Vehicle system dynamics

~ Project ~

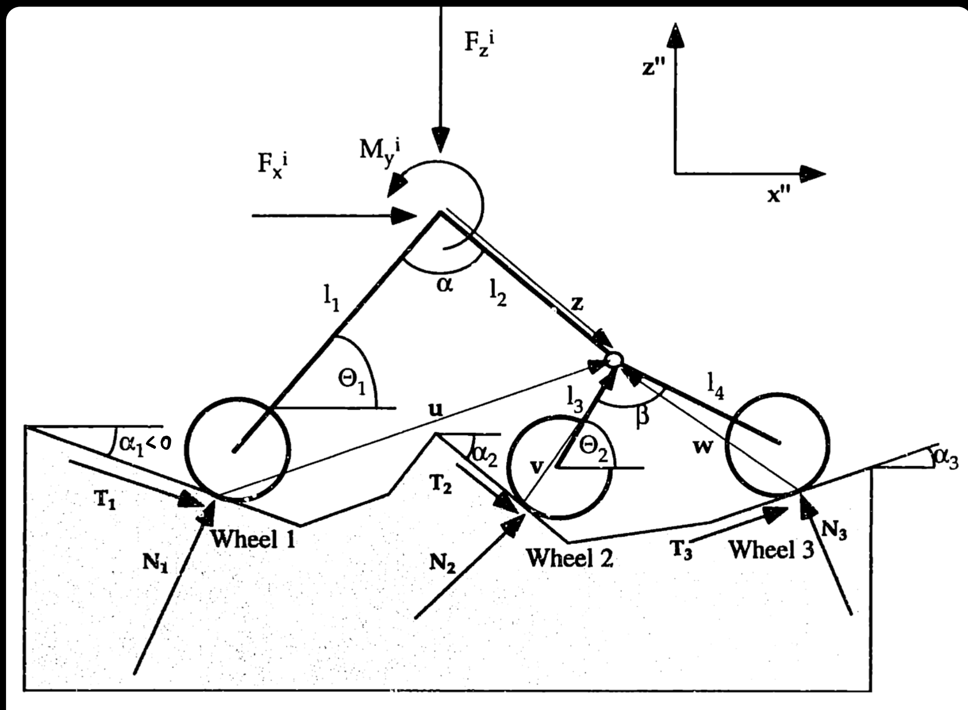


Figure 3-6: Rover Planar Analysis.

STATIC EQUILIBRIUM EQUATIONS

$$(*)\begin{cases}
(1) & T_1 C_1 + T_2 C_2 + T_3 C_3 - N_1 S_1 - N_2 S_2 - N_3 S_3 + F_x = 0 \\
(2) & T_1 S_1 + T_2 S_2 + T_3 S_3 + N_1 C_1 + N_2 C_2 + N_3 C_3 - F_z = 0 \\
(3) & T_1 C_1 u_y - T_1 S_1 u_x - N_1 S_1 u_y - N_1 C_1 u_x + M_y + F_x z_y + F_z z_x = 0 \\
(4) & T_2 C_2 v_y - T_2 S_2 v_x - N_2 S_2 v_y - N_2 C_2 v_x + \\
 & T_3 C_3 w_y - T_3 S_3 w_x - N_3 S_3 w_y - N_3 C_3 w_x = 0
\end{cases}$$

$$A \begin{bmatrix} T_1 & T_2 & T_3 & N_1 & N_2 & N_3 \end{bmatrix}^T = \begin{bmatrix} -F_x & F_z & -M_y - F_x z_y - F_z z_x & 0 \end{bmatrix}^T$$

$$A = \left[\begin{array}{ccc|ccc}
C_1 & C_2 & C_3 & -S_1 & -S_2 & -S_3 \\
S_1 & S_2 & S_3 & C_1 & C_2 & C_3 \\
\overset{\gamma_1}{C_1 u_y - S_1 u_x} & 0 & 0 & \overset{\gamma_2}{-S_1 u_y - C_1 u_x} & 0 & 0 \\
0 & \underset{\gamma_3}{C_2 v_y - S_2 v_x} & \underset{\gamma_4}{C_3 w_y - S_3 w_x} & 0 & \overset{\gamma_5}{-S_2 v_y - C_2 v_x} & \overset{\gamma_6}{-S_3 w_y - C_3 w_x}
\end{array}\right]$$

