Forward and Inverse Kinematics Model for Our 6 DOF Robotic Arm Manipulator

Abstract— *The kinematics of manipulators is a central problem in the automatic control of robot manipulators The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. The kinematic equations of motion are derived using Denavit - Hartenberg (DH) representation. In this model, an analytical solution for the inverse kinematic of Our 6 DOF Robotic Arm Manipulator is presented, to analyze the movement of arm from one point in space to another point. In this solution the only decision variables are the coordinates of the origin and the destination points in space besides the geometric parameters of the robot arm. Our 6 DOF Robotic Arm Manipulator is a vertical articulated robot, with six revolute joints. This versatile system allows us to gain theoretical and practical experience in robotics, automation and control systems. The MATLAB R2018a is used to solve this mathematical model for a set of joint parameter. The kinematics solution of the MATLAB program was found to be identical with the robot arm's actual reading.*

I. INTRODUCTION

Analytical prediction of the behavior of physical systems in many key situations is either extremely complicated or even impossible. Driven with the constraints to prototype a physical system, modeling finds enormous motivations to study and investigate the performance of a system. Modeling a robot involves study of its kinematic behavior. A kinematic model is concerned with the robot’s motion without considering forces producing the motions. The kinematics of a robotic arm deals with the study of the geometric and time based properties of the motion and in particular how various links of a robot move with respect to one another and with time. It provides an analytical description of the spatial movements of a robot i.e. a relationship between position and orientation of robot endeffector and its joint variables. The problem of kinematic modeling is usually categorized into two sub-problems. First is the forward or direct kinematics, which is the problem of solving the Cartesian position and orientation of a mechanism, given the knowledge of the kinematic structure and the joint coordinates. The second sub problem is Inverse Kinematics (IK), which computes the joint variables using the given information of a robot's end-effector position and orientation. In case of serial robotic arms, IK problem is more complex than direct kinematic problem . Kinematic modeling of robots also benefits the industrial automation processes by making them semi autonomous or even fully autonomous. Because of the task nature and operational environment, the industrial robots are usually composed up of series of rigid links mounted on a base. They operate in a manner similar to that of the human arm. A 6 Degree Of Freedom (DOF) robotic arm manipulator is widely used in the industry. The most common applications of industrial robots include Spot welding, Spraying, Assembling and Manufacturing. Many of these applications actually require accomplishment of pick and place task from one point in space to another point. Implementation of this task essentially necessitates having the kinematic model of the robotic arm being employed. Hence Forward kinematics is defined as transformation from joint space to cartesian space whereas Inverse kinematics is defined as transformation from cartesian space to joint space. General methods do exist for solving forward kinematics. So Main objective of this model is to present an analytical solution for the inverse kinematics of Our 6 DOF Robotic Arm Manipulator, to analyze the movement of arm from one point in space to another point. In this model the only decision variables are the coordinates of the origin and the destination points in space besides the geometric parameters of the robot. For the given set of parameter, a program in MATLAB R2018a is made and its output is compared with the experimental result.

We first presents kinematic model of the robot in Section II. The robot has been modeled for its forward kinematics as well as IK. Section III discusses validation of inverse kinematic model using MATLAB toolbox for robotics while Implementation of IK model on the real robot is discussed in Section IV. Finally Section **V** comments on conclusion.

II. KINEMATIC MODEL

It is a vertical articulated robot with six revolute joints. It has a stationary base(waist Θ_1), shoulder (Θ_2) ,elbow(Θ_3) are used to move the end-effector to its desired position and wrist(Θ_4, Θ_5 ,Θ_6) adjusts the orientation of the end-effector. The arm is fully-actuated with each DOF achieved.

