International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 3, March 2012)

Analytical Solution for Inverse Kinematics of SCORBOT-ER-Vplus Robot

Vivek A Deshpande[1], Dr. P M George[2]

[1]*Associate Professor in Mechanical Engineering,*
G. H. Patel College of Engineering & Technology, Vallabh Vidyanagar, Anand, Gujarat – 388120, India.

[2]*Professor & Head, Mechanical Engineering Department,*
Birla Vishwakarma Mahavidyalaya College of Engineering, Vallabh Vidyanagar, Anand Gujarat – 388120, India.

[1]vivekdeshpande@gcet.ac.in
[2]pmgeorge02@yahoo.com

***Abstract*— The kinematics of manipulators is a central problem in the automatic control of robot manipulators. The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. The kinematic equations of motion are derived using Denavit - Hartenberg (DH) representation. In this paper, an analytical solution for the inverse kinematics of SCORBOT ER-Vplus robot arm is presented, to analyze the movement of arm from one point in space to another point. In this solution the only decision variables are the coordinates of the origin and the destination points in space besides the geometric parameters of the robot arm. SCORBOT-ER Vplus is a vertical articulated robot, with five revolute joints. It is a dependable and safe robotic system designed for laboratory and training applications. This versatile system allows students to gain theoretical and practical experience in robotics, automation and control systems. The MATLAB 8.0 is used to solve this mathematical model for a set of joint parameter. The kinematics solution of the MATLAB program was found to be identical with the robot arm's actual reading.**

***Keywords*—Robot manipulator, SCORBOT ER-Vplus robot arm, Inverse kinematics**

I. INTRODUCTION

The mathematical modeling of robot kinematics is motivated by the complexity of robotic systems, which possess highly nonlinear characteristics. The kinematic models are needed for off-line and on-line program generation and for tracking functional trajectories. Since the computational procedures strongly depend of the formulation of the equations specifying robot kinematics, efficient formulations are of crucial importance. Inverse kinematics modeling has been one of the main problems in robotics research. The most popular method for controlling robotic arms is still based on look-up tables that are usually designed in a manual manner [1-3].

Main objective of this paper is to present an analytical solution for the inverse kinematics of SCORBOT ER-Vplus robot arm, to analyze the movement of arm from one point in space to another point [4]. In this model the only decision variables are the coordinates of the origin and the destination points in space besides the geometric parameters of the robot arm.

The kinematics solution of any robot manipulator consists of two sub problems forward and inverse kinematics. Forward kinematics will determine where the robot's manipulator hand will be if all joints are known whereas inverse kinematics will calculate what each joint variable must be if the desired position and orientation of end-effector is determined Hence Forward kinematics is defined as transformation from joint space to cartesian space whereas Inverse kinematics is defined as transformation from cartesian space to joint space. General methods do exist for solving forward kinematics [5-9]. So in this paper analytical solution for inverse kinematics of the SCORBOT ER-Vplus educational robot arm is studied. For the given set of parameter, a program in MATLAB 8.0 is made and its output is compared with the experimental result.

II. KINEMATIC MODEL OF SCORBOT-ER-VPLUS ROBOT

For the research work, SCORBOT-ER V plus robot in our robotics laboratory is being used [10]. It is a vertical articulated robot, with five revolute joints. It has a stationary base, shoulder, elbow, tool pitch and tool roll. Figures 1 & 2 identify the joints and links of the mechanical arm.

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 3, March 2012)

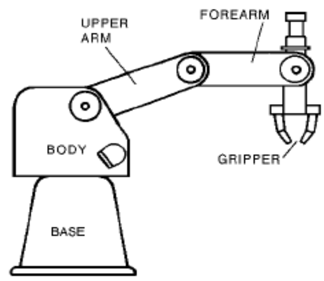

Figure I: Robot Arm Links

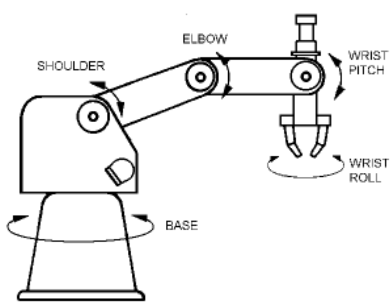

Figure II: Robot Arm Joints

In this study, the standard Denavit-Hartenberg (DH) [11, 12] convention and methodology are used to derive its kinematics. The coordinate frame assignment and the DH parameters are depicted in Figure 3 and listed in Table 1, respectively.

