Minimum Spanning Trees

Hengfeng Wei

Institute of Computer Software
Nanjing University

December 8, 2016

Minimum Spanning Trees

1. The MST Problem
2. The Generic MST Algorithm
3. The Algorithms of Kruskal and Prim

Minimum Spanning Trees

Minimum Spanning Tree

Definition (MST)

Given:

- connected, undirected, weighted graph $G = (V, E)$
- edge weight $w(e)$

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MST: Mimimize $w(T)$ over all possible STs

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MST Example

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Wrong divide-and-conquer algorithm for MST

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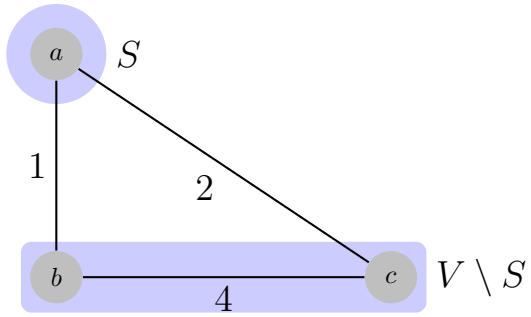

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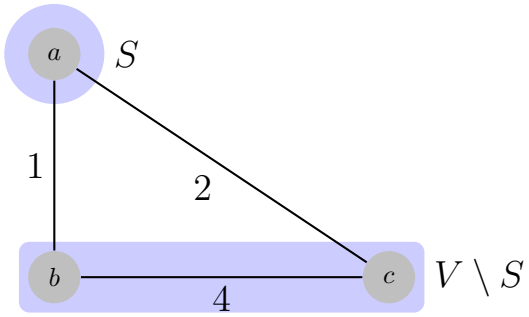
Conquer: T_1: an MST of S; T_2: an MST of $V \setminus S$



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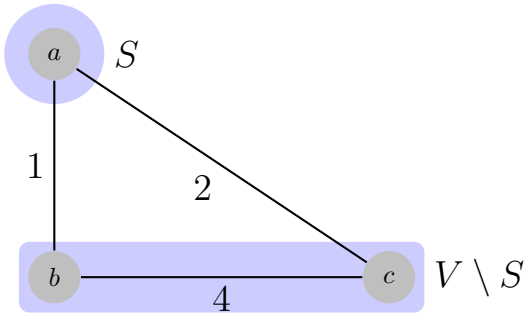
Combine: $T_1 + T_2 + \{e\}$: e is a *lightest* edge across $(S, V \setminus S)$



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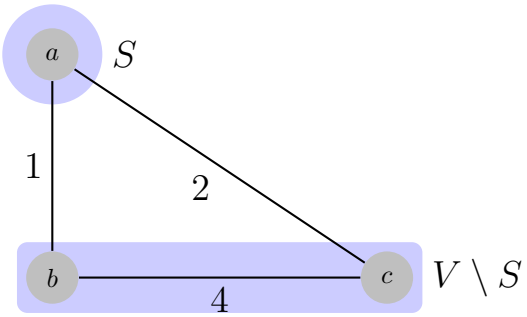
What is wrong?



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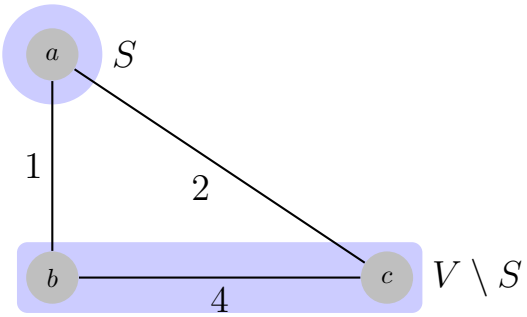

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What if:

Invariant: Manages a set of edges A which is a subset of *some* MST.

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