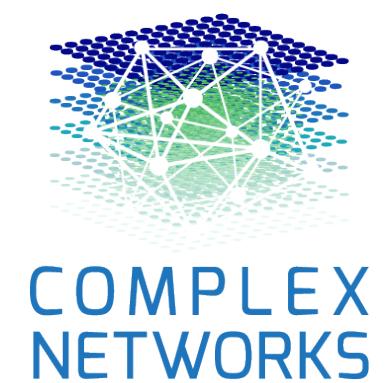
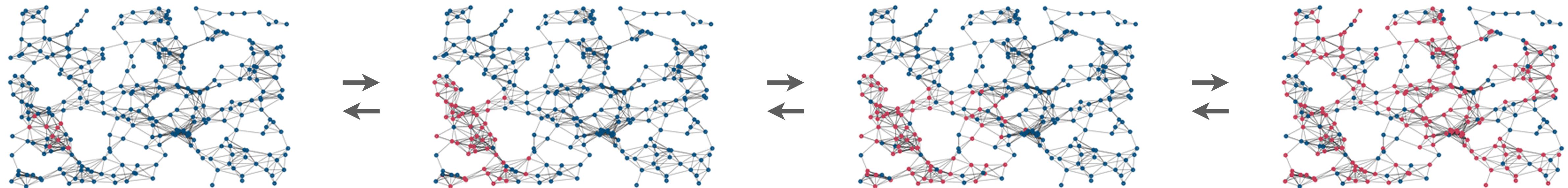


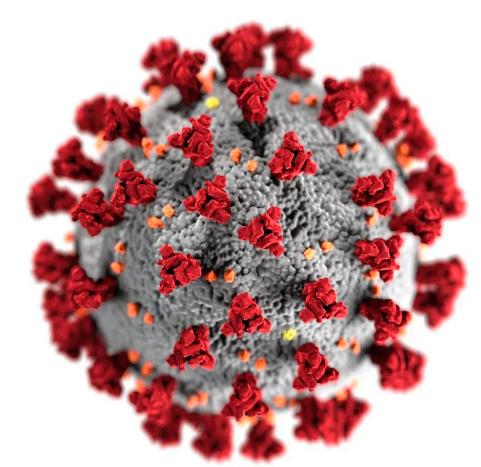
Birth-Death Processes Reproduce the Epidemic Footprint

Gerrit Großmann Michael Backenköhler





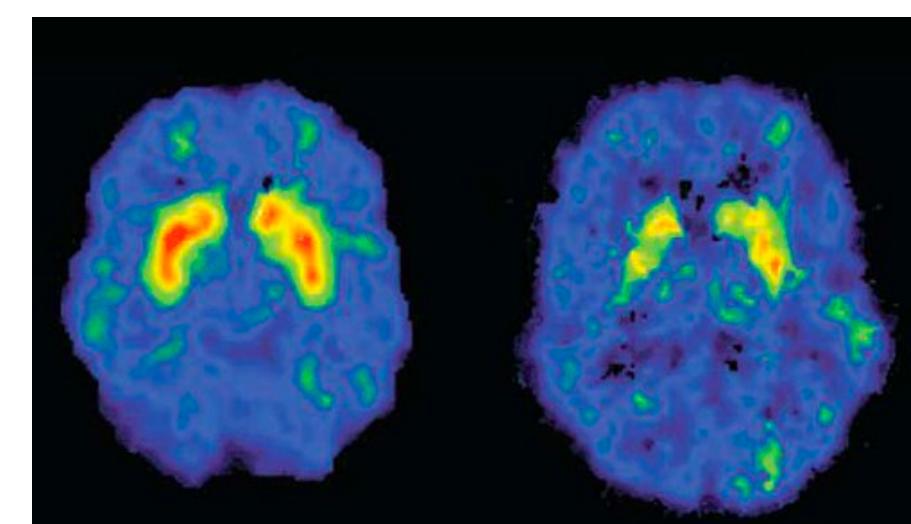
SIS Model: Information diffusion on networks



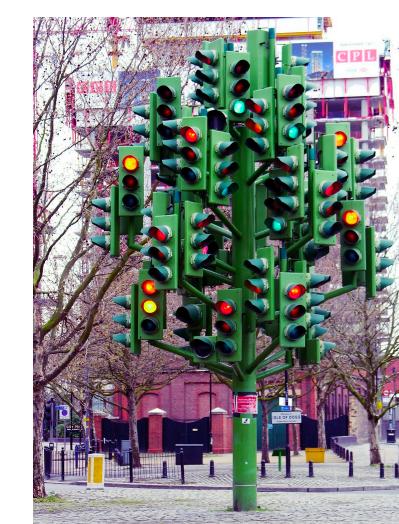
Epidemics



Fake news

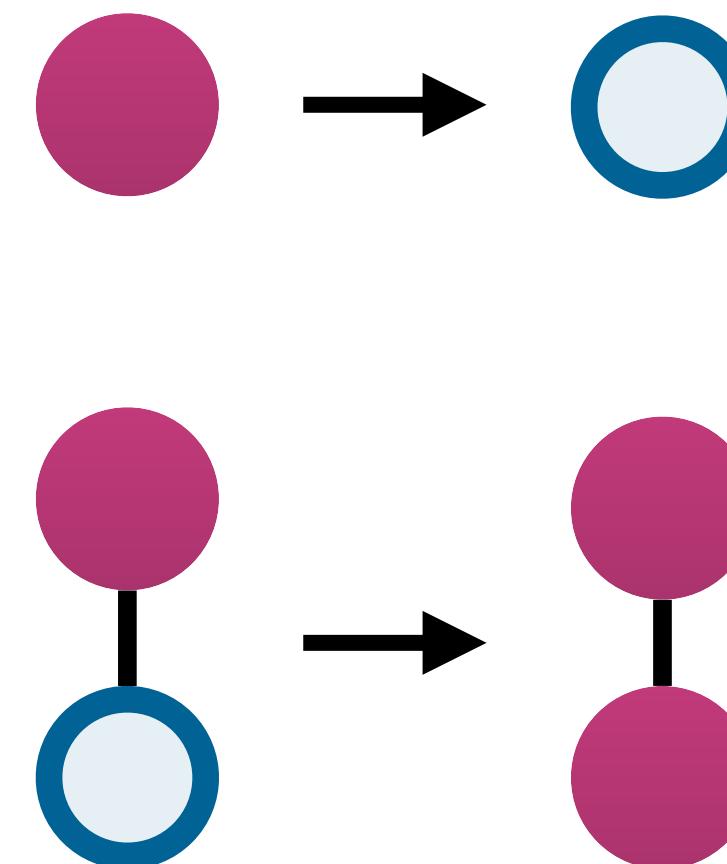
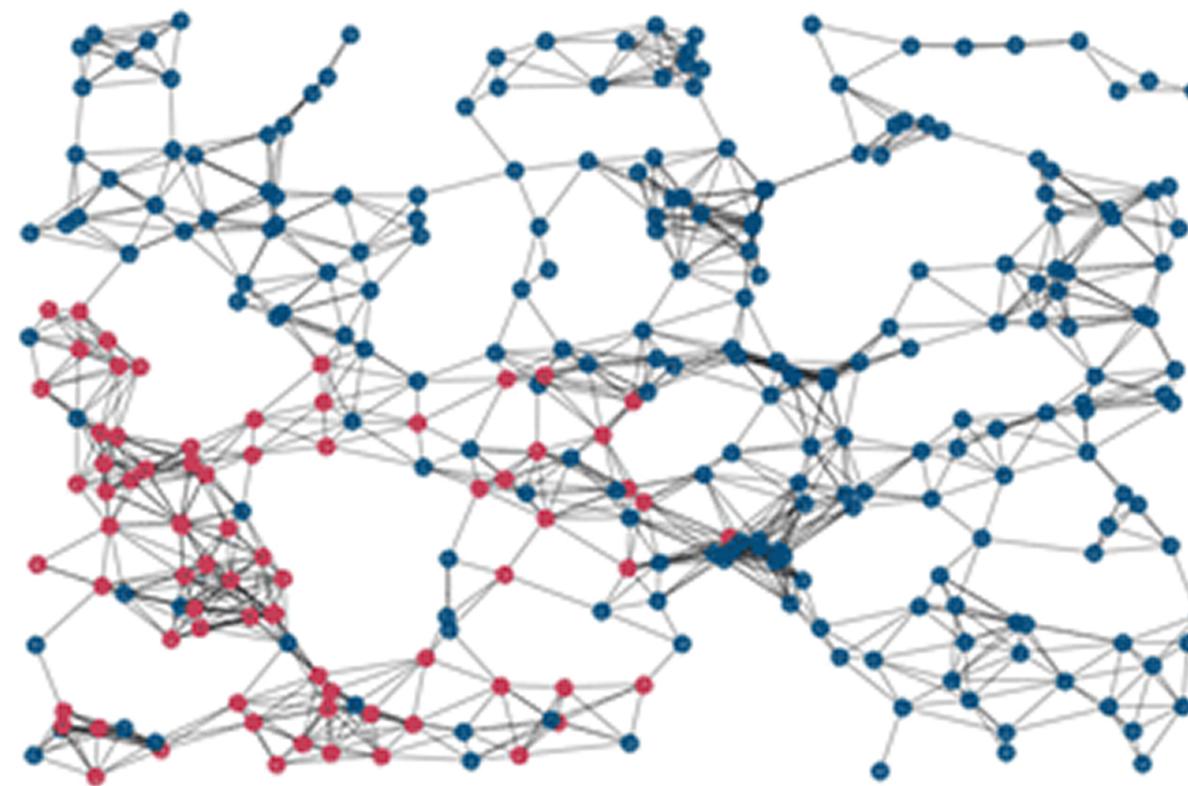


Brains



Traffic

SIS Model



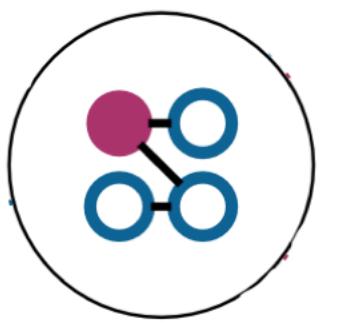
Infected nodes spontaneously recover with rate α .

Infected nodes infect **susceptible** neighbors with rate β .

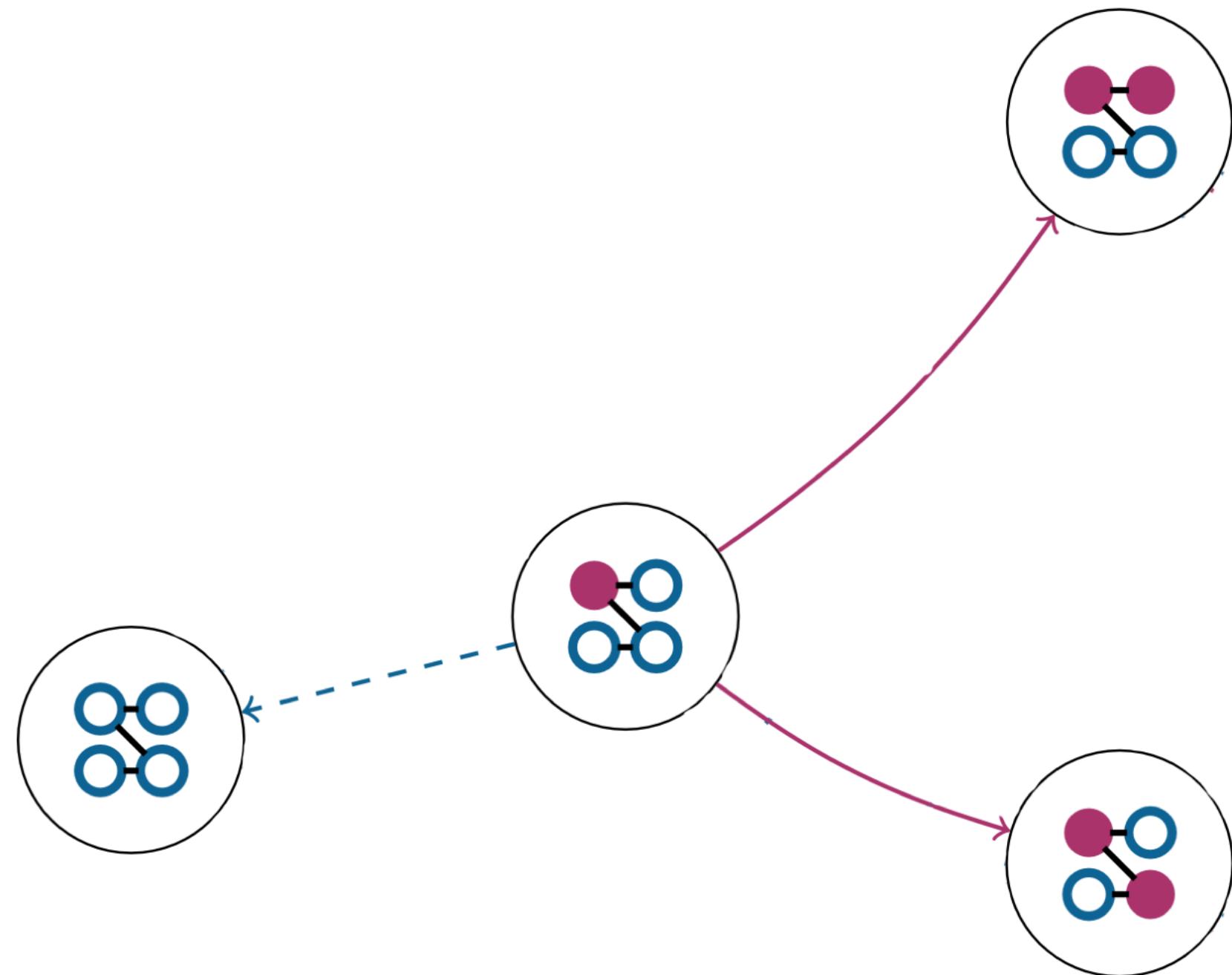
- stochastic
- discrete state-space
- jumps in continuous-time
- CTMC semantics

