Minsky Simulation Engine

Version 3.4.0-beta.4

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See also Modelling with Minsky.

Chapter 1

Introduction to Minsky

Minsky is a system-dynamics program. If you are familiar with these programs, please go to *If you are experienced in system dynamics* §(1.2). If you are not, please go to *If you are new to system dynamics* §(1.1)

1.1 If you are new to system dynamics

Minsky is one of many "system dynamics" programs[1]. These programs allow a dynamic model of a system to be constructed, not by writing mathematical equations or computer code, but by laying out a model in a block diagram, which can then be simulated. These programs are now one of the main tools used by engineers to design complex products, and by management consultants to advise on corporate management, product marketing, local government projects, etc.

Minsky has many features in common with these programs, and adds another unique means to create dynamic equations—the "Godley Table"—that is superior to block diagrams for modelling monetary flows. 1.2.1.

The main advantages of the block diagram representation of dynamic equations over a list of equations are:

- They make the causal relationships in a complex model obvious. It takes a specialized mind to be able to see the causal relations in a large set of mathematical equations; the same equations laid out as diagrams can be read by anyone who can read a stock and flow diagram—and that's most of us;
- The block diagram paradigm makes it possible to structure a complex model via groups. For example, the fuel delivery system in a car can be treated as one group, the engine as another, the exhaust as yet another. This reduces visual complexity, and also makes it possible for different components of a complex model to be designed by different teams, and then "wired together" at a later stage.

[1] https://en.wikipedia.org/wiki/System_dynamics

Though these programs differ in appearance, they all work the same way: variables in a set of equations are linked by wires to mathematical operators. What would otherwise be a long list of equations is converted into a block diagram, and the block diagram makes the causal chain in the equations explicit and visually obvious. They are also explicitly tailored to producing numerical simulations of models.

1.1.1 Block diagram example

One of the very first models of a dynamic system was developed independently by the mathematicians Lokta[2] and Volterra[3] and is now known as the Lotka-Volterra model[4]. This model simulates interacting populations of predators and prey, which had been seen to display fluctuations that either could not be explained by exogenous shocks, or which displayed counterintuitive responses to exogenous shocks—such as the observed increase in the proportion of predators (sharks, rays, etc.) in the fishing catch in the Adriatic Sea during WWI, when biologists had expected the proportion of predators to fall[5].

The basic dynamics of the predator-prey model are that the number of prey is assumed to grow exponentially in the absence of predators, while the number of predators is assumed to fall exponentially in the absence of prey. Using Rabbits as our example of prey and Foxes as our example of predators, the initial dynamic equations are :

$$\frac{d}{dt}Rabbits = r \times Rabbits$$
$$\frac{d}{dt}Foxes = -f \times Foxes$$

When interaction between predators and prey is considered, the simplest assumption is that predators reduce the growth rate of prey (r) by some constant (ρ) times how many predators there are, while prey reduce the death rate of predators (f) by another constant (ϕ) times how many prey there are:

$$\frac{d}{dt}Rabbits = (r - \rho \times Foxes) \times Rabbits$$
$$\frac{d}{dt}Foxes = (-f + \phi \times Rabbits) \times Foxes$$

The non-zero equilibrium of this system is easily calculated by setting the differential equations to zero:

$$Rabbit_{Eq} = \frac{f}{\phi}$$
$$Fox_{Eq} = \frac{r}{\rho}$$

Mathematicians initially expected that this model would converge to this equilibrium over time, but this was not the case:

[2] https://www.jstor.org/stable/84156
[3] https://www-nature-com.libproxy.ucl.ac.uk/articles/118558a0
[4] https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations
[5] https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations#History

> Periodic phenomena play an important role in nature, both organic and inorganic ... it appeared, from the nature of the solution obtained, improbable that undamped, permanent oscillations would arise ... It was, therefore, with considerable surprise that the writer, on applying his method to certain special cases, found these to lead to undamped, and hence indefinitely continued, oscillations[6].
>
> The mathematical reason for this phenomenon is fairly easily derived by mathematical stability analysis. For a system as simple as this, the main advantage of a system dynamics program is that it enables the numerical simulation of this model. To do this, this system *of ordinary differential equations* needs to be converted into *integral equations,* since numerical integration is a more accurate process than numerical differentiation. In mathematical form, these integral equations are:

$$Rabbits = Rabbits_0 + \int ((r - \rho \times Foxes) \times Rabbits)$$
$$Foxes = Foxes_0 + \int ((-f + \phi \times Rabbits) \times Foxes)$$

Here $Rabbits_0$and $Foxes_0$ represent the initial number of Rabbits and Foxes.

The next figure shows this model in *Minsky*, with the initial conditions set up so that they differ slightly from the equilibrium. To see how to build a model like this step by step, see Minsky model building §(1.2.3):

Programs like Vensim and Simulink have been in existence for decades, and they are now mature products that provide everything their user-base of management consultants and engineers want for modeling and analyzing complex dynamic systems. So why has *Minsky* been developed? The key reason is its unique feature of Godley Tables §(1.2.1), which allow dynamic equations to be developed for monetary transactions via double-entry bookkeeping tables.

1.2 If you are experienced in system dynamics

As an experienced system dynamics user (or if you've just read the Introduction to Minsky §(1)), the most important thing you need to know is what *Minsky* provides that other system dynamics programs do not. That boils down to one feature: The Godley Table.

1.2.1 Godley Tables

Godley tables are a unique feature of *Minsky*. They are based on what could be called the world's first GUI ("Graphical User Interface"), the accountant's double-entry bookkeeping table.

Double-entry was invented in 15th century Italy[7] to enable accurate recording of financial transactions. The essence of this method is (a) to classify all of

[6] https://www.jstor.org/stable/84156

[7] https://www.janegleesonwhite.com/double

an entity's accounts as either Assets or Liabilities, with the difference between them representing the Equity of the entity; and (b) to record every transaction twice, once as a Credit (CR)and once as a Debit (DR), where the definitions of Credit and Debit entries for Assets, Liabilities and Equity are designed to ensure that the transaction was accurately recorded. Minsky uses this GUI to generate stock-flow consistent models of financial flows, but by default uses plus ($+$) and minus ($-$) operators (though the accountant's convention of Credit (CR)and Debit (DR) can be chosen via the Options menu). This system guarantees that, no matter how complex the model is, the equations are internally consistent.

Minsky uses this GUI to generate systems of ordinary differential equations to model financial flows. The columns specify stocks, while the entries in rows are flows. *The symbolic sum of a column is thus the rate of change of the associated stock. Minsky* takes a table like the next figure:

And converts it into a set of dynamic equations, which by construction are "stock-flow consistent":

$$
\begin{aligned}
\text{Consume} &= 0 \\
\text{Wages} &= 0 \\
\text{Tax} &= 0 \\
\text{Spend}_G &= 0 \\
\text{Spend}_B &= 0 \\
\text{Int}_L &= 0 \\
\text{Int}_{B^{\text{NB}}} &= 0 \\
\text{Int}_{B^B} &= 0 \\
\text{Credit} &= 0 \\
\text{BondSale}_{T^B} &= 0 \\
\text{BondSale}_{B^{\text{NB}}} &= 0 \\
\text{Banks}_E(0) &= 0 \\
\frac{d\text{Banks}_E}{dt} &= \text{Int}_L + \text{Int}_{B^B} - \text{Spend}_B \\
\text{Bonds}_B(0) &= 0 \\
\frac{d\text{Bonds}_B}{dt} &= \text{BondSale}_{T^B} - \text{BondSale}_{B^{\text{NB}}} \\
\text{Consumers}(0) &= 0 \\
\frac{d\text{Consumers}}{dt} &= \text{Wages} + 0.6 \times \text{Spend}_G + 0.2 \times \text{Int}_{B^{\text{NB}}} - \\
&\quad (0.7 \times \text{Tax} + 0.2 \times \text{BondSale}_{B^{\text{NB}}} + \text{Consume}) \\
\text{Firms}(0) &= 0 \\
\frac{d\text{Firms}}{dt} &= \text{Credit} + 0.4 \times \text{Spend}_G + 0.8 \times \text{Int}_{B^{\text{NB}}} + \text{Consume} + \text{Spend}_B -
\end{aligned}
$$

$$
\begin{aligned}
& (\text{Int}_L + \text{Wages} + 0.3 \times \text{Tax} + 0.8 \times \text{BondSale}_{B^{\text{NB}}}) \\
\text{Loans}(0) &= 0 \\
\frac{d\text{Loans}}{dt} &= \text{Credit} \\
\text{Reserves}(0) &= 0 \\
\frac{d\text{Reserves}}{dt} &= \text{Spend}_G + \text{Int}_{B^B} + \text{Int}_{B^{\text{NB}}} - (\text{Tax} + \text{BondSale}_{T^B})
\end{aligned}
$$

The model is completed by defining the flows on the canvas.

In addition, because one entity's financial asset is another's financial liability, *Minsky* enables the construction of a multi-sectoral view of financial transactions. The wedge next to every account name is used on other tables to search for Assets that have not yet been recorded as a Liability, and vice versa. For example, the previous Table recorded the financial system from the point of view of the Banking Sector, for which Reserves—the deposit accounts of private banks at the Central Bank–are an Asset. Reserves are also a liability of the Central Bank, and that can be shown in *Minsky* by adding an additional Godley Table, naming it *Central Bank*, and recording Reserves as a Liability:

At this stage, the financial transactions have been entered only once—against the Central Bank's Liability of Reserves. Each transaction has to be entered a second time to complete its record, but at present there are no available accounts in which to record the transactions a second time.

Bond purchases by the Central Bank indicate the need for to show bonds owned by the Central Bank as an asset, while the other transactions indicate the need to show the Treasury's account at the Central Bank, the "Consolidated Revenue Fund", as an additional liability:

This is an inherently better way to generate a dynamic model of financial flows than the flowchart method used by all other system dynamics programs, for at least four reasons:

- All financial transactions are flows between entities. The tabular layout captures this in a very natural way: each row shows where a flow originates, and where it ends up;
- The program imposes the rules of double-entry bookkeeping, in which entries on each row balance to zero according to the *accounting equation* ($Assets = Liabilities + Equity$). If you don't ensure that each flow starts somewhere and ends somewhere, then the program will identify your mistake;
- The double-entry perspective assists in the completion of a model, since the requirement of a matching entry for each transaction indicates the accounts that are needed to complete the accounting; and
- Double-entry bookkeeping acts as a prohibition against recording invalid transactions.

Minsky thus adds an element to the system dynamics toolkit which is essential for modeling the monetary flows that are an intrinsic aspect of a market economy.

1.2.2 *Minsky*'s unusual system dynamics features

Minsky differs from conventional system dynamics models in various ways:

- Most programs store their mathematical formulas within a text block, which must be opened to see its equation. The mathematics of a *Minsky* model is shown explicitly on the design canvas;

- Most programs require every entity used to build an equation to be wired to the block in which the equation is defined, while each entity is shown only once on the design canvas. This generates the familiar system dynamics "spaghetti diagram", as even practitioners—rather than merely critics—often describe their models. *Minsky* enables entities to be shown multiple times on the design canvas, which substantially reduces the number of wires needed to build a model. *Minsky*'s equations therefore resemble small "spider webs", rather than large bowls of spaghetti. This makes it easier to understand what a model does; and

- Most programs show the end results of a simulation. *Minsky* shows simulations dynamically, and the numerical values of parameters and variables are shown on the parameters and variables themselves. Parameters can be altered during a simulation, which enables them to be used as a "control panel" for simulation of policy proposals, etc.

These unusual features are best illustrated by building a simple model in *Minsky*. The next section explains how to build a simpler version of the predator-prey model §(1.1).

1.2.3 Building a predator-prey model in Minsky

The first step in building the Rabbits and Foxes model in *Minsky* is to insert two integral blocks on the canvas—one for Rabbits and the other for Foxes. An integral block is inserted by clicking on the  operator §(4.3). To alter its name from the generic "int1", either double-click on the widget, which brings up the context menu for integral blocks, or hover the mouse over the block and click the right-mouse button, and choose "Edit" to bring up the edit form (another method is to choose "Rename all instances" from the context menu). Name the two variables Rabbits and Foxes respectively, and you will have the following objects on your canvas:

As is typical of dynamic models, the rates of change of the variables Rabbits and Foxes depends on their current values. You therefore need to copy the variable names, which you do using the context menu command "Copy item". Do this and place both on the canvas:

Next add the parameters that determine the rates of growth and death of the two species. The non-zero equilibrium is set by the zeros of $(r - \rho \times Foxes)$ and $(-f + \phi \times Rabbits)$.

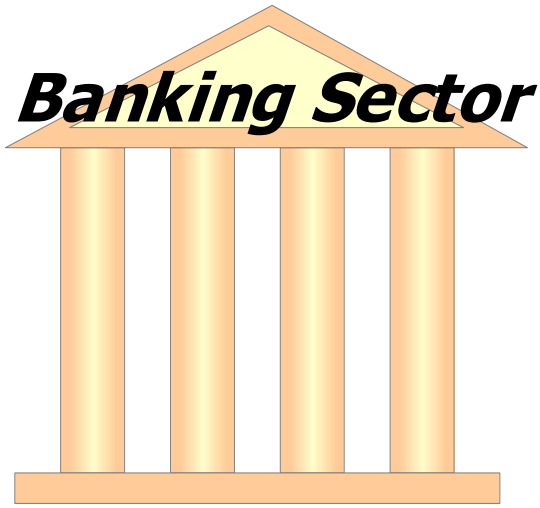
Finally, the equations for the rates of growth of the two species are wired up, and the model can be simulated.

If you don't want to display the complete equations on the canvas, you can use grouping to hide the complexity. The next figure illustrates several ways to use groups, and the feature that groups become transparent (and can be edited *in situ*) as the zoom level increases.

1.2.4 Building monetary models in *Minsky*

Monetary models are actually simpler to build in *Minsky* than block diagram models, because all you have to do is to name the accounts in the Godley Table, and then define the flows on the canvas. *Minsky* automatically generates the differential equations for you.

Building the model starts with placing a Godley Table on the canvas by clicking on the Godley Table operator  . The figure below shows a single Godley Table on the canvas, which has been labelled "Banking Sector" using the context menu "Title" command.



Next either double-click on the icon, or choose "Open Godley Table" from the context menu, to bring up the Godley Table form:

We'll build a model of private money creation in this first example.

Modelling private money creation

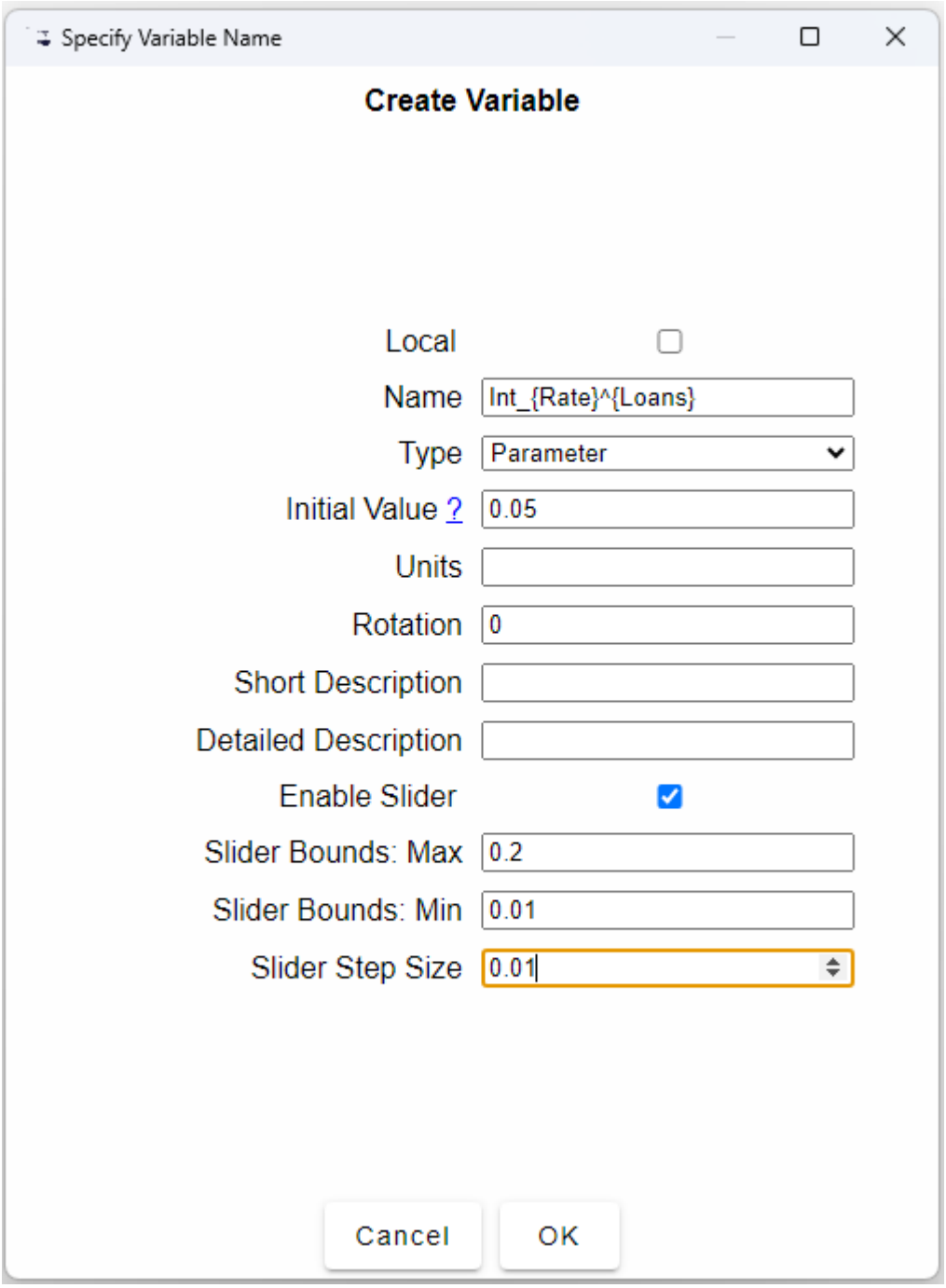
Add four Stocks to this table: *Reserves* and *Loans* as Assets, *Deposits* as a Liability, and $Banks_{Equity}$ as the Equity of the Banks (type Banks_{Equity} to subscript the word Equity). Go back to the Design canvas and choose "Editor mode" from the context menu, and you will see the table displayed as follows (you will need to resize the table using the Resizing arrows, which are visible when your mouse is hovering over the table)

Now enter numbers into the "Initial Conditions" row. Put 90 in *Reserves*, 10 in *Loans*, 80 in *Deposits*, and 20 in *BanksEquity*.

Then add three flows to the model: Net Bank Loans (which can be negative if people in the aggregate are paying their debts down rather than taking on new debt), Interest Payments, and Bank Spending. Call Net Bank Loans "Credit", Interest Payments "Interest_L" (the subscript is to distinguish interest on loans from interest on bonds, which we'll introduce later), and Bank Spending "Spend_B" (the subscript is to distinguish Bank Spending from Government Spending,which we'll introduce later).

Next, return to the canvas and use the Godley Table context menu commands *Copy flow variables* and *Copy stock variables* to copy all flows and stocks and place them on the canvas, where they can be defined.

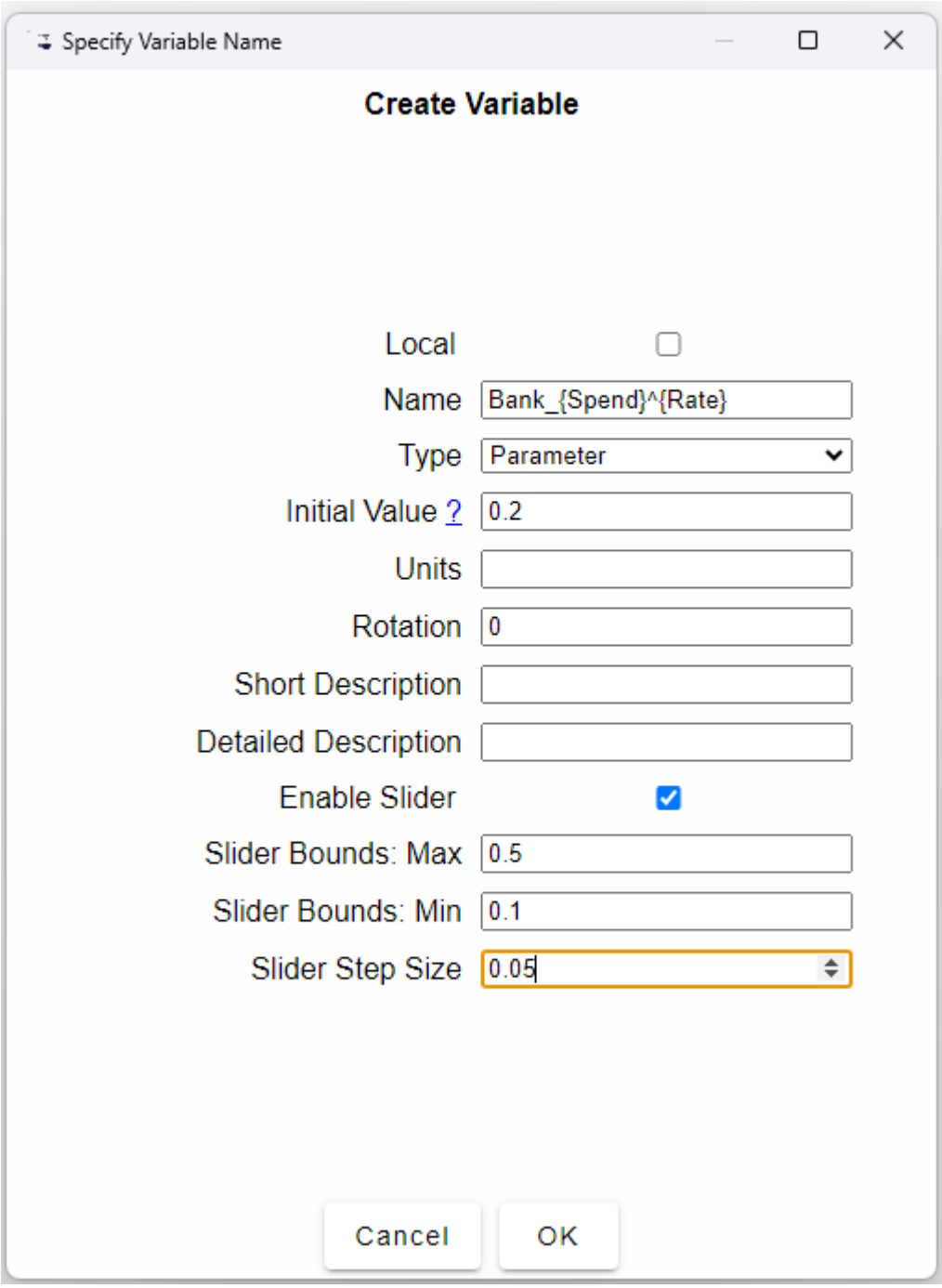
The easiest flow to define is *Interest*, since this is the rate of interest times *Loans*. To define this, first define the parameter ${Int_{Rate}}^{Loans}$: just type Int_{Rate}^{Loans} on the canvas and, once you've finished entering the text, the Variable/Parameter definition form will appear. Put 0.05 as the Initial Value, 0.2 as the *Slide Bounds: Max*, 0.02 as the *Min*, and 0.01 as the *Slider Step Size*.



Wire this parameter and *Loans* up to a multiply block §(4.2.8) and attach it to the flow Interest, and you have defined the first of three flows in

this extremely simple model.

Now define $Spend_B$ as a multiple $Bank_{Spend}{}^{Rate}$ of the $Banks_{Equity}$. Give the parameter a small Initial Value—say 0.2—which means that Banks spend 20% of their equity into the economy every year—this represents paying dividends, wages and bonuses to households, buying goods and services off firms, etc. Set Max to 0.5, Min to 0.1, and the Step Size to 0.05.



Your canvas should now look like this:

This is sufficient to build a model of just financial stocks and flows, if you edit

Credit to give it a Max, Min and Step Size in keeping with the other magnitudes in the model–say a Max of 10, Min of -10, and Step Size of 1. Add two Plots  §(4.6), one for Stocks and one for Flows, attach the Stocks and Flows to their inputs, put a copy of *Credit* at the top of the model for ease of access, click on the Run button [ifgeo]100, and vary the value of *Credit* to see what happens:

To link this very simple model to economic concepts, make the "Friedmanite" assumption that GDP equals Money times Velocity. Define Money as the sum of Deposits plus $Banks_{Equity}$, and define *Velocity* as a parameter with an Initial Value of 2, Max of 4, Min of 0.5, and Step Size of 0.1. Define a parameter $Credit_{Ratio}$ and give it an Initial Value of 0, Max of 0.1, Min of minus 0.1, and Step Size of 0.01. Also define the percentage growth rate as the differential of GDP divided by itself and put through a percentage operator. Run the model and vary $Credit_{Ratio}$ using the arrow keys as you run a simulation.

Much more sophisticated models can be built using *Minsky*, including models that mix the flowchart method with Godley Tables to generate models of both the physical and monetary systems.

