

Vehicle system dynamics

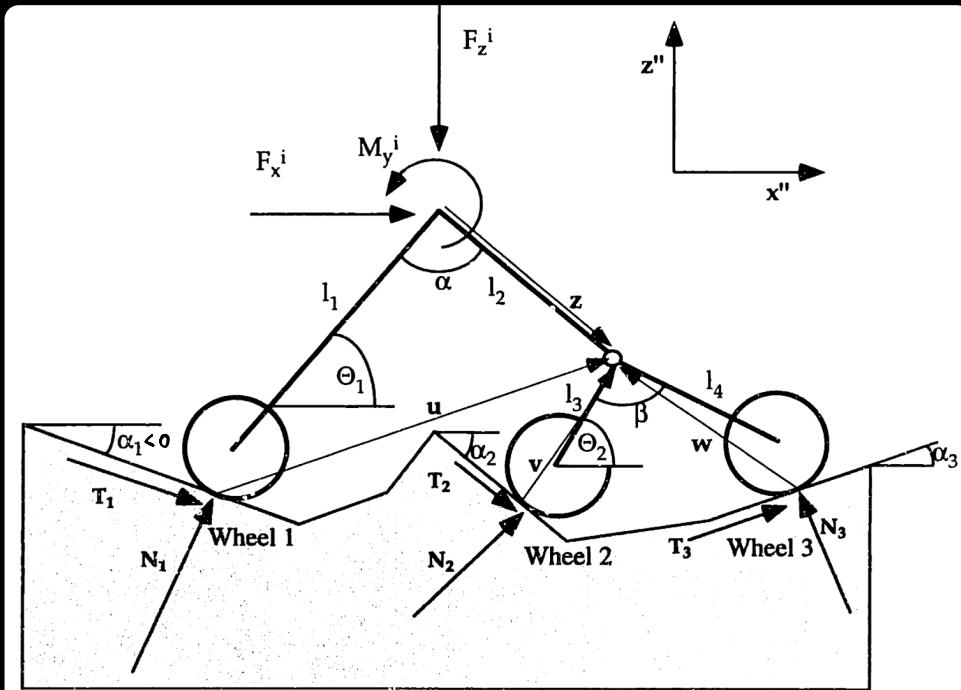


Figure 3-6: Rover Planar Analysis.

STATIC EQUILIBRIUM EQUATIONS

$$\left\{ \begin{array}{l}
 (1) \quad T_1 c_1 + T_2 c_2 + T_3 c_3 - N_1 s_1 - N_2 s_2 - N_3 s_3 + F_x = 0 \\
 (2) \quad T_1 s_1 + T_2 s_2 + T_3 s_3 + N_1 c_1 + N_2 c_2 + N_3 c_3 - F_z = 0 \\
 (3) \quad T_1 c_1 u_y - T_1 s_1 u_x - N_1 s_1 u_y - N_1 c_1 u_x + M_y + F_x z_y + F_z z_x = 0 \\
 (4) \quad T_2 c_2 v_y - T_2 s_2 v_x - N_2 s_2 v_y - N_2 c_2 v_x + \\
 \qquad T_3 c_3 w_y - T_3 s_3 w_x - N_3 s_3 w_y - N_3 c_3 w_x = 0
 \end{array} \right.$$

$$A \begin{bmatrix} T_1 & T_2 & T_3 & N_1 & N_2 & N_3 \end{bmatrix}^T = \begin{bmatrix} -F_x & F_z & -M_y & -F_x z_y & -F_z z_x & 0 \end{bmatrix}^T$$

$$A = \begin{bmatrix}
 c_1 & c_2 & c_3 & -s_1 & -s_2 & -s_3 \\
 s_1 & s_2 & s_3 & c_1 & c_2 & c_3 \\
 c_1 u_y - s_1 u_x & 0 & 0 & -s_1 u_y - c_1 u_x & 0 & 0 \\
 0 & c_2 v_y - s_2 v_x & c_3 w_y - s_3 w_x & 0 & -s_2 v_y - c_2 v_x & -s_3 w_y - c_3 w_x
 \end{bmatrix}^T$$