Only $\frac{\beta}{\alpha}$ is relevant => set $\alpha = 1$ and vary β

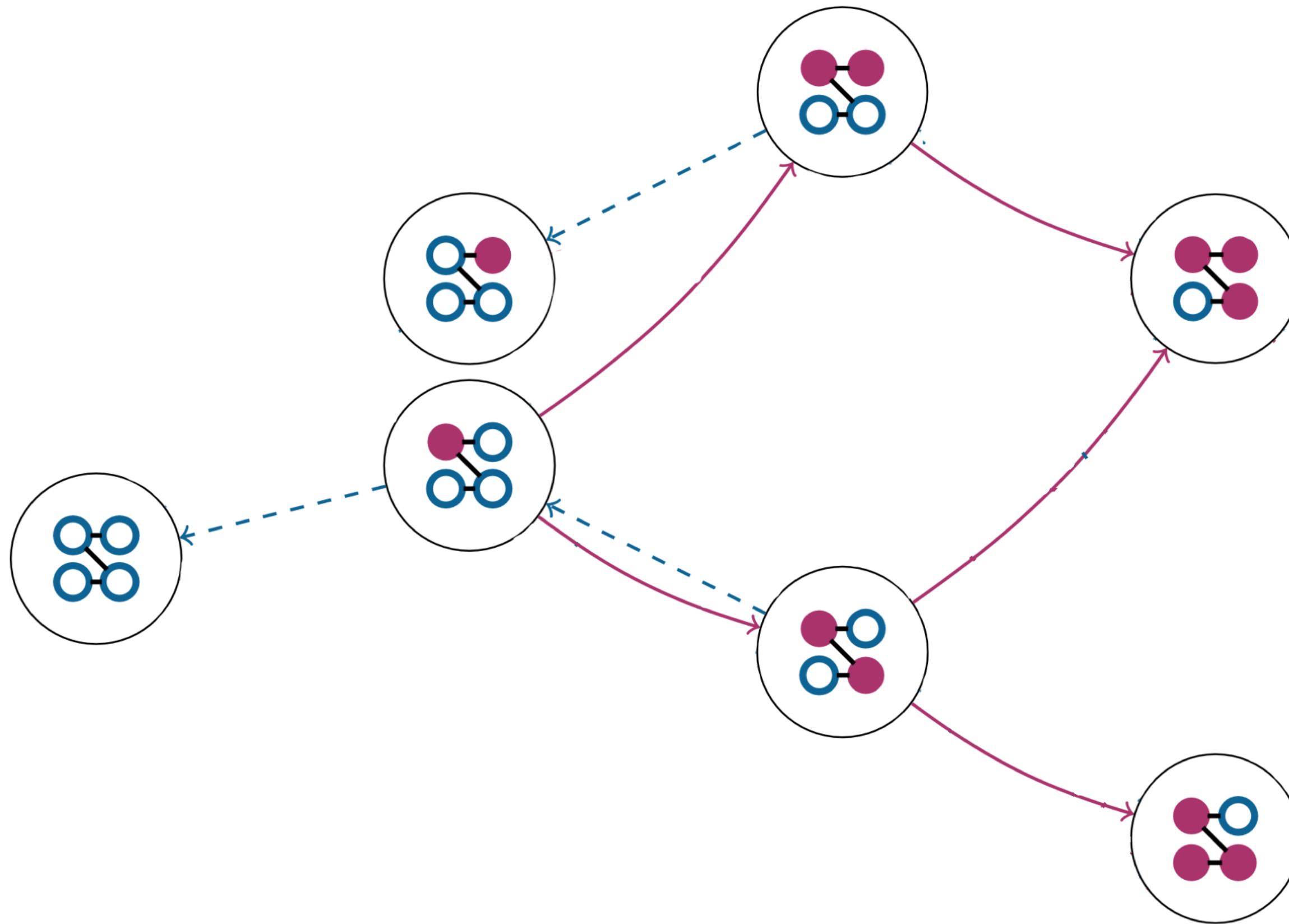
CTMC Semantics



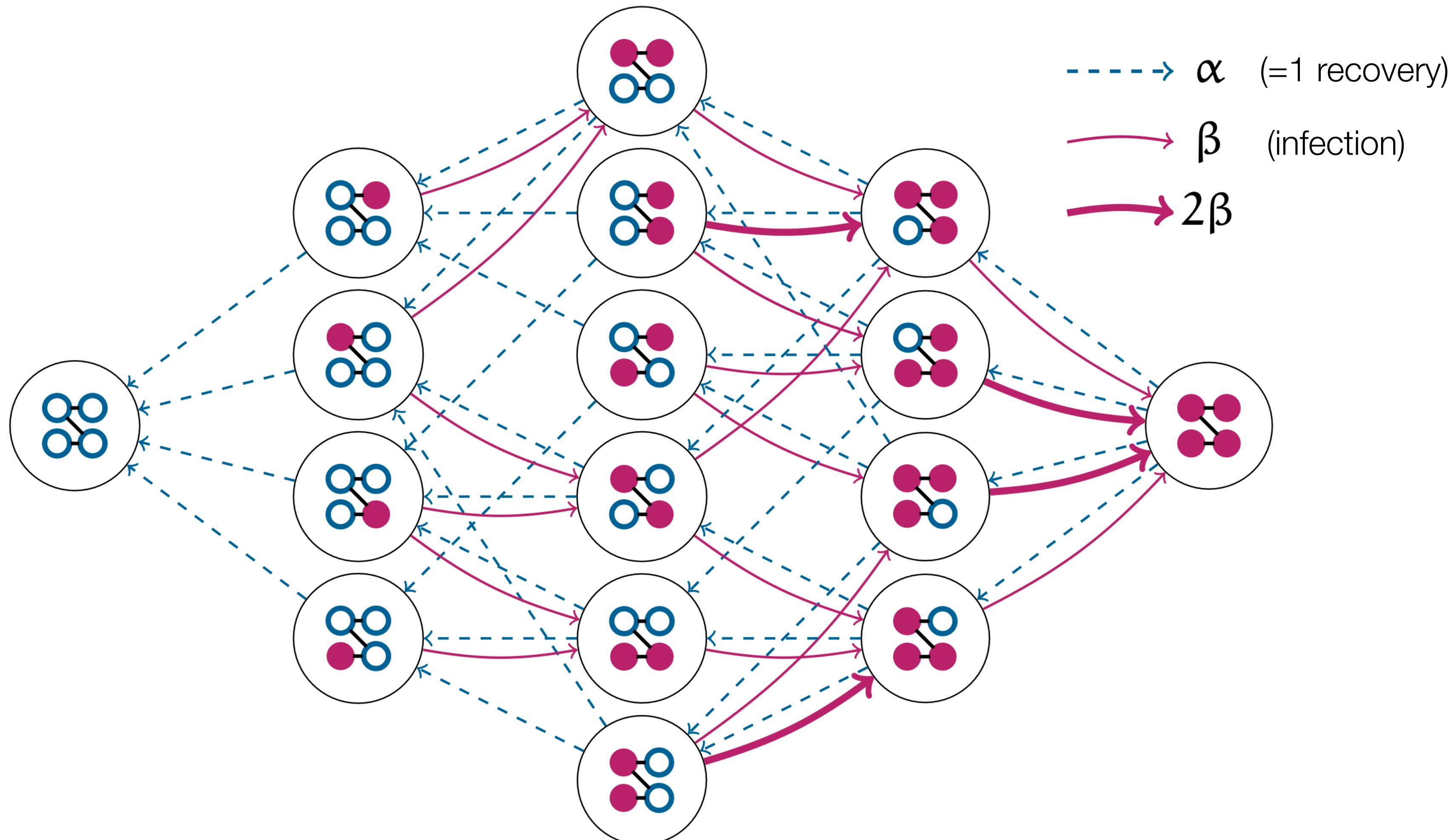
CTMC Semantics



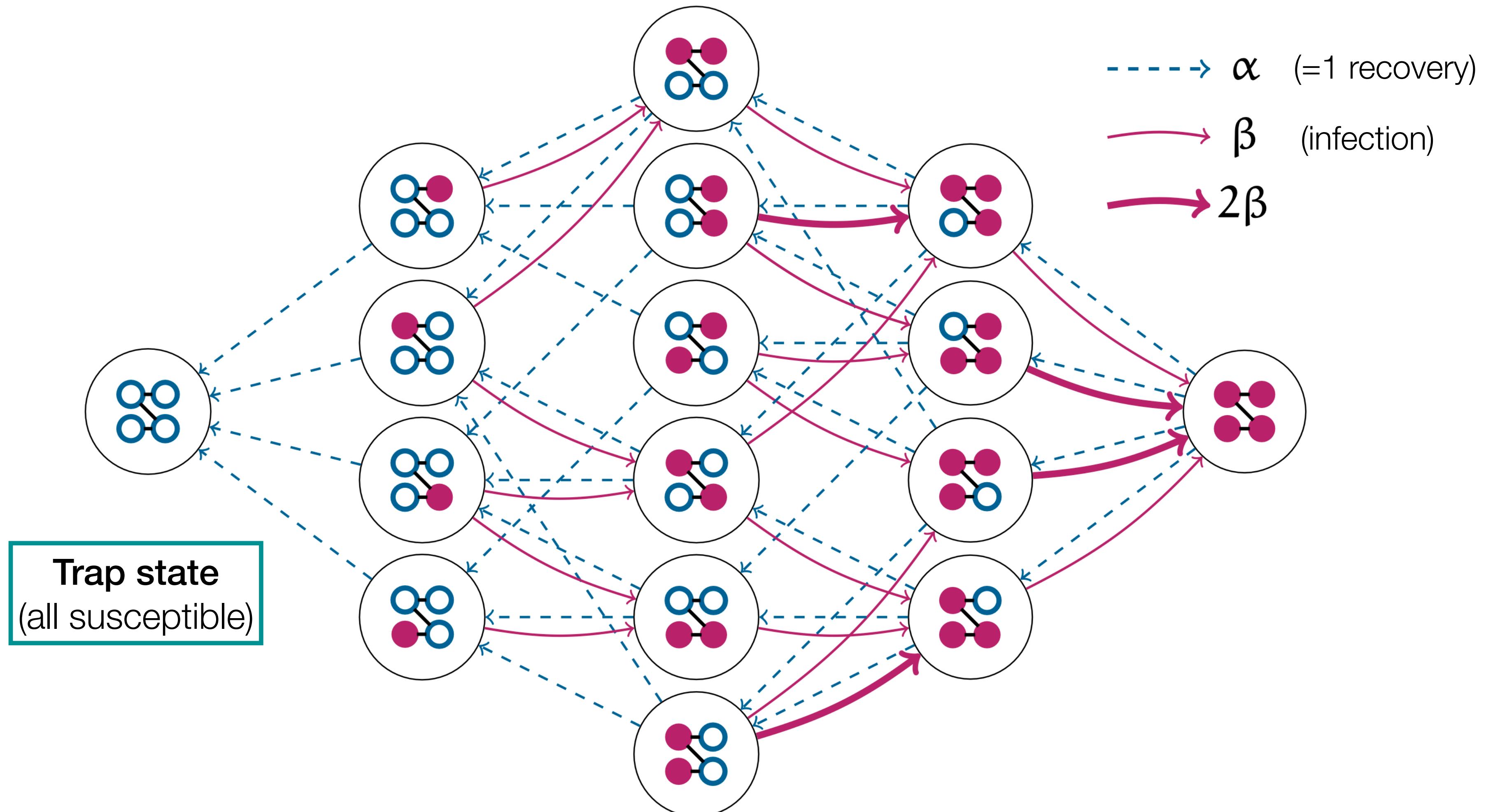
CTMC Semantics



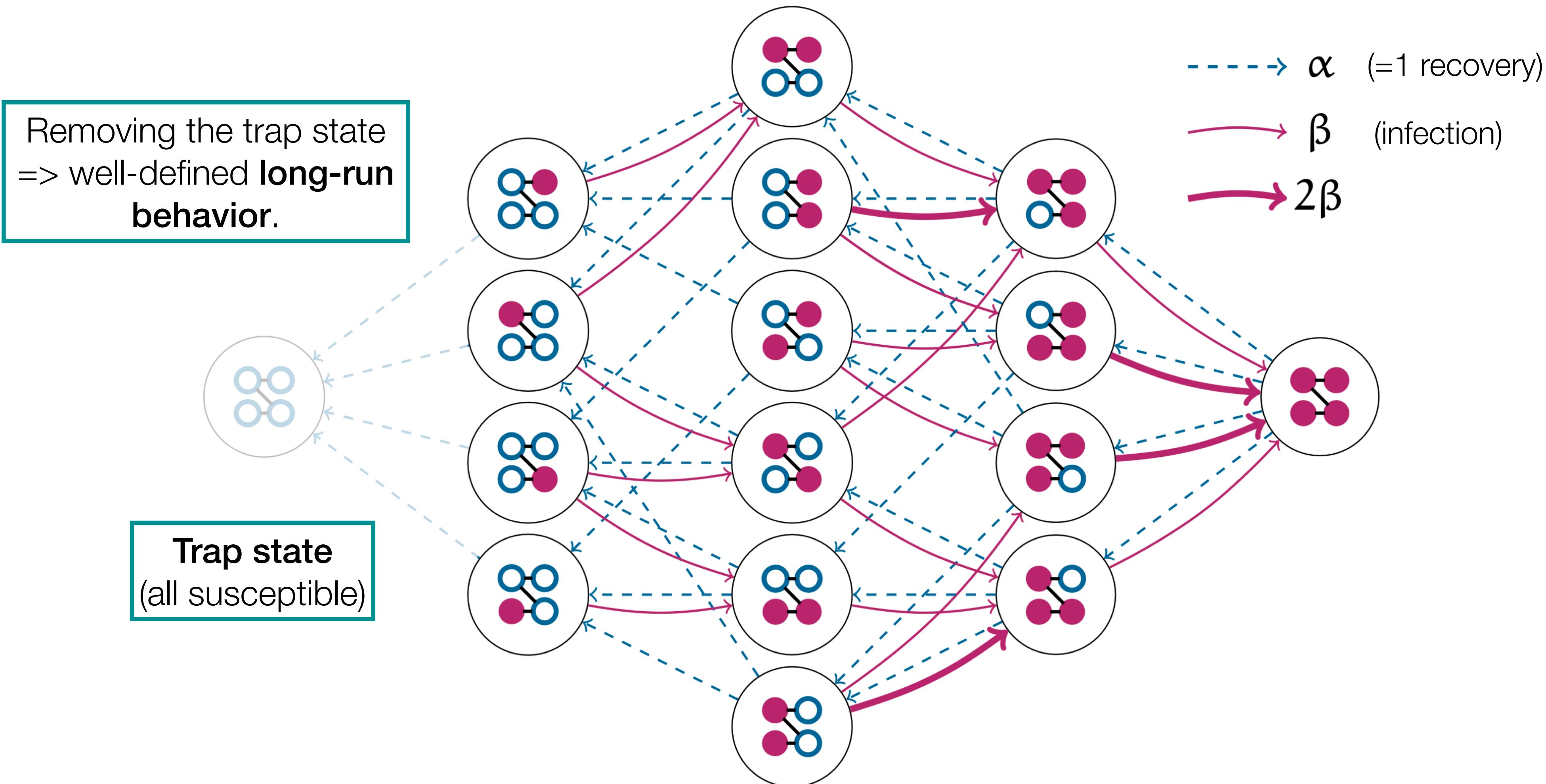
CTMC Semantics



CTMC Semantics



CTMC Semantics



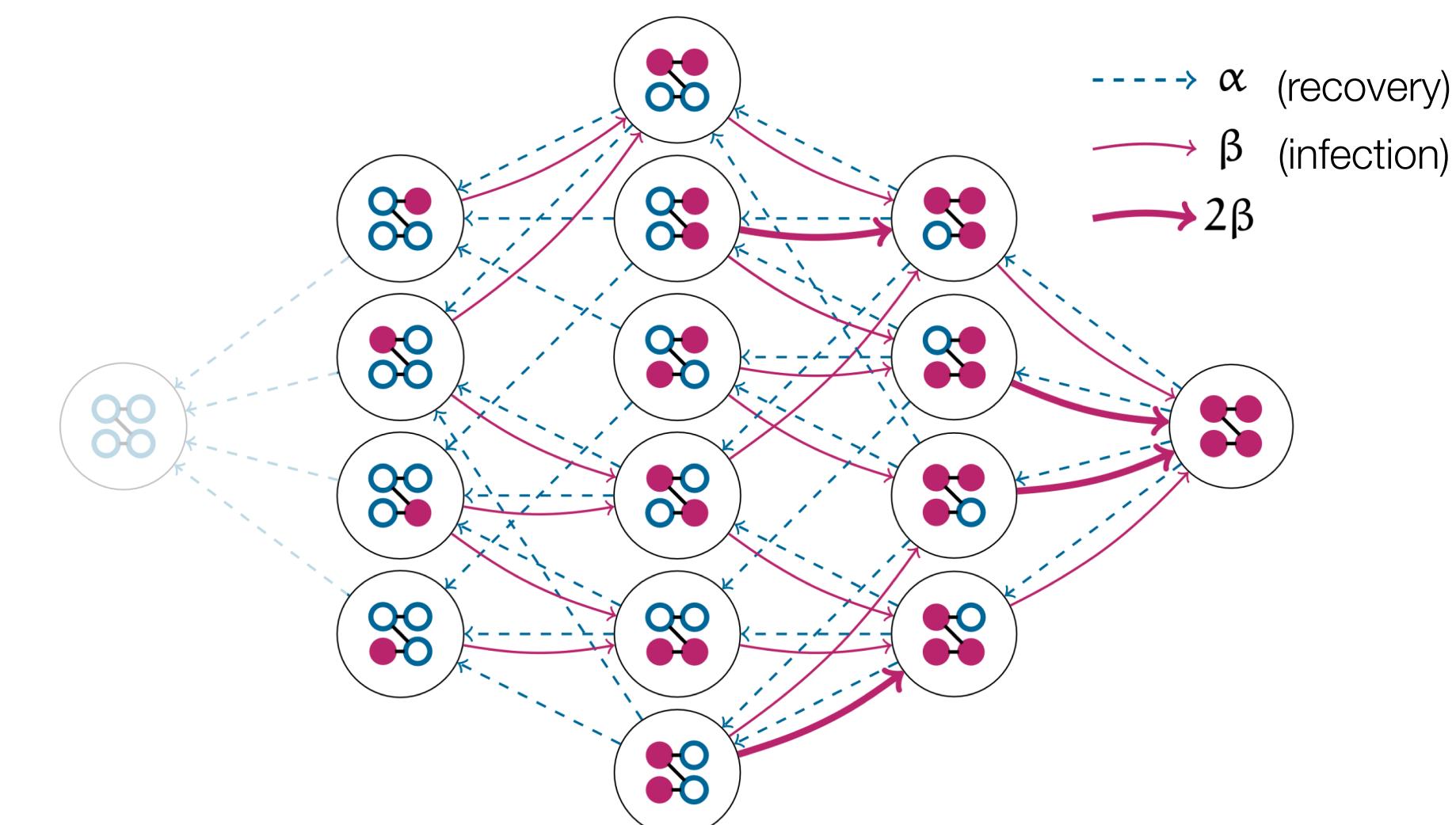
