TD 6

Exercises on: 06: Spectral Representation

6.1 Exercise 1

Find the innovations filter of the process $X(t)$ if

$$S_X(\omega) = \frac{\omega^4 + 64}{\omega^4 + 10\omega^2 + 9}$$

6.2 Exercise 2

Given a non-periodic WSS process $X(t)$, we form the sum $\hat{X}(t) = \sum\limits_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$, and set

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} X(t) e^{-jn\omega_0 t} dt, \quad \beta_n(\alpha) = \frac{1}{T}\int_{-T/2}^{T/2} R(\tau - \alpha) e^{-jn\omega_0 \tau} d\tau$$

where, $\beta_n(\alpha)$ are the coefficients of the Fourier expansion of $R(\tau - \alpha)$ in the interval $(-T/2, T/2)$. Show that

a) For $|t| < T/2, \quad E\{|X(t) - \hat{X}(t)|^2\} = 0$

b) $E\{c_n c_m^*\} = \frac{1}{T}\int_{-T/2}^{T/2} \beta_n(t) e^{-jm\omega_0 t} dt$

c) If T is sufficiently large, then $E\{c_n c_m^*\} \approx \frac{S(n\omega_0)}{T}\delta(n-m)$

6.3 Exercise 3

If $E\{X_n X_k\} = \sigma_n^2 \delta[n-k]$, $X(\omega) = \sum\limits_{n=-\infty}^{\infty} X_n e^{-jn\omega}$ and $E\{X_n\} = 0$, Find $E\{X(\omega)\}$ and $E\{X(u)X^*(v)\}$.

6.4 Exercise 4

Show that, if the process $X(\omega)$ is white noise with zero mean and auto-covariance $Q(u)\delta(u-v)$, then its inverse Fourier transform $X(t)$ is WSS with power spectrum $Q(\omega)/2\pi$.