Vehicle system dynamics

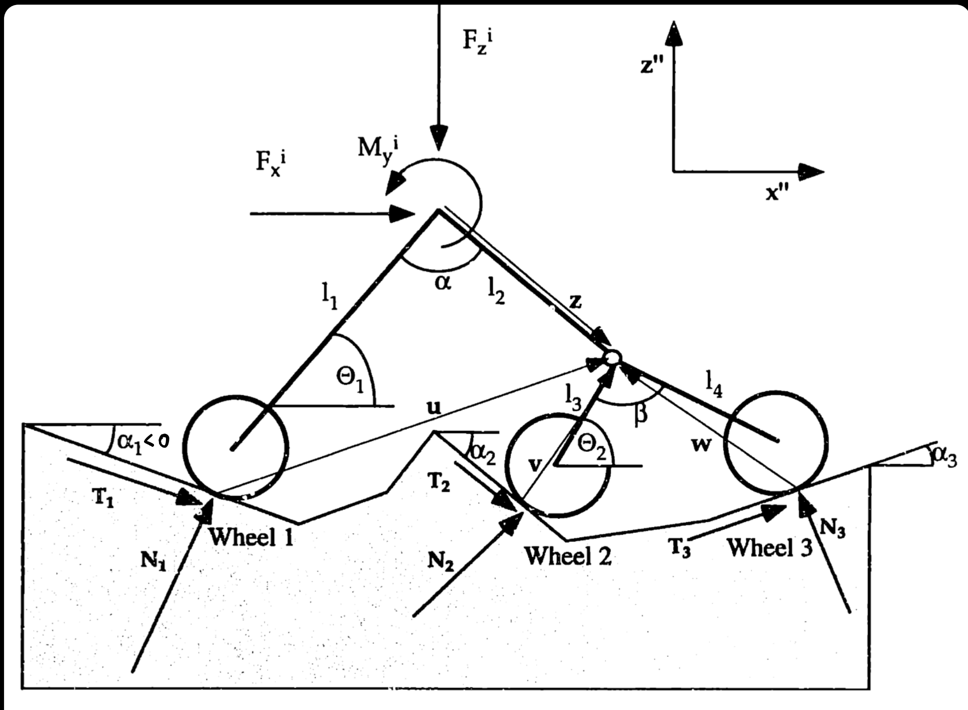


Figure 3-6: Rover Planar Analysis.

STATIC EQUILIBRIUM EQUATIONS

$$(*)\left\{\begin{array}{ll}
(1) & T_1 C_1 + T_2 C_2 + T_3 C_3 - N_1 S_1 - N_2 S_2 - N_3 S_3 + F_x = 0 \\
(2) & T_1 S_1 + T_2 S_2 + T_3 S_3 + N_1 C_1 + N_2 C_2 + N_3 C_3 - F_z = 0 \\
(3) & T_1 C_1 U_y - T_1 S_1 U_x - N_1 S_1 U_y - N_1 C_1 U_x + M_y + F_x z_y + F_z z_x = 0 \\
(4) & T_2 C_2 V_y - T_2 S_2 V_x - N_2 S_2 V_y - N_2 C_2 V_x + \\
 & T_3 C_3 W_y - T_3 S_3 W_x - N_3 S_3 W_y - N_3 C_3 W_x = 0
\end{array}\right.$$

$$A \begin{bmatrix} T_1 & T_2 & T_3 & N_1 & N_2 & N_3 \end{bmatrix}^T = \begin{bmatrix} -F_x & F_z & -M_y - F_x z_y - F_z z_x & 0 \end{bmatrix}^T$$

$$A = \left[\begin{matrix} C_1 & C_2 & C_3 \\ S_1 & S_2 & S_3 \\ \overbrace{C_1 U_y - S_1 U_x}^{\eta_1} & 0 & 0 \\ 0 & \underbrace{C_2 V_y - S_2 V_x}_{\eta_3} & \underbrace{C_3 W_y - S_3 W_x}_{\eta_4} \end{matrix} \;\middle|\; \underbrace{\begin{matrix} -S_1 & -S_2 & -S_3 \\ C_1 & C_2 & C_3 \\ \overbrace{-S_1 U_y - C_1 U_x}^{\eta_2} & 0 & 0 \\ 0 & -S_2 V_y - C_2 V_x & -S_3 W_y - C_3 W_x \end{matrix}}_{B} \right]$$

