Newton-Cooling Tests

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Contents

1 Classic Newton cooling in a bar

Without fluids, mechanical deformation and sinks, the heat equation is

$$\rho_R C_R (1-\phi)\dot{T} = \nabla\lambda\nabla T \ , \tag{1.1}$$

where ϕ is the porosity, ρ_R is the rock grain density (kg.m^{-3}), C_R is the rock grain specific heat capacity (J.kg^{-1}.K^{-1}), T is the temperature, and λ is the tensorial thermal conductivity of the porous material (J.s^{-1}.K^{-1}.m^{-1}).

In section 2, the dynamics of this equation is explored, while this section concentrates on the steady-state situation. Consider the one-dimensional case where a bar sits between $x = 0$ and $x = L$ with a fixed temperature at $x = 0$:

$$T(x = 0, t) = T_0 \ , \tag{1.2}$$

and a sink flux at the other end:

$$\text{sink strength (J.m}^{-2}\text{.s}^{-1}) = \lambda \left.\frac{\partial T}{\partial x}\right|_{x=L} = -C\,(T - T_e)_{x=L} \ . \tag{1.3}$$

Here T_e is a fixed quantity ("e" stands for "external"), and C is a constant conductance (J.m^{-2}.s^{-1}.K^{-1}).

The solution is the linear function

$$T = T_0 + \frac{T_e - T_0}{\lambda + CL} Cx \ . \tag{1.4}$$

The heat sink in Eqn (1.3) is a linear function of T, so the `PorousFlowPiecewiseLinearSink` may be employed.

The simulation is run in MOOSE using $C = 1$, $L = 100$, $\lambda = 100$, $T_0 = 2$ and $T_e = 1$. The solution is shown in Figure 1.1

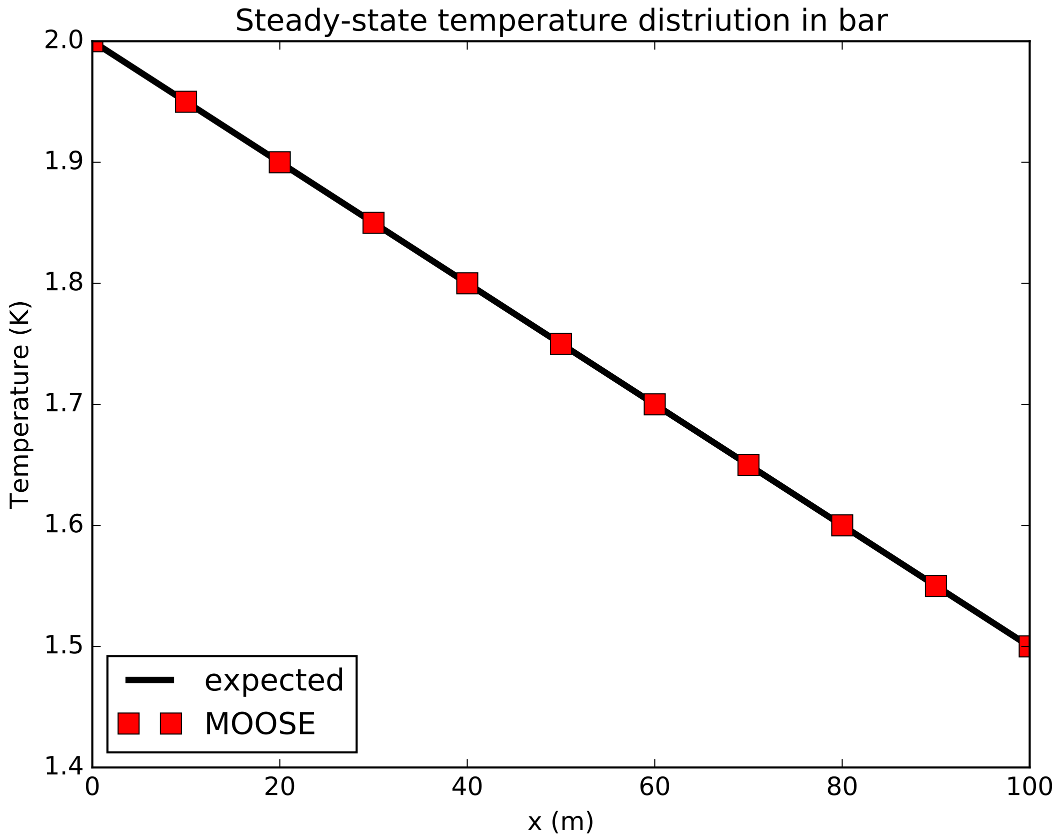

Figure 1.1: The steady-state temperature in the bar. MOOSE agrees well with theory illustrating that piecewise-linear heat sinks/sources and heat conduction are correctly implemented in MOOSE.