\mathcal{B}

Consider T_1, T_2, T_3 as inputs :

$$\begin{bmatrix} -S_1 & -S_2 & -S_3 \\ C_1 & C_2 & C_3 \\ \alpha_2 & 0 & 0 \\ 0 & \gamma_5 & \gamma_6 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

$$(1) \rightarrow -S_1 N_1 - S_2 N_2 - S_3 N_3 = -T_1 c_1 - T_2 c_2 - T_3 c_3 - F_x$$

$$(2) \rightarrow C_1 N_1 + C_2 N_2 + C_3 N_3 = -T_1 S_1 - T_2 S_2 - T_3 S_3 + F_z$$

$$(3) \rightarrow \alpha_2 N_1 = -T_1 \gamma_1 - M_y - F_x z_y - F_z z_x$$

$$(4) \rightarrow \gamma_5 N_2 + \gamma_6 N_3 = -T_2 \gamma_3 - T_3 \gamma_4$$

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \overbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}}^{B^{\#}} \left(T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \right)$$

N_1, N_2, N_3 are known as functions of $\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3$ and COM

(see "Introduction to the mechanics of space robots" eqns 5.237 - 5.238)

From (*) follows :

$$(*) \quad \begin{cases} F_x = -T_1 \cos \alpha_1 - T_2 \cos \alpha_2 - T_3 \cos \alpha_3 + N_1 \sin \alpha_1 + N_2 \sin \alpha_2 + N_3 \sin \alpha_3 \\ F_z = T_1 \sin \alpha_1 + T_2 \sin \alpha_2 + T_3 \sin \alpha_3 + N_1 \cos \alpha_1 + N_2 \cos \alpha_2 + N_3 \cos \alpha_3 \\ M_y = -T_1 \gamma_1(\alpha_1, u_x, u_y) + N_1 \gamma_2(\alpha_1, u_x, u_y) - F_x z_y - F_z z_x \end{cases}$$

$$\text{where } \gamma_1(\alpha_1, u_x, u_y) = \cos \alpha_1 u_y - \sin \alpha_1 u_x$$

$$\gamma_2(\alpha_1, u_x, u_y) = \sin \alpha_1 u_y + \cos \alpha_1 u_x$$

$$\dot{\omega}_i = \frac{T_i}{I_m} = \frac{r T_i}{I_m}$$

T_i	motor Torque
r	Wheel radius
T_i	traction force
I_m	inertia of the motor

$$(*) \quad \left\{ \begin{array}{l} \ddot{x} = \frac{1}{m} \left[-\dot{\omega}_1 \frac{I_m}{r} \cos \alpha_1 - \dot{\omega}_2 \frac{I_m}{r} \cos \alpha_2 - \dot{\omega}_3 \frac{I_m}{r} \cos \alpha_3 + N_1 \sin \alpha_1 + N_2 \sin \alpha_2 + N_3 \sin \alpha_3 \right] \\ \ddot{z} = \frac{1}{m} \left[\dot{\omega}_1 \frac{I_m}{r} \sin \alpha_1 + \dot{\omega}_2 \frac{I_m}{r} \sin \alpha_2 + \dot{\omega}_3 \frac{I_m}{r} \sin \alpha_3 + N_1 \cos \alpha_1 + N_2 \cos \alpha_2 + N_3 \cos \alpha_3 \right] \\ \dot{\omega}_y = \frac{1}{I_y} \left[-\dot{\omega}_1 \frac{I_m}{r} \gamma_1(\alpha, u_x, u_y) + N_1 \gamma_2(\alpha_1, u_x, u_y) - m (\ddot{x} z - \ddot{z} x) \right] \end{array} \right.$$

STATE VARIABLES $\xi = [x, v_x, z, v_z, \theta_y, \omega_y]^T$

ACCELERATION INPUTS $U = [\dot{\omega}_1 \ \dot{\omega}_2 \ \dot{\omega}_3]^T$

OUTPUTS $h = [x \ z \ \theta_y]^T$

$$\left\{ \begin{array}{l} \dot{\xi} = f(\xi) + \sum_i g_i(\xi) u_i \\ y = h(\xi) \end{array} \right.$$

$$\dot{\xi}_1 = \xi_2$$

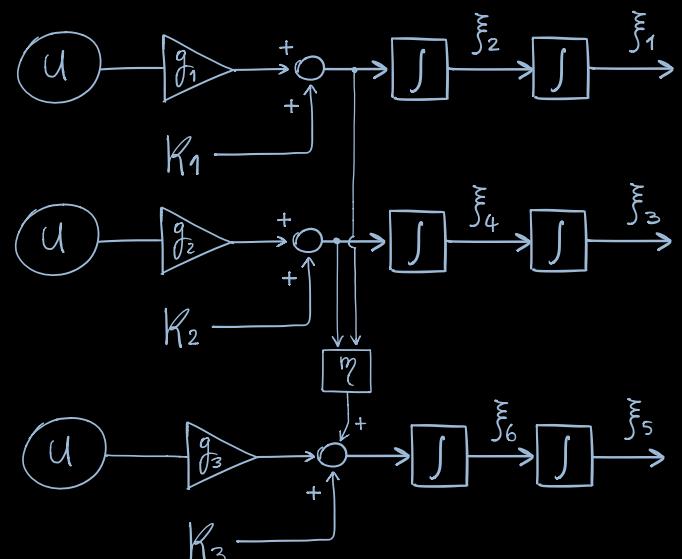
$$\dot{\xi}_2 = g_{11} u_1 + g_{12} u_2 + g_{13} u_3 + k_1$$