Consider T_1, T_2, T_3 as inputs :

$$\begin{bmatrix} -S_1 & -S_2 & -S_3 \\ C_1 & C_2 & C_3 \\ \eta_2 & 0 & 0 \\ 0 & \eta_5 & \eta_6 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

$(1) \longrightarrow \quad -S_1 N_1 - S_2 N_2 - S_3 N_3 = -T_1 C_1 - T_2 C_2 - T_3 C_3 - F_x$

$(2) \longrightarrow \quad C_1 N_1 + C_2 N_2 + C_3 N_3 = -T_1 S_1 - T_2 S_2 - T_3 S_3 + F_z$

$(3) \longrightarrow \quad \eta_2 N_1 = -T_1 \eta_1 - M_y - F_x z_y - F_z z_x$

$(4) \longrightarrow \quad \eta_5 N_2 + \eta_6 N_3 = -T_2 \eta_3 - T_3 \eta_4$

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \overbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}}^{B^{\#}} \left(T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \right)$$

N_1, N_2, N_3 are known as a functions of $\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3$ and CoM

(see "Introduction to the mechanics of space robots" eqns 5.237 - 5.238)

From $(*)$ follows :

$$(*) \begin{cases} F_x = -T_1 \cos\alpha_1 - T_2 \cos\alpha_2 - T_3 \cos\alpha_3 + N_1 \sin\alpha_1 + N_2 \sin\alpha_2 + N_3 \sin\alpha_3 \\ F_z = T_1 \sin\alpha_1 + T_2 \sin\alpha_2 + T_3 \sin\alpha_3 + N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + N_3 \cos\alpha_3 \\ M_y = -T_1 \eta_1(\alpha_1, u_x, u_y) + N_1 \eta_2(\alpha_1, u_x, u_y) - F_x z_y - F_z z_x \end{cases}$$

where $\eta_1(\alpha_1, u_x, u_y) = \cos\alpha_1 u_y - \sin\alpha_1 u_x$

$\eta_2(\alpha_1, u_x, u_y) = -\sin\alpha_1 u_y - \cos\alpha_1 u_x$

$$\dot{\omega}_i = \frac{\tau_i}{I_m} = \frac{r\,T_i}{I_m} \qquad \begin{cases} \tau_i & \text{motor torque} \\ r & \text{wheel radius} \\ T_i & \text{traction force} \\ I_m & \text{inertia of the motor} \end{cases}$$

$$\tau_i = \dot{\omega}_i I_m$$

$$(*)\begin{cases} F_x = -\tau_1 \frac{1}{r}\cos\alpha_1 - \tau_2 \frac{1}{r}\cos\alpha_2 - \tau_3 \frac{1}{r}\cos\alpha_3 + N_1 \sin\alpha_1 + N_2 \sin\alpha_2 + N_3 \sin\alpha_3 \\ F_z = \tau_1 \frac{1}{r}\sin\alpha_1 + \tau_2 \frac{1}{r}\sin\alpha_2 + \tau_3 \frac{1}{r}\sin\alpha_3 + N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + N_3 \cos\alpha_3 \\ M_y = -\tau_1 \frac{1}{r}\eta_1(\alpha, u_x, u_y) + N_1\,\eta_2(\alpha_1, u_x, u_y) - m\left(F_x Z_y - F_z Z_x\right) \end{cases}$$

STATE VARIABLES $\xi = \left[\xi_1 \ \xi_2 \ \xi_3 \ \xi_4 \ \xi_5 \ \xi_6\right]^T = \left[P_x \ F_x \ P_z \ F_z \ P_y \ M_y\right]^T$