2 Porepressure sink in a bar

These tests demonstrate that MOOSE behaves correctly when a simulation contains a sink. The sink is a piecewise linear function of pressure.

Darcy's equation for (single-phase) flow through a fully saturated medium without gravity and without sources is

$$\frac{\partial}{\partial t}\phi\rho = \nabla_i \left(\frac{\rho\kappa_{ij}}{\mu} \nabla_j P \right) \ , \tag{2.1}$$

with the following notation:

- ϕ is the medium's porosity;
- ρ is the fluid density;
- κ_{ij} is the permeability tensor;
- μ is the fluid viscosity;
- $\partial/\partial t$ and ∇_i denote the time and spatial derivatives, respectively.

Using $\rho \propto \exp(P/B)$, where B is the fluid bulk modulus, Darcy's equation becomes

$$\frac{\partial}{\partial t}\rho = \nabla_i \alpha_{ij} \nabla_j \rho \ , \tag{2.2}$$

with

$$\alpha_{ij} = \frac{\kappa_{ij} B}{\mu\phi} \ . \tag{2.3}$$

Here the porosity and bulk modulus are assumed to be constant in space and time.

Consider the one-dimensional case where a bar sits between $x = 0$ and $x = L$ with initial pressure distribution so $\rho(x, t = 0) = \rho_0(x)$. Maintain the end $x = 0$ at constant pressure, so that $\rho(x = 0, t) = \rho_0(0)$. At the end $x = L$, prescribe a sink flux

$$\left.\frac{\partial \rho}{\partial x}\right|_{x=L} = -C\left(\rho - \rho_e\right)_{x=L} \ , \tag{2.4}$$

where ρ_e is a fixed quantity ("e" stands for "external"), and C is a constant conductance. This corresponds to the flux

$$\left.\frac{\partial P}{\partial x}\right|_{x=L} = -CB\left(1 - e^{(P_e - P)/B}\right)_{x=L} \ , \tag{2.5}$$

which can easily be coded into a MOOSE input file: the flux is $\rho\kappa\nabla P/\mu = -CB\kappa(e^{P/B} - e^{P_e/B})/\mu$, and this may be represented by a piecewise linear function of pressure.

The solution of this problem is well known and is

$$\rho(x,t) = \rho_0(0) - \frac{\rho_0(0) - \rho_e}{1 + LC} Cx + \sum_{n=1}^{\infty} a_n \sin\frac{k_n x}{L} e^{-k_n^2 \alpha t/L^2} \ , \tag{2.6}$$

where k_n is the n^{th} positive root of the equation $LC \tan k + k = 0$ (k_n is a little bigger than $(2n-1)\pi/2$), and a_n is determined from

$$a_n \int_0^L \sin^2 \frac{k_n x}{L}\,\mathrm{d}x = \int_0^L \left(\rho_0(x) - \rho_0(0) + \frac{\rho_0(0) - \rho_e}{1 + LC} Cx\right) \sin\frac{k_n x}{L}\,\mathrm{d}x \ , \tag{2.7}$$

which may be solved numerically (Mathematica is used to generate the solution in Figure 2.1).

The problem is solved in MOOSE using the following parameters:

Bar length	100 m
Bar porosity	0.1
Bar permeability	10^{-15} m^2
Gravity	0
Water density	1000 kg.m^{-3}
Water viscosity	0.001 Pa.s
Water bulk modulus	1 MPa
Initial porepressure P_0	2 MPa
Environmental pressure P_e	0
Conductance C	0.05389 m^{-1}

This conductance is chosen so at steadystate $\rho(x = L) = 2000$ kg.m^{-3}.

