

## TD 4

# Exercises on: Frequency-domain characteristics of Random signal

### 4.1 Exercise 1

The process  $X(t)$  is normal with zero mean and  $R_X(\tau) = e^{-a|\tau|} \cos \beta \tau$ . Show that if  $Y(t) = X^2(t)$ , then  $C_Y(\tau) = e^{-2a|\tau|}(1 + \cos 2\beta \tau)$ . Find  $S_Y(\omega)$ .

**Hint:** if the random variables  $X$  and  $Y$  are jointly normal with zero mean. then  $E\{X^2 Y^2\} = E\{X^2\}E\{Y^2\} + 2E^2\{XY\}$ .

### 4.2 Exercise 2

Consider the power spectrum  $S(\omega)$  is impulsive, such that,

$$S(\omega) = \delta(\omega + \omega_0) + \delta(\omega - \omega_0)$$

find the autocorrelation by using integrated spectrum.

### 4.3 Exercise 3

If the process  $X(t)$  is MS continuous, show that its mean is continuous.

### 4.4 Exercise 4

Given a complex process  $X(t)$  with autocorrelation  $R_X(\tau)$ , show that if  $R_X(\tau_1) = R_X(0)e^{j\omega_0\tau_1}$ ,  $Y(t) = e^{-j\omega_0 t} X(t)$ , then

$$R_X(\tau) = e^{j\omega_0 \tau} \omega(\tau)$$

where  $\omega(\tau)$  is a periodic function with period  $\tau_1$  and  $Y(t)$  is an MS periodic process with the same period.

### 4.5 Exercise 5

Given  $R_X(m) = a^{|m|}$ ,  $|a| < 1$ , find  $S_X(\omega)$ .