Algorithm Design 2022-23

- Randomness → design
  - algorithms
  - applications

- Hardness → computationally difficult problems:
  CRITICAL RESOURCE
  - Time
    - approximation
    - exact exp.
      parameterization
  - Space
    - streaming

- game theory

Randomization we need:

- Expectation $E[X] = \sum_{x \in} x \cdot Pr(x)$

linearity $E[aX + bY] = aE[X] + bE[Y]$

random variables

- Indicator variables

LED ON p
OFF $1-p$

$$X_i = \begin{cases} 1 & \text{event } i \text{ occurs } (p) \\ 0 & \text{otherwise } (1-p) \end{cases}$$

Expect. is $Pr = 1$

$$E[X_i] = 1 \cdot p + 0 \cdot (1-p) = p$$

$$X = \sum_{i=1}^{t} X_i \quad \rightsquigarrow \quad \underline{E[X]} = \sum_{i=1}^{t} E[X_i] = \sum_{i=1}^{t} \underline{Pr[X_i = 1]}$$

$$= t \cdot p$$

q_1

Headphone: n stores

- buy the best headphone in the store
- one store at a time
- best among stores

strategy: buy from store 1 → current
for store = 2, 3, ..., n
if the best in store > current
$\boxed{\text{sell current}}$

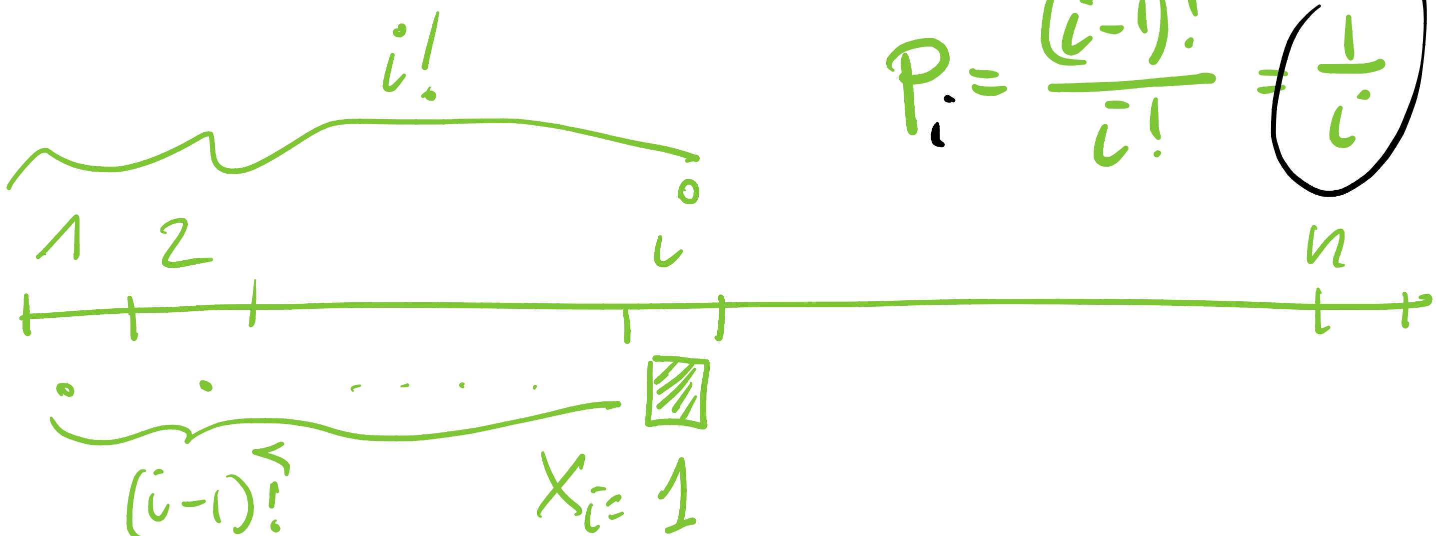
Headphones: 9 3 4 2 5 6 1 7 8
1 2 3 4 5 6 7 8 9

BEST
WORST

Randomized strategy: like the previous one, except that stores are chosen in a <u>random</u> and <u>uniform</u> way

Indicator variable

$$X_i = \begin{cases} 1 & \text{if store } i \text{ has better headphone than previous one} \quad P_i \\ 0 & \text{o.w.} \quad 1 - P_i \end{cases}$$



$$P_i = \frac{(i-1)!}{i!} = \frac{1}{i}$$

$\# \bullet$'s =

$$X = \sum_{i=1}^{n} X_i$$

$$E[X] = \sum_{i=1}^{n} \frac{1}{i}$$

$$\sim \ln n$$

Harmonic Series