Pressure-Pulse Tests

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1 Presssure-pulse in 1D

Richards' equation for flow through a fully saturated medium without gravity and without sources is just Darcy's equation

$$\frac{\partial}{\partial t}\phi\rho = \nabla_i\left(\frac{\rho\kappa_{ij}}{\mu}\nabla_j P\right) , \tag{1.1}$$

with notation described in the Theory Manual. Using $\rho \propto \exp(P/K)$, where K is the fluid bulk modulus, Darcy's equation becomes

$$\frac{\partial}{\partial t}\rho = \nabla_i\alpha_{ij}\nabla\rho , \tag{1.2}$$

with

$$\alpha_{ij} = \frac{\kappa_{ij}B}{\mu\phi} . \tag{1.3}$$

Here I've assumed the porosity and bulk modulus are constant in space and time.

Consider the one-dimensional case were the spatial dimension is the semi-infinite line $x \geq 0$. Suppose that initially the pressure is constant, so that

$$\rho(x, t = 0) = \rho_0 \quad \text{for} \quad x \geq 0 . \tag{1.4}$$

Then apply a fixed-pressure Dirichlet boundary condition at $x = 0$ so that

$$\rho(x = 0, t > 0) = \rho_\infty \tag{1.5}$$

The solution of the above differential equation is well known to be

$$\rho(x,t) = \rho_\infty + (\rho_0 - \rho_\infty)\,\mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) , \tag{1.6}$$

where Erf is the error function.

This is verified by using the following tests on a line of 10 elements.

1. Steady state 1-phase analysis to demonstrate that the steady-state of $\rho = \rho_\infty$ is achieved.
2. Transient 1-phase analysis.
3. Transient 1-phase, 3 component analysis to check that the components diffuse at the same rate.
4. Transient 2-phase analysis, with the "water" state fully saturated.

An example verification is shown in Figure 1.1. These are part of the automatic test suite.

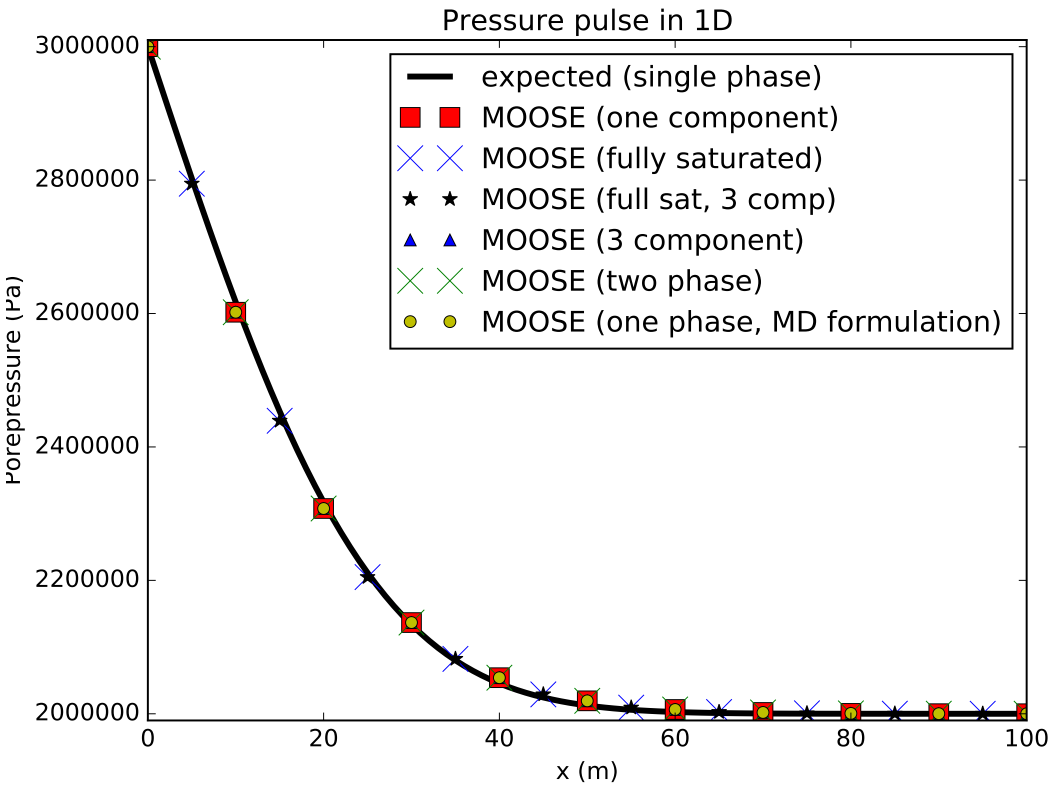

Figure 1.1: Comparison between the MOOSE result (in dots), and the exact analytic expression given by Eqn (1.6). This test had 10 elements in the x direction, with $0 \le x \le 100\,\mathrm{m}$, and ran for a total of 10^4 seconds with 10 timesteps. The parameters were $B = 2\,\mathrm{GPa}$, $\kappa_{xx} = 10^{-15}\,\mathrm{m}^2$, $\mu = 10^{-3}\,\mathrm{Pa.s}$, $\phi = 0.1$, with initial pressure $P = 2\,\mathrm{MPa}$, and applied pressure $P = 3\,\mathrm{MPa}$ at $x = 0$. For greater spatial resolution and smaller timesteps the agreement increases. Both the multi-component single-phase simulation (using the fully-saturated non-upwinding Kernels and the partially-saturated full-upwinding Kernels) and the 2-phase fully-water-saturated simulation give identical results for the water porepressure.