Chapter 2

Getting Started with Minsky

2.1 System requirements

Minsky is an open source program available for Windows, Mac OS X, and various Linux distributions, as well as compilable on any suitable Posix compliant system. Go to our SourceForge page to download the version you need. Linux packages are available from the OpenSUSE build service.

2.2 Getting help

Press the F1 key, or select "help" from the context menu. Help is context-sensitive.

2.3 Components of the Program

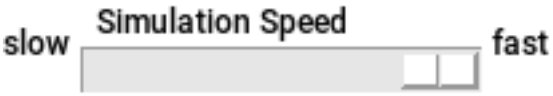
There are 6 components to the Minsky interface:

1. The menus.

   File Edit Insert Options Simulation Help

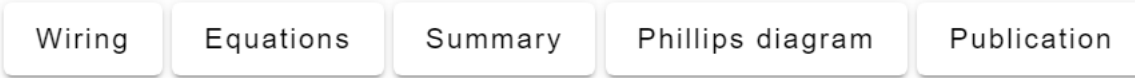
2. The Run buttons

3. The simulation speed slider



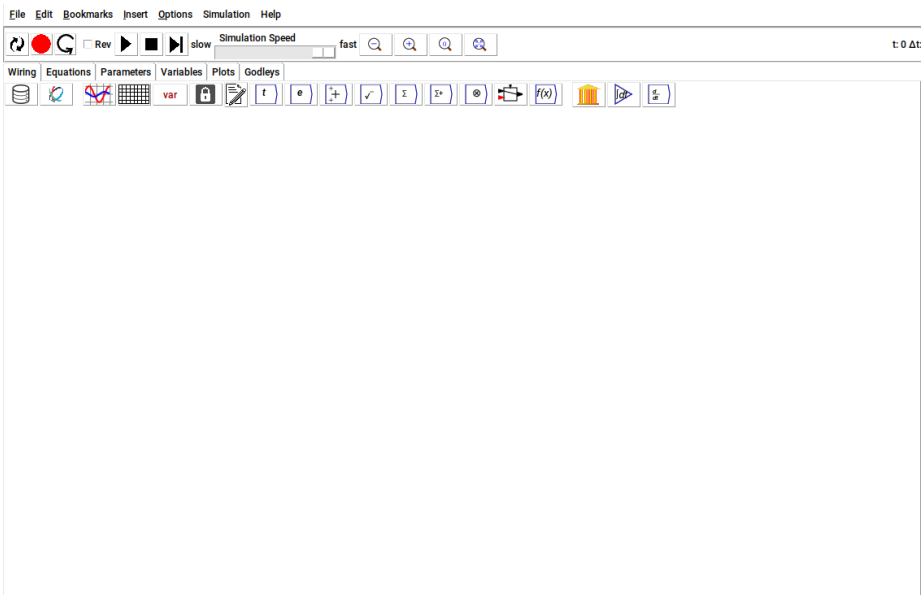
4. The Zoom buttons

5. The Wiring and Equation tabs



6. The design icons

7. And finally the Design Canvas–the large drawing area beneath the buttons and icons.



2.3.1 Menu

File Edit Insert Options Runge Kutta Help

The menu controls the basic functions of saving and loading files, default settings for the program, etc. These will alter as the program is developed; the current menu items are:

File

About Minsky Tells you the version of Minsky that you are using.

New System Clear the design canvas.

Open Open an existing Minsky file (Minsky files have the suffix of "mky").

Recent Files Provides a shortcut to some of your previously opened Minsky files.

Library Opens a repository of models for the Minsky simulation system.

Save Save the current file.

Save As Save the current file under a new name.

Insert File as Group Insert a Minsky file directly into the current model as a group §(4.5)

Export Canvas Export the current canvas into *svg, *pdf, *eps, *tex, or *m format. If using LaTeX (*tex), produce the set of equations that define the current system for use in documenting the model, for use in LaTeX compatible typesetting systems. If your LaTeX implemention doesn't support breqn, untick the wrap long equations option §(2.3.1), which can be found in the preferences panel under the options menu. If using a MatLab function this can be used to simulate the system in a MatLab compatible system, such as MatLab[1] or Octave[2].

Log simulation Outputs the results of the integration variables into a CSV data file for later use in spreadsheets or plotting applications.

Recording Record the states of a model as it is being built for later replay. This is useful for demonstrating how to build a model, but bear in mind that recorded logs are not, in general, portable between versions of Minsky.

Replay recording Replay a recording of model states.

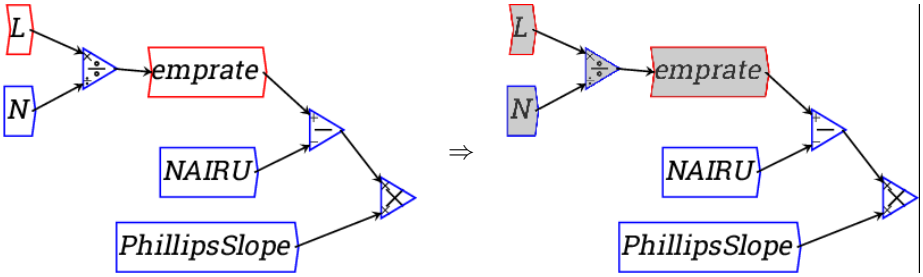
Quit Exit the program. Minsky will check to see whether you have saved your changes. If you have, you will exit the program; if not, you will get a reminder to save your changes.

Debugging use Items under the line are intended for developer use, and will not be documented here. Redraw may be useful if the screen gets messed up because of a bug.

[1] https://en.wikipedia.org/wiki/MATLAB
[2] http://www.gnu.org/software/octave/

Edit

- Undo and Redo allow you to step back and forward in your editing history. If you step back a few steps, and then edit the model, all subsequent model states will be erased from the history.
- Cut/copy/paste. Selecting, or lassoing a region of the canvas will select a group of icons, which will be shaded to indicate the selected items. Wires joining two selected items will also be selected. Note that, compatible with X-windows, selecting automatically performs a copy, so the copy operation is strictly redundant, but provided for users familiar with systems where an explicit copy request is required. Cut deletes the selected items. Paste will paste the items in the clipboard as a group §(4.5) into the current model. At the time of writing, copy-pasting between different instances of Minsky, or into other applications, may not work on certain systems. Pasting the clipboard into a text-based application will be a Minsky schema XML document.



- Create a group §(4.5) using the contents of the selection. Groups allow you to organise more complicated systems specification into higher level modules that make the overall system more comprehensible.

Insert

This menu contains a set of mathematical operator blocks §(4.2) for placement on the Canvas. You can get the same effect by clicking on the Design Icons. Also present are entries for Godley table items §(4.2.17) and Plots §(4.6).

Options

The options menu allows you to customise aspects of Minsky.

Preferences

- Godley table show values. When ticked, the values of flow variables are displayed in the Godley table whilst a simulation is running. This will tend to slow down the simulation somewhat.
- Godley table output style — whether +/− or DR/CR (debit/credit) indicators are used.

- Enable multiple equity columns - whether Godley table have more than one equity columns.
- Number of recent files to display — affects the recent files §(2.3.1) menu.
- Wrap long equations in LaTeX export. If ticked, use the breqn package to produce nicer looking automatically line-wrapped formulae. Because not all LaTeX implementations are guaranteed to support breqn, untick this option if you find difficulty.
- select a font for variable names etc.

Background colour — select a colour from which a colour scheme is computed.

Simulation

¡¡¡¡¡¡¡ HEADTime unit allows one to specify units for the time dimension for dimensional analysis §(4.12). eg "year", "s" etc. ======= Time unit allows one to specify units for the time dimension for dimensional analysis. eg "year", "s" etc. ¿¿¿¿¿¿¿ f85cdaddca17eacc1fd534ee1eccdb509c5807e0 Controls aspect of the adaptive Runge-Kutta equation solver, which trade off performance and accuracy of the model. Note a first order explicit solver is the classic Jacobi method, which is the fastest, but least accurate solver. The algorithm is adaptive, so the step size will vary according to how stiff the system of equations is. Specifying a minimum step size prevents the system from stalling, at the expense of accuracy when the step size reaches that minimum. Specifying a maximum step size is useful to ensure one has sufficient data points for smooth plots. An iteration is the time between updates to plots, increasing the number of solver steps per iteration decreases the overhead involved in updating the display, at the expense of smoothness of the plots. Screen refresh is the period between screen updates, in ms. If an iteration takes less than this time, the screen refresh is postponed until the time has expired. 100ms is fast enough for a smooth animation of the simulation - increasing this value will improve simulation performance at the cost of a jerky animation of the simulation. Start time is the value of the system t variable when the system is reset. Run until time can be used to pause the simulation ince t reaches a certain value. Setting this to "Infinity" causes the simulation to run indefinitely, or until some arithmetic error occurs.

Help

Provides an in-program link to this manual. Note that pressing F1 will also launch help windows in a cintext sensitive way, ie it will open the relevant help section for where ever the mouse is over. Similarly, each item on the canvas has a help menu item in the context menu relevant for that item.

2.3.2 Record/Replay Buttons

These buttons control the recording / replay mode of Minsky. You can record your interactions with Minsky, and replay those interactions for demonstration/presentation purposes.

1. Record a session of building/modifying a model. Note that replaying a recorded session always starts from a blank canvas, so if you're recording the modification of a model, ensure that the first thing recorded is to load the model being modified. This button is a toggle button, so clicking it again finishes the session, and closes the file.

2. Simulate/Replay button. Pressing this button changes Minsky into replay mode, and asks for a recording file. You may use the run buttons (run/pause,stop and step), as well as the speed slider, to control the replay. This button is a toggle button, so clicking it again returns Minsky back to the default simulation mode.

2.3.3 Recalculate button

The recalculate button computes the values of all variables at the start of simulation. It is particularly useful for recalculating the state of the model with tensor valued data.

2.3.4 Run Buttons

The Run buttons respectively:

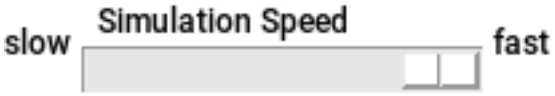
1. Start a simulation–when started the button changes to a pause icon, allowing you to pause the simulation  .

2. Stop a simulation and reset the simulation time to zero

3. Step through the simulation one iteration at a time.

4. Reverse checkbox changes the simulation time direction. Bear in mind that a simulation will eventually diverge from its original trajectory due to chaotic effects.

2.3.5 Speed slider



The speed slider controls the rate at which a model is simulated. The default speed is the maximum speed your system can support, but you can slow this down to more closely observe dynamics at crucial points in a simulation.

2.3.6 Zoom buttons

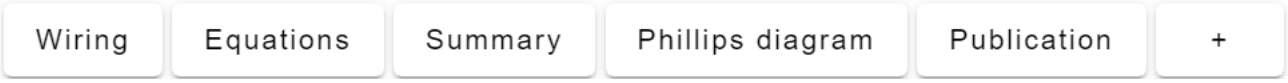
The Zoom buttons zoom in and out on the wiring canvas. The same functionality is accessed via the mouse scroll wheel. The reset zoom button resets the zoom level to 1, and also recentres the canvas. It can also be used to recentre the equation view.

The zoom to fit button zooms the model so that it just fits in the current canvas window.

2.3.7 Simulation time

In the right hand top corner is a textual display of the current simulation time t, and the current (adaptive) difference between iterations Δt.

2.3.8 Wiring and Equations Tabs



This allows you to switch between the visual block diagram wiring view and the more mathematical equations view.

2.3.9 Design Icons

These are the "nuts and bolts" of any system dynamics program. The number of icons will grow over time, but the key ones are implemented now:

Import data Opens an import CSV file dialog, allowing the creation of a parameter from a CSV data file. See Importing CSV files §(4.15).

Ravel See Ravel §(4.14).

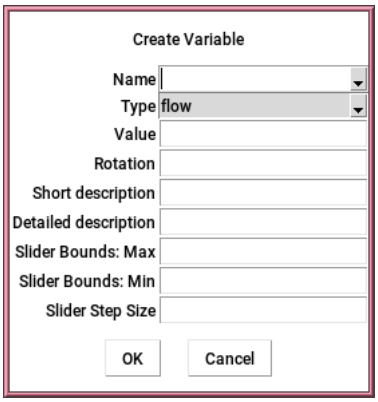
Plot widget Add plots §(4.6) to the canvas.

Sheet widget Add a sheet §(4.7) to the canvas.

Variable .

This is a pull down menu, giving access to creating variables, constants and parameters.

Variables are entities whose value changes as a function of time and its relationship with other entities in your model. Click on it and a variable definition window will appear:



The only essential step here is providing a name for the Variable. You can also enter a value for it (and a rotation in degrees), but these can be omitted. In a dynamic model, the value will be generated by the model itself, provided its input is wired.

When you click on OK (or press Enter), the newly named variable will appear in the top left hand corner of the Canvas. Move the mouse cursor to where you want to place the variable on the Canvas, click, and it will be placed in that location.

Constants are entities whose value is unaffected by the simulation or other entities in the model. Click on it and a constant definition window will appear:

The only essential element here is its value. You can also specify its rotation on the Canvas in degrees. This lets you vary a parameter while a simulation is running—which is useful if you wish to explore a range of policy options while a model is running.

A constant is just a type of variable, which also include parameters (named constants), flow variables, stock variables and integration variables. In fact there is no real conceptual difference between creating a constant or creating a variable, as you can switch the type using the type field.

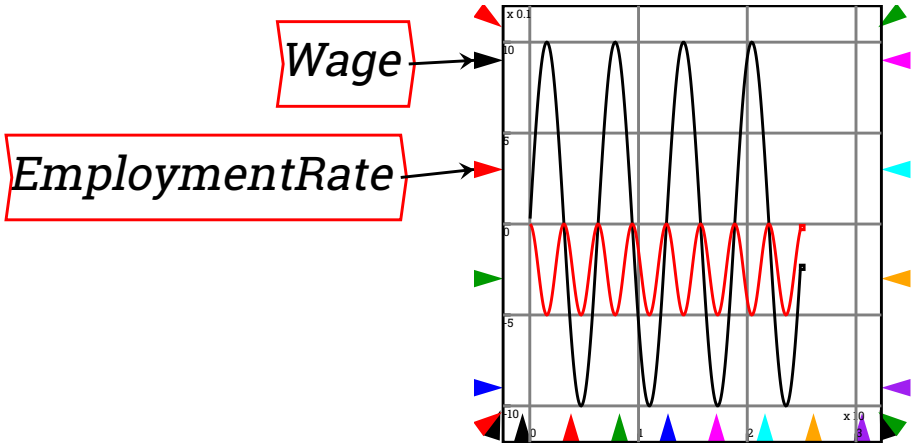
Like the variable and constant button, the parameter button creates a variable defaulting to the parameter type. Parameters differ from flow variables in not having an input port, and differ from constants in having a name and being controllable by a slider during simulation.

Lock Lock widgets are used with Ravels §(4.14).

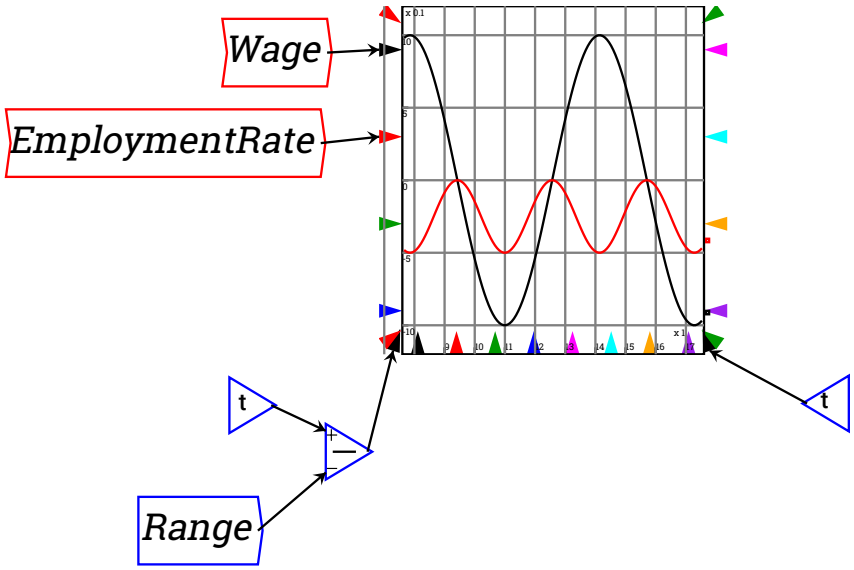
Notes Add textual annotations §(4.8)

Time embeds a reference to the simulation time on the Canvas. This is not necessary in most simulations, but can be useful if you want to make a time-dependent process explicit, or control the appearance of a graph.

For example, by default a graph displays the simulation time on the horizontal axis, so that cycles get compressed as a simulation runs for a substantial period:



If a Time block is added to the marker for the x-axis range, you can control the number of years that are displayed. This graph is set up to show a ten year range of the model only:



Unary functions These are a fairly standard complement of mathematical functions.

Binary operations . These execute the stated binary mathematical operations. Each input can take multiple wires as well—so that to add five

numbers together, for example you can wire 1 input to one port on the Add block, and the other four to the other port.

Min & Max Functions These take the minimum and maximum values, respectively. These also allow multiple wires per input.

Pow and log. These are binary operations (taking two arguments). In the case of the power operation, the exponent is the top port, and the argument to be raised to that exponent is the bottom port. This is indicated by the x and y labels on the ports. In the case of logarithm, the bottom port (labelled b) is the base of the logarithm.

Logical Operators $<$ $\leq$, $=$, $\wedge$ $\vee$ $\neg$ (and, or, not)] These return 0 for false and 1 for true.

Reduction operations This menu contains operations that reduce a vector to a scalar, or reduce the rank of a tensor. Typically sum, product, any, all etc.

Scans These are running sums and the difference operator

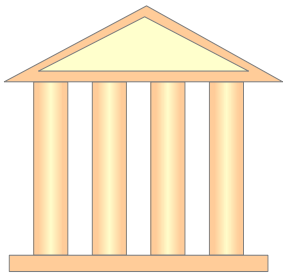
Miscellaneous tensor operations Any other tensor function not covered elsewhere.

Switch Add a piecewise-defined function block §(4.2.15) to the canvas. Also known as a hybrid function.

User defined function You can define your own function §(4.2.16) using an algebraic expression, such as `exp(-x^2y)+`.

Godley Table . This is the fundamental element of Minsky that is not found (yet) in any other system dynamics program.

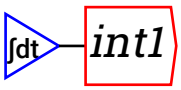
Clicking on it and placing the resulting Bank Icon on the Canvas enters a Godley table into your model:



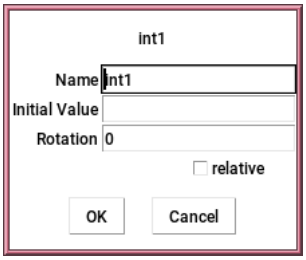
Double-click on the Bank Icon (or right-click and choose “Open Godley Table” from the context menu) and you get a double-entry bookkeeping table we call a Godley Table, which looks like the following onscreen:

Use this table to enter the bank accounts and financial flows in your model. We discuss this later in the Tutorial (Monetary) §(3.2).

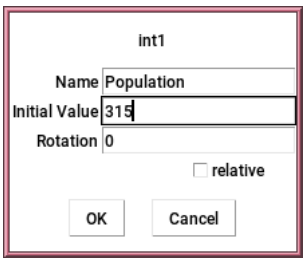
Integration . This inserts a variable whose value depends on the integral of other variables in the system. This is the essential element for defining a dynamic model. Click on it and the following entity will appear at the top left hand side of the canvas (and move with your mouse until you click to place it somewhere:



"int1" is just a placeholder for the integration variable, and the first thing you should do after creating one is give it a name. Double-click on the "int1", or right click and choose Edit. This will bring up the following menu:



Change the name to something appropriate, and give it an initial value. For example, if you were building a model that included America's population, you would enter the following:



The integrated variable block would now look like this:

To model population, you need to include a growth rate. According to Wikipedia, the current US population growth rate is 0.97 percent per annum. Expressed as an equation, this says that the annual change in population, divided by its current level, equals 0.0097:

$$\frac{1}{\text{Population}(t)} \cdot \left(\frac{d}{dt}\text{Population}(t) \right) = 0.0097$$

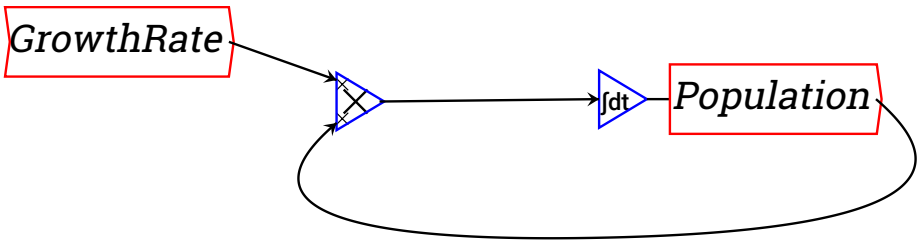
To express this as an integral equation, firstly we multiply both sides of this equation by Population to get:

$$\frac{d}{dt}\text{Population}(t) = 0.0097 \cdot \text{Population}(t)$$

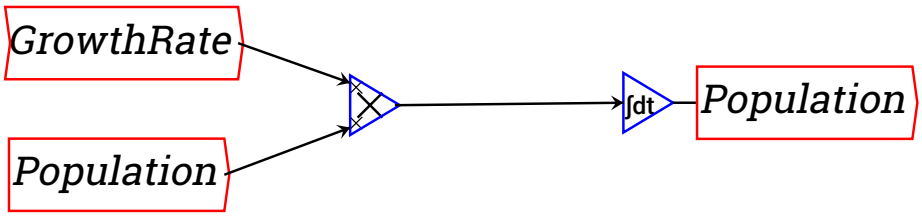
Then we integrate both sides to get an equation that estimates what the population will be T years into the future as:

$$\text{Population}(T) = 315 + \int_0^T 0.0097 \cdot \text{Population}(t)dt$$

Here, 315 (million) equals the current population of the USA, the year zero is today, and T is some number of years from today. The same equation done as a block diagram looks like this:



Or you can make it look more like the mathematical equation by right-clicking on "Population" and choosing "Copy Var". Then you will get another copy of the Population variable, and you can wire up the equation this way:



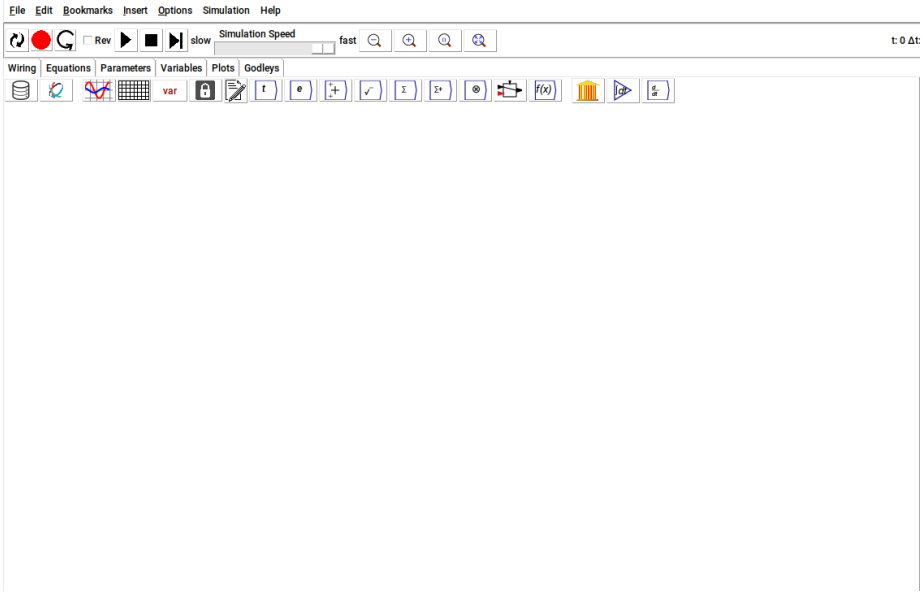
Either method can be used. I prefer the latter because it's neater, and it emphasizes the link between the simple formula for a percentage rate of change and a differential equation.

Derivative Operator This operator symbolically differentiates its input, provided the input is differentiable. An error is generated if the input is not differentiable.

2.3.10 Design Canvas

The Design Canvas is where you develop your model. A model consists of a number of blocks—variables, constants and mathematical operators—connected by wires.

The canvas is *zoomable*, either via the zoom buttons on the toolbar, or via the mouse scroll wheel. It is also *pannable*, either via the scroll bars on the right and bottom, or by holding the shift key and first mouse button together. The canvas is effectively unlimited, however the scroll bars treat the canvas as a 10000 pixels in size.



2.3.11 Equations tab

This displays the mathematical representation of the model

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2.3.12 Summary Tab

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2.3.13 Parameters tab

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This tab provides a summary table of all variables in the system, in a heirarchical fashion that can be navigated by expanding or hiding sections by toggling the caret. Each variable shows its name, it definition, dimensions (for tensor-valued variables), initial expression, units and current value.