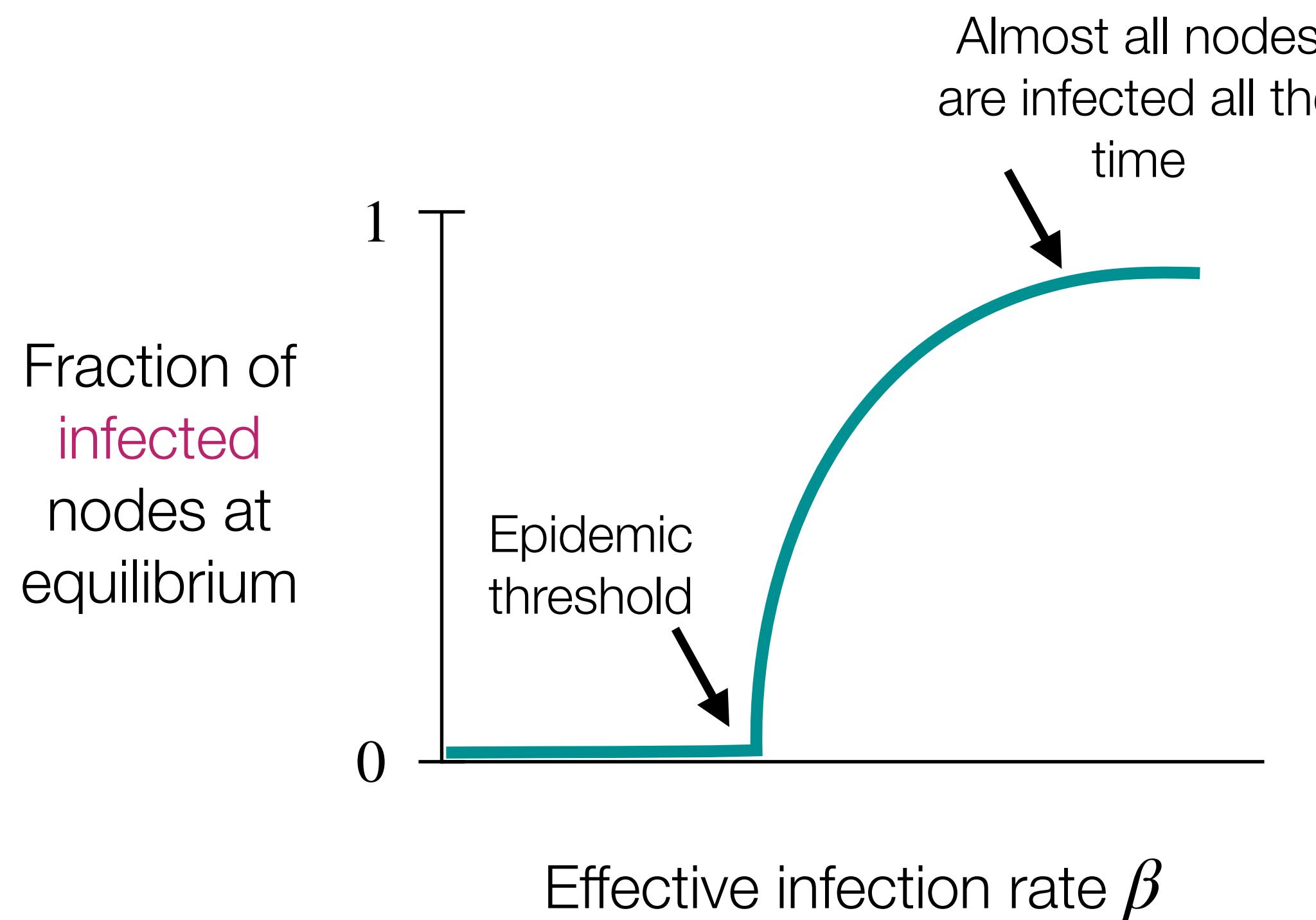
Birth-Death Processes Reproduce the Epidemic Footprint



The **expected fraction** of infected nodes after an infinitely long run as a function of the **infection rate** β .

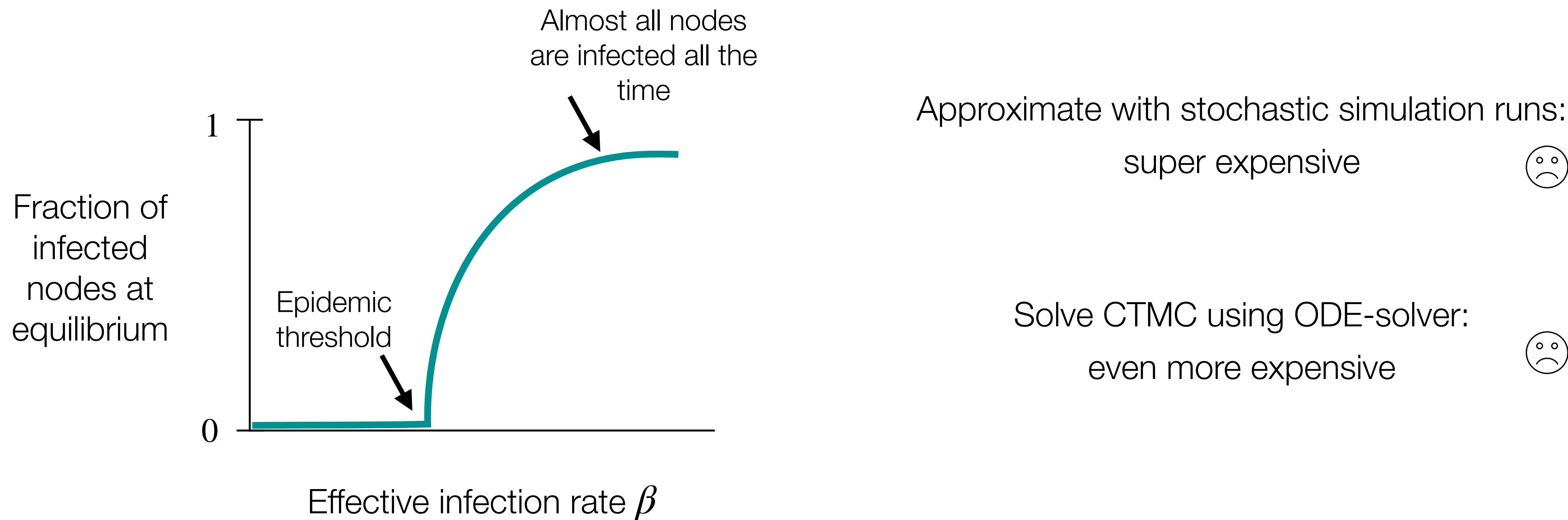
Epidemic Footprint

The expected fraction of infected nodes after an infinitely long run as a function of the infection rate β .



