DiracKernel Tests

CSIRO

January 6, 2017

Contents

1 Geometric tests

The test suite contains tests that demonstrate:

- when a point sink is placed at a node, it withdraws fluid (or heat) from only that node;
- when a point sink that is proportional to mobility (or relative permeability, etc) is placed in an element where some nodes have zero mobility (or relative permeability, etc), then fluid (or heat) is not extracted from those nodes.

2 Peaceman borehole fluxes

The automatic test suite contains four tests that check that the Peaceman flux

$$f(P_i, x_i) = W\,|C(P_i - P_{\mathrm{bh}})|\,\frac{k_{\mathrm{r}}\rho}{\mu}(P_i - P_{\mathrm{bh}}) \tag{2.1}$$

is correctly implemented. A vertical borehole is placed through the centre of a single element, and fluid flow to the borehole as a function of porepressure is measured. The tests are

- A production borehole with $P_{\mathrm{bh}} = 0$, with a fully-saturated medium.
- An injection borehole with $P_{\mathrm{bh}} = 10\,\mathrm{MPa}$, with a fully-saturated medium.
- A production borehole with $P_{\mathrm{bh}} = -1\,\mathrm{MPa}$, with an unsaturated medium.
- An injection borehole with $P_{\mathrm{bh}} = 0$, with an unsaturated medium.

The parameters common to these tests are:

Element size	$2 \times 2 \times 2\,\mathrm{m}^3$
Borehole radius	$0.1\,\mathrm{m}$
Permeability	$10^{-12}\,\mathrm{m}^2$
Gravity	0
Unit fluid weight	0
Fluid reference density	$1000\,\mathrm{kg.m}^{-3}$
Fluid bulk modulus	$2\,\mathrm{GPa}$
Fluid viscosity	$10^{-3}\,\mathrm{Pa.s}$
Van Genuchten α	$10^{-5}\,\mathrm{Pa}$
Van Genuchten m	0.8
Residual saturation	0
FLAC relperm m	2

It is remotely possible that the MOOSE implementation *applies* the borehole flux incorrectly, but *records* it as a Postprocessor correctly as specified by Eqn (2.1). Therefore, these four simulations also record the fluid mass and mass-balance error in order to check that the fluid mass is indeed being correctly changed by the borehole. Figure 2.1 demonstrates that Eqn (2.1) is indeed correctly implemented in MOOSE.

