Dynamics and Control of a Six-wheeled Rover with Rocker-Bogie Suspension

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1 Introduction

The rocker-bogie system is the suspension arrangement developed in 1988 for use in NASA's Mars rover Sojourner, and which has since become NASA's favored design for rovers. It has been used in the 2003 Mars Exploration Rover mission robots Spirit and Opportunity, on the 2012 Mars Science Laboratory (MSL) mission's rover Curiosity, and the Mars 2020 rover Perseverance.

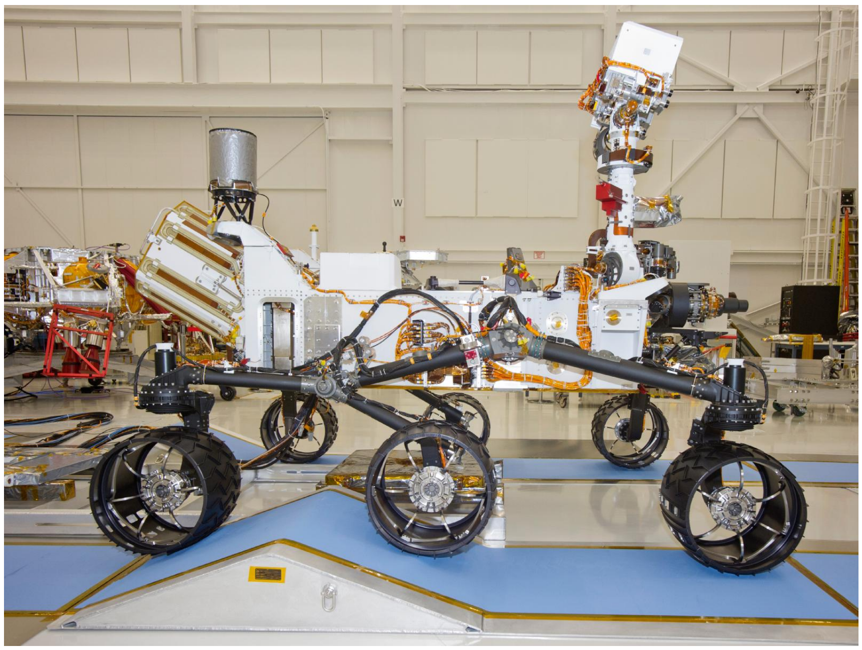

Figure 1: Lateral view of the Curiosity rover by NASA

The **rocker** part of the suspension comes from the rocking aspect of the larger, body-mounted linkage on each side of the rover. These rockers are connected to each other and the vehicle chassis through a differential. Relative to the chassis, the rockers will rotate in opposite directions to maintain approximately equal wheel contact. The chassis maintains the average pitch angle of both rockers. One end of a rocker is fitted with a drive wheel, and the other end is pivoted to the bogie.

The **bogie** part of the suspension refers to the smaller linkage that pivots to the rocker in the middle and which has a drive wheel at each end. Bogies were commonly used as load wheels in the tracks of army tanks as idlers distributing the load over the terrain, and were also quite commonly used in trailers of semi-trailer trucks. Both tanks and semi-trailers now prefer trailing arm suspensions.

The rocker-bogie design has no springs or stub axles for each wheel, allowing the rover to climb over obstacles (such as rocks) that are up to twice the wheel's diameter in size while keeping all six wheels on the ground. As with any suspension system, the tilt stability is limited by the height of the center of gravity. Systems using springs tend to tip more easily as the loaded side yields. Based on the center of mass, the Curiosity rover of the Mars Science Laboratory mission can withstand a tilt of at least 45 degrees in any direction without overturning, but automatic sensors limit the rover from exceeding 30-degree tilts. The system is designed to be used at slow speed of around 10 centimetres per second (3.9 in/s) so as to minimize dynamic shocks and consequential damage to the vehicle when surmounting sizable obstacles.