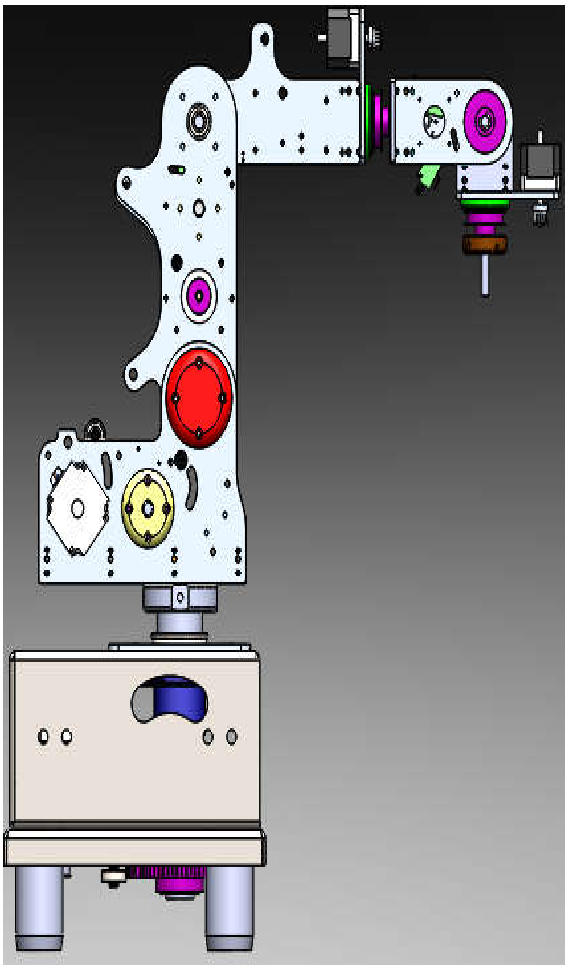
in this study, the standard Denavit-Hartenberg (DH) convention and methodology are used to derive its kinematics:

Z axis must be axis of rotation for revolute joint.

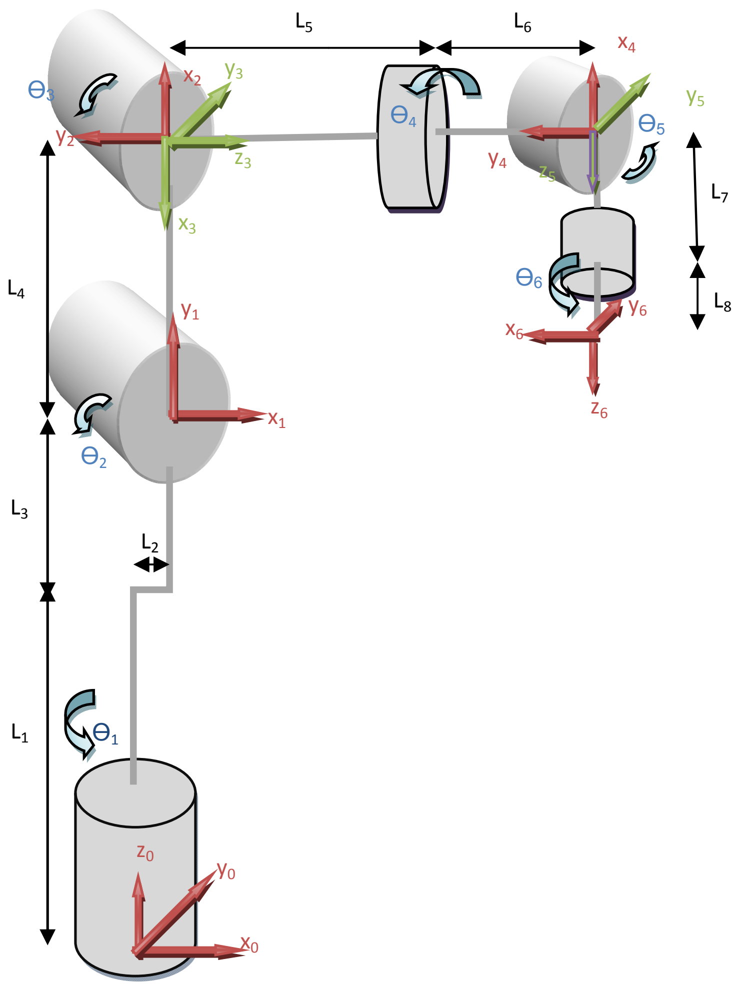
X^n axis must be perpendicular to [Z^n & Z^{n-1}] axises.

all frames follow right_hand rule.

X^n axis must intersect the Z^{n-1} axis.