¡¡¡¡¡¡¡ HEAD Most of these fields are editable, and usually do the obvious thing. Changing a variable's name will do a *replace all instances* operation to update all variables of the same name. Changing a variable's definition will replace the wiring graph leading into the variable by a user defined function §(4.2.16) containing your edited string. At some future point, functionality will be added to convert a user defined function into a wiring graph. =======

2.3.14 Variables tab

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2.3.15 Phillips diagram tab

¡¡¡¡¡¡¡ HEAD This tab implement's Minsky's take on a Phillips diagram showing the stocks and flows in a monetary economy. The Phillips tab shows all the information contained in the Godley tables of the model - if there aren't any, this tab will be blank. =======

2.3.16 Plots tab

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Initially, all the stocks will be arranged around a circle, with the flows shown as connecting arrows showing the current direction of the flow. You can move and rotate the stocks, and bend the flows to make a pleasing layout. The stocks will be coloured as though filled with a fluid like Bill Phillip's original analogue computer, and dynamically updated as the simulation proceeds.

¡¡¡¡¡¡¡ HEAD Publication tabs allow the creation of dashboards to emphasise certain aspects of a simulation. For example, you may wish to focus on a particular plot or Godley table when running the simulation. =======

2.3.17 Godleys tab

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Multiple publication tabs can be created by clicking the '+' tab.

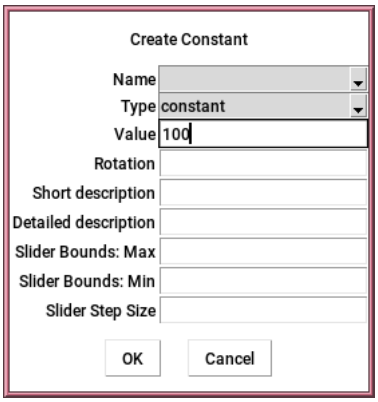
Any item from the wiring tab can be added to a publication tab, and then moved, resized or rotated independently of the item on the wiring tab. For items that dynamically update, the publication tab will be updated dynamically during the simulation.

Textual annotation can be added to the publication tab, independently of any annotations on the wiring tab.

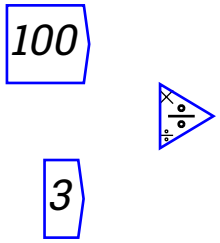
Wires cannot be added to the publication tab, however you can insert arrows (eg →) by typing the LaTeX text `\rightarrow`, and then rotate and scale the arrow to connect parts of the dashboard together.

2.3.18 Wires

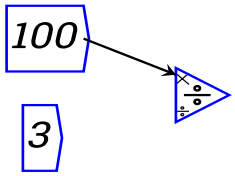
The wires in a model connect blocks together to define equations. For example, to write an equation for 100/33, you would place a  on the canvas, and give it the value of 100:



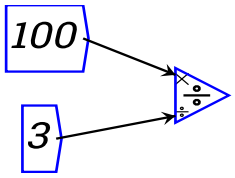
Then do the same for 33, and place a divide block on the canvas:



Then click on the right hand edge of  and drag to extend the wire to the numerator (×) port of the divide operation.



Finally, add the other wire.



2.4 Working with Minsky

2.4.1 Components in Minsky

There are a number of types of components in Minsky

1. Mathematical operators such as plus (+), minus (-)

2. Constants (or parameters, which are named constants) which are given a value by the user

3. Variables whose values are calculated by the program during a simulation and depend on the values of constants and other variables; and

4. Godley Tables, which define both financial accounts and the flows between them. In the language of stock and flow modelling, the columns of a Godley table are the stocks, which are computed by integrating over a linear combination of flow variables.

5. Integrals — represent a variable computed by integrating a function forward in time.

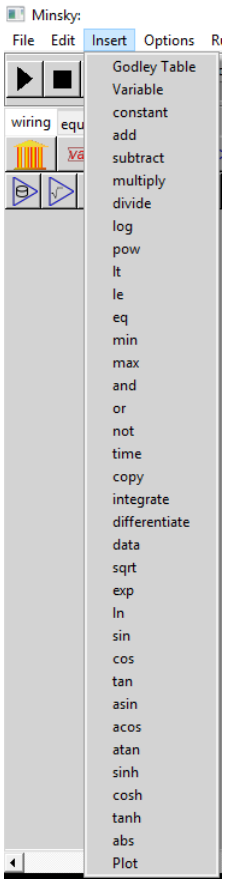
6. Groups, which allow components to be grouped into modules that can be used to construct more complex models.

2.4.2 Inserting a model component

There are five ways to insert a component of a model onto the Canvas:

1. Click on the desired Icon on the Icon Palette, drag the block onto the Canvas and release the mouse where you want to insert it

2. Choose Insert from the menu and select the desired block there



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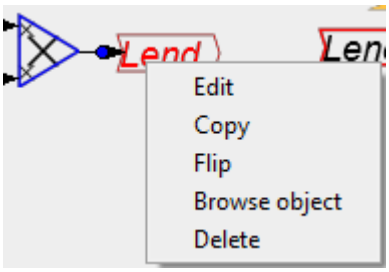
3. Variables can be inserted by typing the variable name on the canvas, and constants enetered by typing the numerical value. Similarly, operations can be inserted by typing the operator name (eg `sin`, or `*`). Notes can be inserted by starting the note with a `#` character.

4. Variables can also be picked from the Variable Browser §(4.3.5) and placed on the canvas. =======

5. Right-click on an existing block and choose copy. Then place the copy where you want it on the palette.



6. Variables can be inserted by typing the variable name on the canvas, and constants enetered by typing the numerical value. Similarly, operations can be inserted by typing the operator name (eg `sin`, or `*`). Notes can be inserted by starting the note with a `#` character.

7. Variables can also be picked from the r**??**e]Variable BrowserVariable Browser-VariableBrowser and placed on the canvas. ¿¿¿¿¿¿¿ f85cdaddca17eacc1fd534ee1eccdb509c5807e0

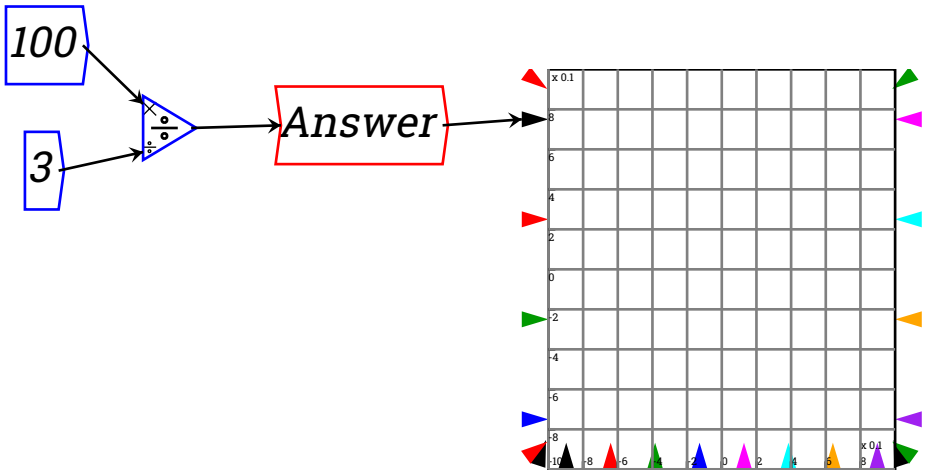
2.4.3 Creating an equation

Equations are entered in Minsky graphically. Mathematical operations like addition, multiplication and subtraction are performed by wiring the inputs up to the relevant mathematical block. The output of the block is then the result of the equation.

For example, a simple equation like

$$100/3 = 33.3$$

is performed in Minsky by defining a constant block with a value of 100, defining another with a value of 3, and wiring them up to a divide-by block. Then attach the output of the divide block to a variable, and run the model by clicking on ▶ :



If you click on the equation tab, you will see that it is:

$$\text{Answer} = \frac{100}{3}$$

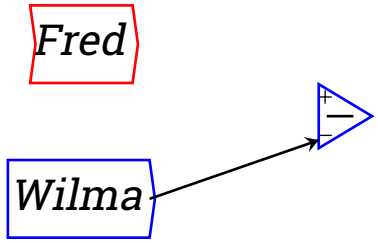
Very complex equations—including dynamic elements like integral blocks and Godley Tables—are designed by wiring up lots of components, with the output of one being the input of the next. See the tutorial for examples.

2.4.4 Wiring components together

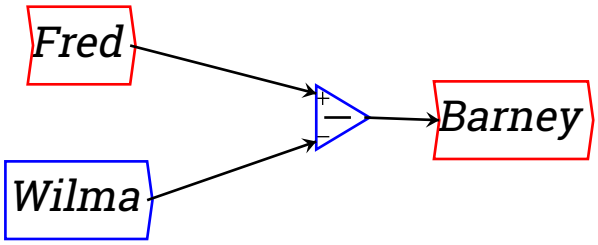
A model is constructed by wiring one component to another in a way that defines an equation. Wires are drawn from the output port of one block to the input port of another. Ports are circles on the blocks to which wires can be attached, which can be seen when hovering the pointer over the block. Variables have an input and an output port; constants and parameters only have an output port. A mathematical operator has as many input ports as are needed to define the operation.

To construct an equation, such as Fred - Wilma = Barney:

Click the mouse near the output port of one block and drag the cursor to the input port of another while holding the mouse button down. An arrow extends out from the output port. Release the mouse button near the required input port of the operator. A connection will be made.



The equation is completed by wiring up the other components in the same way.



2.4.5 Creating a banking model

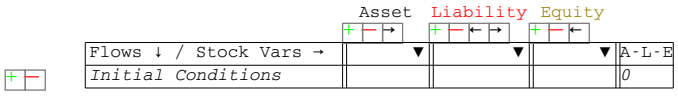
Creating a bank

The first step in creating a model with a banking sector is to click on the Godley Table Icon in the Icon Palette, and place the block somewhere on the Canvas.

Entering accounts

Double click or right click on the Godley table block to bring up the Godley Table. The table is divided up into sections representing the different accounting asset classes: Asset, Liability and Equity. Assets represent what you have to hand at any point in time, and should always be the sum of liabilities and equity. Liabilities represent amounts that are owed to other parties, and equity the amount of capital owned. The column `A-L-E` represents the *accounting equation* (Assets−Liabilities−Equity), and a properly formatted Godley table adhering to *double entry accounting conventions* will have this column zero for all rows.

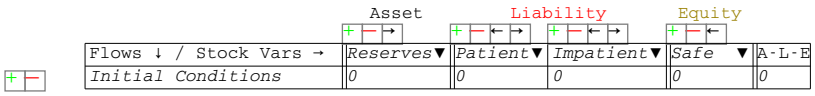
When a Godley Table is first loaded, each accounting class has room for one account (also known as a *stock*) to be defined. To create an additional accounts, click on the '+' button above the first account. One click then adds another column in which an additional account can be defined. Note that the table will delete excess blank accounts, so you should name them as you go. You can chancge the asset class of an account by moving it into the appropriate sector using the ← and → buttons, or by clicking and dragging the column variable name (the first row of the column).



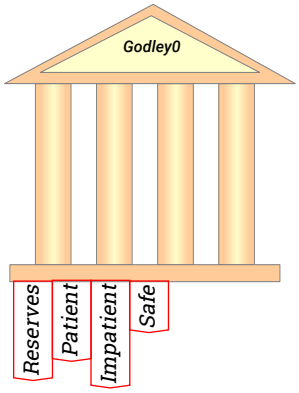
A column can be deleted by clicking on the '–' button above the column.

To define bank accounts in the system you enter a name into the row labeled "Flows ↓ / Stock Variables →". For example, if you were going to define a banking sector that operated simply as an intermediary between "Patient" people

and "Impatient" people—as in the Neoclassical "Loanable Funds" model–you might define the following accounts:

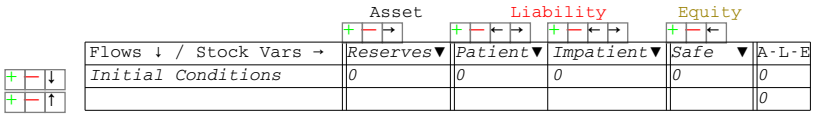


As you enter the accounts, they appear at the bottom of the Bank block on the canvas:

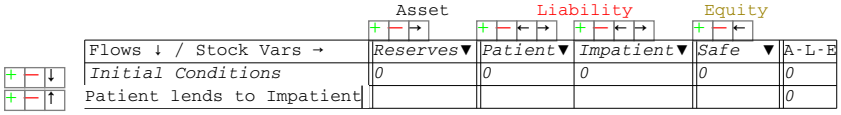


Entering flows between accounts

Flows between accounts are entered by typing text labels in the accounts involved. The source label is entered as a simple name—for example, if Patient is lending money to Impatient, the word "Lend" could be used to describe this action. Firstly you need to create a row beneath the "Initial Conditions" row (which records the amount of money in each account when the simulation begins). You do this by clicking on the '+' key on the Initial Conditions row. This creates a blank row for recording a flow between accounts.



The cell below "Initial Conditions" is used to give a verbal description of what the flow is:



The flows between accounts are then recorded in the relevant cells underneath the columns. Here we will start with putting the label "-Lend" into the Patient column. It is negative, because Patient is lending to Impatient.

		Asset	Liability		Equity	
	Flows ↓ / Stock Vars →	Reserves	Patient	Impatient	Safe	A-L-E
	Initial Conditions	0	0	0	0	0
	Patient lends to Impatient		-Lend			Lend

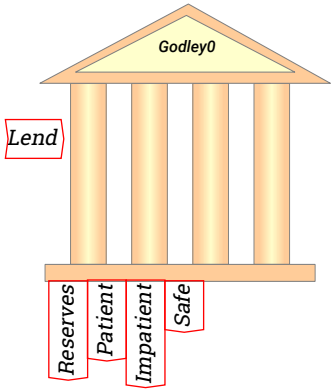
Notice that the program shows that the Row Sum for this transaction is currently "Lend", when it should be zero to obey the double-entry bookkeeping rule that all rows must balance. This is because a destination for "Lend" has not yet been specified. Please note that different asset class columns follow different +ve and -ve rules, so an asset and a liability with the same value might need to both be +ve or both -ve to sum to zero. The destination is Impatient's account, and to balance the row to zero this part of the transaction must be entered as "Lend":

		Asset	Liability		Equity	
	Flows ↓ / Stock Vars →	Reserves	Patient	Impatient	Safe	A-L-E
	Initial Conditions	0	0	0	0	0
	Patient lends to Impatient		-Lend	Lend		0

The accounting equation also applies to the Initial Conditions (the amount of money in each of the accounts prior to the flows between accounts): the Initial Conditions must balance. This requires that there are entries on the Asset side of the Banking ledger that exactly match the sum of Liabilities and Equity:

		Asset	Liability		Equity	
	Flows ↓ / Stock Vars →	Reserves	Patient	Impatient	Safe	A-L-E
	Initial Conditions	120	100	0	20	0
	Patient lends to Impatient		-Lend	Lend		0

As you enter flows, these appear on the left hand side of the bank block:



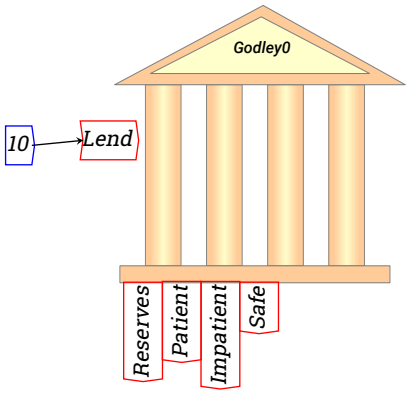
Defining flows

The entries in the Godley Table represent flows of money, which are denominated in money units per unit of time. The relevant time dimension for an

economic simulation is a year (whereas in engineering applications, the relevant time dimension is a second), so whatever you enter there represents a flow of money per year.

You define the value of flows by attaching a constant or variable to the input side of the flow into the bank as shown on the Canvas. For example, you could assign Lend a value of 10 (which would represent a loan of $10 per year by Patient to Impatient) by:

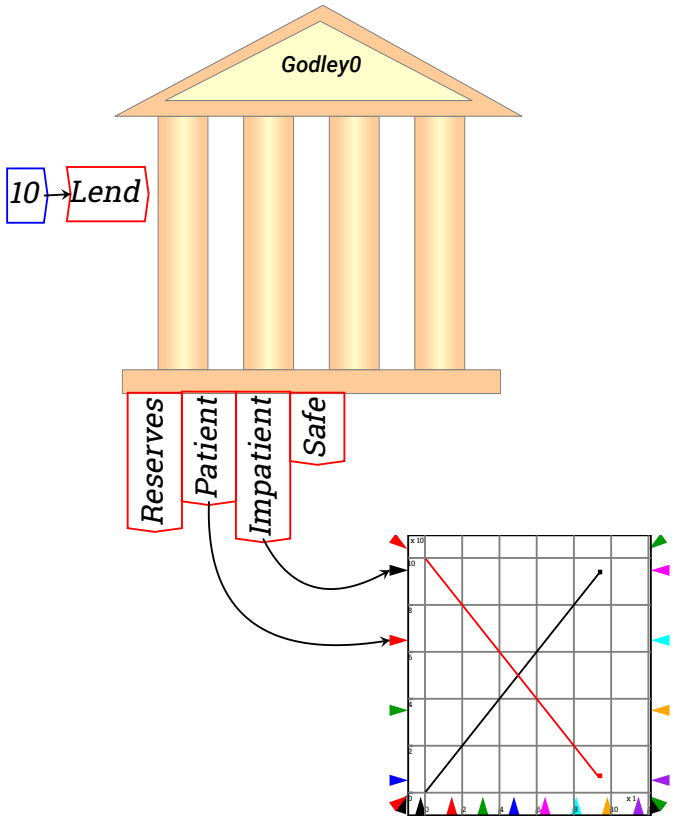
Create a constant with a value of 10, and attaching this to the input side of Lend:



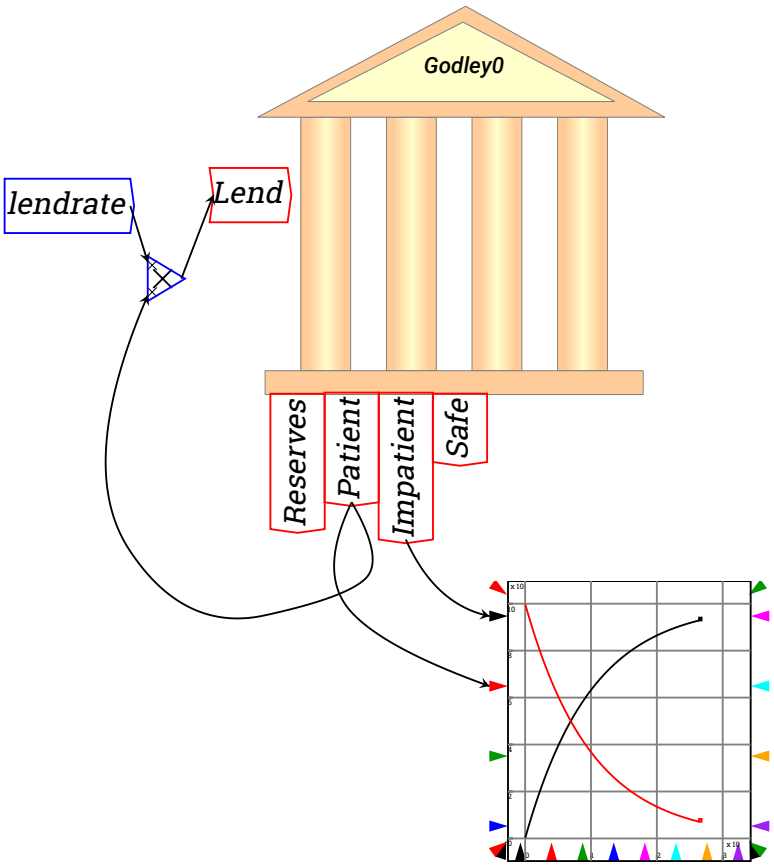
What you have now defined is an annual flow from Patient to Impatient of $10. In the dynamic equations this model generates, Minsky converts all amounts in accounts to positive sums—it shows the financial system from the point of the overall economy, rather than from the point of view of the bank:

$$\begin{aligned} \text{Lend} &= 10 \\ \frac{d\text{Impatient}}{dt} &= \text{Lend} \\ \frac{d\text{Patient}}{dt} &= -\text{Lend} \\ \frac{d\text{Reserves}}{dt} &= \\ \frac{d\text{Safe}}{dt} &= \end{aligned}$$

If you attach a graph to the accounts at the bottom of the bank block, you will see the impact of this flow over time on the balances of the two accounts. Patient's account begins at $100 and falls at $10 per year, while Impatient's account begins at $0 and rises by $10 per year.



Obviously this will result in a negative total worth for Patient after 10 years, so it is not a realistic model. A more sensible simple model would relate lending to the amount left in Patient's account (and a more complex model would relate this to many other variables in the model). That is done in the next example, where a constant "lendrate" has been defined and given the value of 0.1, and Lend is now defined as 0.1 times the balance in Patient's account. This now results in a smooth exponential decay of the amount in the Patient account, matched by a rise in the amount in Impatient account.



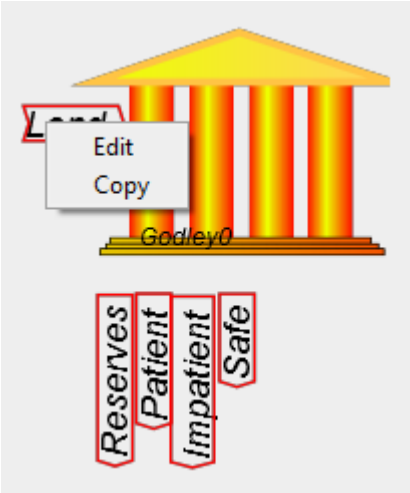
This is because the equation you have defined is identical to a radioactive decay equation, with the amount in the Patient account falling at 10 percent per year:

$$\begin{aligned} \text{Lend} &= \text{lendrate} \times \text{Patient} \\ \frac{d\text{Impatient}}{dt} &= \text{Lend} \\ \frac{d\text{Patient}}{dt} &= -\text{Lend} \end{aligned}$$

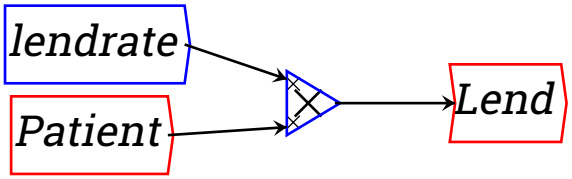
Note however that there are now wires crossing over other wires? There is a neater way to define flows.