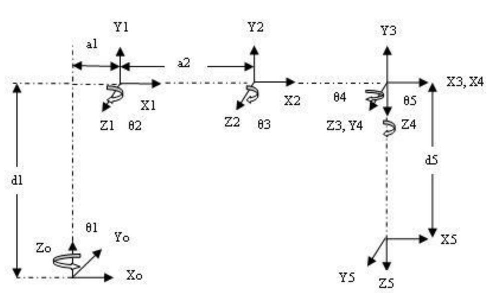

Figure III: Coordinate frame assignment

Table I: D-H Parameter for Scorbot ER-Vplus

Joint i	α_i (deg)	a_i (mm)	d_i (mm)	θ_i (deg)	Operating range
1	π/2	101.25	334.25	38.15°	-155° to +155°
2	0	220	0	-30°	-35° to +130°
3	0	220	0	45°	-130° to +130°
4	π/2	0	0	-63.54°	-130° to +130°
5	0	0	137.35	0°	-570° to +570°

Based on the DH convention, the transformation matrix from joint *i* to joint *i+1* is given by:

$$^{i-1}T_i = \begin{bmatrix} C\theta i & -S\theta iC\alpha i & S\theta iS\alpha i & aiC\theta i \\ S\theta i & C\theta iC\alpha i & -C\theta iS\alpha i & aiS\theta i \\ 0 & S\alpha i & C\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $S\theta_i=\sin\theta_i$, $C\theta_i=\cos\theta_i$, $S\alpha_i=\sin\alpha_i$, $C\alpha_i=\cos\alpha_i$, $S_{ijk}=\sin(\theta_i+\theta_j+\theta_k)$, $C_{ijk}=\sin(\theta_i+\theta_j+\theta_k)$

Multiplying all $^{i-1}T_i$ for i = 1 to 5 ($^0T_1$, $^1T_2$, $^2T_3$, $^3T_4$, $^4T_5$), we get $^0T_5$.

$^0T_5$ =

$$\begin{bmatrix} C_1C_{234}C_5+S_1S_5 & C_1C_{234}S_5+S_1C_5 & C_1S_{234} & C_1(d_5S_{234}+a_3C_{23}+a_2C_2+a_1) \\ S_1C_{234}C_5-C_1S_5 & -S_1C_{234}S_5-C_1C_5 & S_1S_{234} & S_1(d_5C_{234}+a_3S_{23}+a_2C_2+a_1) \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & C_1(d_5+a_3C_{23}+a_2C_2+a_1) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Te = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where T_e is end-effector transformation matrix.

Now solving $^0T_5$ = Te by equating individual terms of both matrices, we get the inverse solution. The following equations will be used to obtain the solution for the inverse kinematics problem [3]:

$$T_e = {}^0T_1 * {}^1T_2 * {}^2T_3 * {}^3T_4 * {}^4T_5 \quad (1)$$
$$({}^0T_1)^{-1}\, T_e = {}^1T_2 * {}^2T_3 * {}^3T_4 * {}^4T_5 \quad (2)$$
$$({}^3T_4)^{-1}\, ({}^2T_3)^{-1}\, ({}^1T_2)^{-1}\, ({}^0T_1)^{-1}\, T_e = {}^4T_5 \quad (3)$$

Equating entries (1, 4) & (2, 4) in the matrix equality (1), we obtain

px = c1*(s234*d5+a3*c23+a2*c2+a1) (4)

py = s1*(s234*d5+a3*c23+a2*c2+a1) (5)