ACTUATED JOINTS AT THEIR ZERO DEGREE



KENIMATICS DIAGRAM

A. Denavit and Hartenberg put forwards to a matrix method to build the attached coordinate system on each link in the joint chains of the robot to describe the relationship of translation or rotation between the contiguous links way back in 1955. This robot kinematic model is based on the D-H Coordination system:

ϴ the rotation around Z^{n-1} axis to match X^{n-1} to X^n axis.
α the rotation around X^n axis to match Z^{n-1} to Z^n axis.
r the distance between frame (n) and frame (n-1) along X^n axis.
d the distance between frame (n) and frame (n-1) along Z^{n-1} axis.

DH works with quadruple (α, d, r,ϴ) represents twist angle, link length, link offset and joint angle respectively.

	ϴ	α	r	D
1				L_1+L_3
2		0		0
3			0	0
4			0	L_5+L_6
5			0	0
6		0	0	L_7+L_8

DH PARAMETER TABLE

The transformations between each two successive joints can be written by simply substituting the parameters from the parameters table into the following matrix:

```
H(n_1)_n = [cos(th(n))   -sin(th(n))*cos(alpha(n))   sin(th(n))*sin(alpha(n))   r_n*cos(th(n));
            sin(th(n))    cos(th(n))*cos(alpha(n))  -cos(th(n))*sin(alpha(n))   r_n*sin(th(n));
            0             sin(alpha(n))              cos(alpha(n))              d_n           ;
            0             0                          0                          1             ];
```

the 3X3 matrix comprising of first three rows and first three columns is the rotation while the last column represents the position (x , y, z).

```
H0_1 = [ cos(th1)   0          sin(th1)   l2*cos(th1) ;
         sin(th1)   0         -cos(th1)   l2*sin(th1) ;
         0          1          0          l1+l3       ;
         0          0          0          1           ];

H1_2 = [-sin(th2)  -cos(th2)   0         -l4*sin(th2) ;
         cos(th2)  -sin(th2)   0          l4*cos(th2) ;
         0          0          1          0           ;
         0          0          0          1           ];

H2_3 = [-cos(th3)   0          sin(th3)    0          ;
        -sin(th3)   0         -cos(th3)    0          ;
         0         -1          0           0          ;
         0          0          0           1          ];

H3_4 = [-cos(th4)   0         sin(th4)     0          ;
        -sin(th4)   0        -cos(th4)     0          ;
         0         -1          0           l5+l6      ;
         0          0          0           1          ];

H4_5 = [-sin(th5)   0         -cos(th5)    0          ;
         cos(th5)   0         -sin(th5)    0          ;
         0         -1          0           0          ;
         0          0          0           1          ];

H5_6 = [ cos(th6) -sin(th6)    0           0          ;
         sin(th6)  cos(th6)    0           0          ;
         0         0           1           l7+l8      ;
         0         0           0           1          ];
```

At the base of the robot, it can be started with the first joint and then transform to the second joint, then to the third until to the arm-end of the robot, and eventually to the end effectors. The total transformation between the base of the robot and the hand is a homogeneous transformation matrix consists of the 3X3 rotation matrix and the position(x, y, z) for the end-effector with reference to the base.

```
H0_6 = H0_1 * H1_2 * H2_3 * H3_4 * H4_5 * H5_6;
```

B. by using the IK, The corresponding variables of each joint could found with the given location requirement of the end of the manipulator in the given references coordinates system. Inverse kinematic analysis is done by multiplying each matrix of H matrices on the right side of above equation and then equalizing the corresponding elements of the equal matrices of both ends.

```
H0_1 * H1_2 * H2_3 * H3_4 * H4_5 * H5_6 = [Rxyz(1,1) Rxyz(1,2) Rxyz(1,3) X;
                                           Rxyz(2,1) Rxyz(2,2) Rxyz(2,3) Y;
                                           Rxyz(3,1) Rxyz(3,2) Rxyz(3,3) Z;
                                           0         0         0         1];
```

Hence Rxyz is the given position and orientation for end-effector with regard to base to achieve(user requirement)... Known.

H0_6 is The total transformation between the base in the given references coordinates system(function of thetas)...thetas are required.