2 Static equilibrium equations

The static equilibrium equations are described by:

$$T_1 \cos\alpha_1 + T_2 \cos\alpha_2 + T_3 \cos\alpha_3 - N_1 \sin\alpha_1 - N_2 \sin\alpha_2 - N_3 \sin\alpha_3 = -F_x \tag{1}$$

$$T_1 \sin\alpha_1 + T_2 \sin\alpha_2 + T_3 \sin\alpha_3 + N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + N_3 \cos\alpha_3 = F_z \tag{2}$$

$$T_1 \sin\alpha_1 e_x - T_1 \cos\alpha_1 e_y + N_1 \sin\alpha_1 e_y + N_1 \cos\alpha_1 e_x = M_y + F_x z_y + F_z z_x \tag{3}$$

$$\begin{aligned} &T_2 \cos\alpha_2 c_y - T_2 \sin\alpha_2 c_x - N_2 \sin\alpha_2 c_y - N_2 \cos\alpha_2 c_x + \\ &+ T_3 \cos\alpha_3 d_y - T_3 \sin\alpha_3 d_x - N_3 \sin\alpha_3 d_y - N_3 \cos\alpha_3 d_x = 0 \end{aligned} \tag{4}$$

Where the traction forces T_1, T_2, T_3 are considered to be the inputs to the system, while F_x (in the O^R reference frame) is variable to be controlled.
Equations (1), (2), (3), (4) can be further developed by introducing the *motor torque* τ_i as input, and expressing the acceleration of the wheels as

$$\dot{\omega}_i = \frac{r\, T_i}{I_m} = \frac{\tau_i}{I_m} \iff T_i = \frac{\tau_i}{r} \quad i = 1, 2, 3 \tag{5}$$

Where r is the *wheel radius* and I_m the *motor inertia* (assumed to be equal for each motor).

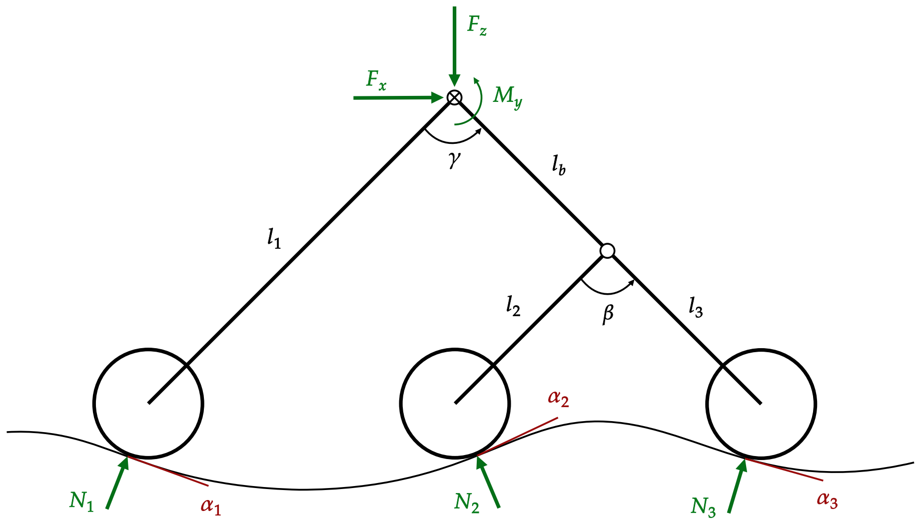

Figure 2: 6-wheeled rover

Previous equations can be expressed as:

$$F_x = -\frac{\tau_1}{r}\cos\alpha_1 - \frac{\tau_2}{r}\cos\alpha_2 - \frac{\tau_3}{r}\cos\alpha_3 + N_1\sin\alpha_1 + N_2\sin\alpha_2 + N_3\sin\alpha_3 \tag{6}$$

$$F_z = \frac{\tau_1}{r}\sin\alpha_1 + \frac{\tau_2}{r}\sin\alpha_2 + \frac{\tau_3}{r}\sin\alpha_3 + N_1\cos\alpha_1 + N_2\cos\alpha_2 + N_3\cos\alpha_3 \tag{7}$$