B

Consider T_1, T_2, T_3 as inputs :

$$\begin{bmatrix} -s_1 & -s_2 & -s_3 \\ c_1 & c_2 & c_3 \\ \gamma_2 & 0 & 0 \\ 0 & \gamma_5 & \gamma_6 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

(1) $\longrightarrow$ $-s_1 N_1 - s_2 N_2 - s_3 N_3 = -T_1 c_1 - T_2 c_2 - T_3 c_3 - F_x$

(2) $\longrightarrow$ $c_1 N_1 + c_2 N_2 + c_3 N_3 = -T_1 s_1 - T_2 s_2 - T_3 s_3 + F_z$

(3) $\longrightarrow$ $\gamma_2 N_1 = -T_1 \gamma_1 - M_y - F_x z_y - F_z z_x$

(4) $\longrightarrow$ $\gamma_5 N_2 + \gamma_6 N_3 = -T_2 \gamma_3 - T_3 \gamma_4$

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} \overbrace{b_{11} \; b_{12} \; b_{13} \; b_{14}}^{B^{\#}} \\ b_{21} \; b_{22} \; b_{23} \; b_{24} \\ b_{31} \; b_{32} \; b_{33} \; b_{34} \end{bmatrix} \left(T_1 \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + T_2 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + T_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \right)$$

N_1, N_2, N_3 are known as a functions of $\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3$ and CoM

(see "Introduction to the mechanics of space robots" eqns 5.237 - 5.238)

From (*) follows :

$$(*) \begin{cases} F_x = -T_1 \cos\alpha_1 - T_2 \cos\alpha_2 - T_3 \cos\alpha_3 + N_1 \sin\alpha_1 + N_2 \sin\alpha_2 + N_3 \sin\alpha_3 \\ F_z = T_1 \sin\alpha_1 + T_2 \sin\alpha_2 + T_3 \sin\alpha_3 + N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + N_3 \cos\alpha_3 \\ M_y = -T_1 \gamma_1(\alpha_1, u_x, u_y) + N_1 \gamma_2(\alpha_1, u_x, u_y) - F_x z_y - F_z z_x \end{cases}$$

where $\gamma_1(\alpha_1, u_x, u_y) = \cos\alpha_1 \, u_y - \sin\alpha_1 \, u_x$

$\gamma_2(\alpha_1, u_x, u_y) = \sin\alpha_1 \, u_y + \cos\alpha_1 \, u_x$

$$\dot{\omega}_i = \frac{\tau_i}{I_m} = \frac{r\,T_i}{I_m}$$

$$\begin{cases} \tau_i & \text{motor torque} \\ r & \text{wheel radius} \\ T_i & \text{traction force} \\ I_m & \text{inertia of the motor} \end{cases}$$

$$(*)\begin{cases} a_x = \frac{1}{m}\left[-\dot{\omega}_1 \frac{I_m}{r}\cos\alpha_1 - \dot{\omega}_2 \frac{I_m}{r}\cos\alpha_2 - \dot{\omega}_3 \frac{I_m}{r}\cos\alpha_3 + N_1 \sin\alpha_1 + N_2 \sin\alpha_2 + N_3 \sin\alpha_3\right] \\ a_z = \frac{1}{m}\left[\dot{\omega}_1 \frac{I_m}{r}\sin\alpha_1 + \dot{\omega}_2 \frac{I_m}{r}\sin\alpha_2 + \dot{\omega}_3 \frac{I_m}{r}\sin\alpha_3 + N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + N_3 \cos\alpha_3\right] \\ \dot{\omega}_y = \frac{1}{I_y}\left[-\dot{\omega}_1 \frac{I_m}{r}\gamma_1(\alpha, u_x, u_y) + N_1\gamma_2(\alpha_1, u_x, u_y) - m\left(a_x Z_y - a_z Z_x\right)\right] \end{cases}$$