Dividing (4) by (5), we obtain the first angle

$$\boldsymbol{\theta_1 = \tan^{-1}(py/px)} \quad (6)$$

Equating entries (1, 4) & (2, 4) in the matrix equality (2), we obtain

c1*px+s1*py = s234*d5+a3*c23+a2*c2 (7)

px = -c234*d5+a3*s23+a2*s2 (8)

Rearranging the two equations, squaring them and then adding the squares gives,

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 3, March 2012)

c3 = ((c1*px+s1*py – s234*d5)^2 + (pz+ c234*d5)^2 - (a2)^2 - (a3)^2) / (2*a2*a3) (9)

Knowing that, s3 = ± $\sqrt{1+C_3^{\,2}}$, we can say that,

θ$_3$ = tan^{-1} (s3/c3) (10)

To find **θ$_3$**, we need to find **θ$_{234}$**, which is solved later.

Now referring to the Equations (7) & (8) and rearranging them, we get

c1*px + s1*py - s234*d5 = a3*c23 + a2*c2 (11)

px + c234*d5 = a3*s23 + a2*s2 (12)

Treating this as a set of two equations in two unknowns and solving for c2 and s2, we get

c2=

$$\frac{C_1 p_x + S_1 p_y - S_{234} d_5)(a_3 C_3 + a_2) + (p_z + C_{234} d_5) S_3 a_3}{(a_3 C_3 + a_2)^2 + s_3^{\,2} a_3^{\,2}} \tag{13}$$

s2 =

$$\frac{(C_1 p_x + S_1 p_y - S_{234} d_5)(a_3 S_3) + (p_z + C_{234} d_5)(a_3 C_3 + a_2)}{(a_3 C_3 + a_2)^2 + S_3^{\,2} a_3^{\,2}} \tag{14}$$

θ$_2$ = tan^{-1}(s2/c2) (15)

Since joints 2, 3 &4 are parallel, pre-multiply by the inverse of $^0T_1$ through $^3T_4$, results in equation (3).

Equating entries (1, 4) & (2, 4) in the matrix equality (3), we obtain

θ$_{234}$ = - tan^{-1} ((c1*ax+s1*ay)/az) (16)

For the known values of ax, ay & az, we can find **θ$_{234}$.**

θ$_4$ = θ$_{234}$ - θ$_2$ - θ$_3$ (17)

III. RESULT

For given coordinates of the origin and the destination point's of the end-effector and known values of geometric parameters of the robot find out the joint angles.

Points	px (mm)	py (mm)	pz (mm)	Pitch(θ$_4$)	Roll(θ$_5$)
1	169.04	0	504.33	-63.54^o	0.0^o
2	315.48	247.88	190.27	-63.54^o	0.0^o

Knowing the following values:

a1 = 101.25 mm; a2 = 220 mm; a3 = 220 mm; d1 = 334.25 mm; d5 = 137.35 mm;

θ$_4$ = -63.54^o; θ$_5$ = 0^o; ax = -0.5893; ay = -0.4629; az = -0.6621;

Case Study: (Analytical Solution)

θ$_1$ = tan^{-1} (py/px) = **38.15^o**

θ$_{234}$ = - tan^{-1} ((c1*ax+s1*ay)/az) = **-48.53^o**

θ$_3$ = tan^{-1}(s3/c3) = **± 45^o**

θ$_2$ = tan^{-1}(s2/c2) = **- 30^o & 60^o**

θ$_4$ = θ$_{234}$ - θ$_2$ - θ$_3$ = -63.54^o

θ$_{23}$ = θ$_{234}$ - θ$_4$ = **15^o**

The Final matrix 0T5 is:

[0.5212 0.6177 -0.5887 315.48]
[0.4094 -0.7863 -0.4625 247.88]
[-0.7487 0.0000 -0.6628 190.27]
[0.0000 0.0000 0.0000 1.0000]