$F = \frac{d}{dt} P$ quantità di moto

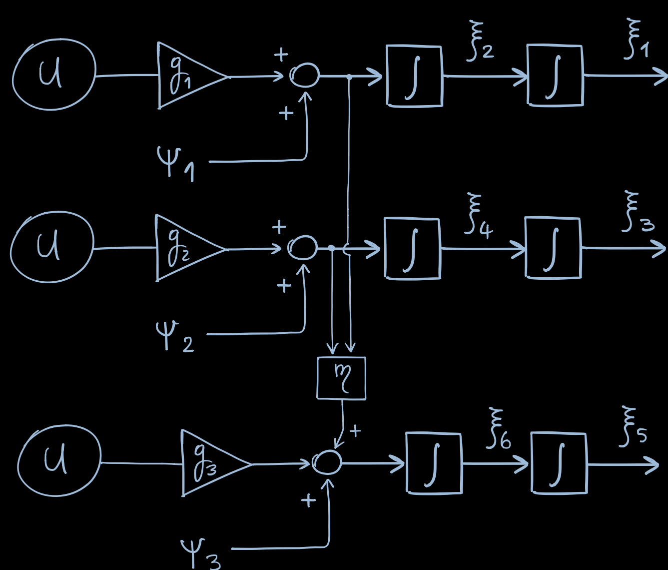
ACCELERATION INPUTS $u = \left[\tau_1 \ \tau_2 \ \tau_3\right]^T$

OUTPUTS $h = \left[x \ z \ \theta_y\right]^T$

$$\begin{cases} \dot{\xi} = \sum_i g_i(\xi)\, u_i + \Psi \\ y = h(\xi) \end{cases}$$

depending on normal forces $N_1 \ N_2 \ N_3$

$$\dot{\xi}_1 = \xi_2$$
$$\dot{\xi}_2 = g_{11} u_1 + g_{12} u_2 + g_{13} u_3 + \Psi_1$$
$$\dot{\xi}_3 = \xi_4$$
$$\dot{\xi}_4 = g_{21} u_1 + g_{22} u_2 + g_{23} u_3 + \Psi_2$$
$$\dot{\xi}_5 = \xi_6$$
$$\dot{\xi}_6 = g_{31} u_1 + \Psi_3 + g(\dot{\xi}_2, \dot{\xi}_4)$$



can be neglected ?
otherwise inputs
must be at the Jerk level

$$\dot{\xi} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \Psi(\dots) \\ 0 \\ \Psi(\dots) \\ 0 \\ \Psi(\dots) \end{pmatrix}$$

