

# Integrated crew management for rail freight

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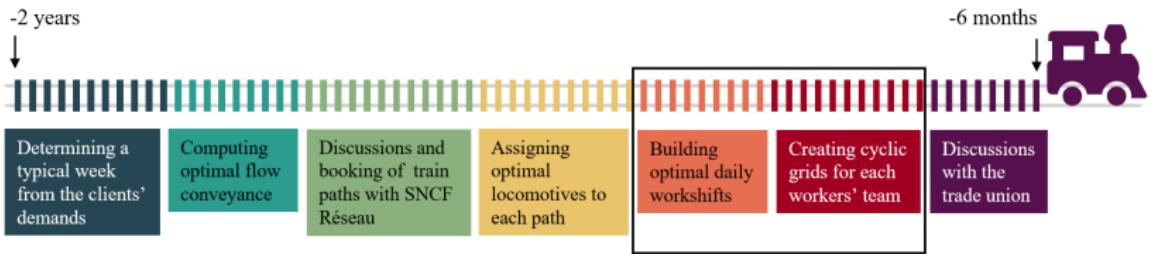
20 november 2024



# Rail freight in France

- 18% of rail freight in Europe (10% in France)
- Between 1800 and 2000 trains per week
- Many differences with passengers transportation...
  - Priority goes to passengers
  - Trains mostly at night
  - Client satisfaction: delivery on time
  - Key goal: reducing operational costs

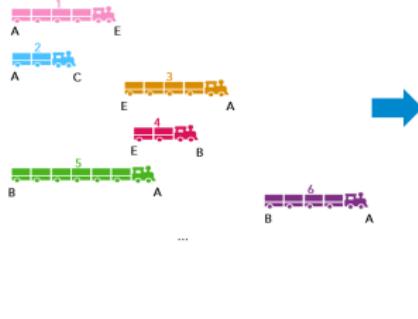
# Resource planning



# Problem definition

**Input:** Trains on a typical week

**Output:** Covering of trains by “blocks” with minimum cost, each block assigned to a team (each block then placed in a grid)



Team A:

|         | sat    | sun  | mon  | tue |
|---------|--------|------|------|-----|
| WS 5    | WS 6   | WS 2 | WS 3 |     |
| 12h 21h | d      | R    | R    | R   |
| A A     |        |      |      |     |
| WS 2    | WS 3   |      |      |     |
| 12h 19h | 5h 11h | R    | R    | R   |
| A B     | B A    |      |      |     |

Team B:

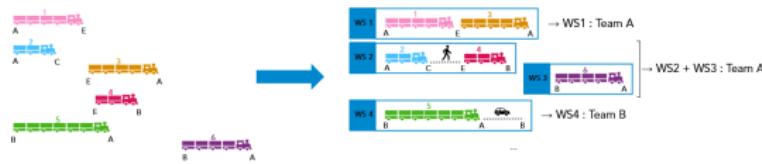
|         | mon     | tue    | wed | thu | fri | sat | sun |
|---------|---------|--------|-----|-----|-----|-----|-----|
| WS 4    | WS 8    | WS 9   |     |     |     |     |     |
| 12h 20h | 11h 17h | 5h 12h | R   | R   | R   | R   |     |
| B B     | B E     | E B    |     |     |     |     |     |

# Heuristic: sequential approach

## ① Building shifts from trains

**Input:** Trains on a typical week

**Output:** Covering of trains by shifts with minimum cost, each shift assigned to a team



## ② Building grids from shifts

**Input:** Shifts, each assigned to a team (*output above*)

**Output:** For each team, covering of shifts by blocks with minimum cost

The diagram shows the mapping of shifts to blocks for Team A and Team B. On the left, the shifts for Team A (WS1, WS2, WS3, WS4) and Team B (WS4) are listed. An arrow points to the right, where the shifts are mapped to a grid of blocks. The grid has days of the week (sat, sun, mon, tue) as columns and shifts as rows. The grid for Team A shows the following assignments: WS1 (sat, sun), WS2 (mon, tue), WS3 (tue), WS4 (sat, sun). The grid for Team B shows WS4 (mon, tue, wed).