From our manipulator specification, Links lengths in mm are :

l1 = 215.5;
l2 = 19.58;
l3 = 173;
l4 = 195;
l5 = 182;
l6 = 120.27;
l7 = 62;
l8 = 62;

and ranges of actuated joints angles are:

-180 < th1 < 180
-108 < th2 < 45
-15 < th3 < 194
-180 < th4 < 180
-19 < th5 < 148
-180 < th6 < 180

Now solving this equation by equating individual terms of both matrices, we get the inverse solution. There are may be more than one solution or no solution if the desired point is out of the workspace of the manipulator.

H0_6(1,1) = Rxyz(1,1);
H0_6(1,2) = Rxyz(1,2);
H0_6(1,3) = Rxyz(1,3);
H0_6(1,4) = X;
H0_6(2,1) = Rxyz(2,1);
H0_6(2,2) = Rxyz(2,2);
H0_6(2,3) = Rxyz(2,3);
H0_6(2,4) = Y;
H0_6(3,1) = Rxyz(3,1);
H0_6(3,2) = Rxyz(3,2);
H0_6(3,3) = Rxyz(3,3);
H0_6(3,4) = Z;

Whereas twelve above kinematics equations is a function of (Θ_1, Θ_2, Θ_3, Θ_4, Θ_5, Θ_6), with inverse kinematic solutions , the value of each joint can be determined in order to place the arm at a desired position and orientation.

III. VALIDATION OF INVERSE KINEMATICS

User requirements:
values of joints angles(Θ_1, Θ_2, Θ_3, Θ_4, Θ_5, Θ_6) to get a particular position
[258.8485, 150, 650, 53.9891093531661, 106.99310099067357, 18.51038314678336]
which is equal to Rxyz =

```
[-0.2771    0.0928    0.9563     257.9;
 -0.5469    0.8031   -0.2364    -149.8;
 -0.7900   -0.5886   -0.1718     650.2;
       0         0         0        1];
```

Given:
coordinates of the origin, known values of geometric parameters of the robot, Links lengths and ranges of actuated joints angles.

By substituting for Links lengths then multiplying each matrix of H matrices on the right side of above equation, H0_6 is the result ,then equalizing each element of H0_6 matrix(a function of joint angles) to each element of the required matrix Rxyz(known):

H0_6(1,1) = -0.2771;
H0_6(1,2) = 0.0928;
H0_6(1,3) = 0.9563;

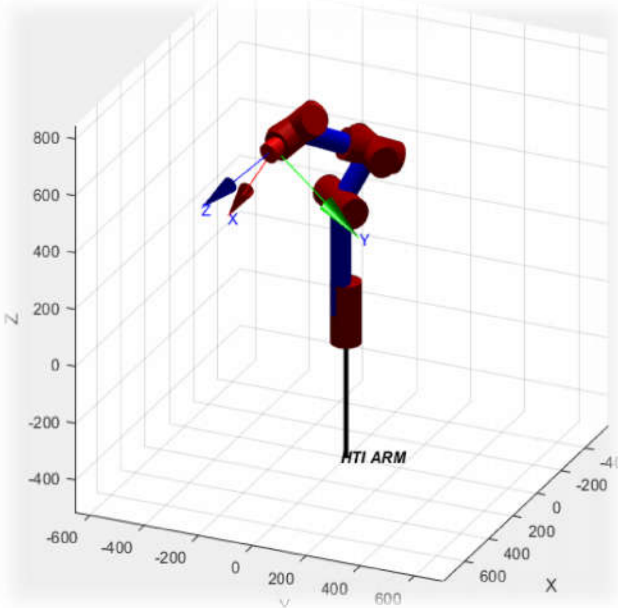
H0_6(1,4) = 257.9;
H0_6(2,1) = -0.5469;
H0_6(2,2) = 0.8031;
H0_6(2,3) = -0.2364;
H0_6(2,4) = -149.8;
H0_6(3,1) = -0.7900;
H0_6(3,2) = -0.5886;
H0_6(3,3) = -0.1718;
H0_6(3,4) = 650.2;

