Plastic Heat Tests

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Contents

1 Plastic deformation heating a porous skeleton

In this section, plastic deformation causes a heat energy-density rate (J.m^{-3}.s^{-1}) of

$$c(1-\phi)\sigma_{ij}\epsilon_{ij}^{\mathrm{plastic}} \ , \tag{1.1}$$

where c is a coefficient (s^{-1}), ϕ is the porosity, σ is the stress, and $\epsilon^{\mathrm{plastic}}$ is the plastic strain.

There is no fluid, and no heat flow is studied: the heat energy released simply heats up the porous skeleton:

$$\frac{\partial}{\partial t}(1-\phi)c_R\rho_R T = c(1-\phi)\sigma_{ij}\epsilon_{ij}^{\mathrm{plastic}} \ . \tag{1.2}$$

The porosity (ϕ) and volumetric heat capacity of the rock grains ($c_R\rho_R$) are chosen to be constant.

Perfect capped weak-plane plasticity is used, so that the admissible zone is defined by

$$\begin{aligned} \sigma_{zz} &\leq S_T \ , && (1.3) \\ \sigma_{zz} &\geq -S_C && (1.4) \\ \sqrt{\sigma_{zx}^2+\sigma_{zy}^2}+\sigma_{zz}\tan\Phi &\leq C \ . && (1.5) \end{aligned}$$

Here S_T is the tensile strength, S_C is the compressive strength, C is the cohesion and Φ is the friction angle. The elastic tensor is chosen to be

$$E_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) \ . \tag{1.6}$$

The parameters in these expressions are chosen to be: $S_T = 1$, $S_C = 1$, $C = 1$, $\Phi = \pi/4$, $\lambda = 1/2$, and $\mu = 1/4$ (all in consistent units).

In each experiment a single finite-element is used.

1.1 Tensile failure

The top of the finite element is pulled upwards with displacement:

$$u_z = zt \ , \tag{1.7}$$

while the displacement in the x and y directions is chosen to be zero. This implies the only non-zero component of total strain is

$$\epsilon_{zz}^{\mathrm{total}} = t \ . \tag{1.8}$$

The constitutive law implies

$$\sigma_{zz} = t \ . \tag{1.9}$$

This stress is admissible for $t \leq 1$, while for $t > 1$ the system yields in tension:

$$\sigma_{zz} = 1 \quad \text{for } t > 1 \ , \tag{1.10}$$

and the plastic strain is,

$$\epsilon_{zz}^{\text{plastic}} = t - 1 \quad \text{for } t > 1 \ . \tag{1.11}$$

This means that the material's temperature should increase as

$$c_R \rho_R \dot{T} = c \quad \text{for } t > 1 \ , \tag{1.12}$$

while the right-hand side is zero for $t \leq 1$.

1.2 Compressive failure

The top of the finite element is pushed downwards with displacement:

$$u_z = -zt \ , \tag{1.13}$$

while the displacement in the x and y directions is chosen to be zero. This implies only non-zero component of total strain is

$$\epsilon_{zz}^{\text{total}} = -t \ . \tag{1.14}$$

The constitutive law implies

$$\sigma_{zz} = -t \ . \tag{1.15}$$

This stress is admissible for $t \leq 1$, while for $t > 1$ the system yields in compression

$$\sigma_{zz} = -1 \quad \text{for } t > 1 \ , \tag{1.16}$$

and the plastic strain is,

$$\epsilon_{zz}^{\text{plastic}} = -(t - 1) \quad \text{for } t > 1 \ . \tag{1.17}$$

This means that the material's temperature should increase as

$$c_R \rho_R \dot{T} = c \quad \text{for } t > 1 \ , \tag{1.18}$$

while the right-hand side is zero for $t \leq 1$.

1.3 Shear failure

The top of the finite element is sheared with displacement:

$$u_x = zt \ , \tag{1.19}$$

while the displacement in the y and z directions is chosen to be zero. This implies only non-zero component of total strain is

$$\epsilon_{xz}^{\text{total}} = t \ . \tag{1.20}$$

The constitutive law implies

$$\sigma_{xz} = t/4 \ . \tag{1.21}$$

This stress is admissible for $t \leq 4$, while for $t > 4$ the system yields in shear:

$$\sigma_{xz} = 1 \quad \text{for } t > 4 \ , \tag{1.22}$$

and the plastic strain is,

$$\epsilon_{xz}^{\text{plastic}} = t - 4 \quad \text{for } t > 4 \ . \tag{1.23}$$

This means that the material's temperature should increase as

$$c_R \rho_R \dot{T} = c \quad \text{for } t > 4 \ , \tag{1.24}$$

while the right-hand side is zero for $t \leq 4$.