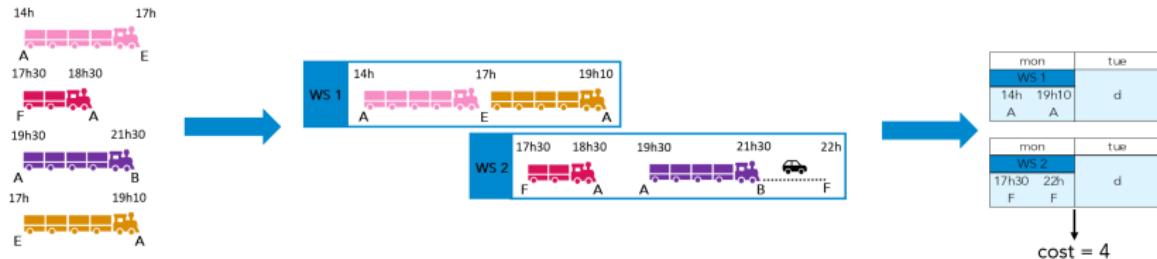
|     | sat | sun | mon | tue |
|-----|-----|-----|-----|-----|
| WS1 |     |     |     |     |
| WS2 |     |     |     |     |
| WS3 |     |     |     |     |
| WS4 |     |     |     |     |

|     | sat | sun | mon | tue |
|-----|-----|-----|-----|-----|
| WS1 |     |     |     |     |
| WS2 |     |     |     |     |
| WS3 |     |     |     |     |
| WS4 |     |     |     |     |

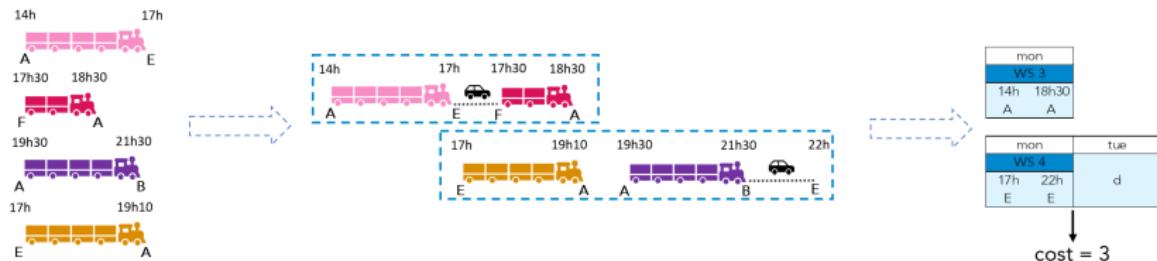
|     | sat | sun | mon | tue |
|-----|-----|-----|-----|-----|
| WS1 |     |     |     |     |
| WS2 |     |     |     |     |
| WS3 |     |     |     |     |
| WS4 |     |     |     |     |

# Sub-optimal approach

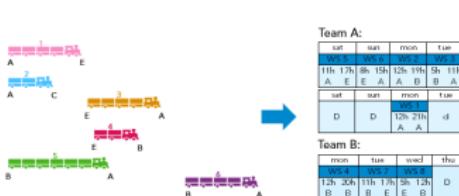
## Sequential approach:



## Optimal approach:



# Model



$$\begin{aligned}
 & \text{Min} \quad \sum_{i \in I} \sum_{b \in B_i} c_b x_{b,i} \\
 & \text{s.t.} \quad \sum_{i \in I} \sum_{\substack{b \in B_i \\ t \in b}} x_{b,i} \geq 1 \quad \forall t \in T \\
 & \quad x_{b,i} \in \{0, 1\} \quad \forall i \in I, \forall b \in B_i
 \end{aligned}$$

$B_i$  = set of feasible “blocks” for the team  $i$

→ combinatorial explosion

National input:

1 800 trains → 200 000 shifts →  $\sim 10^{25}$  blocks

**Column Generation:** standard resolution methodology of a **linear program** when the number of variables is large

# Column Generation

Generation of variables throughout the resolution → solving the pricing sub-problem

**Key:** Solving the pricing sub-problem **quickly**

**For our problem:**

Pricing sub-problem = Shortest path with constraints (**Resource Constrained Shortest Path**)

→ **NP-hard** problem

## Shortest path with constraints

**Input:** Graph with vertices  $o$  and  $d$ , cost function and feasibility function on each path

**Ouput:** Feasible  $o - d$  path with minimum cost

Feasibility is expressed as: “resources” accumulated along the path are under a given threshold

ex.: *shortest  $o - d$  path under energy budget*

→ *resources: accumulated arc energy*

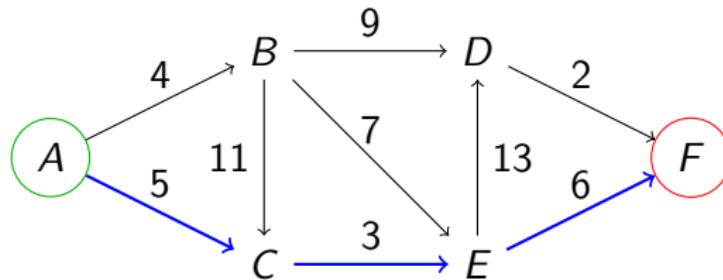
*minimum-cost block under shifts and block feasibility constraints*