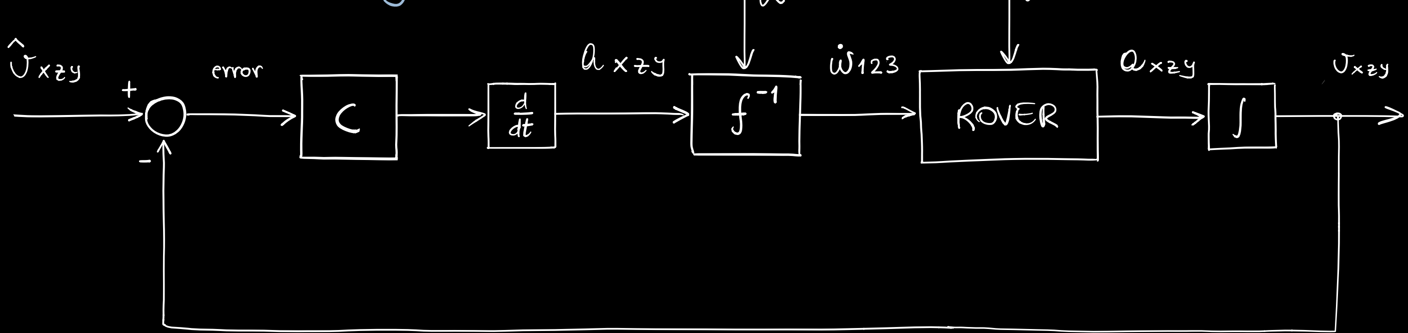
DIRECT / INVERSE DYNAMICS

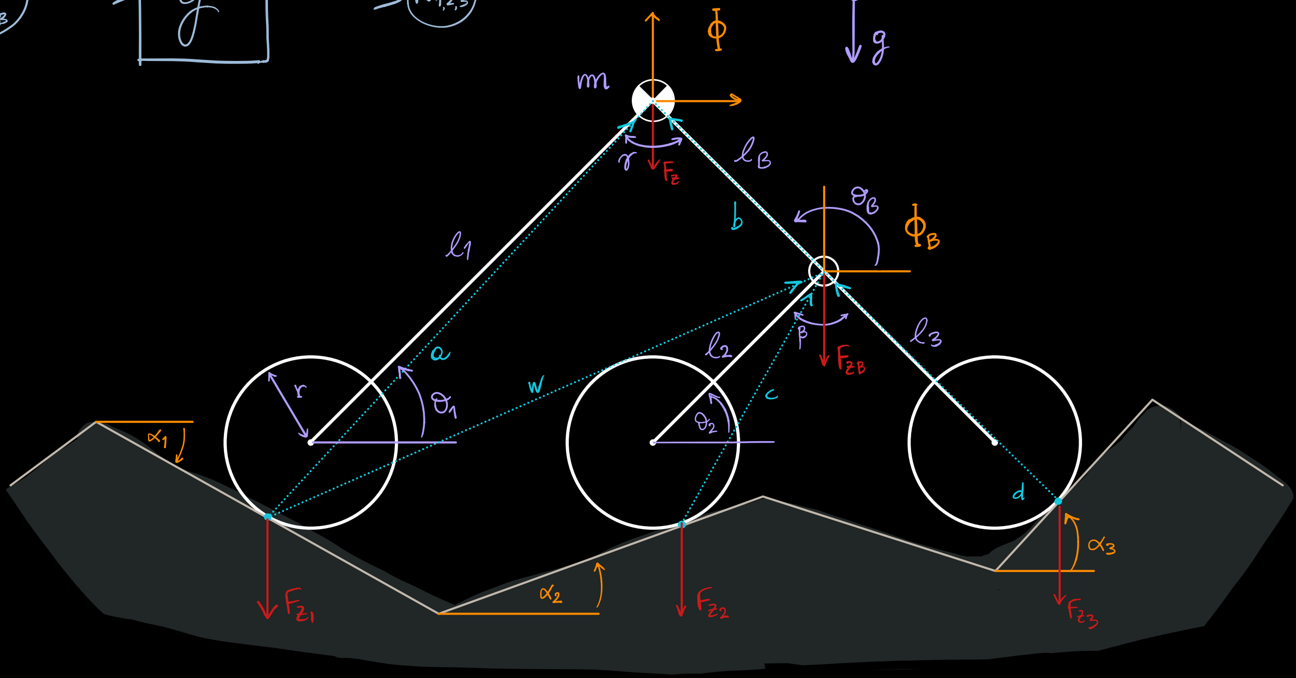
$$\begin{tikzcd} u & F & a \\ \boxed{\begin{matrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{matrix}} \arrow[r, bend left, "f"] & \boxed{\begin{matrix} F_x \\ F_z \\ M_y \end{matrix}} \arrow[l, bend left, "f^{-1}"] \arrow[r] & \begin{matrix} a_x \\ a_z \\ \dot{\omega}_y \end{matrix} \end{tikzcd}$$

acceleration of the CoM

$$F = Gu + \Psi \longleftrightarrow u = G^{\#}(F - \Psi)$$

Control with velocity inputs





$$\theta_1 = \frac{\pi - \gamma}{2} \qquad \theta_2 = \frac{\pi - \beta}{2}$$

$$\theta_B = \frac{\pi + \gamma}{2} \qquad \theta_3 = \frac{\pi + \beta}{2}$$

GROUND-TO-PIVOT Vectors

$$a = R(\phi)\begin{pmatrix} l_1 \cos\theta_1 - r\sin\alpha_1 \\ l_1 \sin\theta_1 + r\cos\alpha_1 \end{pmatrix} \qquad b = R(\phi)\begin{pmatrix} l_B \cos\theta_B \\ l_B \sin\theta_B \end{pmatrix}$$

$$c = R(\phi)\,R(\phi_B)\begin{pmatrix} l_2 \cos\theta_2 - r\sin\alpha_2 \\ l_2 \sin\theta_2 + r\cos\alpha_2 \end{pmatrix}$$

$$d = R(\phi)\,R(\phi_B)\begin{pmatrix} l_3 \cos\theta_3 - r\sin\alpha_3 \\ l_3 \sin\theta_3 + r\cos\alpha_3 \end{pmatrix}$$