Copying Godley Table input & outputs

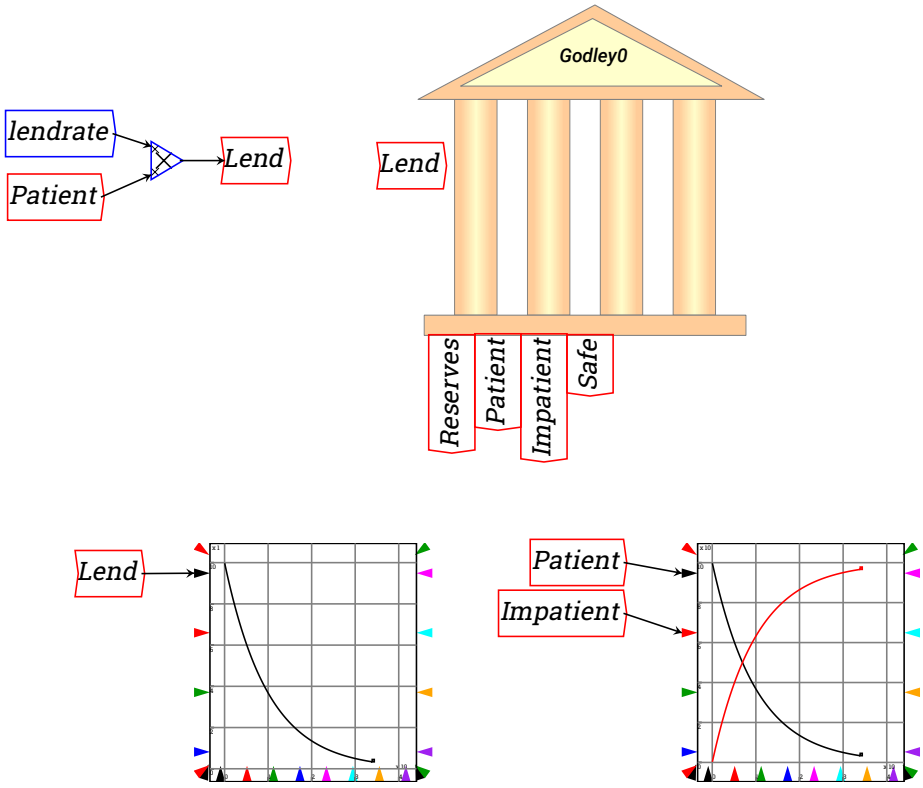
Right-click on the inputs and outputs of a Godley Table and choose “copy” from the drop-down menu:



Place the copied flows and accounts and place them away from the table. Then wire up your definition there:



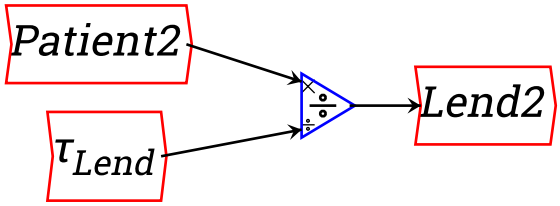
This now results in a much neater model. The same process can be used to tidy up graphs as well:



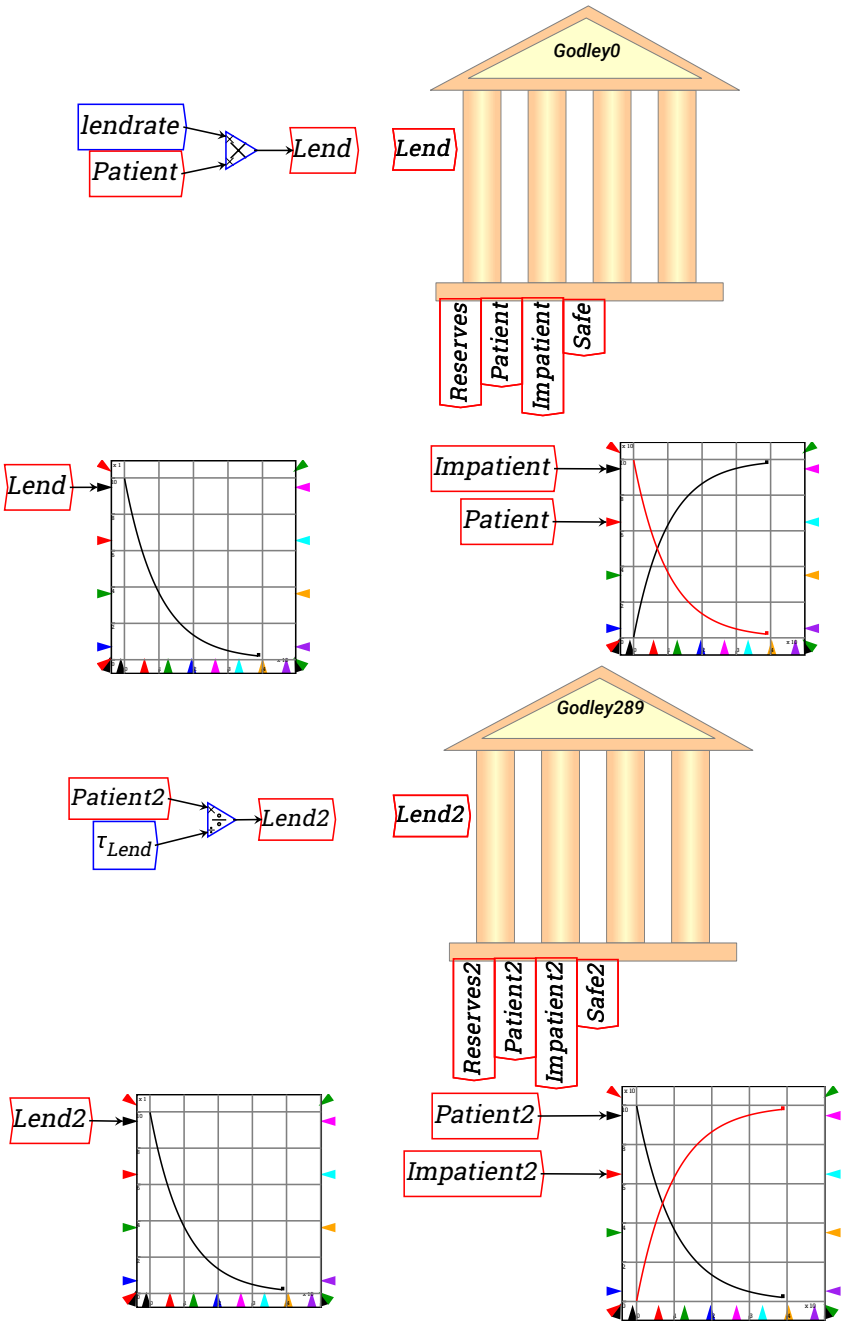
A more complex model would have many more flows, and these in turn would depend on other entities in the model, and be time-varying rather than using a constant "lendrate" as in this example—see the Tutorial on a Basic Banking Model §(3.2) for an example. This example uses the engineering concept of a "time constant" §(2.4.5), which is explained in the next section. Please note that right-clicking godley table variables and selecting "copy flow variables" creates a new group, which, when clicked and selecting "open in canvas", changes the canvas to show just that group. The normal canvas can be brought back by right-clicking and selecting "open master group".

Using "Time Constants"

The value of 0.1 means that the amount of money in the Patient account falls by one tenth every year (and therefore tapers towards zero). An equivalent way to express this is that the "time constant" for lending is the inverse of 1/10, or ten years. The next model uses a variable called τ_{Lend}, and gives it a value of 10:



As the simulation shows, the two models have precisely the same result numerically:

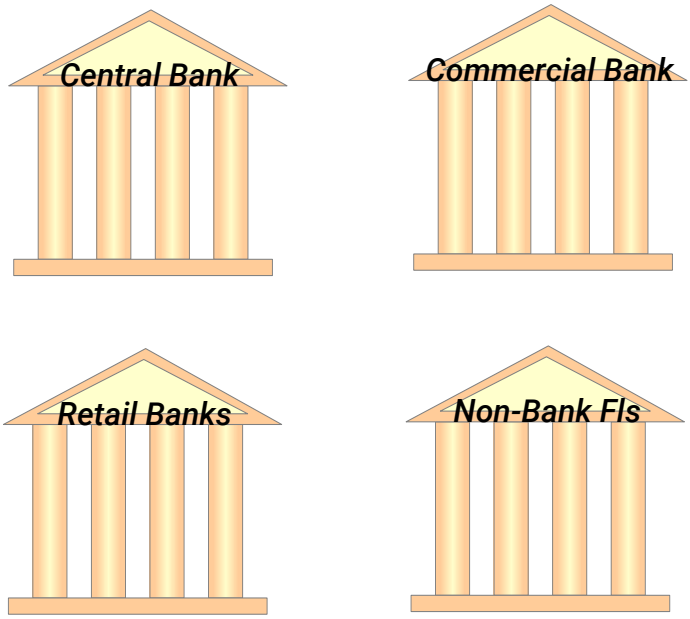


The advantage of the time constant approach is that it is defined in terms of the time that a process takes. A time constant of 10 says that, if this rate of lending was sustained (rather than declining as the account falls), then in ***precisely*** 10 years, the Patient account would be empty. The advantages of

this formulation will be more obvious in the tutorial §(**??**).

Multiple banks

There can be any number of Godley Tables—each representing a different financial institution or sector in an economy—in the one diagram. The name of the institution can be altered by clicking on the default name ("Godley0" in the first one created) and altering it. Here is an example with 4 such institutions/sectors defined:



If there are interlocking accounts in these banks—if one lends to another for example—then what is an asset for one must be shown as a liability for the other.

Godley tables may be further placed in *groups*, which allows scoping of the flow variables and their defining equations, whilst still allowing the tables to be coupled via global variables.

Chapter 3

Minsky Tutorial

3.1 Basic System Dynamics model

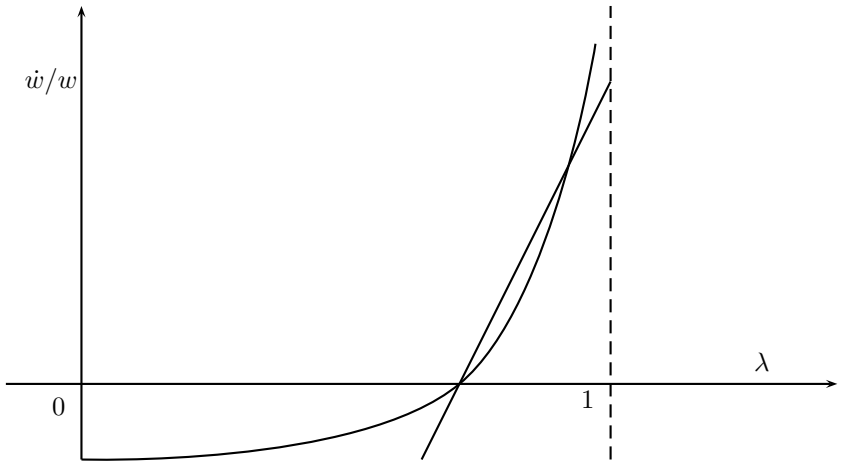
In 1965, Richard Goodwin, the great pioneer of complexity in economics, presented the paper "A Growth Cycle" to the First World Congress of the Econometric Society in Rome. It was later published in a book collection (Goodwin, Richard M. 1967. "A Growth Cycle," in C. H. Feinstein, *Socialism, Capitalism and Economic Growth.* Cambridge: Cambridge University Press, pp. 54–58.); to my knowledge it was never published in a journal.

Goodwin's model has been subjected to much critical literature about implying stable cycles, not matching empirical data, etc., but Goodwin himself emphasized that it was a "starkly schematized and hence quite unrealistic model of cycles in growth rates". He argued however that it was a better foundation for a more realistic model than "the more usual treatment of growth theory or of cycle theory, separately or in combination."

Goodwin emphasized the similarity of this model to the Lokta-Volterra model of interacting predator and prey, which can make it seem as if it was derived by analogy to the biological model. But in fact it can easily be derived from a highly simplified causal chain:

- The level of output (Y) determines the level of employment (L), with $L = Y/a$ where a is a measure of labor productivity;

- Given a population N, the employment rate $\lambda = L/N$ plays a role in determining the ***rate of change*** of the wage w: Goodwin used a linear approximation to a non-linear "Phillips Curve":



His linear approximation was:

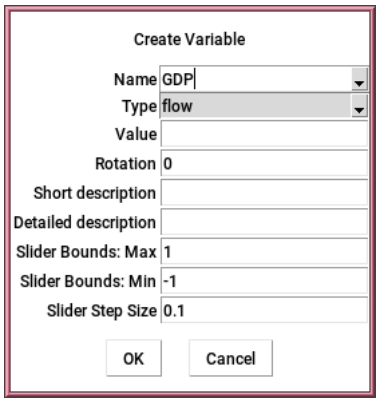
$$\frac{1}{w}\frac{d}{dt}w = -\gamma + \rho \cdot \lambda$$

- In a simple two-class model, profits Π equals the level of output Y minus the wage bill: $\Pi = Y - wL$
- For simplicity, Goodwin assumed that all profits were invested, so that Investment equals profits: $I = \Pi$.
- Investment is the rate of change of the capital stock K;
- The level of output is, to a first approximation, determined by the level of capital stock (K). A simple way of stating this is that Y is proportional to K: $Y = K/v$, where v is a constant (Goodwin notes that this relation "could be softened but it would mean a serious complicating of the structure of the model"); and finally
- Goodwin assumed that labor productivity grew at a constant rate α, while population grew at a constant rate β.

  Goodwin published the model as a reduced form equation in the two system states the employment rate (λ) and the workers' share of output (ω):

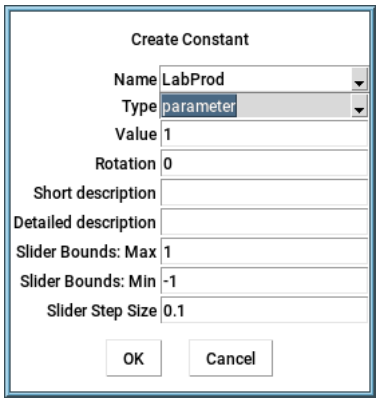
$$\begin{aligned}\frac{d}{dt}\lambda &= \lambda\left(\frac{1-\omega}{v} - \alpha - \beta\right)\\ \frac{d}{dt}\omega &= \omega \cdot (\rho \cdot \lambda - \gamma - \alpha)\end{aligned}$$

This form is useful for analytic reasons, but it obscures the causal chain that actually lies behind the model. With modern system dynamic software, this can be laid out explicitly, and we can also use much more meaningful names. We'll start with defining output (which is a variable). Click on var on the Icon Palette, or click on the Operations menu and choose "Variable". This will open up the "Specify Variable Name" window:



Enter "***GDP***" *into the "Name" field, and leave the other fields blank—since is a variable and we're defining a dynamic system, the value of at any particular point in time will depend on the other entities in the model. Now Click OK (or press "Enter"). The variable will now appear, attached to the cursor. Move to a point near the top of the screen and click, which will place the variable at that location.*

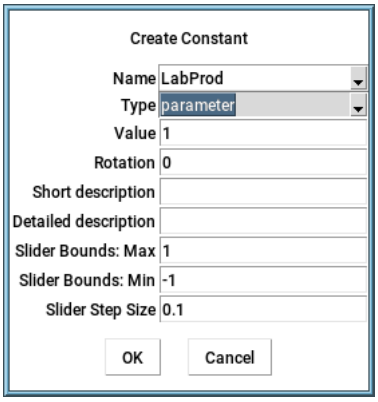
We are now going to write the first part of the model, that Labor (***Labor***) *equals output () divided by labor productivity (). Just for the sake of illustration, we'll make a parameter, which is a named constant (this can easily be modified later). For this we start by clicking on* const *on the Palette, or by choosing Insert/variable from the menu. This will pop-up the Edit Constant window:*



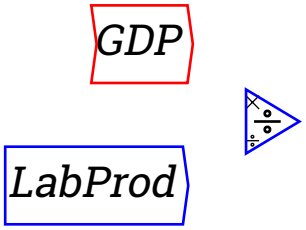
There is actually no real difference between the "Edit constant" dialog and the "Edit variable" dialog. The window's title differs, and the default value

of Type is "constant" instead of "flow". We're going to select "parameter", allowing one to give the parameter a name.

Give the paramter the name "***LabProd***" *and the value of 1 (i.e., one unit of output per worker). Click OK or press Enter and the constant* LabProd *will now be attached to the cursor. Place it below :*

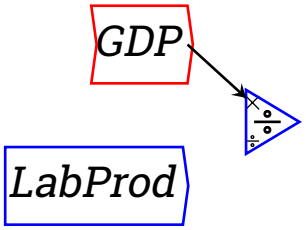


Now we need to divide ***GDP*** *by . Click on the* *symbol on the palette and the symbol will be attached to the cursor. Drag it near the other two objects and click. Your Canvas will now look something like this:*

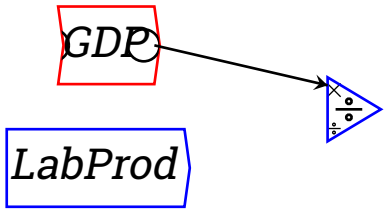


Now to complete the equation, you have to attach ***GDP*** *to the top of the divide block and LabProd to the bottom.*

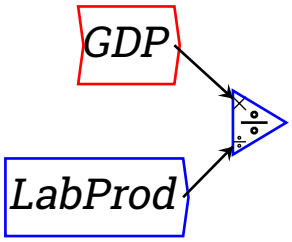
Now move your cursor to the right hand side of GDP and click, hold the mouse button down, and drag. An arrow will come out from GDP . Drag this arrow to the top of the divide block (where you'll see a tiny multiply sign) and release the mouse. You should then see this:



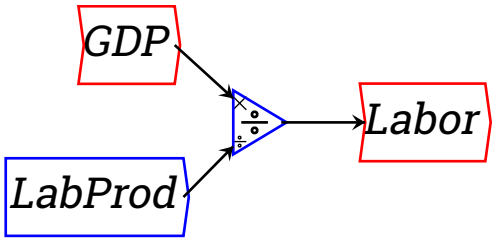
When the mouse hovers over a block, you will then see little circles that identify the input and output ports of the block:



Those are the connection points for wires, so start dragging from one and release on the other. Now wire LabProd to the bottom of the Divide block (where you'll see a miniature divide symbol (blown up below):



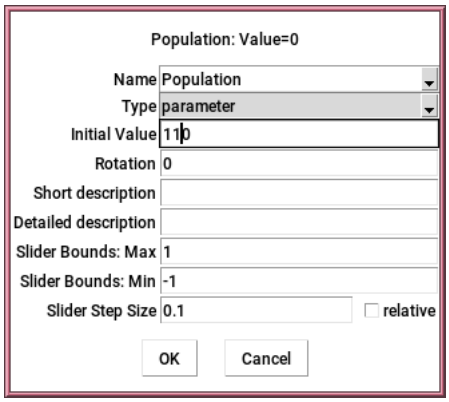
Then click on var in the Design Icons to create a new variable, call it Labor, place it the the right of the Divide block, and wire the output port from the Divide block to the input port for ***Labor***:



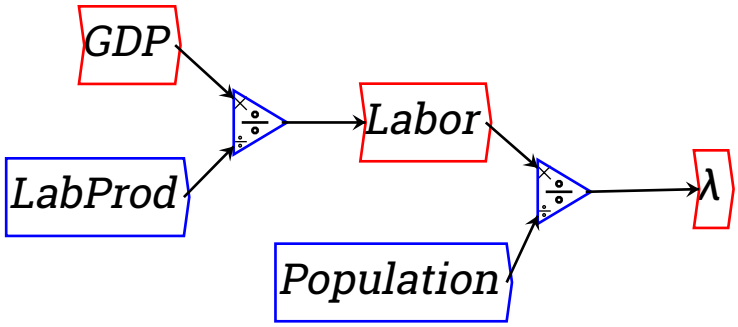
To show the correspondence between the flowchart above and standard modeling equations, click on the equations tab:

$$\begin{aligned} \text{GDP} &= \\ \text{Labor} &= \frac{\text{GDP}}{\text{LabProd}} \end{aligned}$$

Now let's keep going with the model. With ***Labor*** defined, the employment rate will be ***Labor*** divided by ***Population***. Define ***Population*** as a parameter (we'll later change it to a variable), and give it a value of 110.



Add it to the Canvas and you are now ready to define the employment rate—another variable. Click on var , give it the name "\lambda" (be sure to include the backslash symbol), put another Divide block on the canvas, choose Wire mode and wire this next part of the model up. You should now have:



Now switch to the equations tab, and you will see

$$\begin{aligned} \text{GDP} &= \\ \text{Labor} &= \frac{\text{GDP}}{\text{LabProd}} \\ \lambda &= \frac{\text{Labor}}{\text{Population}} \end{aligned}$$

Notice that Minsky outputs a Greek λ in the equation. You can input such characters directly, if your keyboard supports them as unicode characters, however you can also use a subset of the LaTeX language to give your variables more mathematial names.

With the employment rate defined, we are now ready to define a "Phillips Curve" relationship between the level of employment and the ***rate of change*** *of wages. There was far more to Phillips than this (he actually tried to introduce economists to system dynamics back in the 1950s), and far more to his*

employment-wage change relation too, and he insisted that the relationship was nonlinear (as in Goodwin's figure above). But again for simplicity we'll define a linear relationship between employment and the rate of change of wages.

Here we need to manipulate the basic linear equation that Goodwin used:

$$\frac{1}{w}\frac{d}{dt}w = -\gamma + \rho \cdot \lambda$$

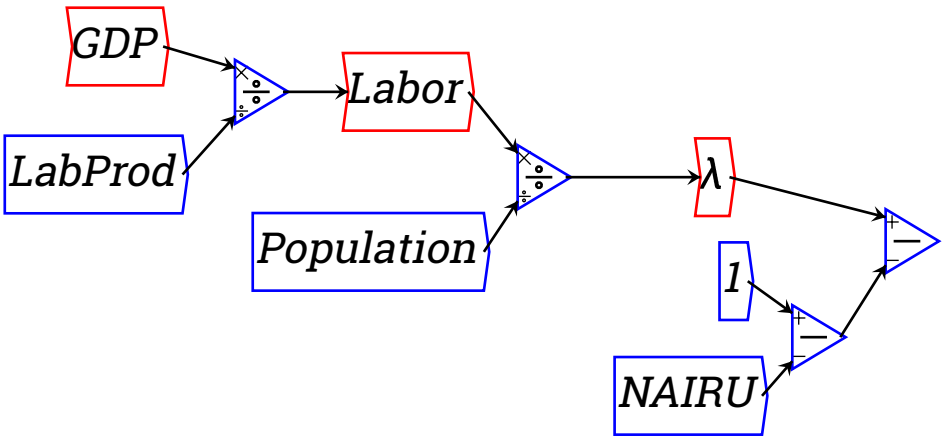
Firstly multiply both sides by w:

$$\frac{d}{dt}w = w \cdot (-\gamma + \rho \cdot \lambda)$$

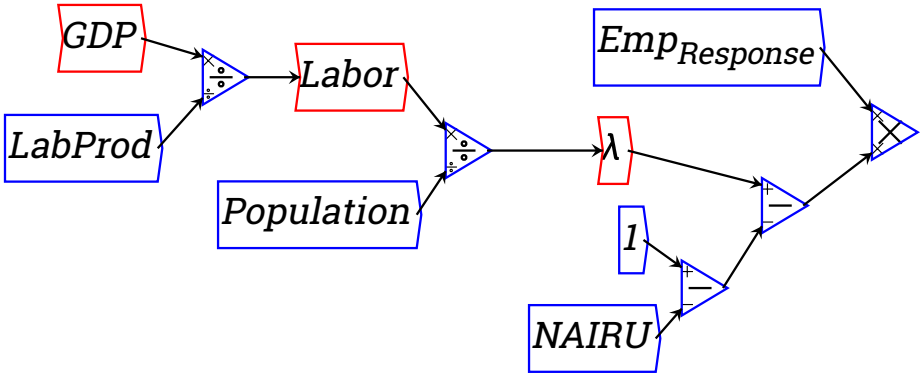
Then integrate both sides (because integration is a numerically much more stable process than differentiation, all system dynamics programs use integration rather than differentiation):

$$w = w_0 + \int w \cdot (-\gamma + \rho \cdot \lambda)$$

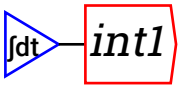
In English, this says that the wage now is the initial wage plus the integral of the wage multiplied by its rate of change function. That's what we now need to add to the Canvas, and the first step is to spell out the wage change function itself. Firstly, since we're using a linear wage response function, the rate of employment has to be referenced to a rate of employment at which the rate of changes is zero. I suggest using Milton Friedman's concept of a "Non-Accelerating-Inflation-Rate-of-Unemployment", or NAIRU. We need to define this constant, subtract it from 1, and subtract the result from the actual employment rate λ. To enter 1, click on const , define a constant and give it a value of 1. Then define another variable NAIRU, and give it a value of 0.05 (5% unemployment). Select "parameter" as the variable type. Subtract this from 1 and subtract the result from λ. You should have the following:



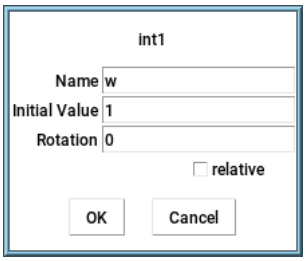
Now we need to multiply this gap between the actual employment rate and the "NAIRE" rate by a parameter that represents the response of wages to this gap. Let's call this parameter ***Emp_{Response}*** (remember to include the underscore and the braces). Define the parameter, give it a value of 10, and multiply (λ minus NAIRE) by it:



Now we are ready to add a crucial component of a dynamic model: the integral block, which takes a flow as its input and has the integral of that flow as the output. The wage rate ***w*** is such a variable, and we define it by clicking on the ∫dt symbol in the Icon Palette (or by choosing Operations/Integrate from the *Insert* menu). This then attaches the following block to the cursor:



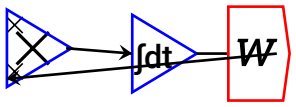
Now we need to rename this from the default name of "int1" to "***w***" *for the wage rate. Either right click or double-click on "int1" and this will bring up the edit window . Rename it to "w" and give it a value of 1:*



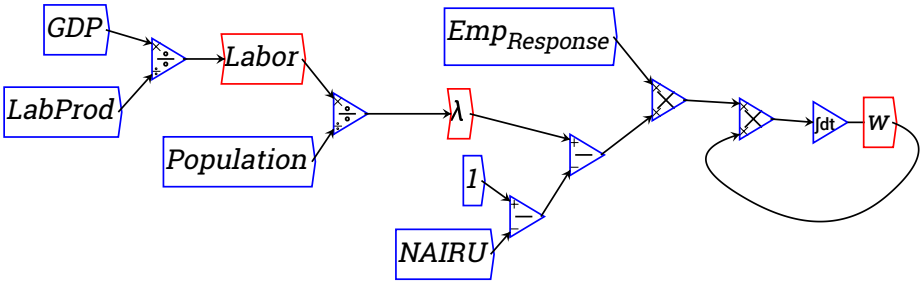
To compete the integral equation, we need to multiply the linear employment response function by the current wage before we integrate it (see the last equation above). There are two ways to do this. First, place a multiply block between the wage change function and the integral block, wire the function up to one input on the multiply block, and then either:

- wire the output of the ***w*** block back to the other input on multiply block; or
- Right-click on ***w***, choose "Copy Var", place that copy before the multiply block, and wire it up.