The problem is solved using 1000 elements along the x direction ($L = 100$ m), and using 100 time-steps of size 10^6 s. Using fewer elements or fewer timesteps means the agreement with the theory is marginally poorer. Two tests are performed: one with transient flow, and one using the steadystate solver. In this case the initial condition is $P = 2 - x/L$ MPa, since the uniform $P = 2$ MPa does not converge. The results are shown in Figure 2.1.

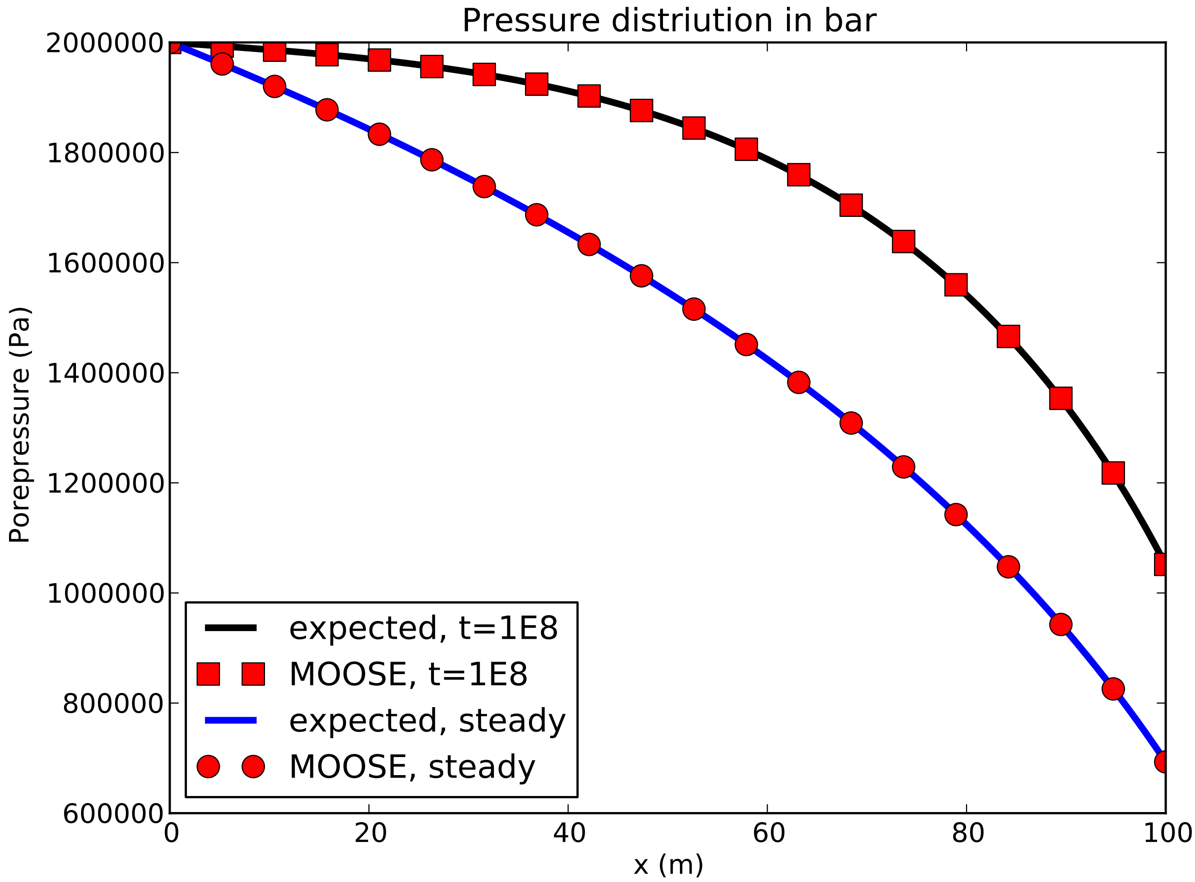

Figure 2.1: The porepressure in the bar at $t = 10^8$ s, and at steadystate. The pressure at $x = 0$ is held fixed, while the sink is applied at $x = 100$ m. MOOSE agrees well with theory demonstrating that piecewise-linear sinks/sources and single-phase Darcy fluid flow are correctly implemented in MOOSE.