$$M_y = \frac{\tau_1}{r}\sin\alpha_1 e_x - \frac{\tau_1}{r}\cos\alpha_1 e_y + N_1\sin\alpha_1 e_y + N_1\cos\alpha 1 e_x - q(F_x, F_z) \tag{8}$$

$$\begin{aligned} 0 = {} & \frac{\tau_2}{r}\cos\alpha_2 c_y - \frac{\tau_2}{r}\sin\alpha_2 c_x - N_2\sin\alpha_2 c_y - N_2\cos\alpha_2 c_x + \\ & + \frac{\tau_3}{r}\cos\alpha_3 d_y - \frac{\tau_3}{r}\sin\alpha_3 d_x - N_3\sin\alpha_3 d_y - N_3\cos\alpha_3 d_x \end{aligned} \tag{9}$$

3 External forces

3.1 Normal forces

In this section we are going to compute normal forces N_1, N_2, N_3. We first define **ground-to-pivots** vectors which are:

$$a = R(\theta)\begin{pmatrix} l_1 \cos\left(\frac{\pi-\gamma}{2}\right) - r\sin(\alpha_1) \\ l_1 \sin\left(\frac{\pi-\gamma}{2}\right) + r\cos(\alpha_1) \end{pmatrix} \tag{10}$$

$$b = R(\theta)\begin{pmatrix} l_b \cos\left(\frac{\pi+\gamma}{2}\right) \\ l_b \sin\left(\frac{\pi+\gamma}{2}\right) \end{pmatrix} \tag{11}$$

$$c = R(\theta)\,R(\theta_B)\begin{pmatrix} l_2 \cos\left(\frac{\pi-\beta}{2}\right) - r\sin(\alpha_2) \\ l_2 \sin\left(\frac{\pi-\beta}{2}\right) + r\cos(\alpha_2) \end{pmatrix} \tag{12}$$

$$d = R(\theta)\,R(\theta_B)\begin{pmatrix} l_2 \cos\left(\frac{\pi+\beta}{2}\right) - r\sin(\alpha_2) \\ l_2 \sin\left(\frac{\pi+\beta}{2}\right) + r\cos(\alpha_2) \end{pmatrix} \tag{13}$$

$$e = a - b = R(\theta)\begin{pmatrix} l_1 \cos\left(\frac{\pi-\gamma}{2}\right) - r\sin(\alpha_1) - l_b\cos\left(\frac{\pi+\gamma}{2}\right) \\ l_1 \sin\left(\frac{\pi-\gamma}{2}\right) + r\cos(\alpha_1) - l_b\sin\left(\frac{\pi+\gamma}{2}\right) \end{pmatrix} \tag{14}$$

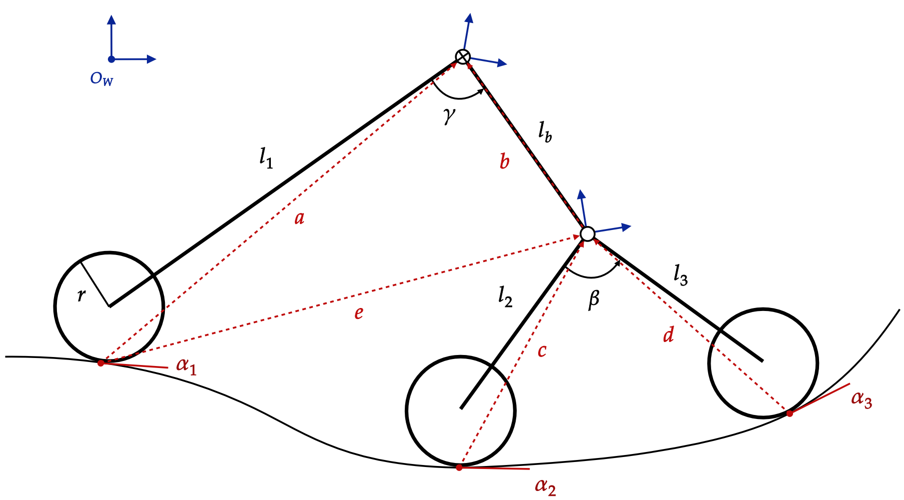

Figure 3: Rover kinematics

The normal forces on the three wheels and on the bogie pivot are computed as

$$N_1 = -mg \, \frac{|b_x|}{|a_x| + |b_x|} \qquad\qquad N_B = -mg \, \frac{|a_x|}{|a_x| + |b_x|} \tag{15}$$

$$N_3 = -mg \, \frac{|d_x|}{|c_x| + |d_x|} \qquad\qquad N_4 = -mg \, \frac{|c_x|}{|c_x| + |d_x|} \tag{16}$$