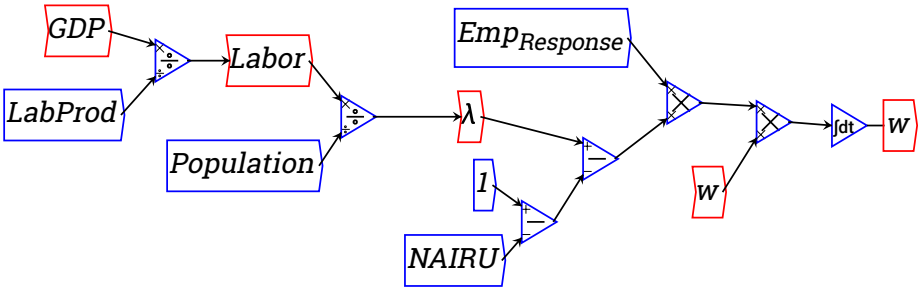
The first method gives you this initial result:



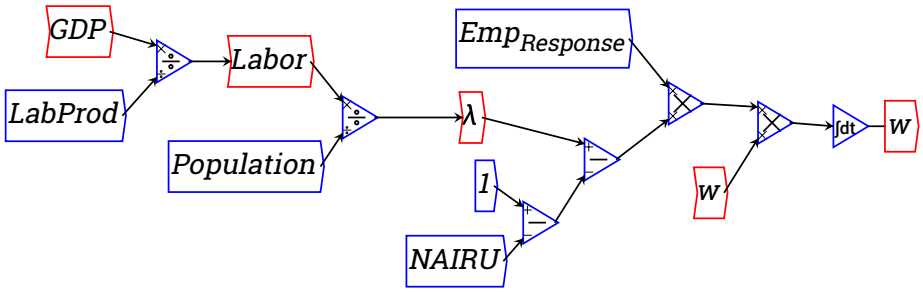
That looks messy, but notice the blue dot on the wire? Click and drag on that and you will turn the straight line connector into a curve:



The second approach, which I personally prefer (it's neater, and it precisely emulates the integral equation), yields this result:

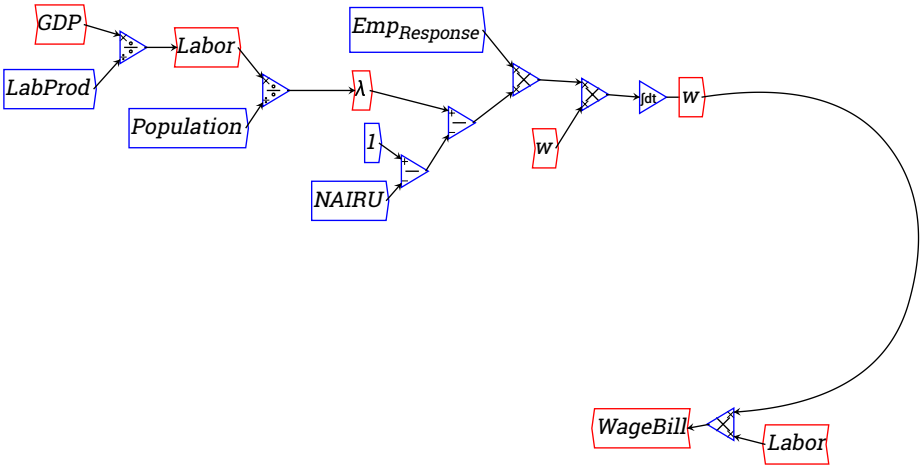


From this point on the model develops easily—"like money for old rope", as one of my maths lecturers used to say. Firstly if we multiply the wage rate ***w*** by ***Labor*** we get the ***Wage Bill***. *To do this, firstly create the variable Wage Bill, and put it well below where w currently is on your diagram:*

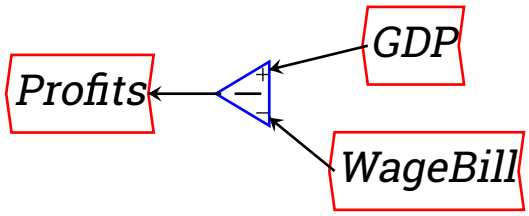


Now right-click on ***WageBill*** and choose "Flip". This rotates the block through 180 degrees (any arbitrary rotation can be applied from the variable definition window itself). Now right-click on ***Labor***, which you've already defined some time ago, and choose "Copy". Place the copy of ***Labor*** to the right of ***WageBill***:

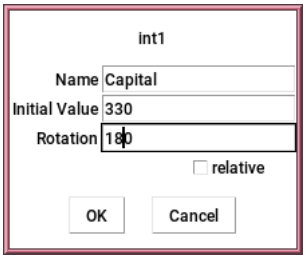
Now insert a multiply block before ***WageBill***, and wire ***w*** and ***Labor*** up to it. Curve the wire from w using the blue dots (you can do this multiple times to create a very curved path: each time you create a curve, another 2 curve points are added that you can also manipulate, as I have done below:



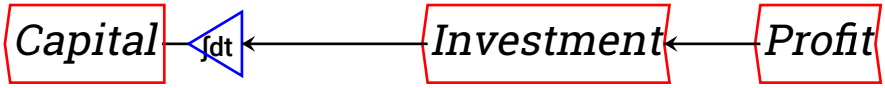
The next step is to subtract the ***WageBill*** from ***GDP*** to define ***Profits***. Take a copy of ***GDP***, insert it above ***WageBill***, insert a subtract block, and wire it up to define the variable ***Profits***:



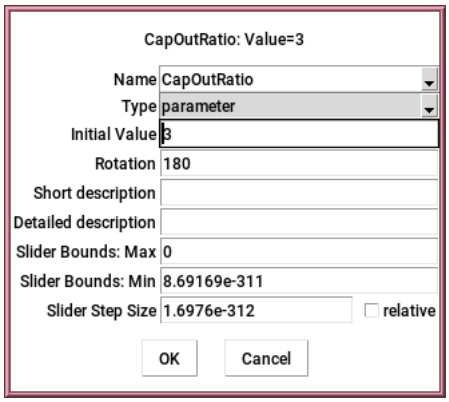
In the simple Goodwin model, all Profits are invested, and investment of course is the rate of change of the capital stock Capital. Create a variable called Investment, wire this up to Profits, and then create a new integral variable *Capital* using the ∫dt icon. Right-click or double-click on it to rename *int2* to *Capital*, and give it an initial value of 300:



Wire this up to Investment:

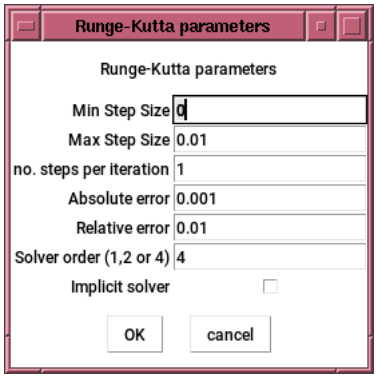


Now there's only one step left to complete the model: define a parameter CapOutputRatio and give it a value of 3:

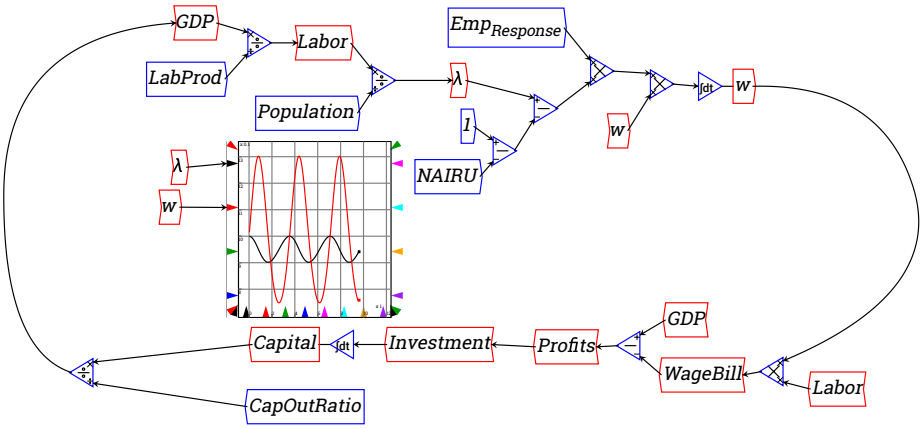


Divide Capital by this, and wire the result up to the input on GDP. You have now built your first dynamic model in Minsky:

Before you attempt to run it, do two things. Firstly from the *Runge Kutta* menu item, change the Max Step Size to 0.01—to get a smoother simulation.



Secondly, add some graphs by clicking on the  icon, placing the graph in the middle of the flowchart, and wiring up λ and w to two of the four inputs on the left hand side. You will now see that, rather than reaching equilibrium, the model cycles constantly:



If you click on the equations tab, you will see that you have defined the following system of equations:

$$
\begin{aligned}
\text{GDP} &= \frac{\text{Capital}}{\text{CapOutRatio}} \\
\text{Investment} &= \text{Profits} \\
\text{Labor} &= \frac{\text{GDP}}{\text{LabProd}} \\
\text{Profits} &= \text{GDP} - \text{WageBill} \\
\text{WageBill} &= w \times \text{Labor} \\
\lambda &= \frac{\text{Labor}}{\text{Population}} \\
\omega &= \frac{\text{WageBill}}{\text{GDP}} \\
\frac{dw}{dt} &= \text{Emp}_{\text{Response}} \times (\lambda - (1 - \text{NAIRU}) \times w \\
\frac{d\text{Capital}}{dt} &= \text{Investment}
\end{aligned}
$$

At this level of complexity, the equation form—if you're accustomed to working in equations—is as accessible as the block diagram model from which it was generated. But at much higher levels of complexity, the block diagram is far easier to understand since it displays the causal links in the model clearly, and can be structured in sub-groups that focus on particular parts of the system.

3.2 Basic Banking model

If you haven't yet read the section on Creating a Banking Model §(2.4.5), do so now. This tutorial starts from the skeleton of the "Loanable Funds" model

built in that section, and using time constants §(2.4.5) to specify how quickly lending occurs.

3.2.1 Loanable Funds

Our model begins with the single operation of Patient lending to Impatient at a rate that, if kept constant at its initial level of of $10 per annum, would empty the Patient account in 10 years. Because the rate of outflow declines as the Patient account declines, the money in the account decays towards zero but never quite reaches it.

	Asset	Liability		Equity	
Flows ↓ / Stock Vars →	Reserves▼	Patient▼	Impatient▼	Safe ▼	A-L-E
Initial Conditions	120	100	0	20	0
Patient lends to Impatient		-Lend	Lend		0

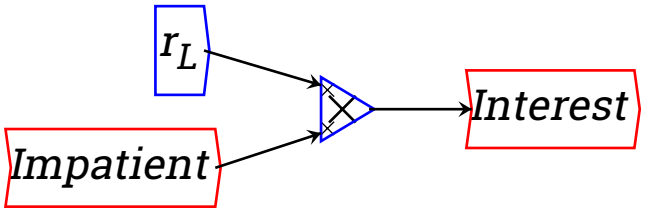
Many more actions need to be added to this model to complete it. For a start, Impatient should be paying interest to Patient on the amount lent. Add an additional row to the Godley Table by clicking on the '+' key next to "Patient lends to Impatient" to create a blank row:

	Asset	Liability		Equity	
Flows ↓ / Stock Vars →	Reserves▼	Patient▼	Impatient▼	Safe ▼	A-L-E
Initial Conditions	120	100	0	20	0
Patient lends to Impatient		-Lend	Lend		0
					0

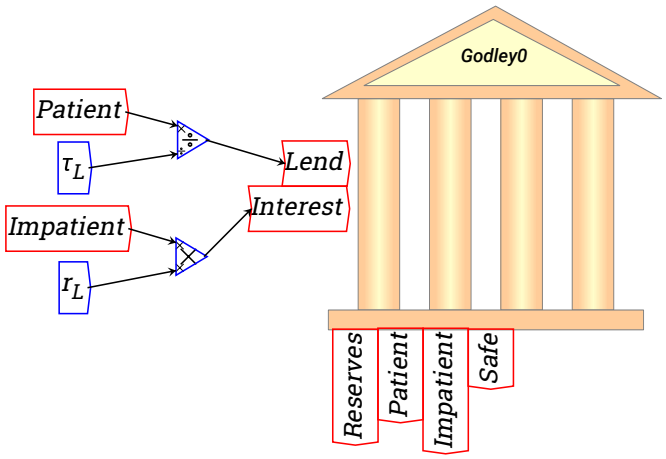
Then label this flow "Impatient pays interest" and make the entry "Interest" into the cell for Patient on that row. Make the matching entry "-Interest" in the cell for Impatient. The flow "Interest" now appears on the input side of the Godley Table on the Canvas:

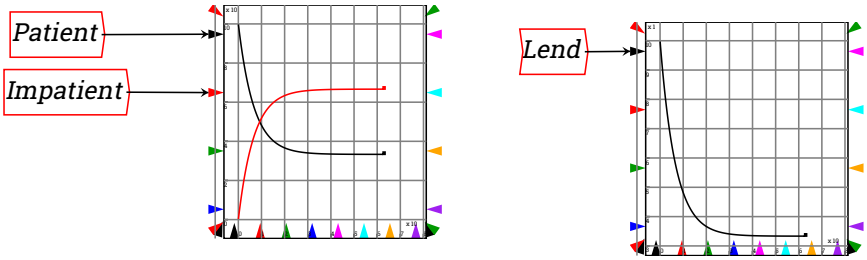
	Asset	Liability		Equity	
Flows ↓ / Stock Vars →	Reserves▼	Patient▼	Impatient▼	Safe ▼	A-L-E
Initial Conditions	120	100	0	20	0
Patient lends to Impatient		-Lend	Lend		0
Impatient pays interest		Interest	-Interest		0

Interest now has to be defined. It will be the amount in Impatient's account (since this began at zero) multiplied by the rate of interest charged by Patient:



With that definition, the dynamics of the model change: rather than the Patient account falling to zero and Impatient rising to 100, the two accounts stabilize once the outflow of new loans by Patient equals the inflow of interest payments by Impatient:





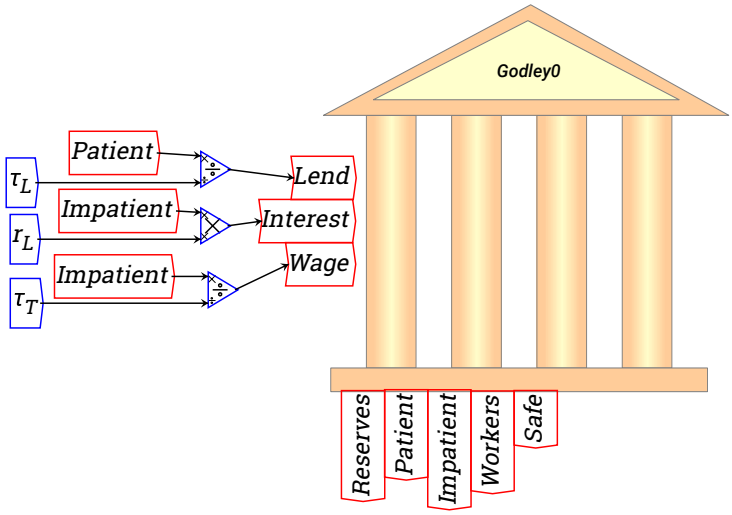
Though it stabilizes, this is is still a very incomplete model: neither Patient nor Impatient are doing anything with the money apart from lending it and paying interest. I am now going to assume that Impatient is borrowing the money in order to hire workers to work at a factory and produce output for sale. So we now need another account called Workers, and a payment from Impatient to Workers called Wage:

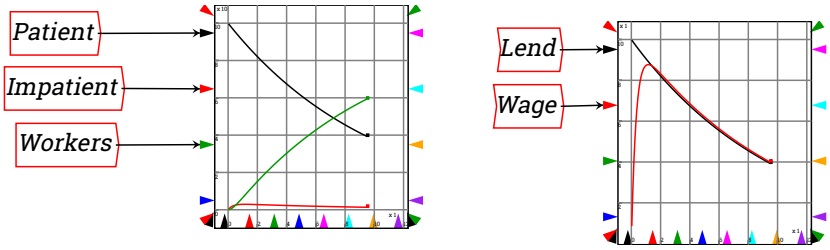
		Asset	Liability			Equity	
	Flows ↓ / Stock Vars →	Reserves▼	Patient▼	Impatient▼	Workers▼	Safe ▼	A-L-E
	Initial Conditions	120	100	0	0	20	0
	Patient lends to Impatient		-Lend	Lend			0
	Impatient pays interest		Interest	-Interest			0
				-Wage	Wage		0

In a more complex model, the Wage bill could be related to the current rate times the number of workers in employment. In this simple model I will regard

the wage as a function of the amount of money in Impatient's account turning over several times a year in the payment of wages. Using a time constant, I will assume that the amount in Impatient's account turns over 3 times a year paying wages, so that the time constant τ_T is 1/3rd of a year:

The dynamics of this incomplete model are very different again: very little money turns up in the Impatient account, and all of the money ends up in the Workers account. However economic activity also ceases as both lending and the flow of wages falls towards zero:





This is because wages are being paid to workers, but they are doing nothing with it. So we need to include consumption by workers–and by Patient as well. Here the reason time constants are useful may be more obvious. The time constant for consumption by Workers is given the very low value of 0.05—or 1/20th of a year—which indicates that if their initial rate of consumption was

maintained without any wage income, they would reduce their bank balances to zero in 1/20th of a year or about 2.5 weeks.

Chapter 4

Reference

4.1 Tabs

Ravel has six Tabs:

- *Wiring*, which houses the Operators and the Design canvas on which data analysis (*Ravel*) and dynamic models (*Minsky*) are constructed;
- *Equations*, which shows the equations created by the analysis or model on the Wiring Tab;
- *Summary*, which shows a hierarchical overview of all the definitions of variables and parameters in the model;
- *Phillips Diagram*, a *Minsky*-specific view of the financial flows in a dynamic economic model;
- *Publication*, which enables selected items from a Wiring to be displayed; and
- +, which adds additional Publication Tabs

4.1.1 Wiring

The Wiring Tab consists of:

- The Operations bar 4.2, which contains all of *Ravel*'s mathematical operators; and
- The design canvas, a free-form space on which variables, parameters and operations are combined to perform analysis of data in *Ravel*, and to simulate dynamic systems in *Minsky*.

4.1.2 Equations

The Equation Tab shows the equations in your data analysis or model. To export equations in a format that can be inserted into LaTeX editors or programs like MathType, choose "Export canvas as" from the File menu, and choose the format LaTeX. For details see **??**.

4.1.3 Summary

The Summary Tab shows the equations in your data analysis or model in a structured tabular layout. For details see 2.3.12.

4.1.4 Phillips Diagram

The Phillips Diagram Tab shows the financial flows in a *Minsky* simulation model in a dynamic flowchart format. This is a *Minsky*-specific feature.

4.1.5 Publication

A Publication Tab enables you to place any item on the Wiring canvas in any location. This is useful for distributing the results of a model without having to show all the workings involved in deriving results.

Any item from the wiring tab can be added to a publication tab, and then moved, resized or rotated independently of the item on the wiring tab. To add an item to a Publication Tab, hover the mouse over the item and choose "Add item to a publication tab" from the context menu. The sub-menu lists available Publication tabs. Click on the one you wish to place the item on, and the item will be placed in the upper-left hand corner of that tab. It can then be moved to anywhere else on that tab.

For items that dynamically update, the publication tab will be updated dynamically when selections on the Wiring Tab are altered.

Ravel components can also be manipulated on the Publication tab–for example, a different store can be selected on the "Store" axis of a Ravel documenting retail sales. If a Ravel has a caliper applied to an axis on the Wiring Tab, to, for example, select a range of dates, that caliper can be moved to a different range of dates on the Publication Tab.

Textual annotation can be added to the publication tab, independently of any annotations on the wiring tab.

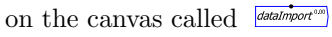
Wires currently cannot be added to the publication tab. However, you can insert text-based arrows (eg →) by typing `#\rightarrow` on the canvas. This item can then be rotated and scaled to emulate the wiring diagram connecting parts of the dashboard together.

4.2 Operations

Except for , which invokes a pop-up sideways menu for defining variables,

constants and parameters, the first 8 icons on the Operations bar—from data importing to the simulation time–are immediately activated by clicking on them. The remaining mathematical and data operations are stored on pop-up sideway menus, which are activated when you click on the menu's icon. For example, clicking on the  symbol brings up the Binary Operators menu:

4.2.1 Import Data

The import data operator  inserts a parameter on the canvas called 

, and invokes the data import form. For more details see CSV import §(4.15).

4.2.2 Ravel

A Ravel is a visual tool for manipulating and analyzing multi-dimensional data. For full details on the concept see Ravel §(4.14).

The Ravel operator  inserts a Ravel onto the Wiring canvas. The first Ravel inserted is in Edit mode, and has sample axis names and a sample data point selected.

Subsequent Ravels are inserted in iconized mode.

To use a Ravel, it must first have a parameter (or variable) attached to it. Normally the parameter will contain data imported from a CSV file—for more details see CSV import §(4.15).

The key feature of a Ravel is its Axes (or "Arrows"). A Ravel has one axis per dimension of the data attached to it. Axes can be:

Rotated The right-pointing axis determines what is shown on the x-axis of a Plot and columns of a Sheet; the down-pointing axis determines what is displayed on the Y-axis of a plot and the rows of a sheet.

Collapsed An axis is collapsed by dragging the arrow head back to the centre of the Ravel. This applies an aggregation method to the axis, which is determined by the setting for "Set next aggregation" on the context menu. The aggregations are Sum, Product, Average, Standard Deviation, Maximum and Minimum. The aggregation type can be changed *post facto* unser the "Axis properties" context menu.

Selected When the selector dot for an Axis is in the centre of the Ravel, the entire Ravel axis is output (though this can be a selection of values on the axis if "Pick axis slices" has been activated). When the selector dot is on a particular value on an axis, only that value is output.

Sorted The "none" sort order of a Ravel is that of the axis as it is attached (usually the order in which it is read from a CSV file). The axis can also be sorted in either a "forward" or "reverse" direction. The sorting comparison depends on the dimension type — numerical in the case of "value", time order in the case of "time" and *lexicographic* ("alphabetical") in the case of "string".

Sorted by value When the output of a Ravel is 1-dimensional, axes can be sorted by the data values along this 1-dimensional axis. The choices are a combination of Forward and Reverse, and Statically and Dynamically. A Static sort applies the order of the data for a given instance—for example, countries could be sorted by the inflation rate for 2022. This sort order is maintained if the selector dot for date is altered. For example, with a static sort, choosing 2000 on the Date axis would show the inflation rate in 2000, with countries sorted according to their inflation rates in 2022. A Dynamic sort applies the sort order for each selection. For example, a reverse dynamic sort of Country by inflation rates would show countries by descending order of their inflation rate in whichever year was chosen on the Date axis.

Picked The context menu option "Pick axis slices" invokes a pop-down menu showing all the entries on that axis—countries for a Country axis, dates for a Time axis, etc. You can select specific values by clicking on them, and select multiple values by control-clicking on them.

Calipered "Toggle axis calipers" brings up a caliper which initially spans the entire data range. The ends of the caliper can be individually adjusted to select a range of the data, while the whole caliper can be moved to select the same number of data points for different segments of the data.

Ravels can be linked with other Ravels that share the same dimension(s). This means that any operation applies to one Ravel is applied to all Ravels in the linked set. For more details see 4.14.6.

4.2.3 Lock

Locks are attached to the output of Ravels. After a Lock is closed (either by double-clicking on the Lock, or choosing Lock from the context menu), alterations can be made to the source Ravel without altering the output of the Lock. This makes it possible to take multiple slices of the data from one Ravel, and to assign those slices to variables so that they can be further analyzed by *Ravel*. For details see 4.14.5

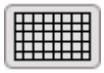
4.2.4 Variable

This operator activates a pop-up menu for defining variables, parameters and constants. For details see 4.3.

4.2.5 Plot

This operator inserts a Plot onto the Wiring canvas. For details see 4.6.

4.2.6 Sheet



This operator inserts a Sheet onto the Wiring canvas. For details see 4.7.

4.2.7 time t

Returns the current value of system time.
The operator can be placed on the canvas in two ways:

- From the time widget on the toolbar  ; or
- By typing the letters "time" on the canvas and then pressing the Enter key.

4.2.8 Binary Operations

add +

Add multiple numbers together.
The input ports allow multiple wires, which are all summed. If an input port is unwired, it is equivalent to setting it to zero.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By pressing the plus key anywhere on the wiring canvas.

subtract −

Subtract two numbers. The input ports allow multiple wires, which are summed prior to the subtraction being carried out. If an input port is unwired, it is equivalent to setting it to zero. Note the small '+' and '−' signs on the input ports indicating which terms are added or subtracted from the result.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or

- By pressing the minus key anywhere on the wiring canvas, *followed by pressing the Enter key, or clicking on OK in the text input window.* The reason for requiring the Enter key to be pressed—rather than immediately placing the minus operator on the keyboard, as with the plus and multiply operators—is that a user may wish to enter a negative number as a constant.

multiply ×

Multiply numbers with each other. The input ports allow multiple wires, which are all multiplied together. If an input port is unwired, it is equivalent to setting it to one.

The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By pressing the multiply (asterisk: *) key anywhere on the wiring canvas.

divide ÷

Divide a number by another. The input ports allow multiple wires, which are multiplied together for each port prior to the division being carried out. If an input port is unwired, it is equivalent to setting it to one. Note the small '×' and '÷' signs indicating which port refers to the numerator and which the denominator.

The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By pressing the divide key (/) anywhere on the wiring canvas.

min

Returns the minimum of x and y. If multiple wires are connected to the ports, the the overall minimum from all inputs is returned.