NORMAL FORCES

$$F_z = mg \qquad F_{z1} = F_z \frac{|b_x|}{|a_x| + |b_x|} \qquad F_{zB} = F_z \frac{|a_x|}{|a_x| + |b_x|}$$

$$F_{z2} = F_B \frac{|d_x|}{|c_x| + |d_x|} \qquad F_{z3} = F_B \frac{|c_x|}{|c_x| + |d_x|}$$

$$w = a - b = \begin{pmatrix} w_x \\ w_y \end{pmatrix} = R(\phi) \begin{pmatrix} l_1 \cos\theta_1 - r\sin\alpha_1 - l_B \cos\theta_B \\ l_1 \sin\theta_1 + r\cos\alpha_1 - l_B \sin\theta_B \end{pmatrix}$$

$$\eta_1 = \cos\alpha_1 \, w_y - \sin\alpha_1 \, w_x$$

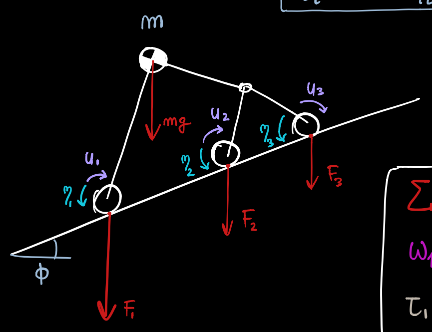
$$\eta_2 = -\sin\alpha_1 \, w_y - \cos\alpha_1 \, w_x$$

$$M_y = -\tau_1 \frac{1}{r} \eta_1 + N_1 \eta_2 - \underbrace{\cancel{m\left[F_x(-b_y) - F_z(-b_x)\right]}}_{\text{can be neglected for small accelerations}} \rightarrow 0$$

ROLLING RESISTANCE

$$\boxed{f_{rr} = C_{rr} \cdot N}$$

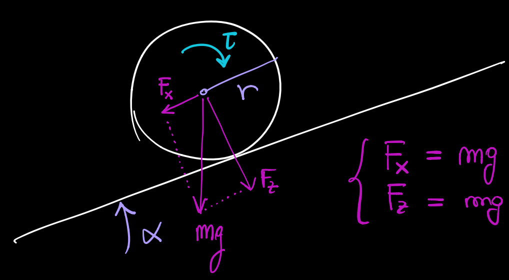
$$\boxed{\tau_i = u_i + \eta_i}$$



$$\sum F_i = mg$$
$$\omega_1 = \omega_2 = \omega_3$$
$$\tau_1 \neq \tau_2 \neq \tau_3$$

$$-F_1 \sin\phi = -m_1 \overbrace{g \sin\phi}^{acc}$$

$$\dot{\omega}_1 = \frac{a}{r} = \frac{g}{r}\sin\phi$$



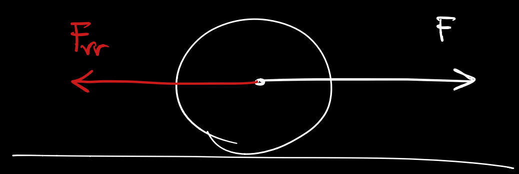
$$\begin{cases} F_x = mg \sin\alpha \\ F_z = mg \cos\alpha \end{cases}$$

$$\boxed{F = Gu + \Psi + F_{rr}}$$

$$\begin{cases} F_{rr_x} = \dfrac{\cos\phi}{r} \sum_i \tau_{rr_i} \\ F_{rr_z} = \dfrac{\sin\phi}{r} \sum_i \tau_{rr_i} \\ M_{rr_y} = 0 \end{cases}$$

$$\tau_{rr_i} = - C_{rr} N_i$$

$$\boxed{\tau = u + \tau_g + \tau_{rr}}$$



$v_{CM} = 0$

$|F| \leq |F_{rr}| \longrightarrow$ no motion

$F = 0$

$v_{CM} = 0$

$|F| > |F_{rr}| \longrightarrow$ motion

$F \neq 0$

$$\boxed{\begin{array}{l} \text{critical case:} \\ v_{CM} = 0 \quad \&\& \quad |F| \leq |F_{rr}| \end{array}} \longrightarrow$$

NO MOTION CASE

$F = 0$

$v_{CM} \longrightarrow$ taken by integration of the output

$F_{rr} \longrightarrow$ computed by τ and ϕ