STATE VARIABLES $\xi = [x, v_x, z, v_z, \theta_y, \omega_y]^T$

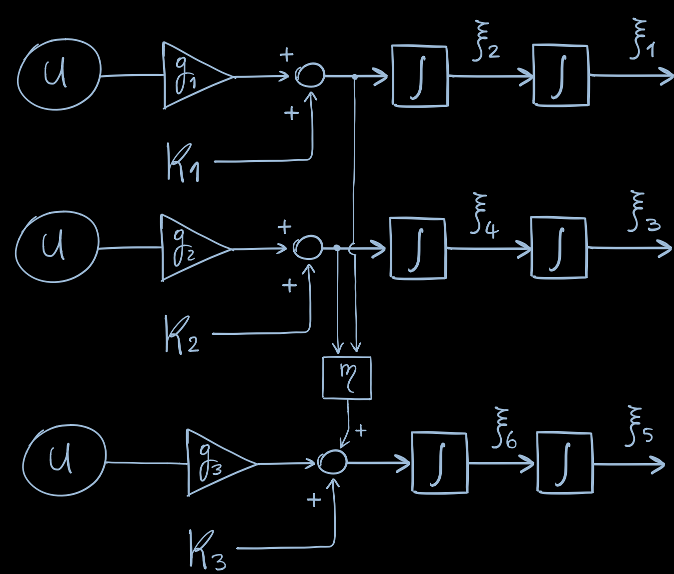
ACCELERATION INPUTS $u = [\dot{\omega}_1\ \dot{\omega}_2\ \dot{\omega}_3]^T$

OUTPUTS $h = [x\ \ z\ \ \theta_y]^T$

$$\begin{cases} \dot{\xi} = f(\xi) + \sum_i g_i(\xi)\, u_i \\ y = h(\xi) \end{cases}$$

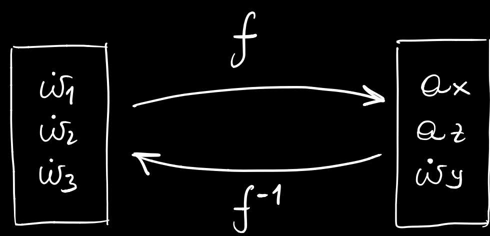
$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= g_{11}u_1 + g_{12}u_2 + g_{13}u_3 + k_1 \\ \dot{\xi}_3 &= \xi_4 \\ \dot{\xi}_4 &= g_{21}u_1 + g_{22}u_2 + g_{23}u_3 + k_2 \\ \dot{\xi}_5 &= \xi_6 \\ \dot{\xi}_6 &= g_{31}u_1 + k_3 + \eta(\dot{\xi}_2, \dot{\xi}_4) \end{aligned}$$