The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "min" on the canvas and then pressing the Enter key.

max

Returns the maximum of x and y. If multiple wires are connected to the ports, the the overall maximum from all inputs is returned.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "max" on the canvas and then pressing the Enter key.

and $\wedge$

Logical and of x and y, where $x \leq 0.5$ means false, and $x > 0.5$ means true. The output is 1 or 0, depending on the result being true (1) or false (0) respectively.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "and_" (the word and, *followed by an underscore*) on the canvas and then pressing the Enter key. The underscore is needed because "and" is a reserved word in *C++,* the programming language in which *Ravel* is written.

or $\vee$

Logical or of x and y, where $x \leq 0.5$ means false, and $x > 0.5$ means true. The output is 1 or 0, depending on the result being true (1) or false (0) respectively.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "or_" (the word or, *followed by an underscore*) on the canvas and then pressing the Enter key. The underscore is needed because "or" is a reserved word in *C++,* the programming language in which *Ravel* is written.

log

Take the logarithm of the x input port, to base b. The base b needs to be specified — if the natural logarithm is desired ($b = e$), use the natural log §(4.2.10) instead.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or

- By typing the word "log" anywhere on the wiring canvas, and then pressing the Enter key.

When you use the direct typing method to enter the Log operator, the text entry window pops up. This allows you to type a variable/parameter name starting with Log (like, for example, "Logical". If you press Enter (or click on OK) with only the word Log in the window, the Log operator will be placed on the canvas.

pow x^y

Raise one number to the power of another. The ports are labelled x and y, referring the the formula x^y.
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "Pow" anywhere on the wiring canvas and then pressing the Enter Key (or clicking on OK).

Polygamma $\psi^{(n)}(x)$

Returns the polygamma function of the first argument x, with the order n being given by the floor of the second argument.

$$\psi^{(n)}(x) = \frac{d^{n+1}}{dx^{n+1}} \ln \Gamma(x)$$

The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "polygamma" on the canvas and then pressing the Enter key.

lt $<$

Returns 0 or 1, depending on whether $x < y$ is true (1) or false (0).
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "lt" on the canvas and then pressing the Enter key.

le $\leq$

Returns 0 or 1, depending on whether $x \leq y$ is true (1) or false (0).
The operator can be placed on the canvas in two ways:

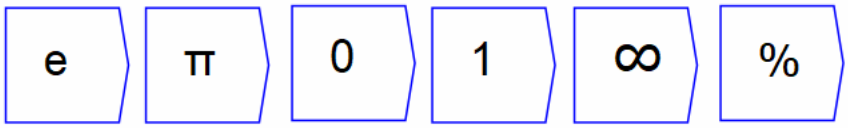
- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "le" on the canvas and then pressing the Enter key.

eq $=$

Returns 0 or 1, depending on whether $x = y$ is true (1) or false (0).
The operator can be placed on the canvas in two ways:

- From the *Binary Operations ("binop")* toolbar; or
- By typing the letters "eq" on the canvas and then pressing the Enter key.

4.2.9 Special constants



Some special constants ($e = 2.72\ldots$, $\pi = 3.14\ldots$, 0, 1, ∞) can be placed on the canvas, via two methods:

- By clicking on the relevant icon on the *Fundamental Constants ("constop")* toolbar; or
- By typing the constant name on the canvas, and pressing Enter (or clicking OK) in the variable definition window. These names are: *Euler* for $e = 2.72\ldots$; *pi* for $\pi = 3.14\ldots$; *inf* for ∞; and the percent symbol for the percent operator (which multiplies the input by 100).

Percent

The percent operator takes one input, and multiplies its elements by 100. It is useful for converting fractions into percentages for display purposes.
The operator can be placed on the canvas in two ways:

- From the *Fundamental Constants ("constop")* toolbar; or
- By pressing the percent key anywhere on the wiring canvas.

4.2.10 Functions/Unary Operators

sqrt $\surd$

This produces the square root of the input value.
For example, connecting the value of 9 with the "sqrt" block will produce the value of 3. As with all Ravel operators, this function accepts multidimensional arguments.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "sqrt" on the canvas and then pressing the Enter key.

exp

Connecting a variable (for example, "time") to this block will produce the exponential function e^x where x is the input variable.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "exp" on the canvas and then pressing the Enter key.

ln

Produces a natural logarithm of the input, to the base of e. This takes the equation $\log_e x$ where x is the input.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "ln" on the canvas and then pressing the Enter key.

sin

Produces a sine function of the input.
For example, connecting a "time" block to this function, and then to a graph, will produce a sine wave.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "sin" on the canvas and then pressing the Enter key.

For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

cos

Produces a cosine function of the input.

The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "cos" on the canvas and then pressing the Enter key.

For example, connecting a "time" block to this function, and then to a graph, will produce a cosine wave. For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

tan

Produces a tangent function of the input.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "tan" on the canvas and then pressing the Enter key.

For example, connecting a "time" block to this function, and then to a graph, will produce a tangent graph. For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

asin

Produces an arc sine function of the input, or the inverse of the sine function.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "asin" on the canvas and then pressing the Enter key.

For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

acos

Produces an arc cosine function of the input, or the inverse of the cosine function.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or

- By typing the letters "acos" on the canvas and then pressing the Enter key.

For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

atan

Produces an arc tangent function of the input, or the inverse of the tangent function.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "atan" on the canvas and then pressing the Enter key.

For further explanation regarding trigonemtric functions, see Wikipedia's page on trigonometric functions.

sinh

hyperbolic sine function $\frac{e^x - e^{-x}}{2}$
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "sinh" on the canvas and then pressing the Enter key.

cosh

hyperbolic cosine function $\frac{e^x + e^{-x}}{2}$
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "cosh" on the canvas and then pressing the Enter key.

tanh

hyperbolic tangent function $\frac{e^x - e^{-x}}{e^x + e^{-x}}$
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "tanh" on the canvas and then pressing the Enter key.

abs $|x|$

absolute value function. Returns the magnitude of any entered variable.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "abs" on the canvas and then pressing the Enter key.

floor $\lfloor x \rfloor$

The greatest integer less than or equal to x.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "floor" on the canvas and then pressing the Enter key.

frac

Fractional part of x, ie $x - \lfloor x \rfloor$.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "frac" on the canvas and then pressing the Enter key.

not $\neg$

The output is 1 or 0, depending on whether $x \leq 0.5$ is true (1) or false (0) respectively.
The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "not_" (the word not, *followed by an underscore*) on the canvas and then pressing the Enter key. The underscore is needed because "not" is a reserved word in *C++,* the programming language in which *Ravel* is written.

Gamma Γ

Returns the Gamma function of its argument:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "gamma" on the canvas and then pressing the Enter key.

Factorial !

Returns the factorial of its argument:

$$\begin{aligned} 0! &= 1 \\ n! &= \prod_{i=1}^{n} i \end{aligned}$$

The operator can be placed on the canvas in two ways:

- From the *Functions ("function")* toolbar; or
- By typing the letters "fact" on the canvas and then pressing the Enter key.

Note:

$$n! = \Gamma(n+1)$$

which is how it is implemented in Minsky.
Minsky provides this function because of its relationship to the derivative of the Gamma function (and factorials).

copy

This just copies its input to its output, which is redundant on wiring diagrams, but is needed for internal purposes.

4.2.11 differentiate d/dt

Symbolically differentiates its input with respect to system time, producing d/dt[input]. For further explanation regarding differentiation, see this wikipedia page.

4.2.12 User defined function

A user defined function is a function defined by an algebraic expression. Support for this feature is courtesy of the wonderful exprtk library developed by Arash Partow.
A user defined function has a *name*, *parameters* and an *expression*. Example expressions are things like `x+y` or `sin(x)`. More details of the sorts of expressions possible can be found in the User Defined Functions §(5) section of the manual.
The parameters are specified as part of the name, so a user defined function adding x and y would be called `useradd(a,y)` and the sin example might be called `mysin(x)`. Functions with up to two arguments can be wired on the canvas. User defined functions can call other user defined functions, so specifiying more than 2 parameters can be a useful thing to do.

4.2.13 Tensor operations

4.2.13

sum $\sum$

Sum along a given axis.
The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "sum" on the canvas and then pressing the Enter key.

The same result can be achieved by collapsing the relevant Ravel axis after choosing Σ from the "Set next aggregation" context menu.

product $\prod$

Multiply along a given axis.
The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar ; or
- By typing the letters "product" on the canvas and then pressing the Enter key.

The same result can be achieved by collapsing the relevant Ravel axis after choosing Π from the "Set next aggregation" context menu.

infimum

Return the least value along a given axis.
The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "infimum" on the canvas and then pressing the Enter key.

supremum

Return the greatest value along a given axis.

The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "supremum" on the canvas and then pressing the Enter key.

any

Return 1 if any value along a given axis is nonzero, otherwise return 0 if all are zero.
The operator can be placed on the canvas in two ways:

- From the Reductions Operations toolbar  ; or
- By typing the letters "any" on the canvas and then pressing the Enter key.

all

Return 1 if all values along a given axis are nonzero, otherwise return 0 if any are zero.
The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "all" on the canvas and then pressing the Enter key.

infindex

Return the index of the least value along a given axis.

The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "infIndex" on the canvas and then pressing the Enter key.

supindex

Return the index of the greatest value along a given axis.

The operator can be placed on the canvas in two ways:

- From the *Reductions Operations ("reduction")* toolbar  ; or
- By typing the letters "supIndex" on the canvas and then pressing the Enter key.

running sum $\sum +$

Computes the running sum of the input tensor along a given axis.
The optional numerical argument (in the "edit" dialog) can be used to specify a window over which the running sum is performed. If negative, the window is over the entire dimension.
The operator can be placed on the canvas in two ways:

- From the *Scan Operations ("scan")* toolbar  ; or
- By typing the letters "runningSum" on the canvas and then pressing the Enter key.

For example, take this rank 2 tensor:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 \\ 8 & 7 & 6 & 5 \end{pmatrix}$$

The running sum of this tensor, along the horizontal dimension, is:

$$\begin{pmatrix} 1 & 3 & 6 & 10 \\ 5 & 9 & 12 & 14 \\ 8 & 15 & 21 & 26 \end{pmatrix}$$

Running sum will normally be applied to a Ravel or a variable defined via a Ravel.

running product $\prod +$

Computes the running product of the input tensor along a given axis.
The optional numerical argument (in the "edit" dialog) can be used to specify a window over which the running sum is performed. If negative, the window is over the entire dimension.
The operator can be placed on the canvas in two ways:

- From the *Scan Operations ("scan")* toolbar ; or
- By typing the letters "runningProduct" on the canvas and then pressing the Enter key.

For example, take this rank 2 tensor:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 \\ 8 & 7 & 6 & 5 \end{pmatrix}$$

The running product of this tensor, along the horizontal dimension, is:

$$\begin{pmatrix} 1 & 2 & 6 & 24 \\ 5 & 20 & 60 & 120 \\ 8 & 56 & 336 & 1680 \end{pmatrix}$$

Running product will normally be applied to a Ravel or a variable defined via a Ravel.

difference Δ^-, Δ^+

Computes the nearest neighbour difference along a given direction.

These operators can be placed on the canvas in two ways:

- From the *Scan Operations ("scan")* toolbar ; or
- By typing the letters "difference" or "differencePlus" on the canvas and then pressing the Enter key.

The optional argument (δ) can be used to specify the number of neighbours to skip in computing the differences. For example if your data is monthly and you want to calculate changes per year, you would set δ to 12. The length of the dimension being differenced is reduced by δ in the result.
It comes in two different forms which differ only in how the resultant x-vector is calculated. $\Delta_i^- = x_i - x_{i-\delta}$, and $\Delta_i^+ = x_{i+\delta} - x_i$, where i refers to the x-vector index.
Where there is more than one dimension to data, the user needs to specify the axis along which the difference operation is calculated.

inner product ·

Calculates the inner product of two tensors.
The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "innerProduct" on the canvas and then pressing the Enter key.

It computes

$$z_{i_1,\ldots,i_{r_x-1},j_1,\ldots,j_{r_y-1}} = \sum_k x_{i_1\ldots,i_{a-1},k,i_{a+1}\ldots,i_{r_x-1}} y_{j_1,\ldots,j_{r_y-1},k},$$

where a is the given axis, and r_x and r_y are the ranks of x and y respectively.

outer product ⊗

Calculates the inner product of two tensors.
The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "outerProduct" on the canvas and then pressing the Enter key.

It computes:

$$z_{i_1,i_2,\ldots,i_{r_x},j_1,\ldots,j_{r_y}} = x_{i_1,,i_2,\ldots,i_{r_x}} y_{j_1,\ldots,j_{r_y}}.$$

where r_x and r_y are the ranks of x and y respectively.

index

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "index" on the canvas and then pressing the Enter key.

Returns the index within the hypecube where the input is true (ie > 0.5). For example, where

$$\begin{aligned} \iota(3,3) &= \begin{pmatrix} 0 & 3 & 6 \\ 1 & 4 & 7 \\ 2 & 5 & 8 \end{pmatrix}, \\ \text{idx}(\iota(3,3) < 5) &= \begin{pmatrix} 0 & 3 & nan \\ 1 & 4 & nan \\ 2 & nan & nan \end{pmatrix}, \end{aligned}$$

Note that the output array has the same shape as the input, with unused values padded with NANs (missing value).
Dimension and argument parameters are unused.

gather

Gather collects the values at index locations of the first argument, indexed by the second argument. If the index argument is a scalar, the output tensor's rank is one less than the first argument's rank, ie $[a_0, \ldots a_{j-1}, a_{j+1}, \ldots a_{ar}]$ where j is the axis along which the gather is performed. If the index argument is a tensor, however, it's shape is ignored, and the output tensor is the same rank as the first argument, and has shape $[i, a_0, \ldots a_{j-1}, a_{j+1}, \ldots a_{ar}]$ where i is the number of elements in the index argument's hypercube, and j is the axis along which the gather is performed.
If the index is not an integer, the gather will linearly interpolate between the values on either side. So, for example, $x[2.5] = 0.5(x[2] + x[3])$. The x-vector elements are also linearly interpolated if the x-vector is a value or time type dimension, or the closest string label if a string type.
If the index value is outside the range of the x-vector §(4.13.1) along the axis being gathered, then NAN is assigned to that tensor element, and the x-vector gets a default assigned element (empty string, NaN or not-a-date).
The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "gather" on the canvas and then pressing the Enter key.

Meld

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "meld" on the canvas and then pressing the Enter key.

The meld operation is used to insert additional values in an existing axis of a Ravel. For example, if you had data from two different time periods and you wished to merge them into one time dimension, you would use Meld to combine them.

Merge

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "merge" on the canvas and then pressing the Enter key.

Merge combines two Ravels sharing n dimensions to produce a new Ravel with $n+1$ dimensions.
For example, the file below creates two 2-dimensional Ravels, one with sales in dollars and the other with sales by each salesperson as a percentage of total sales.
$\text{Sales}_{\text{SalesPerson}^{\$}\text{and SalesPerson}^{\%}}$ share the same dimensions (SalesPerson and Date), and they can be separately displayed as in the next figure.
Using the Merge operator, you can create one new Ravel containing information from both the $ sales and the % of total sales analysis:
The merged Ravel has 3 dimensions, with the extra dimension Analysis now containing the Sales by dollar amount and Sales as a percentage of total sales as entries on the Analysis axis.

Slice

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "slice" on the canvas and then pressing the Enter key.

Slice will cut off a tensor along a given axis by the argument, as configured in the operation edit dialog. For example slice$(\{x_1, x_2, \ldots x_n\}, 3) = \{x_1, x_2, x_3\}$.
If the tensor is rank one (ie a vector), it is not necessary to specify the axis.
If the slice argument is negative, then it refers to the number of elements from the end of that axis. For example slice$(\{x_1, x_2, \ldots x_n\}, -3) = \{x_{n-2}, x_{n-1}, x_n\}$.

Size

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar ⊡ ; or
- By typing the letters "size" on the canvas and then pressing the Enter key.

Size refers to the number of elements along a named dimension given by the operation axisargument—eg a 3×2 rank 2 tensor with named axes "0" and "1", size("1")==2.
If the axis argument is left blank, the size returns the total number of elements present in the tensor. This may be less than the product of axis sizes if the data is sparse—which will normally be the case.

Shape

The operator can be placed on the canvas in two ways:

- From the *Tensor Operations ("tensor")* toolbar  ; or
- By typing the letters "shape" on the canvas and then pressing the Enter key.

Returns a vector of axis sizes. Coupling this operation with a 4.2.13 operation and variable would allow you interactively select axis size.

4.2.14 Statistical Operations

Mean

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar  ; or
- By typing the letters "mean" on the canvas and then pressing the Enter key

Returns the mean (or average) along a named dimension of all elements present. If the dimension is not named, then the mean is over all elements present in the tensor. Note that missing elements are not counted.

Median

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar  ; or
- By typing the letters "median" on the canvas and then pressing the Enter key

Returns the median along a named dimension of all elements present. If the dimension is not named, then the median is over all elements present in the tensor.

Standard Deviation

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar  ; or
- By typing the letters "stdDev" on the canvas and then pressing the Enter key

Returns the standard deviation along a named dimension of all elements present. If the dimension is not named, then the standard deviation is over all elements present in the tensor. Note that missing elements are not counted.

$$\sigma = \frac{1}{N-1}\sum_{i}^{N}(x_i - \langle x \rangle)^2$$

k-th moment

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar  ; or
- By typing the letters "moment" on the canvas and then pressing the Enter key

Returns the k-th moment about the mean along a named dimension of all elements present. k is specified by the numerical argument of the operation, which defaults to 1 (hence the result will be 0 in that case). If the dimension is not named, then the k-th moment is over all elements present in the tensor. Note that missing elements are not counted.

$$\langle \Delta x^k \rangle = \frac{1}{N}\sum_{i}(x_i - \langle x \rangle)^k$$

Also

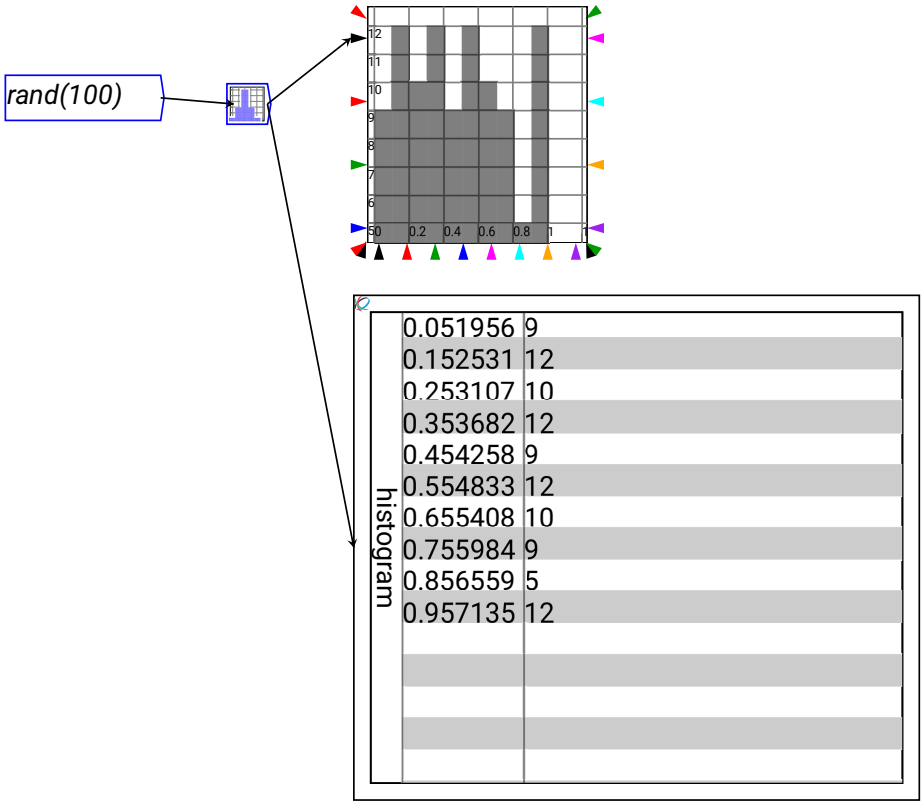
$$\sigma^2 = \frac{N}{N-1}\langle \Delta x^2 \rangle$$

Histogram

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar  ; or
- By typing the letters "histogram" on the canvas and then pressing the Enter key

Computes the histogram along a named dimension of all elements present. If the dimension is not named, then the histogram is over all elements present in the tensor. The number of bins is specified by the numeric argument to the operation.



An example usage of the histogram operation

Covariance

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar ; or
- By typing the letters "covariance" on the canvas and then pressing the Enter key

Computes the covariance of two tensors along named dimension. If the inputs are of rank N and M respectively, the output will be a $(N-1) \times (M-1)$ rank tensor, where the (i, j) element is the covariance of the i-th slice of the first argument along the named dimension, and the j-th slice along the named dimension. As such, it is conformant with the definition of `cov` function in Octave, but not with the equivalently named function in Matlab:

> Compatibility Note:: Octave always treats rows of X and Y as multivariate random variables. For two inputs, however, MATLAB treats X and Y as two univariate distributions regardless of their shapes,

and will calculate covariance whenever the number of elements in X and Y are equal. This will result in a 2x2 matrix. Code relying on MATLAB's definition will need to be changed when running in Octave.

If only a single argument x is supplied to the covariance, then the result is equivalent to $\mathrm{cov}(x, x)$, ie each slice is covaried with each other slice.

The formula for covariance between stochastic variables x and y is

$$\mathrm{cov}(x, y) = \frac{1}{N-1} \sum_i (x_i - \langle x \rangle)(y_i - \langle y \rangle)$$

Correlation coefficient ρ

The operator can be placed on the canvas in two ways:

- From the *Statistics ("statistics")* toolbar ; or
- By typing the letters "correlation" on the canvas and then pressing the Enter key

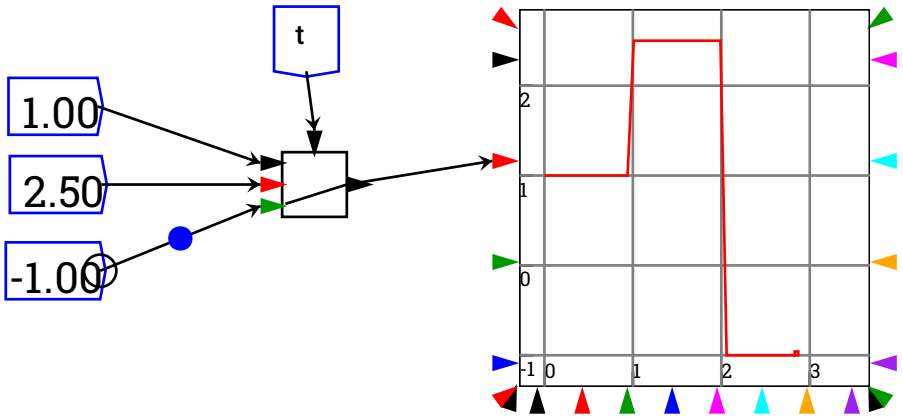
See 4.2.14 for the interpretation of tensor valued arguments. The correlation coefficient is defined as

$$\rho(x, y) = \frac{\mathrm{cov(x, y)}}{\sigma(x)\sigma(y)}$$

4.2.15 Switch

The operator can be placed on the canvas by clicking on its icon on the widget bar.

A switch block (also known as a case block, or select in the Fortran world) is a way of selecting from a range of alternatives according to the value of the input, effectively defining a piecewise function.



An example switch block with 3 cases

The default switch has two cases, and can be used to implement an if/then/else construct. However, because the two cases are 0 and 1, or false and true, a two case switch statement will naturally appear "upside down" to how you might think of an if statement. In other words, it looks like:

`if not` *condition* `then`
`...else`
`...`

You can add or remove cases through the context menu.

4.2.16 User defined function

A user defined function is a functioned defined by an algebraic expression. Support for this feature is courtesy of the wonderful exprtk library developed by Arash Partow.

A user defined function has a *name*, *parameters* and an *expression*. Example expressions are things like `x+y` or `sin(x)`. More details of the sorts of expressions possible can be found in the User Defined Functions §(5) section of the manual.

The parameters are specified as part of the name, so a user defined function adding x and y would be called `useradd(a,y)` and the sin example might be called `mysin(x)`. Functions with up to two arguments can be wired on the canvas. User defined functions can call other user defined functions, so specifiying more than 2 parameters can be a useful thing to do.

4.2.17 Godley Tables

A Godley Table employs double-entry bookkeeping to create dynamic models of monetary transactions. It can also be used to build dynamic models of systems based on transitions between exclusive categories—such as epidemiological systems like SEIRD models of disease transmission.

A Godley Table uses the rules of accounting to create consistent equations to describe financial flows. The essential rules are that:

- Each flow must be entered twice on its row; and
- The sum of $Assets - Liabilities - Equity$ must equal zero on each row.