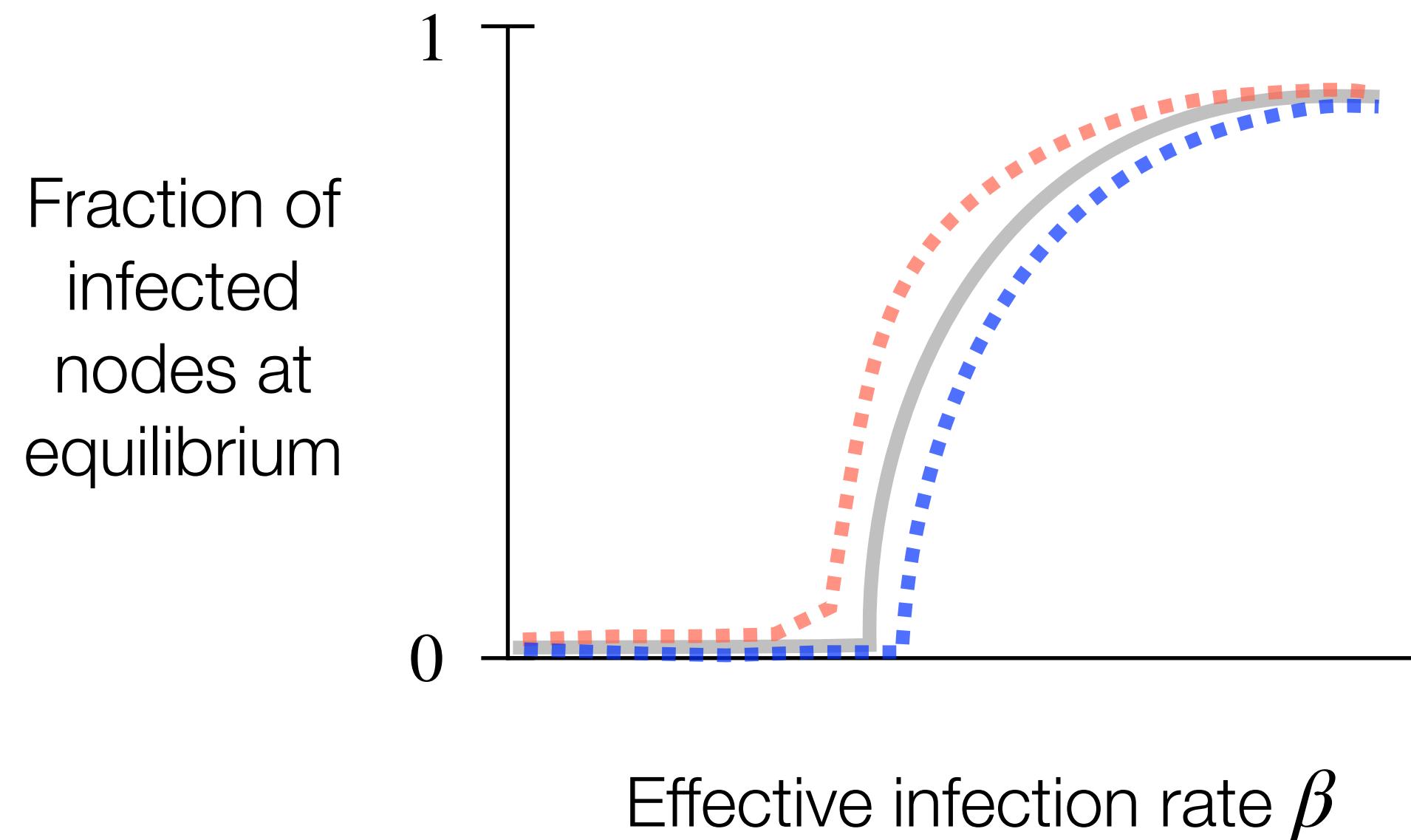
Epidemic Footprint

The expected fraction of infected nodes after an infinitely long run as a function of the infection rate β .



Bounding Epidemic Footprint

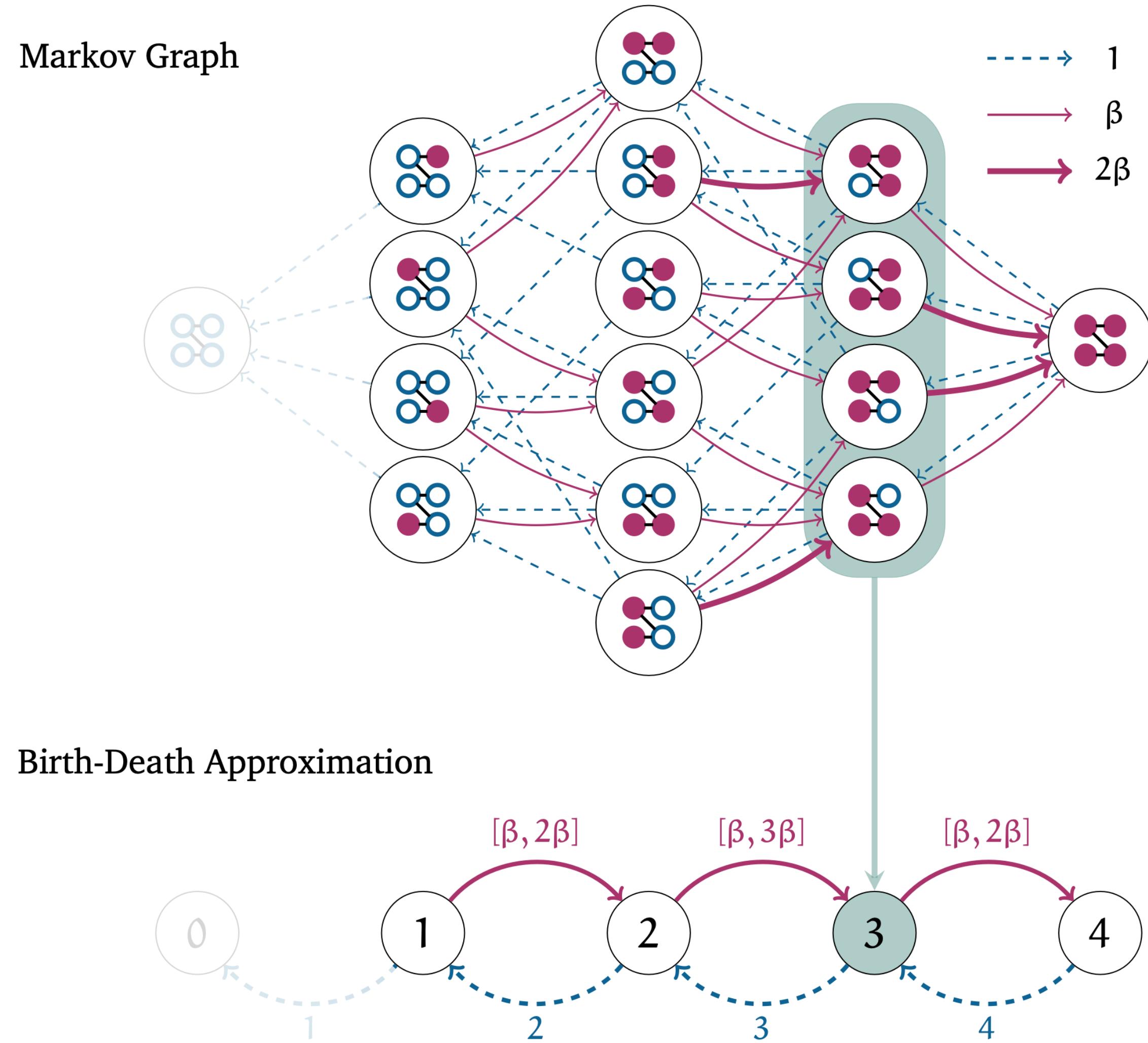
The expected fraction of infected nodes after an infinitely long run as a function of the infection rate β .



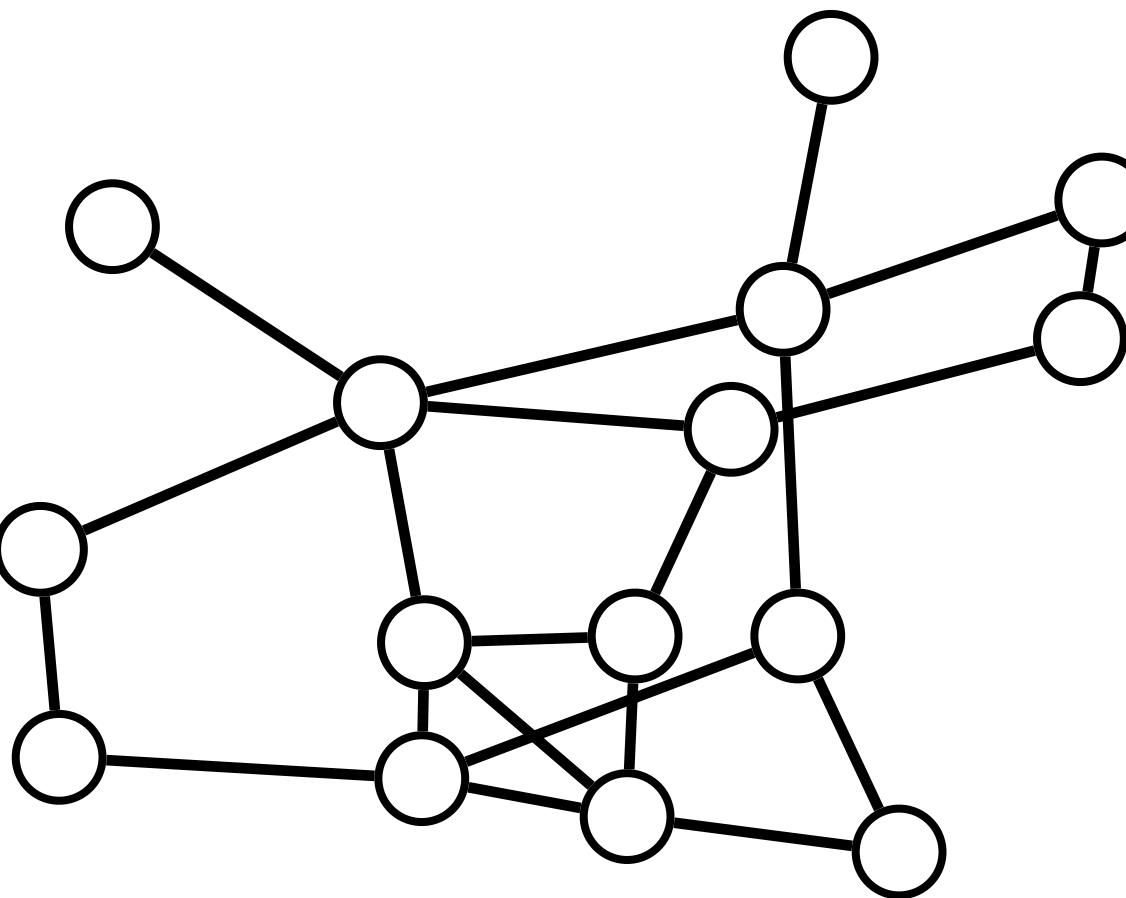
Our Goal:
find cheap **upper** and **lower** bounds

Birth-Death Process

- State identifies number of **I**-nodes.
- Death-rate:**
Number of infected node
- Birth-rate:**
Depends on number of **SI**-edges