can be neglected ?
otherwise inputs
must be at the Jerk level

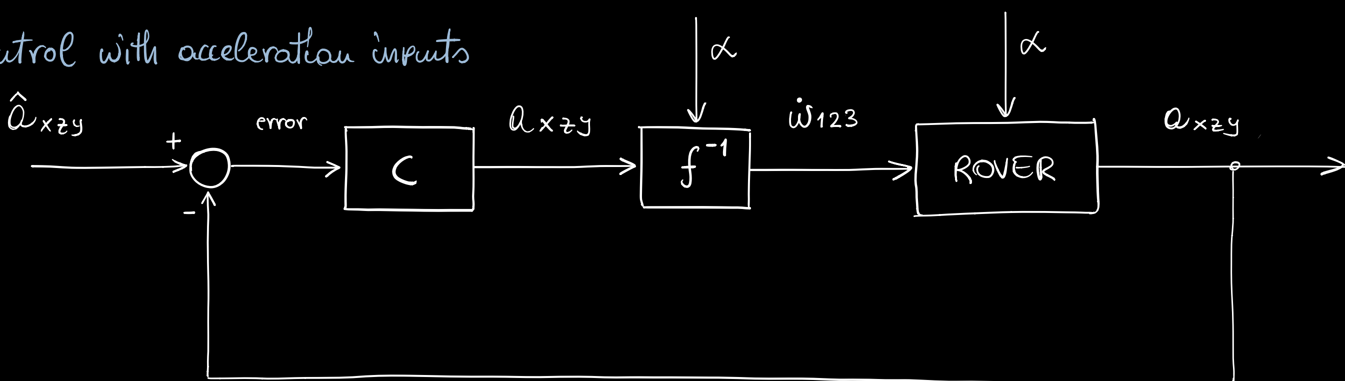


$$
\dot{\xi} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} 0 \\ K_1(\dots) \\ 0 \\ K_2(\dots) \\ 0 \\ K_3(\dots) \end{pmatrix}
$$

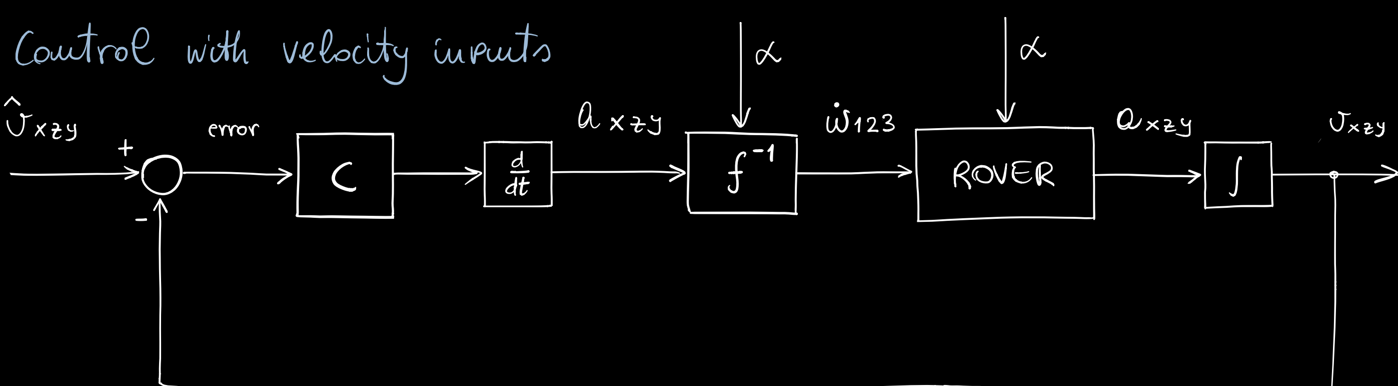
DIRECT / INVERSE DYNAMICS



Control with acceleration inputs



Control with velocity inputs



$$a = f = G u + K \longleftrightarrow u = f^{-1} = G^{-1}(a - K)$$

$$u = \begin{pmatrix} -I_m\left(a_x \cos\alpha_1 + a_z \cos\alpha_2 + \dot{\omega}_y \cos\alpha_3 - k_1 \cos\alpha_1 - k_2 \cos\alpha_2 - k_3 \cos\alpha_3\right)/mr \\ I_m\left(a_x \sin\alpha_1 + a_z \sin\alpha_2 + \dot{\omega}_y \sin\alpha_3 - k_1 \sin\alpha_1 - k_2 \sin\alpha_2 - k_3 \sin\alpha_3\right)/mr \\ -I_m r_1 \left(a_x - k_1\right)/I_y r \end{pmatrix}$$