Using these rules, *Minsky* develops stock-flow consistent models of financial transactions. The row entries are flows in differential equations for the stocks. For example, the Godley Table shown in the next figure:

generates the following set of differential equations:

$$\frac{d}{dt}Loans = Lending - Repay$$
$$\frac{d}{dt}Deposits = Lending - Interest - Repay + Spend_{Banks}$$
$$\frac{d}{dt}Banks_{Equity} = Interest - Spend_{Banks}$$
$$\frac{d}{dt}Reserves = 0$$

Stocks are thus completely defined by the Godley Table itself as equations in a set of coupled differential equations. For this reason, when placed on the canvas, each stock has an output but no input. Flows, on the other hand, have both inputs and outputs, and have to be fully defined on the canvas itself.

For complete details see 1.2.1.

Creating a Godley Table

To insert a Godley Table into a model, click on the Godley icon on the Operations bar. This attaches a Godley Table to your cursor. Click where you wish to place it on the canvas and this object will be inserted:

You can edit this object by either double-clicking on the icon, or by choosing "Edit Godley Table" from its context menu.

Godley Table Context Menu

The Godley-Table-specific commands in the context menu are:

Command	Effect
Open Godley Table	Opens the form for editing Godley Tables
Title	Give the Table a name which is displayed on the icon and edit form
Set currency	If multiple currencies are used in a model, specify the currency for the current table
Editor Mode	Convert the canvas view of the table from an icon to the table itself, which can be edited on the canvas
Row/Col buttons	When Editor Mode is active, show row and column buttons used to add and delete rows and columns
Display Variables	Show the stocks and flows defined in the table as variables attached to the table in icon view
Copy flow variables	Copy all the flow variables in the table to the canvas
Copy stock variables	Copy all the stock variables in the table to the canvas
Export as	Export the table in either CSV or LaTeX format
Rotate GodleyIcon	This command has been deprecated and will be deleted in a future release

Open Godley Table

This commands bring up a dedicated window for editing a Godley Table:

Defining Stocks and Flows A Stock is defined by entering its name in the row on which the label *Flows↓/Stock Vars→* is shown. Normally these stocks will be bank accounts, though Godley Tables can be used to model any dynamic system in which the stocks define exclusive categories—such as a epidemiological model of disease transmission, in which the population is divided into mutually exclusive categories such as Infected or Uninfected, in Hospital or Not in Hospital, etc.

Stocks are classified as either *Assets*, *Liabilities* or *Equity*, and the default form enables the entry of only one Asset, Liability and Equity (similarly, there is only one row for recording flows on the default form). To add more columns for accounts, click on the + key below the relevant class (similarly, additional rows for financial transactions are added by clicking on the + key next to a row). This creates an additional column for entering stock names. The - key deletes a stock, and the arrow keys move stocks right and left as desired. If you press the leftarrow key on the first entry in Liabilities, *Minsky* will warn you that this will change the classification of the stock from a Liability to an Asset.

The final column in the Table, labelled $A - L - E$, checks whether the row sums to zero—which it must do to obey the rules of accounting. The entries in the columns, and the sum itself, are *symbolic*: words are entered rather than numbers. The sum of that row must be zero according to this formula. Normally this will involve two entries of the same word—for example,

Lending—though fractional entries are possible:
$Lending - 0.7Lending - 0.3Lending$ still sums to zero.
Three stocks and flows have been defined in the following figure.

Initial Conditions These are the values which the stocks take at the beginning of a simulation. This row must also sum to zero, to ensure stock-flow consistency.

Editor Mode

This toggle command converts the canvas display of the Godley table from the icon to a double-entry bookkeeping view:

Row/Col buttons

This toggle command adds row and column buttons to a double-entry bookkeeping view of the table on the canvas, so that it can be edited on the canvas as an alternative to editing in the dedicated window.

Display Variables

This toggle command places the stocks (bank accounts) and flows (financial transactions) entered in a Godley Table as variables on the Design canvas, attached to the Godley Table icon itself.

Copy Stock/Flow Variables

These commands copy all Stocks or all Flows from a Godley Table and attach them to the mouse cursor, for placement on the canvas. The flows can then be defined using *Minsky*'s mathematical operators (Stocks are already completely defined by the Godley Table itself). The next figure shows the result of using both these commands to place all stocks and flows on the canvas.

Create multiple related Godley Tables

Minsky uses the fact that one entity's financial asset is another's financial liability to rapidly build interlocked Godley Tables, which describe the financial system from the perspective of every entity or sector modelled in it. The inverted wedge [ifgeo]99 next to the name of every column is used to check for Assets that have not yet been recorded as a Liability, and vice versa. The next figure shows an expanded Godley Table for the Banking Sector, with additional Tables added for the Non-bank private sector, the Central Bank, and the Treasury.
As well as auto-completing much of the additional Godley Tables, the system indicates the need for additional accounts that are not part of the initial Banks Godley Table. Central Bank purchases of bonds from private banks indicate the need for the $Asset\ Bonds_{CB}$, while the deficit indicates the need

to add the Treasury's account at the Central Bank, the "Consolidated Revenue Fund", as an additional liability of the Central Bank.

The inverted wedge next to the Equity column supports *nonfinancial* assets—things like houses, and shares valued in excess of their limited liability. If these are included amongst the assets of an entity, then they can also be recorded as an Equity of that entity (this feature is still under development).

Godley Table Context Menu

The Godley Table Form has a context menu which assists in the rapid completion of a multi-Godley-Table model. Entries can be copied from one cell and pasted into another, and Flows in the model can be inserted into a Godley Table cell as a positive or a negative entry:

It is also possible to record all as-yet-unclassified flows as additions to or substractions from the Equity of the relevant sector—which is often (but not always) what is needed to complete a set of interlocking Godley Tables. In the figure above, $-Interest$ and $+Spend_{Banks}$ are shown in the $A - L - E$ checksum column. Choosing "Balance equity" from the context menu transfers both these entries to the Equity column.

Export Godley Table

This command exports the Godley Table either as a CSV file, or a LaTeX file, which can be imported into other programs for documentation purposes.

4.2.18 integrate $\int dt$

Creates an integration (or stock) variable.

The operator can be placed on the canvas in two ways:

- From the integrate widget on the toolbar; or
- By typing the letters "integrate" on the canvas and then pressing the Enter key.

Integral blocks require two inputs: the flow being integrated, which is attached to the top input on the block, labelled f; and the initial conditions for the integral, which is attached to the bottom input, labelled 0.

Integral Block context menu

The integral-block-specific commands on the context menu are:

Command	Effect
Edit	Bring up the Edit form for this integrals
Copy item	Copy the variable name, rather than the combined integral block plus name
Toggle var binding	Enable the integral block to be separated from the variable name
Rotate IntOp	Rotate the combined integral block plus name

Editable attributes include the variable's name and its initial value at $t = 0$.

The main role of the Edit form is to provide the name for the integrated variable—other details included on the form can be controlled from elsewhere in the program. Type the name you want to give the variable into the Name field and press Enter or click on OK. You can also name the variable via the context menu item "Rename all instances".

The flows to be integrated are connected to the top port of the integral block, labelled 'f'. The bottom port, labelled '0', can optionally be connected to a constant, parameter or variable, which is used to specify the initial value of the integral, following the same rules as the variable initial value field §(4.3.2).

4.2.19 differentiate d/dt

Symbolically differentiates its input with respect to system time, producing d/dt[input]. For further explanation regarding differentiation, see this wikipedia page.

4.3 Variables and Parameters

Variables represent values in a calculation, and come in a number of varieties:

Constants represent an explicit numerical value, and do not have a name. Their graphical representation shows the actual value of the constant.

Parameters are named constants. All instances of a given name represent the same value, as with all other named variables, so changing the value of one parameter, either through its edit menu, or through a slider, will affect all the others of that name.

Parameters are used to import data from a CSV file4.15, which is how external data is imported into Ravel for analysis.

Parameters can also be used to generate arrays of numbers for use in a *Ravel* file using built-in formulas which are placed in the "Initial value" field of the parameter definition form. See 4.3.3.

Flow variables have an input port that defines how the value is to be calculated. Only one flow variable of a given name can have its input port

connected, as they all refer to the same quantity. If no input ports are connected, then flow variables act just like parameters.

Integral variables represent the result of integrating its input over time by means of the differential equation solver. The integrand is represented by the input to an integral operator that is attached to the integral variable.

Stock variables are the columns of Godley tables, and represent the integral over time of the sum of the flow variables making up the column.

Variables may be converted between types in the variable edit menu, available from the context menu, subject to certain rules. For example, a variable whose input is wired anywhere on the canvas cannot be changed from "flow". Stock variables need to be defined in a Godley table, and so on.

4.3.1 Variable names

Variable names uniquely identify variables. Multiple icons on the canvas with the same name at the same level all refer to the same variable. Variable names can have local scope, in which case it is specific to the group §(4.5) in which it is contained, othewise its scope is determined by the innermost containing group that has a local variable of that name. If no such variable exists, its scope is global.

A variable which is local to a group can be used as a formula in several locations in a file. For example, if you use a, b anc c as constants in a polynomial $a + bx + cx^2$ and make them local to a group, then the group can be used in several parts of a document, with the argument x and the values of a,b and c being different in every instance.

4.3.2 Initial conditions

Variable initial conditions can be defined through the "init value" field of the variable edit menu, or in the case of Godley table stock variables, through the initial condition row of the Godley table. An initial value can be a simple number, or it can be a multiple of another named parameter. In case of symbolic definitions, it would be possible to set up a circular reference where the initial value of variable A is defined in terms of the initial value of variable B, which in turn depends on the intial value of A. Such a pathological situation is detected when the system is reset.

4.3.3 Tensor valued initial conditions

Ravel provides a simple functional language which allows for the generation of tensor-valued operations. This enables *Ravel* to work with hypothetical as well as actual data—for example, hypothetical sales volumes for a new product from zero to 500,000 sales per year can be generated using the iota formula by putting the expression "iota(500001)" in the initial value field for the variable

Sales. This generates an axis/dimension named 0 with values from 0 to 500,000.

These functions take the form $func(n_1, n_2, \ldots, n_r)$ where r is the desired rank, and n_1, n_2, etc are the dimensions of the tensor. Available functions include:

name	description
`one`	the tensor is filled with '1'
`zero`	the tensor is filled with '0'
`iota`	the arithmetic sequence $(0, 1, ... \prod_i n_i)$
`eye`	diagonal elements filled with '1', offdiagonal '0'
`rand`	tensor filled with random numbers in the range $[0, 1)$

- `eye` is equivalent to `one` for vectors.

- `rand` generates different random numbers each time the simulation is reset, and uses the clib `rand()` function.

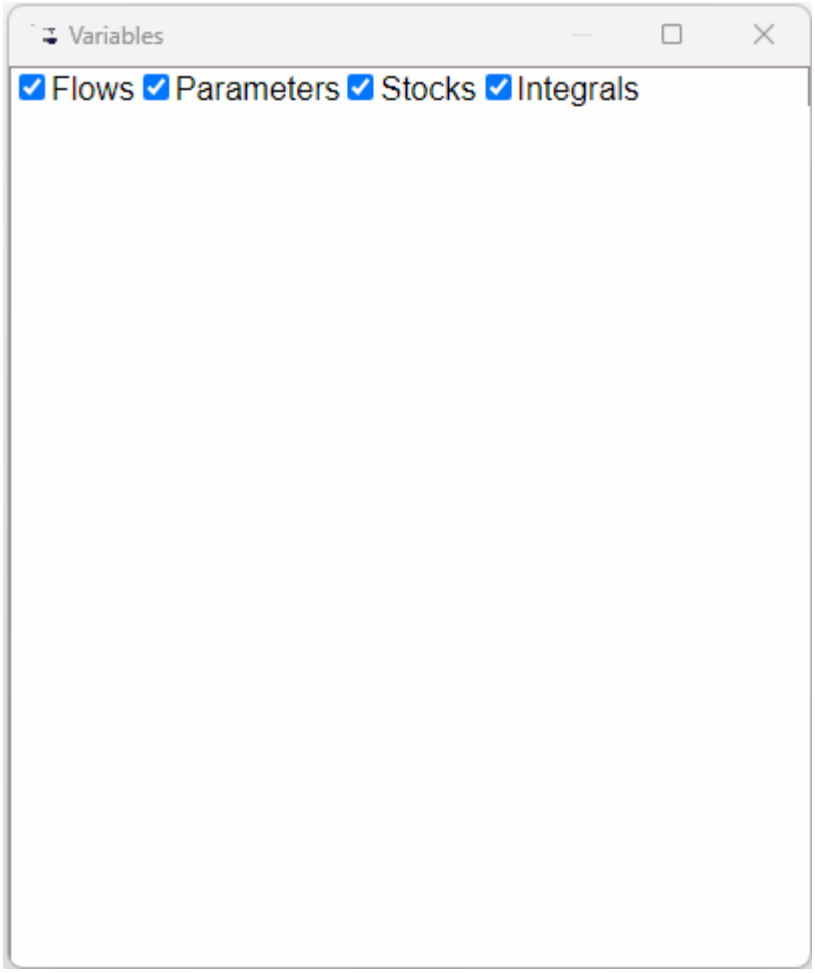
4.3.4 Sliders

Variables have a "slider" feature, which allows the value of the variable (or parameter or constant) to be altered by the user via the mouse or by arrowkeys.

The options Max, Min and Step Size in the Edit form let you control the maximum and minimum values for the variable/parameter, and how much the value changes for each press of an arrowkey. A relative slider means that the step size is expressed as a fraction of max-min.

Adjusting the slider of an integral (or stock) variable while the system is running actually adjusts the present value of the variable. The sliders can also be adjusted using the keyboard arrow keys: uparrow ↑ or rightarrow → increases the value of the parameter by the step size; downarrow ↓or leftarrow ← reduces it.

4.3.5 Variable Browser



The browser can be accessed in two ways:

- From the Insert Menu; or
- From the widget, which brings up a horizontal menu for defining, constants, parameters and variables. Click on the final option on this menu, "Browser", to bring up the window.

The *variable browser* is a popup window that shows all currently defined variables in the system. It is an "always on top" window, which will be visible over any other window including those for *Ravel/Minsky* (and those for other programs which you have open at the time). This facilitates its use as a means to insert multiple variables into a model.
This is a convenience toolbar that allows one to select a variable for insertion into the design canvas, instead of having to type the new variable's name from

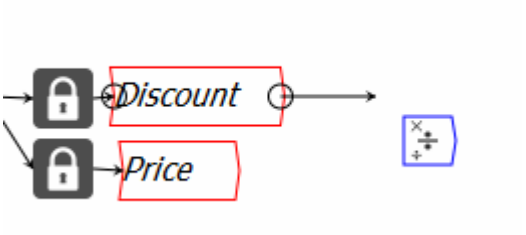
scratch.

At the top of the variable browser are some filter checkboxes, that allow you filter the variables shown by variable type.

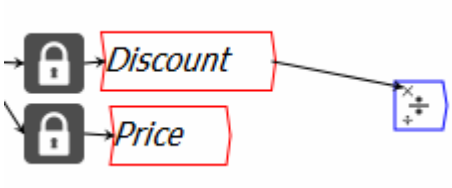
4.4 Wires

4.4.1 Connecting operations with wires

A wire represents the flow of values from one operation to the next. To add a wire to the canvas, click on the output port of an operator or variable (on the right hand side of an icon in its initial unrotated orientation), and then drag it towards an input port (on the left hand side of an unrotated icon); an arrow will be drawn out of the port. The next figure shows an arrow being drawn out of an output port and dragged towards an input port on a divide block.



When the mouse is released, Ravel searches for a nearby input port and attaches the wire there.



When the other variable is wired to the divide operator, the output can be further processed and allocated to another variable for further analysis or display.

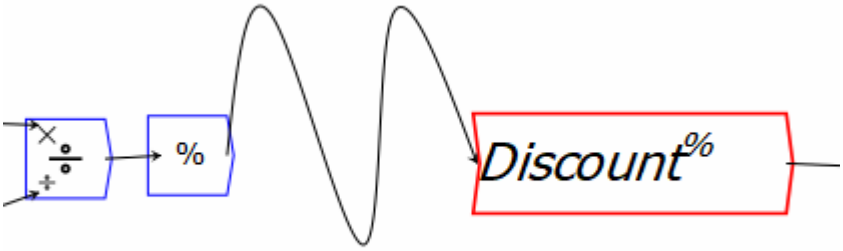
The Equation Tab and LaTeX export shows the equation generated by this wiring: $\text{Discount}^{\%} = \left(\frac{\text{Discount}}{\text{Price}}\right)$.

You can't connect an operator to itself (that would be a loop, which is not allowed, unless passing through an integral), nor in general can an input port have more than one wire attached—the exceptions are mathematical operators such as $+/-$ and $\times/\div$ (and similarly max/min), where the multiple wires are summed or multiplied, respectively.

4.4.2 Curving Wires

A wire can be bent by hovering over it, at which point a blue dots ("handle") will appear. Click on the blue dot—or anywhere along the line—and the line will turn into a curve, with 3 "bezier" points showing the degree of curvature.

Additional curves can be added to a line, which can allow you to connect variables without drawing a line across some other entity.



However, it can be preferable to copy a variable and paste into a document multiple times, rather than having curved wires cluttering up a diagram. Wires can also become excessively kinked by curving them, and there is a context-menu command to straighten a curved line.

4.5 Groups

Grouping gives the capability to create reusable modules, or subroutines that can dramatically simplify more complicated systems. Groups may be created in the following ways:

- by lassoing a number of items to select them, then selecting "group" from the canvas context menu, or the edit menu.
- by pasting the selection. You may "ungroup" the group from the context menu if you don't desire the result of the paste to be a group.
- by copying another group
- by inserting a Minsky file as a group

Groups are "transparent": at a low level of magnification, the group icon is shown, but as you zoom in on a group, it will become transparent, which allows you see and edit its contents.

Groups may be nested heirarchically, which gives an excellent way of zooming in to see the detail of a model, or zooming out to get an overview of it. The group context menu item "Zoom to display" zooms the canvas in just enough for the group's contents to be visible.

You may also select "Open in canvas" from the context menu. This replaces the current canvas contents with the contents of the group, allowing you to edit the contents of the group directly without the distractions of the rest of the model. Select "Open master group" to return to the top level group occupying the canvas.

Around the edges of a group are input or output variables, which allow one to parameterise the group. One can drag a variable and dock it in the I/O area to create a new input or output for the group.

When creating a group, or dragging a variable or operation into or out of a group, if a wire ends up crossing the group boundary, a new temporary variable is added as an I/O variable. You may then edit the I/O variable name to be something more meaningful to your model.

Variable names within groups can be locally scoped to that group. That means that a variable of the same name outside the group refers to a different entity completely. By default, grouped variables refer to entities outside the group scope, but may be marked local by means of context menu option. One can also convert all variables in a group to be local by means of the "Make subroutine" context menu entry.

Nonlocal variables refers to a local variable within an outer scope, going all the way to global scope if no such variable exists. In this way, two groups can share a variable reference to a variable, and you can limit the scope of the shared variable by placing a local variable of the same name in an outer group that both groups are contain within.

A group can also be exported to a file from the context menu. This allows you to build up a library of building blocks. There is a github project "minsky-models" allowing people to publish their building blocks and models for others to use. In the future, we hope to integrate *Minsky* with this github repository, allowing even more seamless sharing of models.

4.6 Plot widget

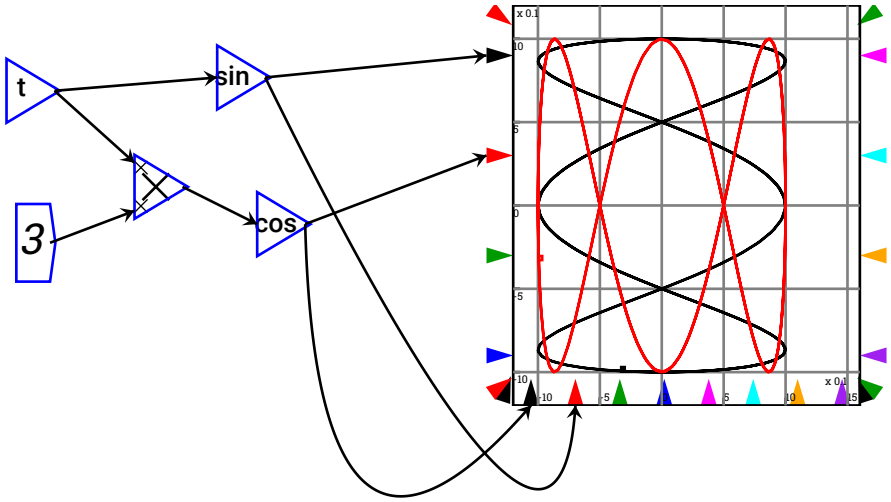
The plot widget embeds a plot into the canvas.

Around the outside of the plot are a number of input ports that can be wired:

- left hand edge Multiple series can be plotted by attaching them as inputs to the black input triangle on the middle of the Y1 axis.

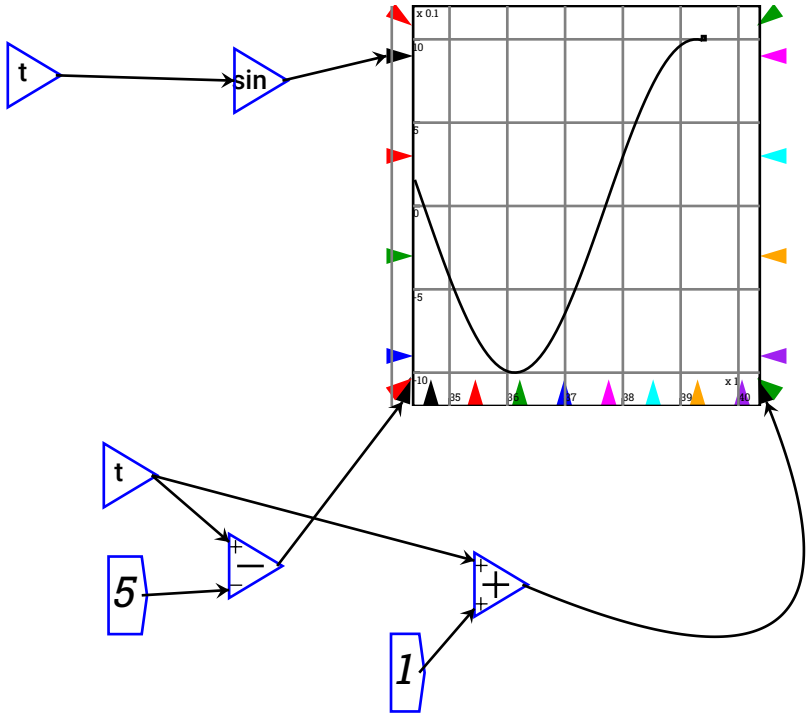
- right hand edge Multiple series can be plotted by attaching them as inputs to the red input triangle on the middle of the Y2 axis. These are shown on a different scale to the left hand Y1 inputs, which allows very different magnitudes to be compared on the one plot.

- bottom edge The default for the X-axis is either time—for a *Minsky* simulation model—or the right-pointing axis of a Ravel. XY plots can be created by attaching an input to the matching input on the X axis: the input to the black triangle will be plotted against the variable(s) inputted on the black Y1 input port; and the input to the red triangle will be plotted against the variable(s) inputted on the red Y2 input port.

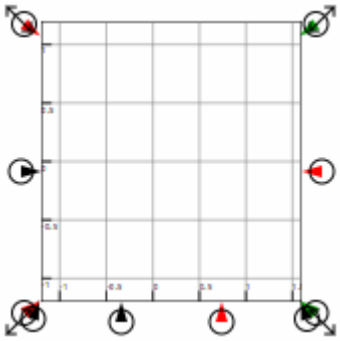


If only one bottom port is connected, then that controls all pens simultaneously, and if no ports are connected, then the simulation time is used to provide the x coordinates

- corners Corner ports control the scale. You can wire up variables controlling minimum and maximum of the x, y and right hand y axes. If left unwired, the scales are determined automatically from the data—or set by fields in the Options form. Corner ports can also be used to, for example, implement a sliding window graph—so that the scale of the graph adapts to the simulation in a *Minsky* model.

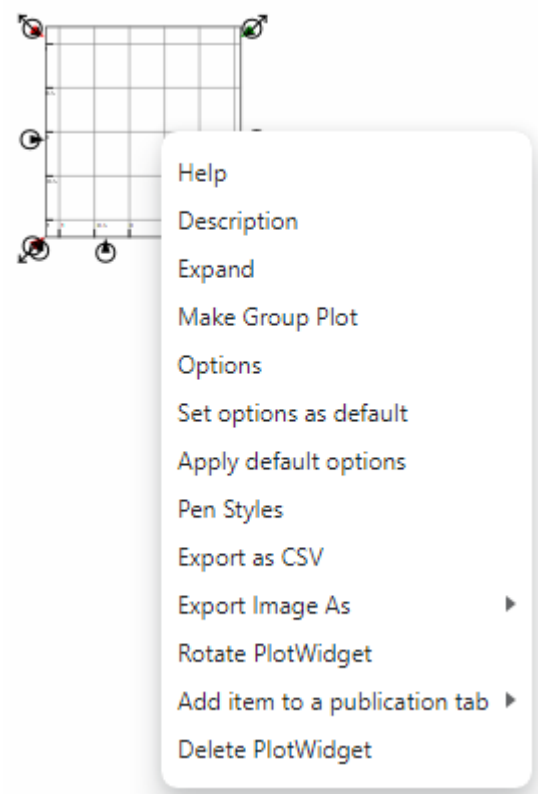


Plots can be resized using the resize arrows that appear when the mouse hovers over a plot. Click on one of these arrows and drag the mouse and the plot will be resized.