3 Porepressure sink in a bar with heat

The simulation of Section 2 is re-run, but this time heat flow is included. In this section it is assumed that the fluid specific enthalpy (J.kg^{-1}) is exactly equal to the fluid internal energy, and that internal energy is ideal:

$$h = \mathcal{E} = C_v T \ . \tag{3.1}$$

This makes the arguments below simple without having to consider real fluids with complicated enthalpy and density expressions.

At the left end of the bar, the temperature is kept fixed:

$$T(x = 0, t) = T_0 \ . \tag{3.2}$$

At the other end of the bar, heat is removed only by the fluid flowing out of the system. That is, there is a heat sink:

$$\text{sink strength (J.m}^{-2}\text{.s}^{-1}) = -C\mathcal{E}\,(\rho - \rho_e)_{x=L} \ . \tag{3.3}$$

No other sinks or sources are applied to the heat equation.

With this setup, the steady-state temperature in the bar must be exactly

$$T(x, t = \infty) = T_0 \ . \tag{3.4}$$

For consider the fluid flowing from $x = 0$ to $x = L$ in order to assume steady-state. At $x = 0$ it must have temperature T_0 because that temperature is fixed at $x = 0$. It advects this temperature with it as it moves, so therefore at $t = \infty$, this temperature has permeated throughout the entire bar. This occurs even without heat conduction, and is independent of the initial temperature of the bar.

MOOSE produces this result exactly.

4 Hot ideal fluid in a bar

This test uses a similar setup to Section 3, except that here an ideal fluid is used. The use of an ideal gas simplifies the equations. Only the steady-state is studied in this section.

The governing equation for the fluid's porepressure P is

$$\nabla \frac{\rho\kappa}{\mu} \nabla P = 0 \ . \tag{4.1}$$

It is assumed that κ and μ are constant, and that

$$\rho = \frac{MP}{RT} \ , \tag{4.2}$$

holds (this is the ideal gas equation of state). In this formula M is the gas molar mass, R is the gas constant and T is the temperature.

The equation governing the temperature is assumed to be just the fluid advection equation

$$\nabla \frac{h\rho\kappa}{\mu} \nabla P = 0 \ . \tag{4.3}$$

As in Section 3, heat conduction could be added, but it is actually irrelevant since the solution to the problem below is constant T. The enthalpy, h, for an ideal gas is

$$h = C_v T + \frac{P}{\rho} = C_v T + \frac{RT}{M} = C_p T \ . \tag{4.4}$$

The boundary conditions at the left-hand end are

$$P(x = 0) = P_0 \quad \text{and} \quad T(x = 0) = T_0 \ . \tag{4.5}$$

Physically these correspond to fluid and heat being removed or added to the left-hand end by some external source in order to keep the porepressure and temperature fixed.

The porepressure boundary condition at the right-hand end of the bar is

$$\text{sink flux (kg.m}^{-2}\text{.s}^{-1}) = \left.\frac{\rho\kappa}{\mu} \nabla P\right|_{x=L} = \left.-C \frac{\rho\kappa}{\mu} (P - P_e)\right|_{x=L} \ . \tag{4.6}$$

Physically this corresponds to the mass-flow through the boundary being proportional to $P - P_e$. Here P_e is a fixed "environmental" porepressure, and this acts as a source or sink of fluid. C is the "conductance" of the boundary. Notice the appearence of $\rho\kappa/\mu$ in the LHS of this equation means that this is truly a flux of fluid mass (measured in kg.m^{-2}.s^{-1}), and the appearence of $\rho\kappa/\mu$ on the RHS means that a `PorousFlowPiecewiseLinearFlux` may be used with `use_mobility=true`.

The temperature boundary condition at the right-hand end of the bar is

$$\text{heat flux (J.m}^{-2}\text{.s}^{-1}) = \frac{h\rho\kappa}{\mu}\nabla P\Big|_{x=L} = -C\frac{h\rho\kappa}{\mu}(P - P_e)\Big|_{x=L} \ . \tag{4.7}$$

Comparing this with Equation 4.6, it is seen that this is exactly the heat loss (or gain) at the boundary corresponding to the loss (or gain) of the fluid. Notice the appearence of $h\rho\kappa/\mu$ in the LHS of this equation means that this is truly a flux of fluid mass (measured in J.m^{-2}.s^{-1}), and the appearence of $h\rho\kappa/\mu$ on the RHS means that a `PorousFlowPiecewiseLinearFlux` may be used with `use_mobility=true` and `use_enthalpy=true`.