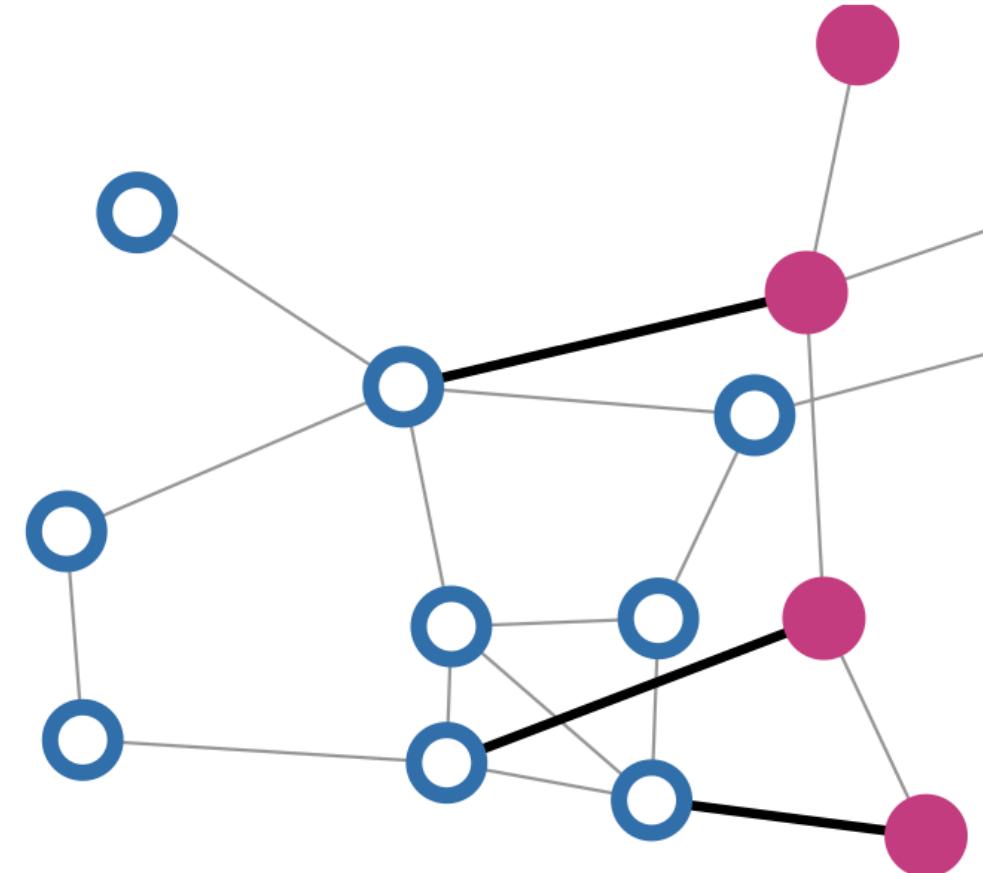
Number of **SI** edges



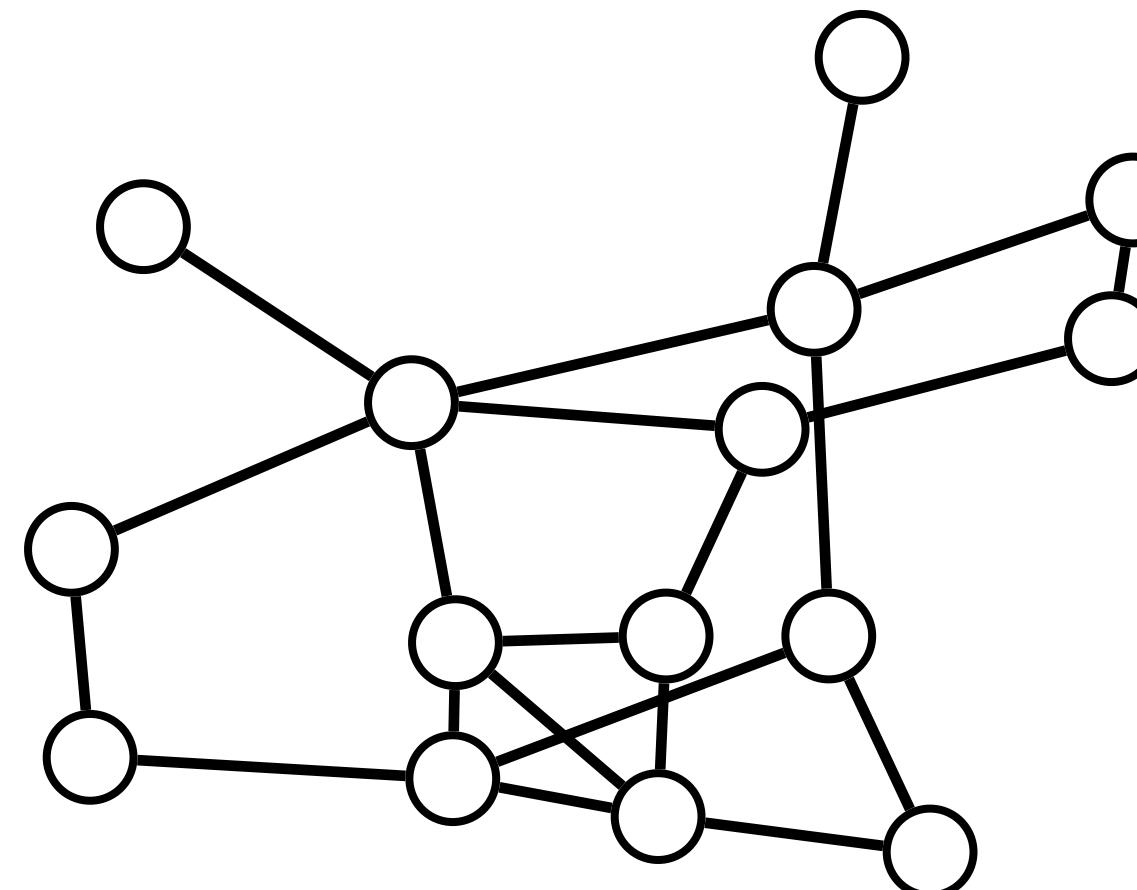
What is the maximal/minimal number of
SI edges if there are **X** infected nodes?

- ▶ We need to solve this for each **X**. 😞
- ▶ Every instance is NP-hard. 😞 😞

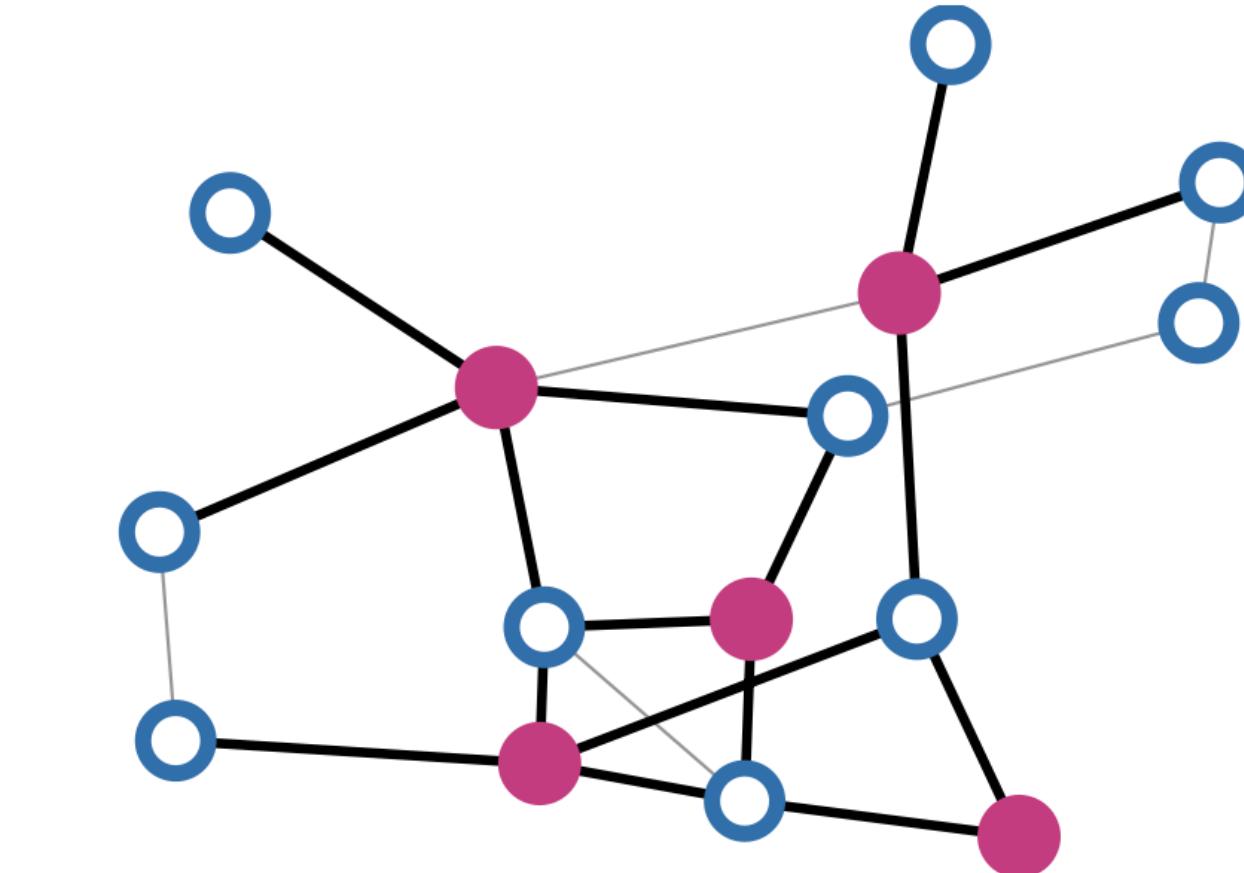
Number of **SI** edges



$X = 5$
minimum of **4 SI** edges



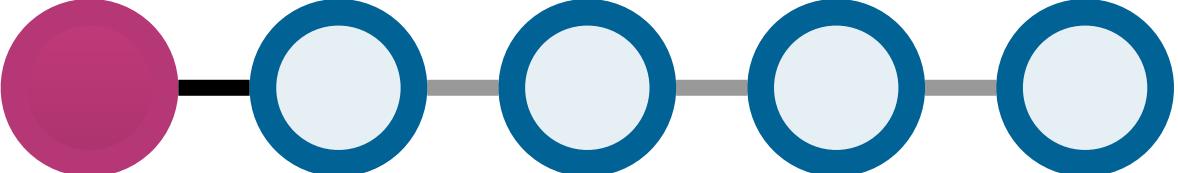
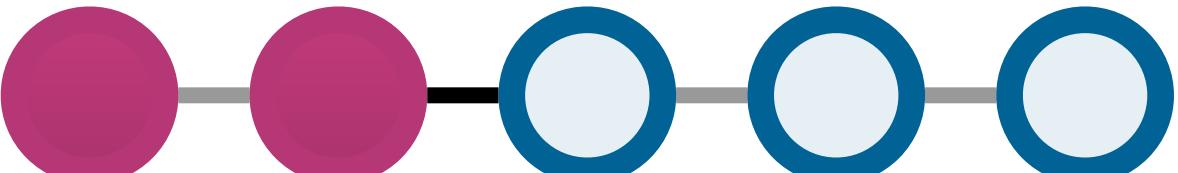
What is the maximal/minimal number of
SI edges if there are **X** infected nodes?



