1}$ to the right, and this advects a temperature front at velocity $1\,\mathrm{m.s}^{-1}$ to the right. The pictures above that the numerical implementation of porous flow within MOOSE (including upwinding) diffuses sharp fronts, but advects them at the correct velocity (notice the centre of the upwinded front is at the correct position in each picture). Less diffusion is experienced without upwinding. Left: temperature $t = 0.1\,\mathrm{s}$. Right: temperature at $t = 0.6\,\mathrm{s}$.