→ *resources: accumulated shift range, number of days in block, etc.*

## Shortest path

### Usual shortest path

**Input:** Graph with vertices  $o$  and  $d$ , non-negative cost on each arc  
**Output:**  $o - d$  path with minimum cost



→ Dijkstra's algorithm  
→ A\* search algorithm

# Shortest path

## A\* search algorithm

**Principle:** Enumeration algorithm with bounds to discard paths

- Bound  $b_v$  under-estimating cost of shortest path from any vertex  $v$  to  $d$
- Discard paths  $P$  from  $o$  to  $v$  with “estimated cost”  $c(P) + b_v$  greater than one of an explored  $o - d$  path

→ with  $b_v = 0$ : Dijkstra's algorithm

ex.: *shortest route on a map*  
→  $b_v = \text{distance as the crow flies}$



# Shortest path with constraints

**Resolution:** Enumeration algorithm using

- Key: order of paths processing
- Bound: under-estimate of the resources and cost to reach  $d$
- [not in  $A^*$ ] Dominance: comparison of resources for paths with same cost

Different algorithms<sup>1</sup>:

- Generalized  $A^*$

Key = “estimated cost”, discard paths using “estimated cost”

- Label dominance

Key = cost of path, discard paths using dominance

- Label correcting

Key = “estimated cost”, discard paths using “estimated cost” and dominance

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<sup>1</sup> synthetized by A.Parmentier in Algorithms for Non-Linear and Stochastic Resource Constrained Shortest Paths, 2017.

## For our problem

- **Vertices** are trains
- **Arcs** can be of different types, mainly:
  - succession of trains within shift
  - change of shifts between trains
- **Resource** contains several indicators, as:
  - block indicators (number of days, etc.)
  - previous shift indicators (range, driving duration, etc.)
  - current shift indicators

# Results

|                     | Sequential approach |      | Our approach     |                               |            |               |              |
|---------------------|---------------------|------|------------------|-------------------------------|------------|---------------|--------------|
|                     | Objective           | Time | Obj. lower bound | Obj. upper bound <sup>2</sup> | Total time | Nb iterations | Nb variables |
| Instance 195 trains | 176                 | 2s   | 149.9            | 151 (-14%)                    | 15min31    | 8             | 128 527      |
| Instance 265 trains | 261                 | 3s   | 227.3            | 229 (-12%)                    | 28min13    | 11            | 175 716      |
| Instance 400 trains | 402                 | 5s   | 344.2            | 347 (-13%)                    | 32min46    | 9             | 108 150      |

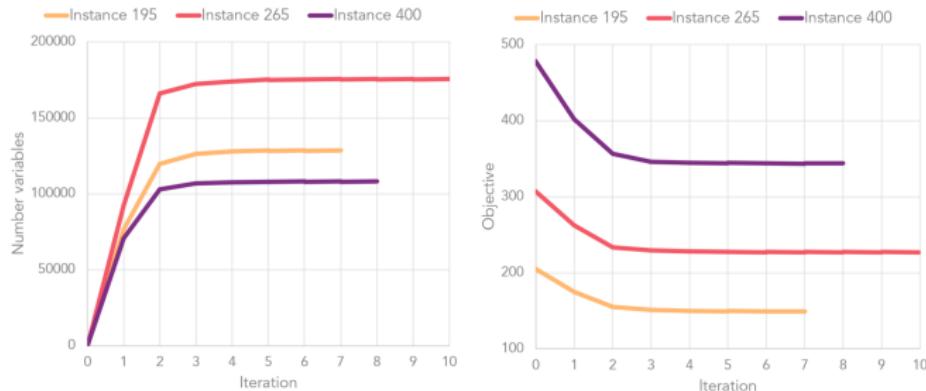


Figure: Column Generation indicators per iteration

<sup>2</sup>solving MILP with CG output variables

## Current limitations

- Overall computation time
  - Preprocessing time: building pricing graph
  - Pricing sub-problem time
- Extend results to a national instance

Thank you!