$X = 5$
maximum of **16 SI** edges

- We need to solve this for each **X**. 😞
- Every instance is NP-hard. 😞 😞

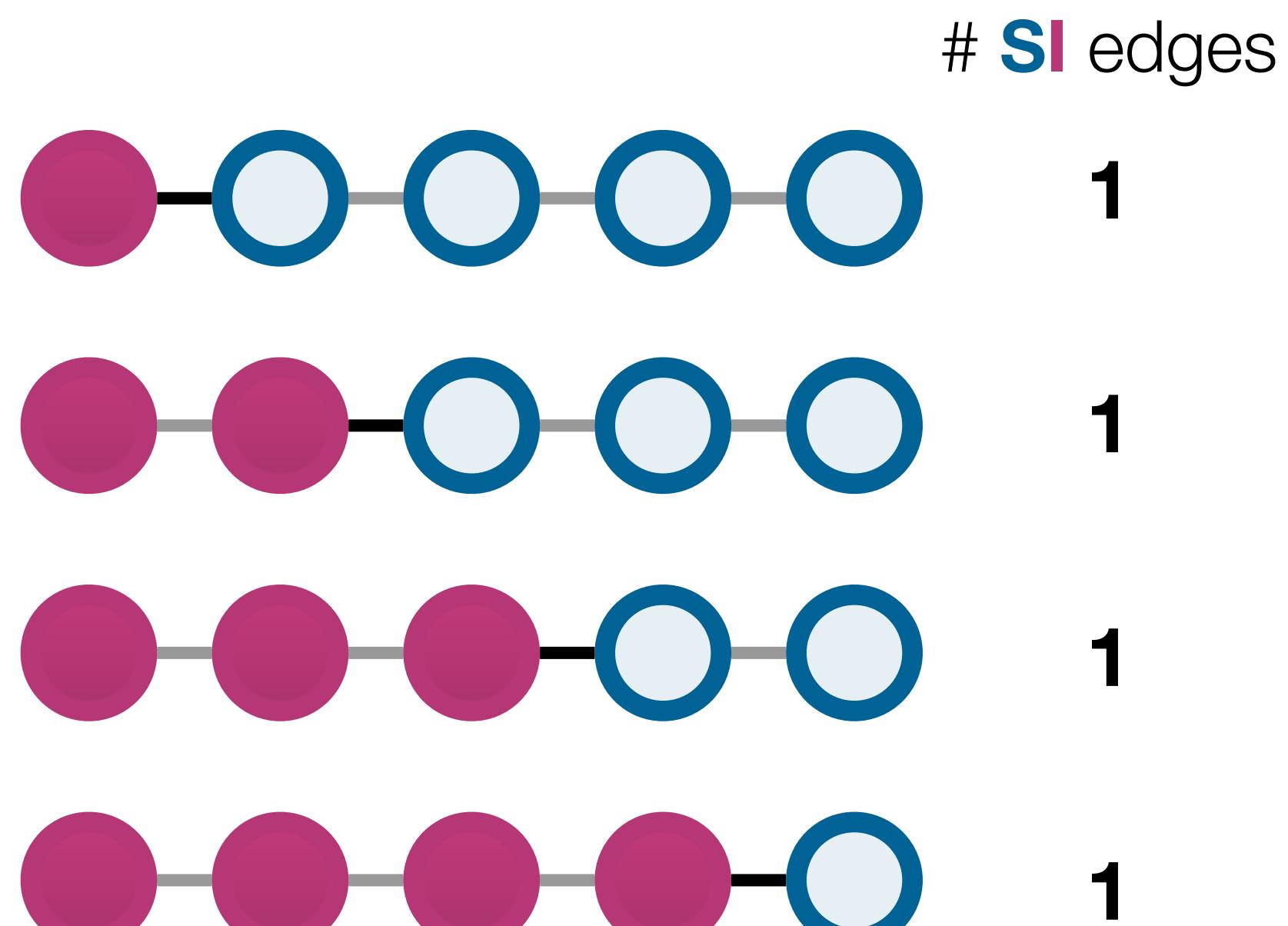
Solution: Monotonization

# SI edges	
1	
1	
1	
1	

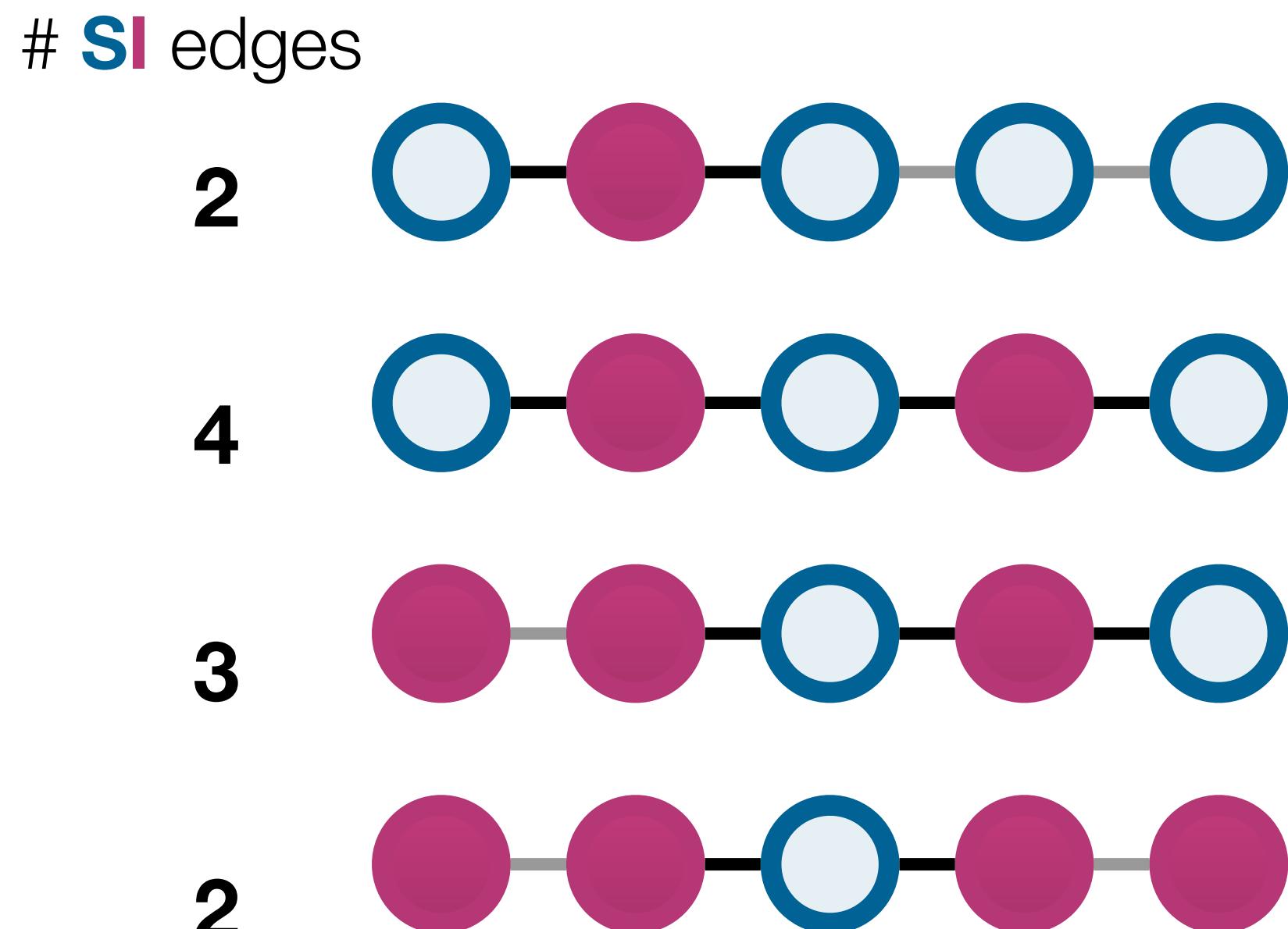
Find a **node ordering** such that successively infecting nodes minimizes/maximizes the number of **SI** edges.

Finding the '*perfect*' node ordering is still computationally difficult. ☹

Solution: Monotonization

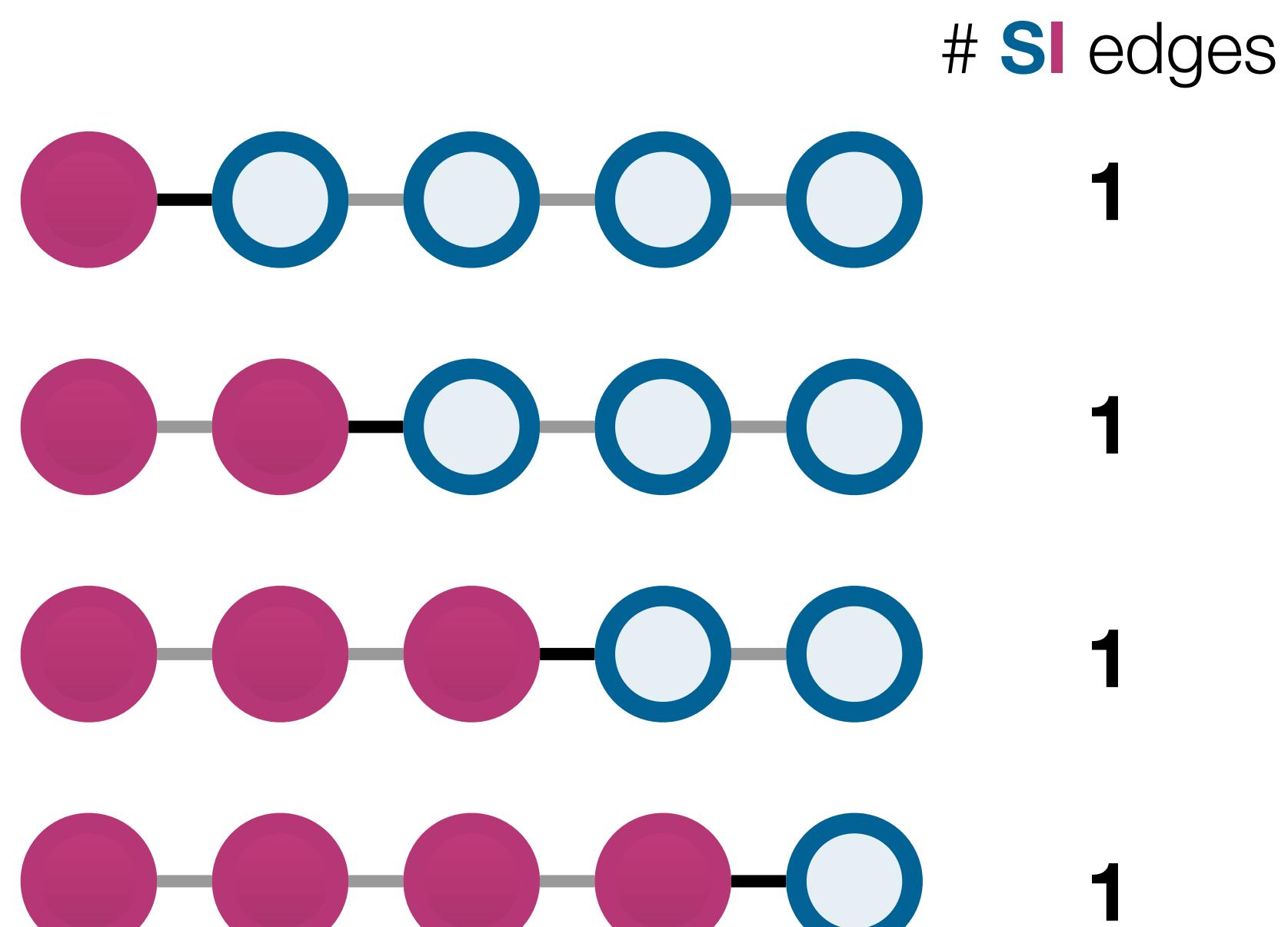


Find a **node ordering** such that successively infecting nodes minimizes/maximizes the number of **SI** edges.