There is a clear similarity between the fluid and heat equations. The heat equation does not actually depend on temperature, and is simply

$$0 = \nabla(P\nabla P) \ , \tag{4.8}$$

which is solved by

$$P^2 = P_0^2 + Ax \ . \tag{4.9}$$

The fluid equation then yields

$$T(x) = T_0 \ . \tag{4.10}$$

The constant A may be determined from the either of the boundary conditions. For the special case of $P_e = 0$ and $2LC = 1$, the solution is

$$P = P_0\sqrt{1 - \frac{x}{2L}} \ . \tag{4.11}$$

MOOSE produces this result exactly, as illustrated in Figure 4.1

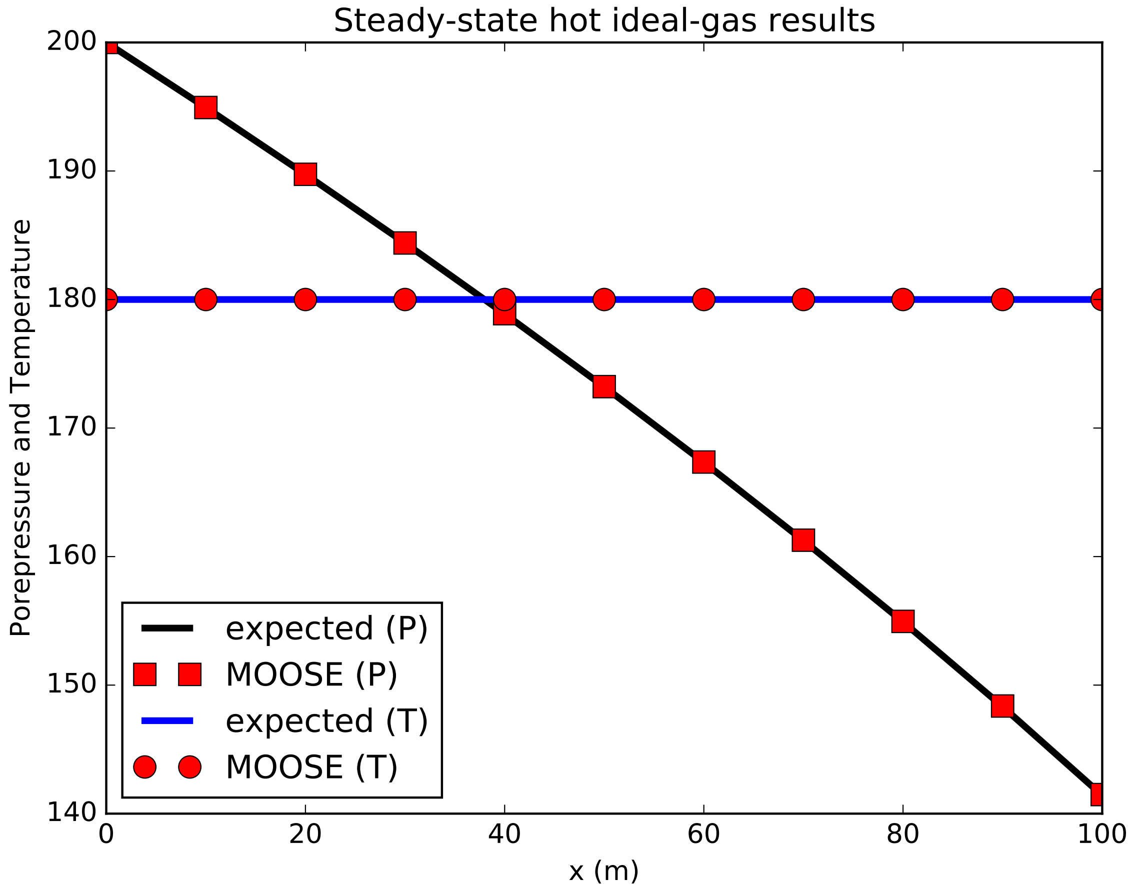

Figure 4.1: The steady-state porepressure and temperature distributions in the bar ($P_0 = 200$ and $T_0 = 180$). MOOSE agrees well with theory illustrating that piecewise-linear fluid and heat sinks/sources as well as ideal fluids are correctly implemented in MOOSE.