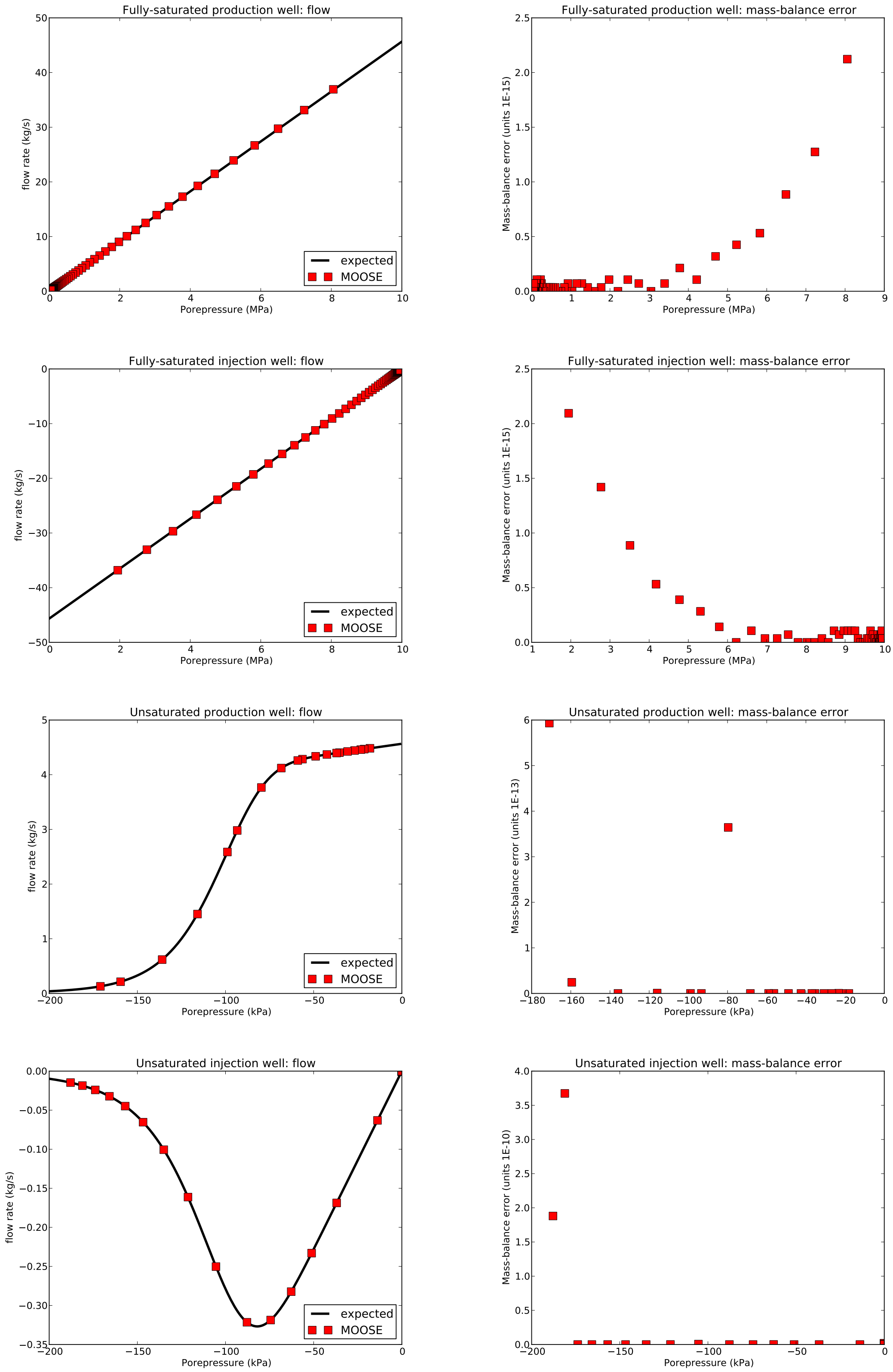

Figure 2.1: Left figures: Comparison between the MOOSE result (in dots), and the expected behaviour of the borehole flux given by Eqn (2.1) (as a line) for the cases listed in the text. Right figures: The mass balances, which are all small.

3 Comparison with analytic solution

The Richards' equation for a fully-saturated medium with $\rho \propto \exp(P/B)$ and large constant bulk modulus B becomes Darcy's equation

$$\frac{\partial}{\partial t}\rho = \nabla_i \alpha_{ij} \nabla_j \rho \tag{3.1}$$

where $\alpha_{ij} = k_{ij}B/(\mu\phi)$, with notation described in the Theory Manual. In the isotropic case (where $k_{ij} = \kappa\delta_{ij}$), the steadystate equation is just Laplace's equation

$$\nabla^2 \rho = 0 \ , \tag{3.2}$$

Place a borehole of radius r_{bh} and infinite length oriented along the z axis. Then the situation becomes 2D and can be solved in cylindrical coordinates, with $\rho = \rho(r, \theta)$ and independent of z. If the pressure at the borehole wall $r = r_{\mathrm{bh}}$ is P_{bh}, then the fluid density is $\rho_{\mathrm{bh}} \propto \exp(P_{\mathrm{bh}}/B)$. Assume that at $r = R$ the fluid pressure is held fixed at P_R, or equivalently the density is held fixed at ρ_R. Then the solution of Laplace's equation is well-known to be

$$\rho = \rho_{\mathrm{bh}} + (\rho_R - \rho_{\mathrm{bh}})\frac{\log(r/r_{\mathrm{bh}})}{\log(R/r_{\mathrm{bh}})} \ . \tag{3.3}$$

This is the fundamental solution used by Peaceman and others to derive expressions for W by comparing with numerical expressions resulting from Eqn (2.1) (see Theory Manual for more details).

