TD 3

Exercises On:Random signal time-domain characteristics & The power spectrum

3.1 Exercise 1

The Process $X(t)$ is SSS. Define a new process

$$Y(t) = \begin{cases} 1 & X(t) \leq x \\ 0 & X(t) > x \end{cases}$$

Where x is a real number. Show that
a) $E\{Y(t)\} = F_X(x)$.
b) $R_Y(\tau) = F_X(x, x, \tau)$.

3.2 Exercise 2

Show that if φ is a random variable with $\Phi(\lambda) = E\{e^{j\lambda\varphi}\}$ and $\Phi(1) = \Phi(2) = 0$, then the process $X(t) = cos(\omega t + \varphi)$ is WSS.

3.3 Exercise 3

Consider the random process:$X(t) = acos(\omega_0 t + \theta)$ where:

- a, ω_0 are numeric constants;
- θ is a random variable uniformly distributed in the interval [0,2π].

a) Evaluate the mean of $X(t)$.

b) Evaluate the autocorrelation of $X(t)$.

c) Determine if the process is wide-sense ergodic.

3.4 Exercise 4

Show that the power spectrum of an SSS process $X(t)$ equals

$$S_X(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 G(x_1, x_2; \omega) dx_1 dx_2$$

where $G(x_1, x_2; \omega)$ is the Fourier transform in the variable τ of the second-order density $f(x_1, x_2; \tau)$ of $X(t)$.

3.5 Exercise 5

The power spectrum of a stationary process $X(t)$ equals

$$S(\omega) = \frac{16}{\omega^4 + 13\omega^2 + 36}$$

Find the auto-correlation and mean square of this process.