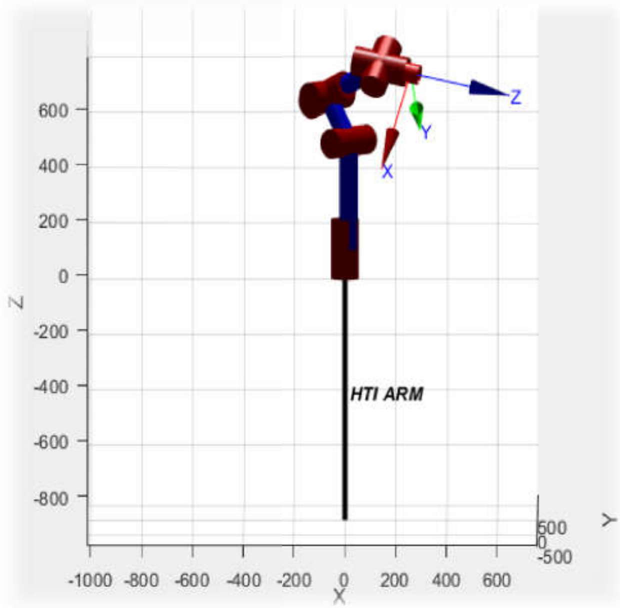
By using MATLAB, joint angles as a result of analytical inverse method to solve above twelve kinematics equations are :

th1 = -40.7096 ;
th2 = 34.6655 ;
th3 = -10.7738 ;
th4 = 40.9022 ;
th5 = 47.2396 ;
th6 = -74.1101;

The inverse kinematic solution has been validated by substituting in the forward model or using Robotics Toolbox for MATLAB. Numerical results together with visual plot of position and orientation of a robot in MATLAB environment gives clear insight of the kinematic behaviour of a robot .Inputs of the test matlab program are previous outs of inverse kinematic matlab program (joints angles values): th1 = -40.709; th2 = 34.6655; th3 = -10.7738; th4 = 40.9022; th5 = 47.2396; th6 =74.1101;
Out of the test matlab program is the our desired matrix:
Rxyz =

```
[-0.2771    0.0928    0.9563     257.9;
 -0.5469    0.8031   -0.2364    -149.8;
 -0.7900   -0.5886   -0.1718     650.2;
       0         0         0        1];
```





MATLAB Plot for joint angle configuration

[40.709,34.6655,10.7738,40.9022,47.2396,74.1101;]

IV. IMPLEMENTATION OF IK MODEL ON OUR 6 DOF ROBOTIC ARM MANIPULATOR

The IK model has been implemented on our 6 DOF robotic arm manipulator. An object has been placed at a known position and orientation. With this known information from a user The task chosen is basically picking an object from one location and placing it onto another. Both the source and destination position and orientation have been given as an input.

Figure (1-a) illustrates the arm at its 'home' position (angles are in their zero degree). Based on the object coordinates given by the user, the robot moves as per computed joint angles (by IK model). Figure (1-b) shows the motion of the robot toward the object. before reaching the target location, it passed through via point 'A' in Figure (2-a) to be ready for picking the object. Sequence of pick-up of the object from its known source position is shown in Figure(2-b). After picking-up the object, it passed through via point 'B' in Figure (2-c) to be ready for moving to the destination point. Figure (3) illustrates the robot then moves toward destination point through via points(C,D) , whose coordinates have also been taught by the user. then finds its way back to the home position in Figure (4).
Both the source and the destination point should lie inside the operational workspace of the robot.

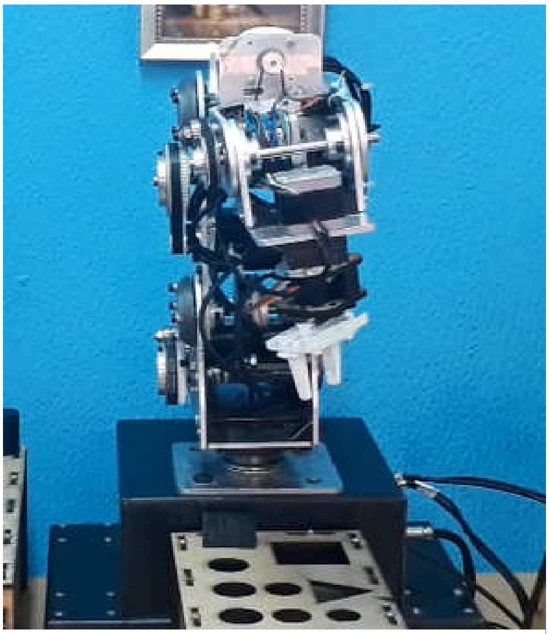


fig.(1-a)