$$\dot{\xi}_3 = \xi_4$$

$$\dot{\xi}_4 = g_{21} u_1 + g_{22} u_2 + g_{23} u_3 + k_2$$

$$\dot{\xi}_5 = \xi_6$$

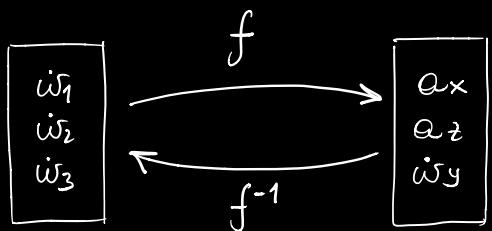
$$\dot{\xi}_6 = g_{31} u_1 + k_3 + \gamma(\dot{\xi}_2, \dot{\xi}_4)$$



can be neglected?
Otherwise inputs
must be at the Jerk level

$$\dot{\xi} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} 0 \\ K_1(\cdot) \\ 0 \\ K_2(\cdot) \\ 0 \\ K_3(\cdot) \end{pmatrix}$$

DIRECT / INVERSE
DYNAMICS

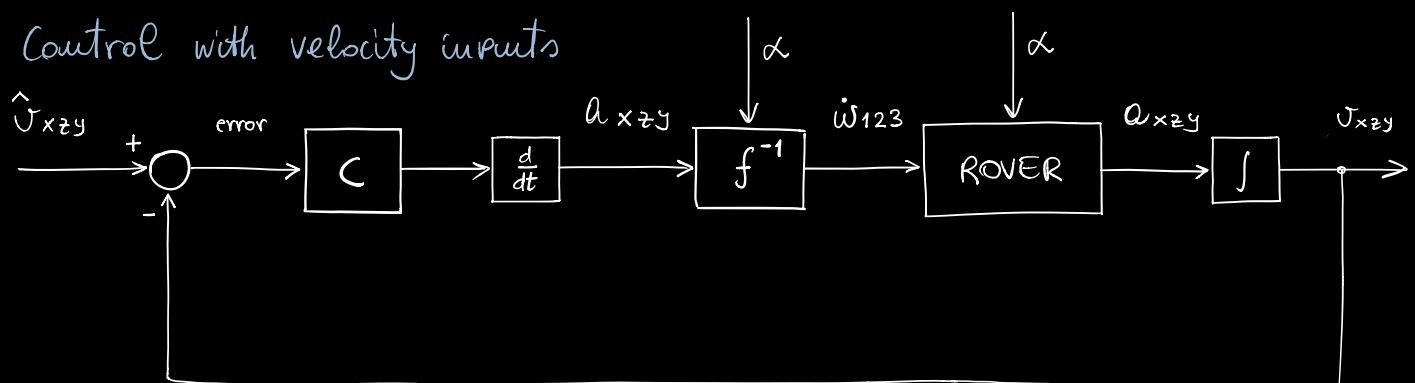


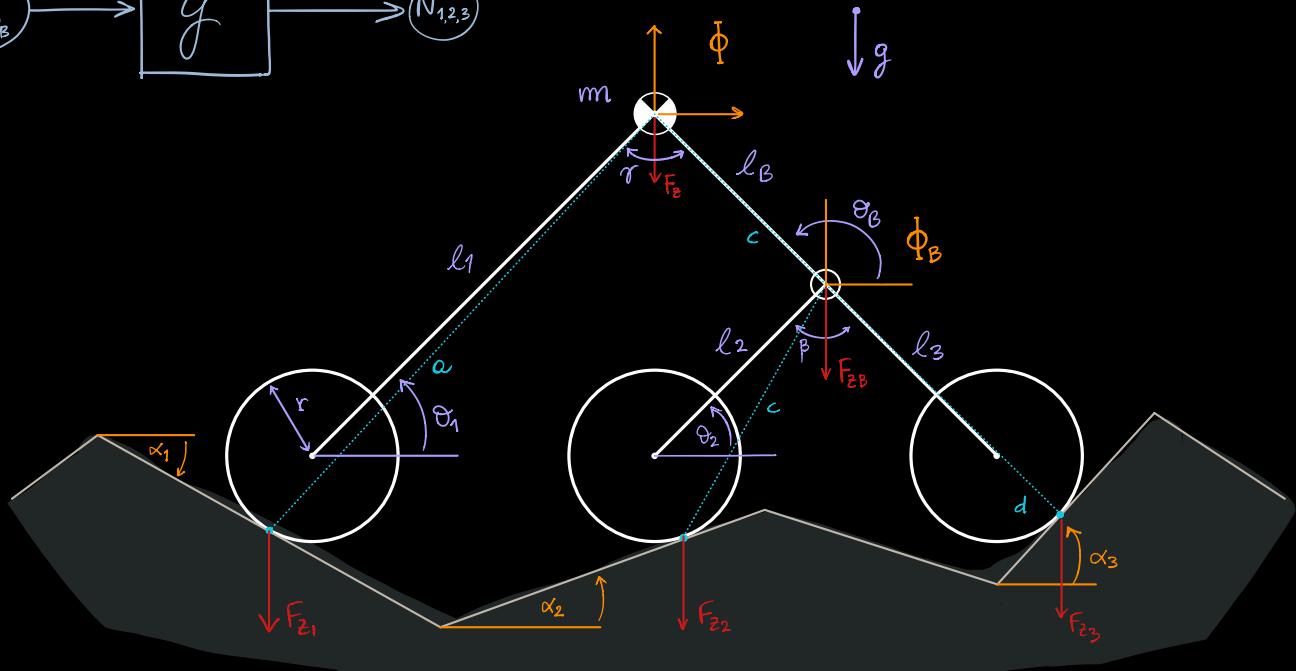
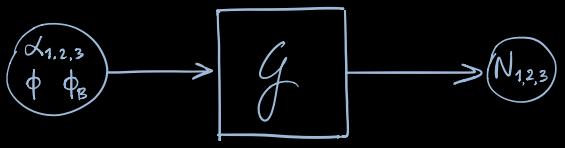
CoM acceleration

$$a = f(u) + k \quad \longleftrightarrow \quad u = f^{-1}(a - k)$$

$$u = \begin{pmatrix} -\text{Im}(\alpha_x \cos \alpha_1 + \alpha_z \cos \alpha_2 + \dot{\omega}_y \cos \alpha_3 - k_1 \cos \alpha_1 - k_2 \cos \alpha_2 - k_3 \cos \alpha_3) / m r \\ \text{Im}(\alpha_x \sin \alpha_1 + \alpha_z \sin \alpha_2 + \dot{\omega}_y \sin \alpha_3 - k_1 \sin \alpha_1 - k_2 \sin \alpha_2 - k_3 \sin \alpha_3) / m r \\ - I_m \gamma_1 (\alpha_x - k_1) / I_y r \end{pmatrix}$$