Case-study: (MATLAB Program Output)

The joint parameters are as follows:

a	alpha1	d	theta
101.2500	90.0000	190.2700	38.1500
220.0000	0.0000	0.0000	-30.0000
220.0000	0.0000	0.0000	45.0000
0.0000	90.0000	0.0000	-63.5400
0.0000	0.0000	137.3500	0.0000

The Final matrix 0T5 is:

[0.5207 0.6177 -0.5893 315.6175]
[0.4090 -0.7864 -0.4629 247.9208]
[-0.7494 0.0000 -0.6621 190.2512]
[0.0000 0.0000 0.0000 1.0000]

The Final matrix values ($^0T_5$) are checked against the physical positions of the robot arm in Table 2.

Table II: Difference between analytical & physical values of the robot

Position Values	$^0T_5$ Values (mm)	Measured Values (mm)	Difference (mm)
px	315.48	315.61	0.13
py	247.88	247.92	0.04
pz	190.27	190.25	-0.02

IV. CONCLUSION

A complete analytical solution to the inverse kinematics of SCORBOT ER-Vplus robot arm is derived in this paper. The derived analytical inverse kinematics model always provides correct joint angles for moving the arm end-effector to any given reachable positions and orientations. This robot arm is used to perform a pick and place task. We found that the difference between the analytical & physical values of this robot is very less. Hence it proves the effectiveness of the analytical solution. Hence, this analytical method can also be used for deriving the inverse kinematics of other types of robotic arms.

International Journal of Emerging Technology and Advanced Engineering
Website: www.ijetae.com (ISSN 2250-2459, Volume 2, Issue 3, March 2012)

V. ACKNOWLEDGEMENT

The authors wish to thank "Sophisticated Instrumentation Centre for Applied Research and Testing" (SICART) for permit us in using the facilities to carry out research work. Authors also wish to thank Prof. Mehul M. Gor (Asst. Professor, Mechanical Engg., GCET) & Prof. R. C. Sanghavi (Asst. Professor, Mathematics Dept., GCET) for their technical help.

References

[1] R. D. Klafter, T. A. Chmielewski and M. Negin. (1989). Robotic Engineering: An Integrated Approach. Prentice Hall.

[2] P. J. McKerrow. (1991). Introduction to Robotics. Addison-Wesley.

[3] S. B. Niku. (2001). Introduction to Robotics: Analysis, Systems, Applications. Prentice Hall.

[4] Baki Koyuncu, and Mehmet Güzel, "Software Development for the Kinematic Analysis of a Lynx 6 Robot Arm", World Academy of Science, Engineering and Technology 30 2007.

[5] Paul, R. P.(1981). Robot Manipulators: Mathematics, Programming and Control. Cambridge, MIT Press.

[6] Featherstone, R. (1983). Position and velocity transformations between robot end-effector coordinates and joint angles. Int. J. Robotics Res. 2, 35-45.

[7] Lee, C. S. G. (1982). Robot arm kinematics, dynamics, and control. Computer, 15, 62-80.

[8] Myers, D. R. & Gordon, D. F. (1982). Kinematic equations for industrial manipulators. Ind. Robot 162-165.

[9] Dr. Anurag Verma & Mehul Gor (2010). Forward Kinematics Analysis of 6 DOF Arc Welding Robot. International Journal of Engineering Science and Technology, Vol. 2(9), pp 4682-4686.

[10] SCORBOT-ER Vplus User's Manual, 3rd edition, Intelitek Inc., Catalog # 100016 Rev. C, February 1996.

[11] Denavit J. & R.S. Hartenberg (1955). A kinematic Notation for Lower- Pair Mechanism Based on Matrices. ASME Journal of Applied Mechanics, 215-221.

[12] Dr. Anurag Verma & Vivek Deshpande (2011). End-effector Position Analysis of SCORBOT-ER Vplus Robot. International Journal of Advanced Science and Technology, Vol. 29. pp 61-66.