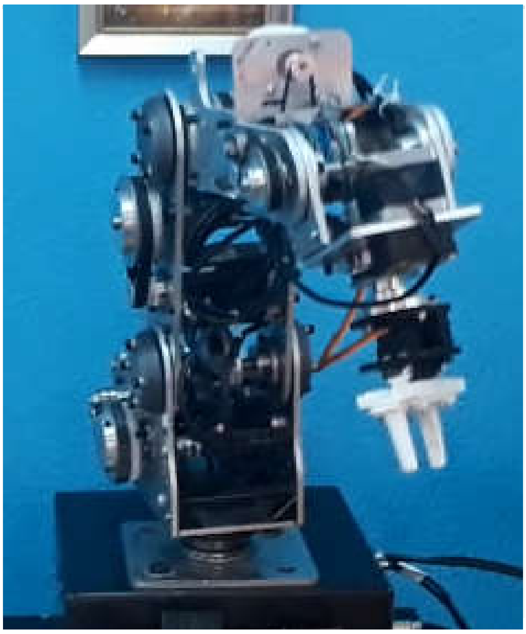


fig.(1-b)

fig.(2-a)

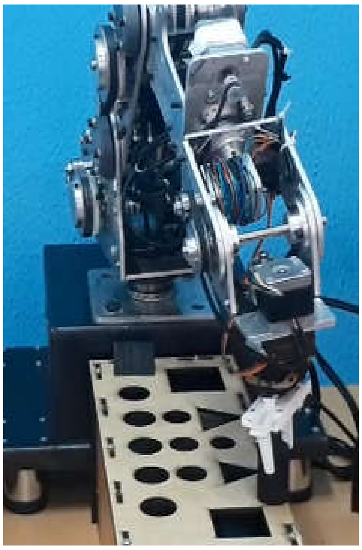


fig.(2-b)

fig.(2-c)

fig.(3-a)

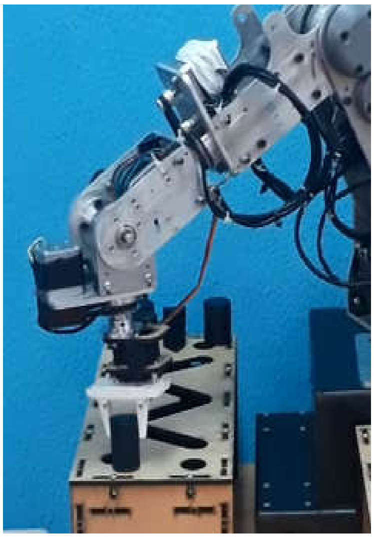


fig.(3-b)

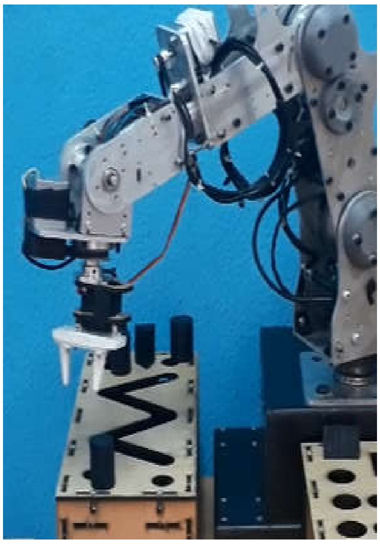


fig.(3-c)

fig.(4-a)

fig.(4-b)

V. CONCLUSION

A complete analytical solution for the inverse kinematics of OUR 6 DOF ROBOTIC ARM MANIPULATOR is derived in this study. The derived analytical inverse kinematics model always provides correct joint angles for moving the arm end-effector to any given reachable positions and orientations. This robot arm is used to perform a pick and place task. We found that the difference between the analytical & physical values of this robot is very less. Hence it proves the effectiveness of the analytical solution. Hence, this analytical method can also be used for deriving the inverse kinematics of other types of robotic arms.