Control with velocity inputs





$$\theta_1 = \frac{\pi - \alpha}{2}$$

$$\theta_2 = \frac{\pi - \beta}{2}$$

$$\theta_B = \frac{\pi + \gamma}{2}$$

$$\theta_3 = \frac{\pi + \beta}{2}$$

GROUND-TO-PIVOT Vectors

$$a = R(\phi) \begin{pmatrix} l_1 \cos \theta_1 - r \sin \alpha_1 \\ l_1 \sin \theta_1 + r \cos \alpha_1 \end{pmatrix} \quad b = R(\phi) \begin{pmatrix} l_B \cos \theta_B \\ l_B \sin \theta_B \end{pmatrix}$$

$$c = R(\phi) R(\phi_B) \begin{pmatrix} l_2 \cos \theta_2 - r \sin \alpha_2 \\ l_2 \sin \theta_2 + r \cos \alpha_2 \end{pmatrix}$$

$$d = R(\phi) R(\phi_B) \begin{pmatrix} l_3 \cos \theta_3 - r \sin \alpha_3 \\ l_3 \sin \theta_3 + r \cos \alpha_3 \end{pmatrix}$$

NORMAL FORCES

$$F_z = mg$$

$$F_{z1} = F_z \frac{|b_x|}{|a_x| + |b_x|}$$

$$F_{zB} = F_z \frac{|a_x|}{|a_x| + |b_x|}$$

$$F_{z2} = F_B \frac{|d_x|}{|c_x| + |d_x|}$$

$$F_{z3} = F_B \frac{|c_x|}{|c_x| + |d_x|}$$