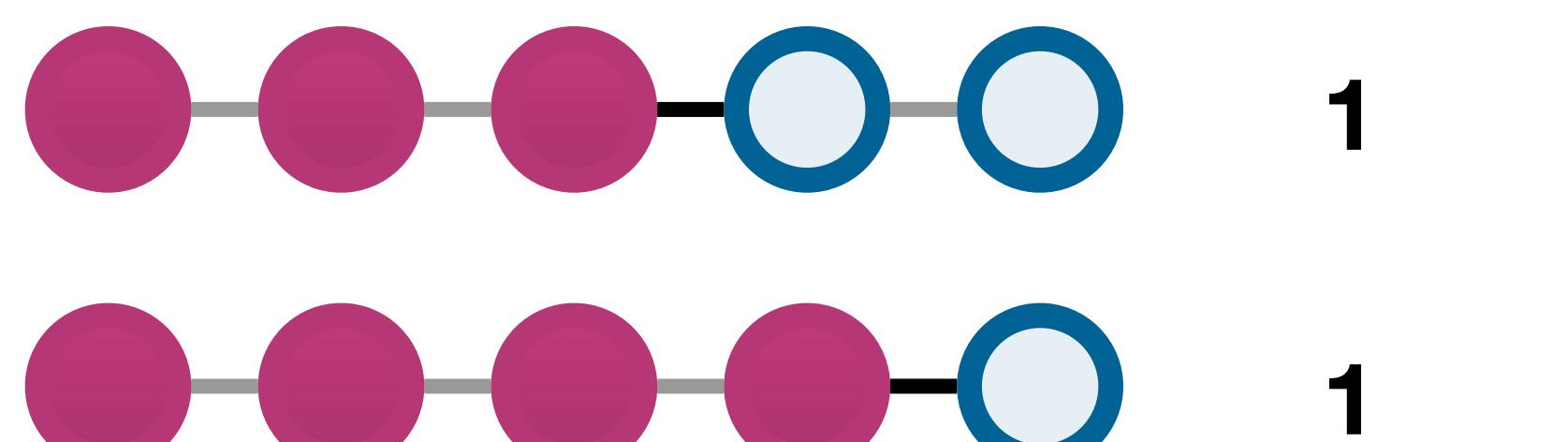


Finding the 'perfect' node ordering is still computationally difficult. ☹

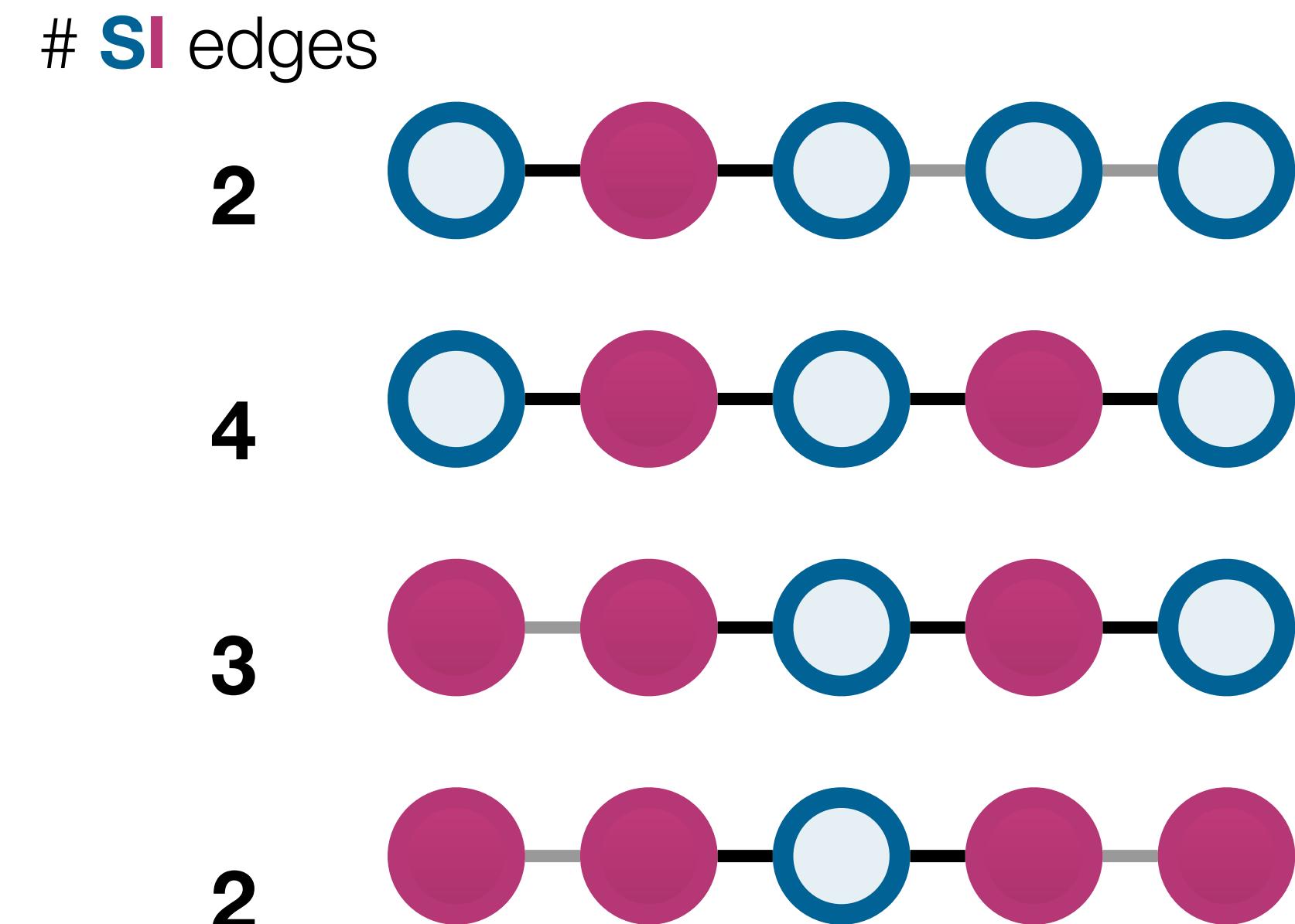
Solution: Monotonization



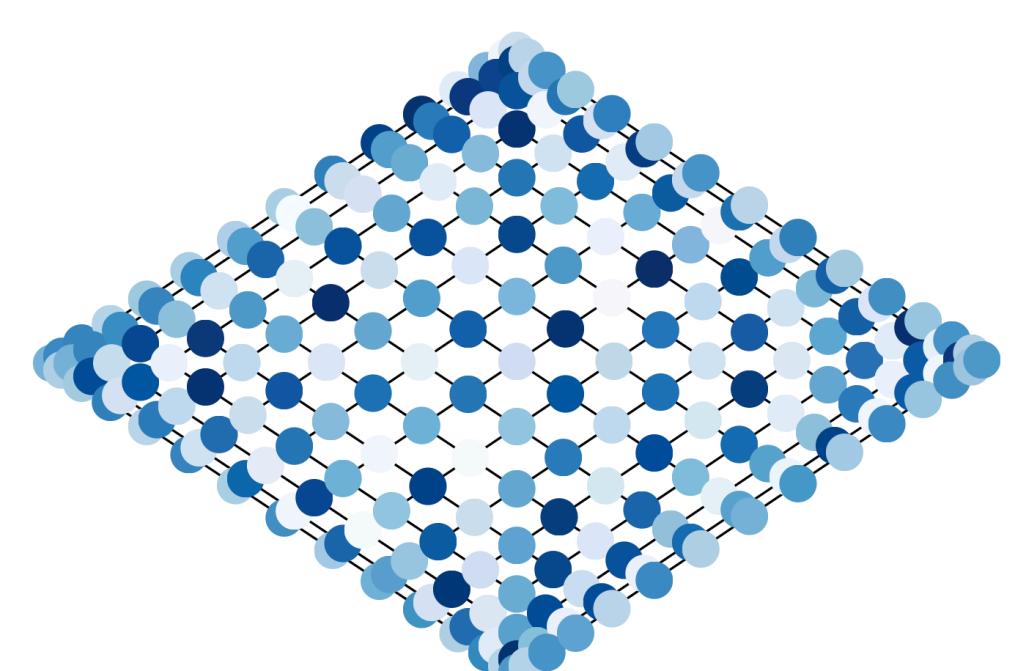
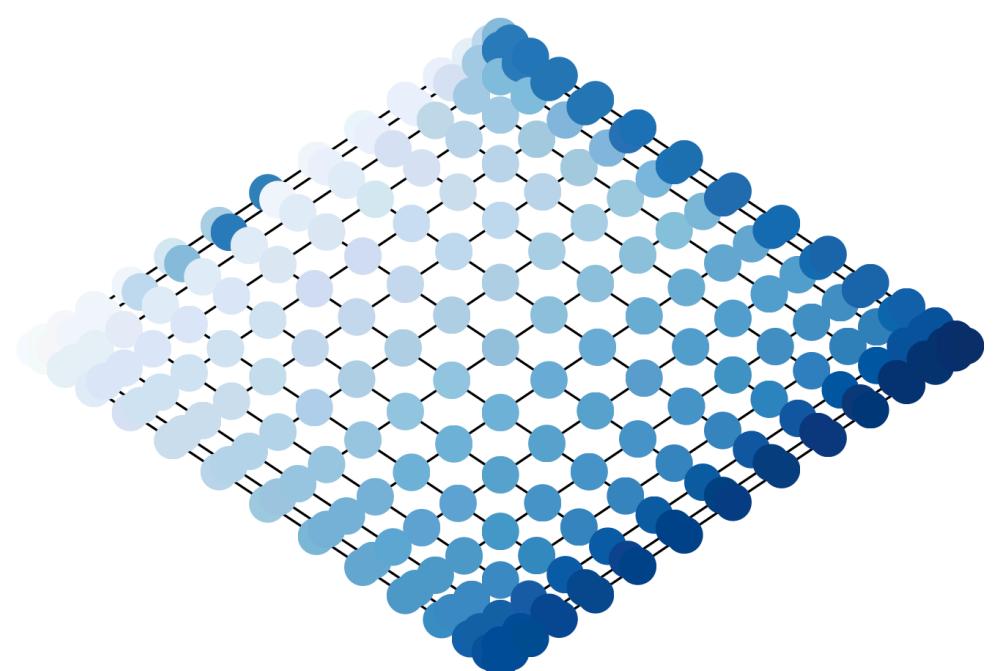
Find a **node ordering** such that successively infecting nodes minimizes/maximizes the number of **SI** edges.



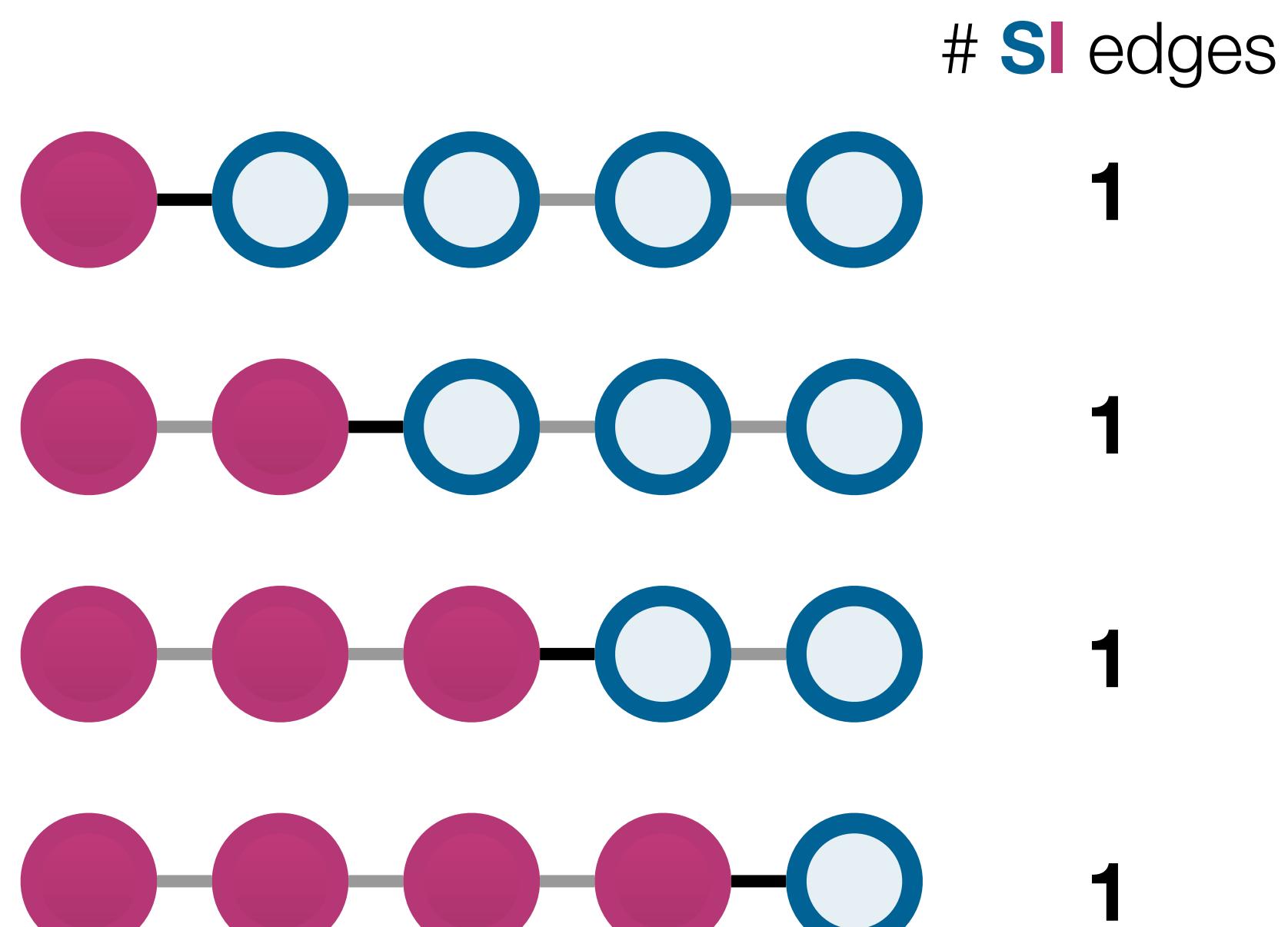
Finding the 'perfect' node ordering is still computationally difficult. ☹



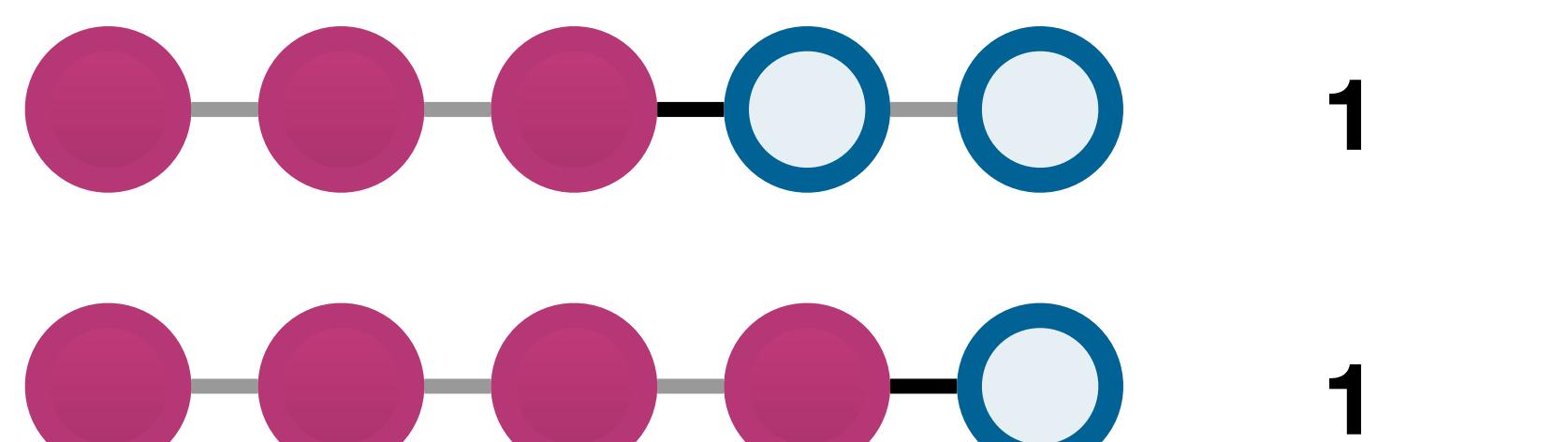
- ▶ Resort to **heuristics**:
 - ▶ Eigenvalues of the graph Laplacian
 - ▶ Greedy
 - ▶ Discrete optimization



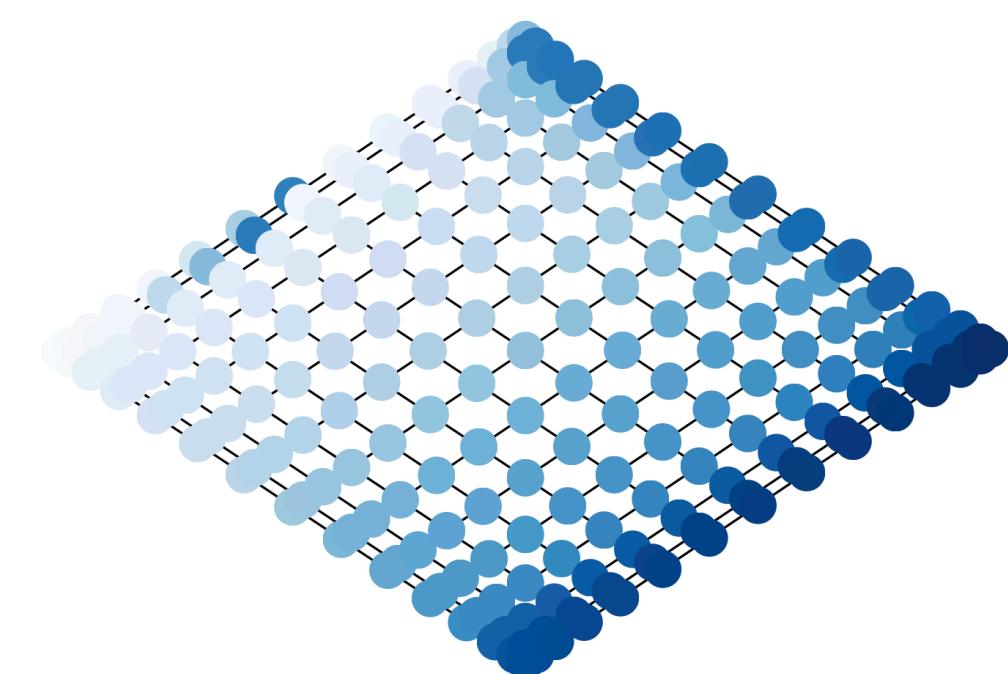
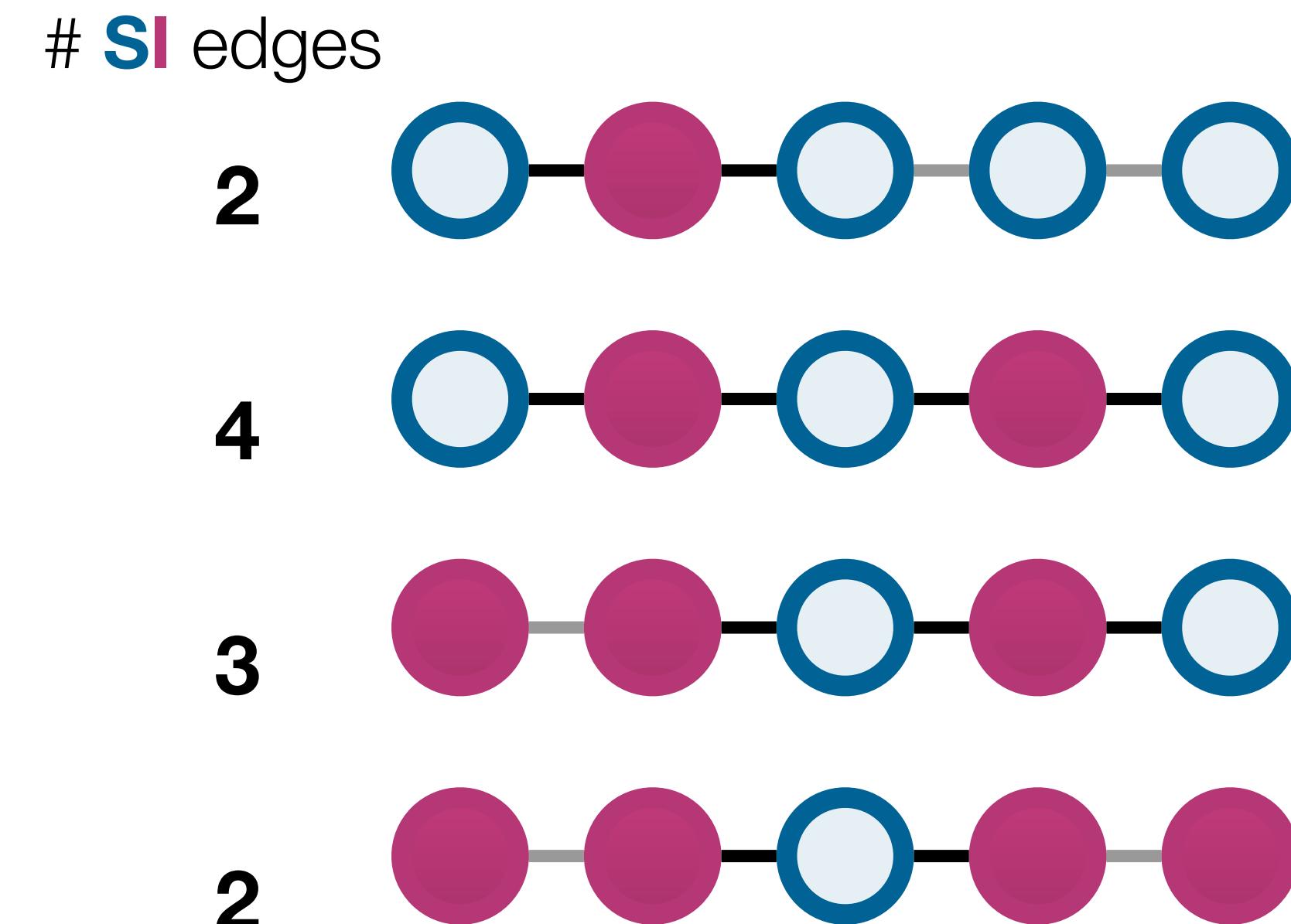
Solution: Monotonization