3.2 Rolling resistance

3.3 Aerodynamic drag

4 State-space representation

We would like to express the systems in classical nonlinear form like:

$$\begin{aligned} \dot{\xi} &= f(\xi) + \sum_i g_i(\xi) u_i + \Psi(\cdot) \\ y &= h(\xi) \end{aligned} \tag{17}$$

To do so, we define the *momenta* p_x and p_z along the x and z axis, and the *angular momenta* L and L_B along the y axis of, respectively, the of the CoM and the boogie. It is now possible to introduce the state variables

$$\xi = [\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \xi_5,\ \xi_6,\ \xi_7,\ \xi_8]^T = [x,\ p_x,\ z,\ p_z,\ \theta,\ L,\ \theta_B,\ L_B]^T \tag{18}$$

the torque inputs

$$u = [u_1,\ u_2,\ u_3]^T = [\tau_1,\ \tau_2,\ \tau_3]^T \tag{19}$$

and the position outputs

$$y = [y_1,\ y_2,\ y_3,\ y_4]^T = [x,\ z,\ \theta,\ \theta_B]^T \tag{20}$$

With this notation in mind, the whole system dynamics can be rewritten as

$$
\begin{aligned}
\dot{\xi}_1 &= \frac{1}{m}\xi_2 \\
\dot{\xi}_2 &= g_{11}\,u_1 + g_{12}\,u_2 + g_{13}\,u_3 + \Psi_1(\cdot) \\
\dot{\xi}_3 &= \frac{1}{m}\xi_4 \\
\dot{\xi}_4 &= g_{21}\,u_1 + g_{22}\,u_2 + g_{23}\,u_3 + \Psi_2(\cdot) \\
\dot{\xi}_5 &= \frac{1}{I}\xi_6 \\
\dot{\xi}_6 &= g_{31}\,u_1 + \cancel{q(\dot{\xi}_2, \dot{\xi}_4)} + \Psi_3(\cdot) \\
\dot{\xi}_7 &= \frac{1}{I_B}\xi_8 \\
\dot{\xi}_8 &= g_{42}\,u_2 + g_{43}\,u_3 + \Psi_4(\cdot) \\
y_1 &= \xi_1 \\
y_2 &= \xi_3 \\
y_3 &= \xi_5 \\
y_4 &= \xi_7
\end{aligned}
\tag{21}
$$

Where Ψ_1, Ψ_2, Ψ_3 and Ψ_4 are nonlinear functions linking the three subsystems through normal forces and taking into account the ground angles $\alpha_{1,2,3}$ (acting, in this context, as nonlinear disturbances).

By neglecting the term $q(\dot{\xi}_2, \dot{\xi}_4)$ (which would require a jerk-controlled system), the rover dynamics (21) can be expressed in matrix form as

$$
\begin{aligned}
\dot{\xi} &= A\,\xi + G(\xi)\,u + \Psi(\cdot) \\
y &= C\,\xi
\end{aligned}
\tag{22}
$$

$$
\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \\ \dot{\xi}_7 \\ \dot{\xi}_8 \end{bmatrix} = \begin{bmatrix} 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/I_B \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \\ g_{31} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & g_{42} & g_{43} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \Psi_1(\cdot) \\ 0 \\ \Psi_2(\cdot) \\ 0 \\ \Psi_3(\cdot) \\ 0 \\ \Psi_4(\cdot) \end{bmatrix}
$$

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/I_B & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix}
$$