Chen and Zhang (see Theory manual) have derived an expression for W in the case where this borehole is placed at a node in a square mesh. This test compares the MOOSE steadystate solution with a single borehole with W defined by Chen and Zhang's formula is compared with Eqn (3.3) to illustrate that the MOOSE implementation of a borehole is correct.

Figure 3.2 shows this comparison. Most parameters in this study are identical to those given in the above table with the following exceptions: the mesh is shown in Fig 3.1; the permeability is $10^{-11}\,\mathrm{m}^2$; the borehole radius is $1\,\mathrm{m}$; the borehole pressure is $P_{\mathrm{bh}} = 0$; the outer radius is $r = 300\,\mathrm{m}$; and the outer pressure is $P_R = 10\,\mathrm{MPa}$.

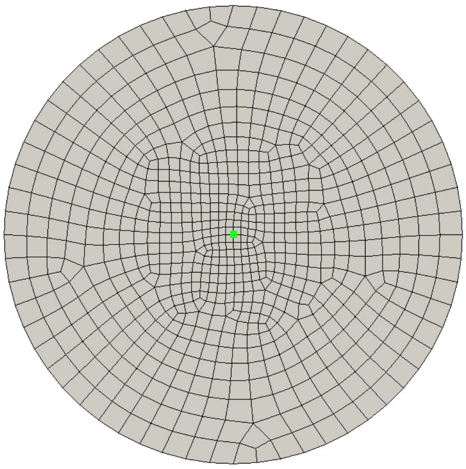

Figure 3.1: The mesh used in the comparison with Eqn (3.3), with the green dot indicating the position of the borehole. The central elements are $10 \times 10\,\mathrm{m}^2$, and the outer boundary is at $r = 300\,\mathrm{m}$.

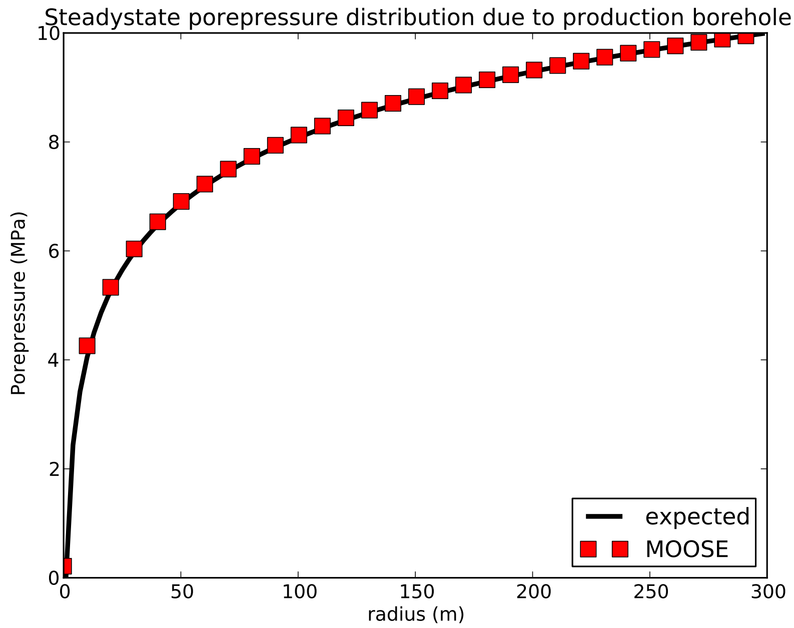

Figure 3.2: Comparison of the MOOSE results (dots) with the analytical solution Eqn (3.3) for the steadystate porepressure distribution surrounding single borehole.