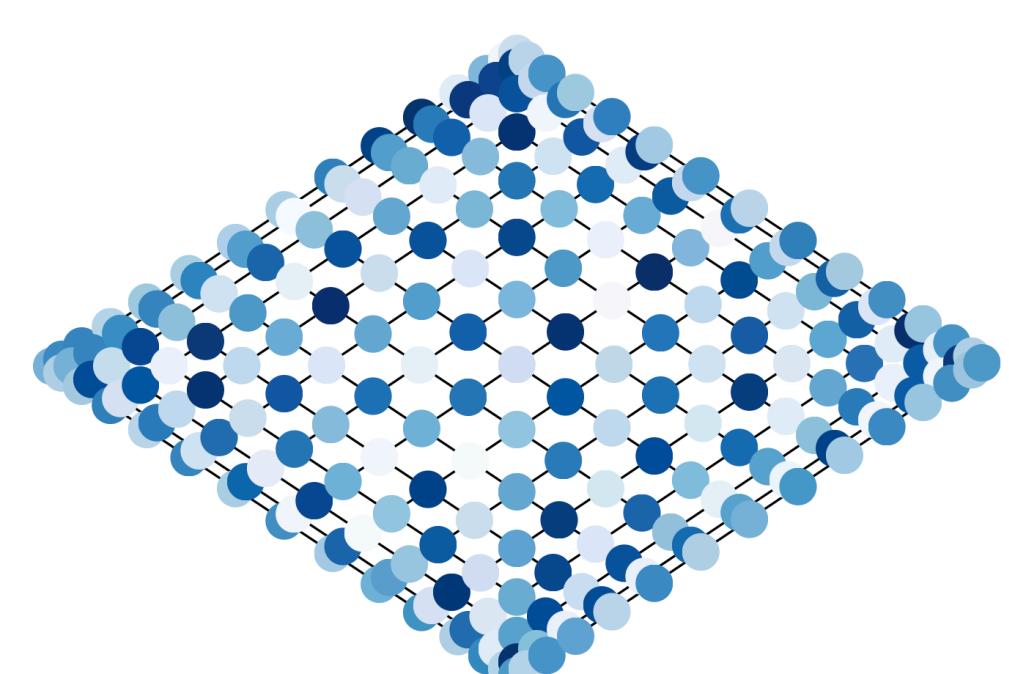
Find a **node ordering** such that successively infecting nodes minimizes/maximizes the number of **SI** edges.



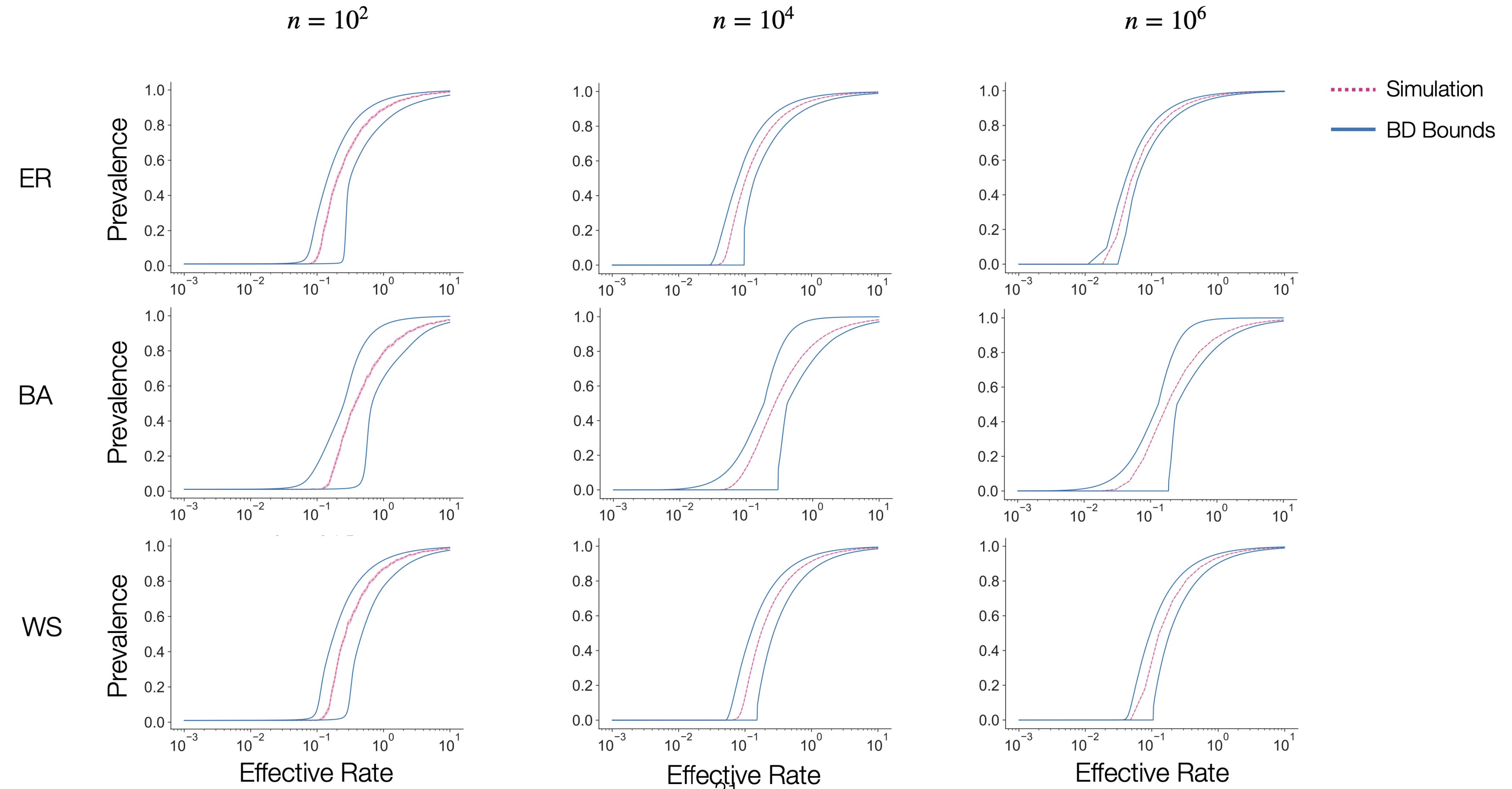
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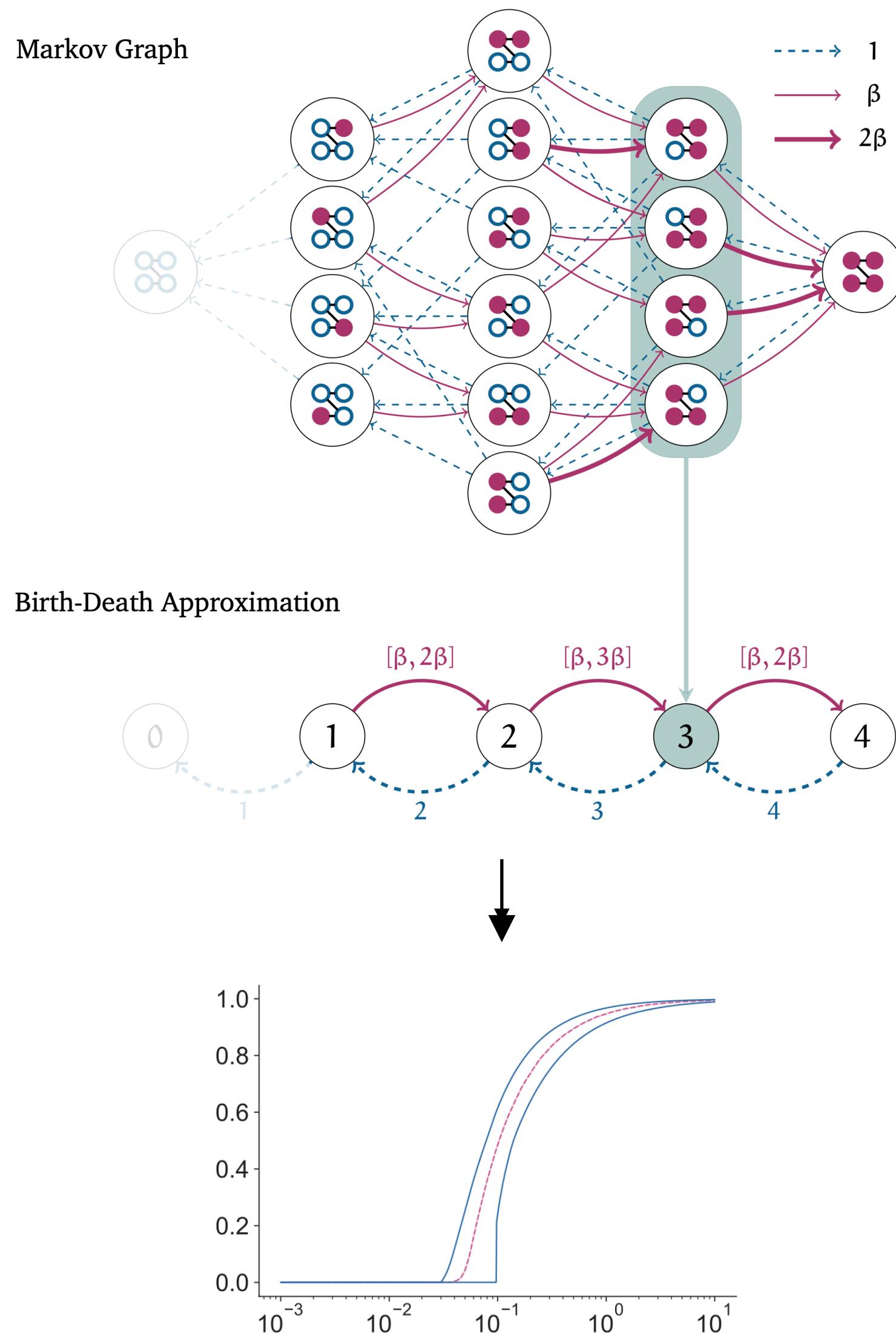
- Resort to **heuristics**:
 - Eigenvalues of the graph Laplacian
 - **Greedy**
 - Discrete optimization



Results



Summary



- ▶ BD processes provide *in principle* correct upper and lower bounds.
- ▶ Heuristic to construct birth-death process
- ▶ Less structure and more nodes lead to tighter bounds.
- ▶ **Future work:** Use graph cuts with guarantees.

Code: github.com/gerritgr/BD-Reduction

Image Credit:

- ▶ <https://www.pexels.com/de-de/foto/mockup-schreibmaschine-papier-text-4160067/>
- ▶ <https://wordpress.org/openverse/image/4f12f8fa-c76c-4886-966c-cef8db491874>
- ▶ <https://wordpress.org/openverse/image/8d091b12-9966-49c9-84e2-4bcdb4384d5a>