4.6.1 Plot context menu

The plot context menu is:



Description

Description bring up a text window §(4.8) into which you can enter text to describe a plot. The Short Description field becomes a "Hint" that is displayed when the mouse hovers over a plot. If you check the Bookmark field, the Short Description becomes a bookmark which can be used to navigate the document.

Expand

This generates a large display window for the plot alone, independent of the *Ravel/Minsky* canvas itself.

Make Group Plot

If this option is checked on a plot within a Group, this plot replaces the default image for a Group. It is a dynamic rendition, so in a *Minsky* model, this can show the plotted variables changing over time.

Options

The format of a plot is controlled by the Plot Window Options form. Titles, axis labels, plot type and other features of a plot are controlled through this form.

Set options as default

This command reproduces the settings of the selected plot in any subsequent plot. This can expedite plot labelling, as well as setting a default appearance for the style of graph applied to the data in each plot.

Apply default options

This applies the default plot options to any selected plot.

Pen Styles

This brings up the pen styles control form. When variables are attached to the input ports, their names will appear above each formatting bar. This menu can also be accessed from the Pen Styles button on the Options form.

Export as CSV

This command exports the data from a Plot as a csv file.

4.7 Sheet Widget

The Sheet widget displays data as in a typical spreadsheet program. The default sheet is a blank square box, as shown above. Once data is attached to the sheet, formatted and hierarchical data will fill the sheet. The example shown below has 4 dimensions—the data is Price, while the dimensions are Date, Salesperson, Suburb and Source. Data for the first two dimensions—the downward and right-pointing Ravel axes—is shown, while the first entry in the third and higher order dimensions are shown at the head of the sheet. You can alter the displayed data by manipulating the Ravel attached to the sheet.

To use the Sheet widget, simply attach a variable or Ravel to it via a wire. The input is on the left-hand side of the sheet widget box. This sheet displays the input data as a number (in the case of a variable being simulated in a *Minsky* model) or an array of data (in the case of a Ravel). Note that only one wire can be connected to a sheet, as the sheet can only display a single input value–however, the *Ravel* operators *meld* §(4.2.13) and *merge* §(4.2.13) enable you to combine data from several Ravels to allow them to be displayed in a single sheet.

By default, a Sheet outputs the beginning of the data series. The context menu allows you to specify that the data displayed comes from the Head of the data (the default), its Tail (the final entries in the data series), or the Head and Tail. The next figure shows the Head and Tail displayed for the rows of this sheet.

4.8 Note Widget

The note widget places the text line "Enter your note here" on the canvas.

Enter your note here

Double-clicking on the text, or choosing Description from the context menu, brings up the note form:
The Short Description field becomes a tool-tip for this note, and also doubles as a Bookmark §(**??**) should you check the Bookmark? box.
Notes allow arbitrary text to be placed on the canvas for explanatory purposes. Anything that can be entered on the keyboard can be placed here, including unicode characters, and LaTeX formatting is supported.

Text can be
multi-line
and supports ^{Superscripts} $_{Subscripts}$
and Greek characters like α, β

4.9 Context Menu

All canvas items have a context menu, accessed by clicking the right-mouse button while hovering over an item, which allow a variety of operations to be applied to the canvas item. Common context menu items are explained here:

Help bring up context specific help for the item

Description Attach an annotation to the item. This is only visible by selecting the description item from the context menu, although whatever is set as the "Short Description" will also appear as a tooltip whenever the mouse hovers over the item.

Port values When running a simulation, you can drill down into the actual values at the input and output ports of the variable or operation, which is a useful aid for debugging models.

Edit set or query various attributes of an item. This function can also be accessed by double clicking on the item. (Plot widgets behave slightly differently: double-clicking on a plot brings up an independent window in which the plot is displayed.).

Copy Creates a copy of an item, retaining the same attributes of the original. This is very useful for creating copies of the same variable to reduce the amount of overlapping wiring in a model. This can substantially improve the readability of a *Ravel/Minsky* model when compared to standard system-dynamics programs.

Flip actually rotates an object through 180°. You can specify aribtrary rotations of objects through the edit menu.

Delete delete the object.

Item specific context menu items:

variables, parameters and constants

Local Make the variable's scope local to its group §(4.5)

Find definition Place a red circle on the variable that defines its value.

Select all instances Select all instances of this variable

Rename all instance Do a global search and replace of this variable name with a new name.

Export as CSV Export the current variable's value as a CSV file.

Add integral attach an integration operation, and convert the variable into an integral type.

integrals

Copy Var copy just the integrated variable, but not the integration operation itself.

Toggle Var Binding Normally, integrals are tightly bound to their variables. By toggling the binding, the integral icon can then be moved independently of the variable it is bound to.

Godley tables

Open Godley Table opens a spreadsheet-like window to allow financial flows to be defined in a Godley Table (these can also be defined directly on the canvas when a Godley Table is displayed in Edit Mode).

Resize Godley Table allows the icon to be resized.

Edit/Copy var allows individual stock and flow variables to be copied or edited.

Export to file export table contents as either CSV data, or as a LaTeX table, for importing into other software, such as a spreadsheet or word processor.

Groups

Zoom to Display Zoom the canvas sufficiently to see the contents of the group.

Resize Resize the group icon on the canvas.

Save group as Save the group in it's own Minsky file.

Flip contents Rotate each item within the group by 180°

Ungroup Ungroup the group, leaving its contents as icons on the canvas.

contentBounds Draws a box on the canvas indicating the smallest bounding box containing the group items.

Plot Widgets

Expand By double-clicking, or selecting "Expand" from the context menu, a popup window is created of the plot, which can be used examine the plotting in more detail.

Resize Allows you to resize the plot icon on the canvas

Options Customize the plot by adding a title, axes labels and control the number of axis ticks and grid lines on the detailed plot. You can also add a legend, which is populated from the names of variables attached to the plot.

4.10 Canvas background and keyboard shortcuts

The canvas is not simply an inert place for the canvas items to exist. There is also a background context menu, giving access to the edit menu functionality such as cut/copy/paste, and also keyboard entry.

	F1	context sensitive help
Special keys:	Shift	enter panning mode
	$\leftarrow, \rightarrow, \uparrow, \downarrow$	adjust sliders, adjust Ravel slicers

The following keystrokes insert an operation

`+`	add
`-`	subtract
`*`	multiply
`/`	divide
`^`	pow
`%`	percent operator
`&`	integral
`=`	Godley table
`@`	plot
`#`	start a text comment, finish with return

Typing any other character, followed by the Return key, will insert an operation (if the name matches), or otherwise a variable with that name.

4.11 Bookmarks

Bookmarks are a useful feature for saving the current position and zoom of the canvas, to be able to come back to that part of the canvas later. This helps to manage more complicated models. To create a new bookmark, click on the "Bookmark" tab in the top left-hand corner (in-between "Edit" and "Insert") and select "Bookmark this position". The program will provide a dialogue box to enter in a name for the new bookmark. After creating the bookmark, all user-created bookmarks can be seen in the Bookmarks menu. To delete a bookmark, simply select "Delete" from the Bookmarks menu and select the desired bookmark. To open an existing bookmark, select one from the menu. You may also create a bookmark from any item on the canvas by selecting the bookmark checkbox in the "description" dialog. This is especially useful with text boxes, since they provide a visual indicator for what is bookmarked at a given location—Parameter definitions, Plots, etc.

4.12 Dimensional Analysis

Dimensional analysis is the idea of attaching units of measurement (eg metre or second) to the quantities being computed. It provides an additional constraint that the system must satisfy, reducing the chance of wiring errors. Two different units being added together will throw up an error — you cannot add 2 metres to 3 kilograms. But it should be possible add 2 metres to 3 feet, and get the correct answer. You may need to explicitly add a multiply operation to convert from one unit to another, for example, dividing the 3 feet by 3.281 before adding it to the 2 metres, providing a total of 2.914 meters.

Using Dimensional Analysis in *Minsky*

To attach units to quantities in *Minsky*, you use the units field of the variable/parameters/constants edit dialog box. Each word typed in this box describes a separate unit. `^` followed by an integer is used to represent a power. Finally, a single "/" indicates that the following units are on the denominator, dividing the first set of units by the second. So to represent the unit of acceleration, you can equivalently type all of the following:

- `m/s^2`
- `m/s s`
- `m/s^-2`

Or spelling it out in full:

- `metre/second^2`

- `metre second^-2`
- `metre / second second`

Note that metre and m are distinctly different units in *Minsky*.
Note – setting the time dimension is done in the simulation menu 4.5.
All function objects require dimensionless inputs. You can use dimensional analysis to avoid, for example, incorrectly feeding a degree measurement into a sine function, by requiring them to be multiplied by a radiansPerDegree parameter.

4.13 Tensor values

Tensors are arrays of data, where the rank of the tensor tells how many dimensions the data has. A single number (a scalar) has a tensor of rank 0. A tensor of rank 1 is a column or row of numbers—a vector. A rank 2 tensor appears is a 2D sequence of numbers (a matrix), for example:

$$\begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

A tensor of rank 3 will appear as a three-dimensional cube, rank 4 as a four-dimensional hypercube, and so on. Tensor variables can be created and used in *Minsky*, but power analysis of multidimensional data requires the installation of *Ravel*.

4.13.1 x-vectors

When two or more tensors are combined with a binary operation (such as addition or multiplication), they must have the same rank. For example, two tensors of rank 2 can be multiplied together, but a tensor of rank 2 and a tensor of rank 3 cannot. They may have differing dimensions, which means the values within each tensor may not necessarily match up 1-to-1 exactly. To understand what happens when a given dimension is mismatched requires understanding the concept of an x-vector.
When Minsky is given tensor values, it sorts the values within each tensor by corresponding dimensions. For example, a rank 2 tensor would have its values sorted into two sets of data. This data can be in the form of numbers, dates (time values), or strings. Minsky will then look at cross-sections of the datasets in order to process the values within. When the dimensions of two tensors match up, for example two rank 2 tensors, the corresponding cross-sections of both tensors should also match up. When they don't, a weighted interpolation of the corresponding values is taken. This involves using an x-vector.
An x-vector is a vector of real values, strings or date/time values. If no x-vector is explicitly provided, then implicitly it consists of the the values $(0, \ldots, n_i - 1)$, where n_i is the dimension size of axis i of the tensor.

For example, if the first tensor consists of three elements (x_0, x_1, x_2) and the second consist of a number of different elements that roughly correspond to the same three elements, these can be added together. The x-vector starts with the first tensor's value of (x_0) and looks for a matching value in the second tensor. If it can't find a direct match, it will search for nearby values which roughly correspond. It can then take those values and interpolate the corresponding value based on where in the tensor it appears. This is weighted, so say there are four values nearby, the program will average those out and find where a value in the middle of those four values would appear, and what that hypothetical value would be. To take another example:
Suppose the first tensor was a vector (x_0, x_1) and had an x-vector (1,3) and the second tensor (y_0, y_1, y_2) had an x-vector (0,2,3), then the resulting tensor will be $(x_0 + 0.5(y_0 + y_1), x_1 + y_2)$. If the x-vector were date/time data, then the tensor values will be interpolated according to the actual time values. If the first tensor's x-vector value lies outside the second tensor's x-vector, then it doesn't result in a value being included in the output. The resultant x-vector's range of values is the intersection of input tensors' x-vector ranges. If both tensor had string x-vectors, then the resultant tensor will only have values where both input tensors have the same string value in their x-vectors. In the above case, where the x-vectors were ('1','3') and ('0','2','3') the resulting tensor will be the scalar $x_1 + y_2$.
It goes without saying that the type of the x-vector for each axis must also match.

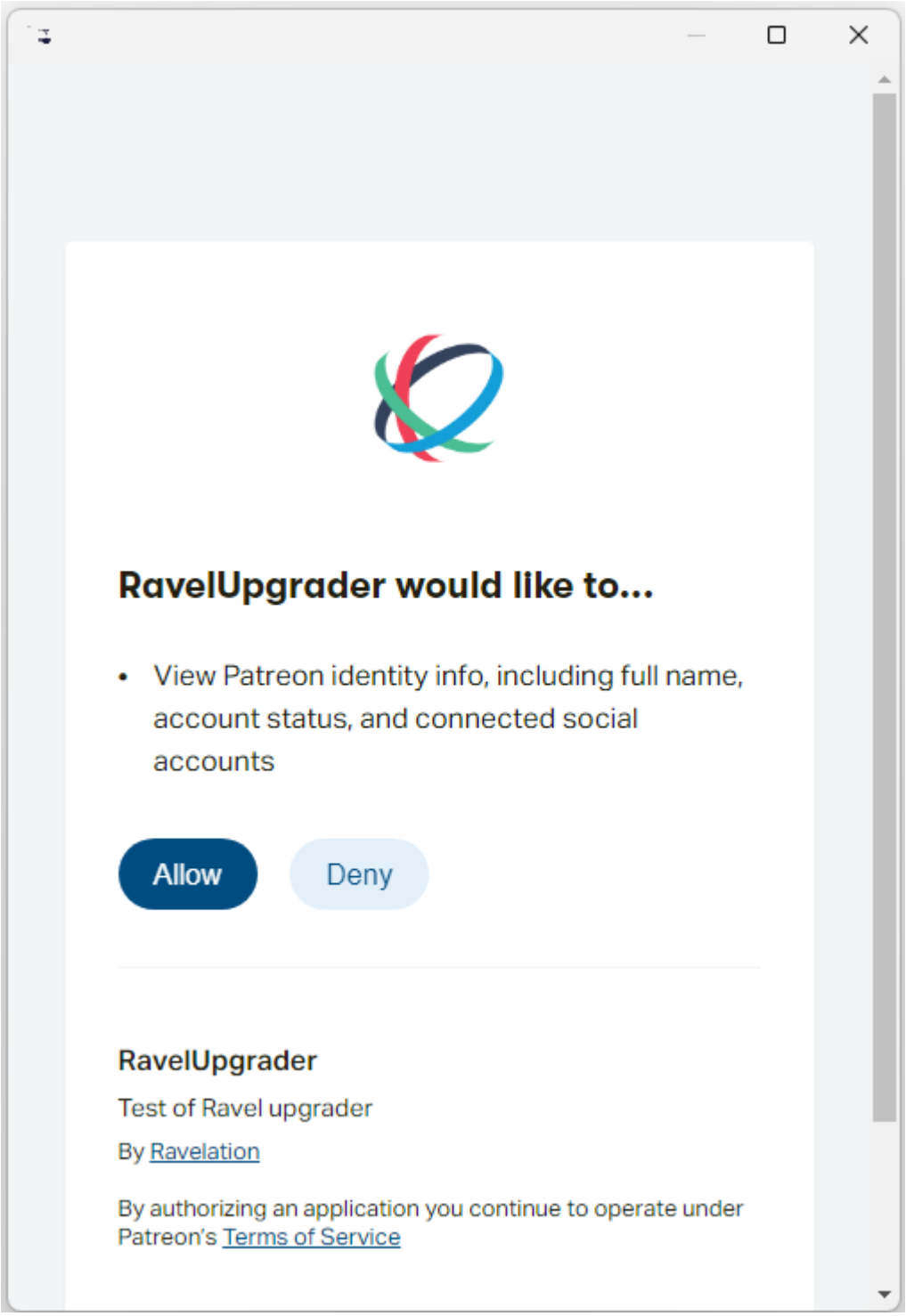
4.14 Ravel

Ravel the program is a commercial data analysis extension to the Open Source system dynamics program *Minsky*. It inherits all of *Minsky*'s features and adds the Ravel™, a graphical object for the manipulation and analysis of multidimensional data. If you are using *Minsky* and would like to check out *Ravel*, go to *Ravel*'s Patreon page[1] and sign up. The monthly fee is US$7, and there is a 7-day trial period during which you can experiment with using *Ravel* before the fee is applied.

4.14.1 Installing Ravel

Once you are a member of *Ravel*'s Patreon group, you have access to the Ravelation Patreon store, and you can download the latest copy of *Minsky*—either from the store or from attachments to posts announcing updated versions.
Download and install *Minsky*, which will launch the program once it is installed. Then choose "Upgrade" from the File menu. The RavelUpgrader form will pop up, asking you for confirmation that you are willing to let the process check your name and account details on Patreon.

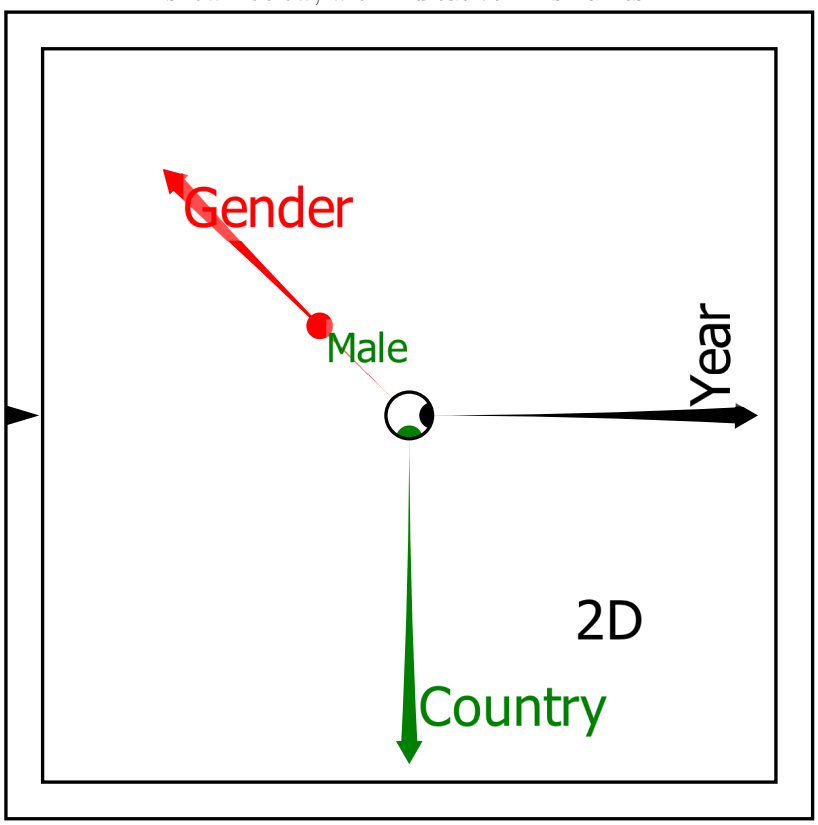
[1] https://www.patreon.com/hpcoder


RavelUpgrader would like to...
View Patreon identity info, including full name, account status, and connected social accounts
Allow
Deny
RavelUpgrader
Test of Ravel upgrader
By Ravelation
By authorizing an application you continue to operate under Patreon's Terms of Service

Click on "Allow" and *Minsky* will check that you are a member, and if so then download and install the *Ravel* extension into *Minsky*. Once this is completed, the latest version of *Ravel* will be installed in your *Minsky* subdirectory, and you will have full access to its commands.

4.14.2 Introduction to the Ravel GUI Object

A Ravel is inserted onto the wiring canvas by clicking on the widget from the toolbar. The first Ravel inserted into a document appears in Edit mode as shown below, with indicative Axis names:



The essential step in using a Ravel is to attach data to it via the triangular input port. See importing CSV files §(4.15) for how to import data. When a data file is attached to the input port of a Ravel, a multi-arrowed object fills the Ravel rectangle. If there are seven dimensions or more to the data, it will appear as shown below, with arrows in a pastel shade and axes labels not shown unless your mouse is hovering over an axis:

With six or fewer dimensions it will appear as shown below, with full-colour axes and axis labels shown:

The arrows are the dimensions of the data; the dots are selectors for specific values on each dimension. When a CSV import parameter §(4.15) is first attached to a Ravel, all the selector dots are in the centre of the Ravel, so that all the data in the Ravel is output to any Variable, Sheet or Plot attached to it. The next figure shows all 4 selectors being used so that the Ravel outputs the annualized rate of inflation for the USA in June 2022.

If the selector dot remains in the centre for one dimension, then a column of data for that dimension is output from the Ravel:

With two selector dots remaining in the centre, an array of data is output. This results in multiple countries' inflation rates being plotted:

Clearly a number of countries had extreme rates of inflation at various times between the 1970s and the mid-1990s. There are Ravel functions which can be used to isolate these countries (see *Operations* §(4.2)), but it is also easy to identify them visually by moving the selector dot along the Reference Area axis. You can do this by either:

- Clicking on the selector dot with the mouse and dragging it along the axis; or
- Hovering the mouse over the axis and using the arrow keys. The up-arrow and right-arrow move you forward along a data axis from the first entry to the last; the down and left arrow keys move you backward.

Eyeballing the data indicates that a 40% annual inflation rate is the dividing line between countries which did and did not experience a hyperinflation in the post-WWII period (defined as from 1955 onwards to exclude WWII-influenced price disturbances—for instance, those associated with the Russian occupation of Austria ending in 1955). That set of countries (Argentina, Brazil, Bulgaria, Chile, Colombia, Croatia, Czechia, Estonia, Iceland, Indonesia, Israel, Latvia, Lithuania, Mexico, North Macedonia, Peru, Philippines, Poland, Romania, Serbia, Slovakia, Slovenia and Turkey) are identified in the next figure as "HyperInflation" countries, while the remainder are "NormalInflation" countries. Except for Iceland, Israel, the Philippines, and Turkey, the hyperinflation countries were either South American, or countries which underwent the transition from socialism to capitalism in the 1990s. This figure also shows the iconized version of a Ravel, which is generated by toggling "Editor mode" on the context menu (also, by default, the first Ravel inserted into a document is in Editor mode, and all subsequent Ravels are inserted in iconized mode).

This selection employed one context-menu feature of a Ravel—"Pick Axis Slices" was used on the Reference area axis to choose countries which had annual inflation rate of above 40% between 1955 and 2024—and one Ravel-specific widget, the Lock §(4.14.5), which creates a subset of the data which can be allocated to a variable. The selection on the Ravel can then be changed without altering the subset selected by the Lock.

Now that the data is sensibly divided, it is feasible to calculate average inflation rates for normal countries. This is done by choosing "Av" on the context menu command "Set next aggregation", and then dragging the Reference Area axis in to the centre of the Ravel.

It is now possible to compare the inflation of individual countries to the average for all normal inflation countries. In the next figure, the USA and Japan were selected using Pick axis slices, and their data is graphed against the average. This shows that Japan's inflation rate exceeded America's—and the average—until 1975, after which Japan's inflation rate has been consistently lower than that of the USA and the average.

4.14.3 Ravel-specific capabilities

Ravels have a range of features controlled by the Ravel itself, the axes of the Ravel, and context menu commands relevant to the Ravel and its axes.

Axis rotation

A Ravel outputs the data in the right-pointing axis as the series for graphing in a plot, and as the rows of a sheet. In the previous figures, Date was the right-pointing axis, which generated a set of time series for plotting (the data shown on the Y1 axis). In the next figure, Reference Area is rotated into the right-pointing axis position in the second plot, which means that Date data now generates the x-axis information. The y-axis information is therefore the rate of inflation on a specified date. The second plot in the figure below shows the annual inflation rates for Japan and the USA at the highest point for the inflation data since WWII. Since this date follows the Oil Embargo[2] imposed by OPEC members after the Yom Kippur War[3], this implies that the main cause of the inflationary surge was the four-fold increase in oil prices (from US$2.50 a barrel to $10) caused by the embargo.

Axis aggregation

Axes can be collapsed as well as rotated, in which case the data on the axis is aggregated according to the choice made on the "Set next aggregation" Ravel context menu. In the next figure, Price is chosen on the Data axis, and all other axes apart from Date are aggregated by the Sum function (the default aggregation method). The Ravel therefore outputs total sales by date.
The available aggregations are: Sum, Product, Average, Minimum, Maximum, and Standard Deviation. The default setting is Sum. The other options can be set via the Context (right-mouse click) menu for Ravels.

[2] https://en.wikipedia.org/wiki/1973_oil_crisis
[3] https://en.wikipedia.org/wiki/Yom_Kippur_War

4.14.4 Context menu for Ravels

Command	Effect
Help	Bring up context help on Ravels
Description	Open a text form for documenting the Ravel
Editor Mode	Toggle between seeing the Ravel in Edit or Icon mode
Export as CSV	Export the data in the Ravel as a CSV file
Set next aggregation	Determine the mathematical operation applied when the next axis is collapsed
Collapse all handles	Aggregate all axes. Ravel will return a single number
Expand all handles	Disaggregate all axes.
Flip	Turn the Ravel around so that the input is on the right and output on the left
Link specific handles	Control over linked Ravels to link only some aspects– orientation, caliper, etc.
Join Link Group	Add this Ravel to an existing set of linked Ravels
Unlink	Remove this Ravel from an existing set of linked Ravels
Toggle Axis Calipers	Turn the caliper on a selected axis on or off
Sort Axis	Sort axis by the value of axis labels.
Sort by value	Sort axis by the value of the data on an axis.
Pick axis slices	Manually choose which axis values are output by the Ravel
Axis properties	Description and Dimension show the dimension name; Set next aggregation
Rotate Ravel	Rotate the Ravel through an arbitrary angle
Add to Publication Tab	Place the Ravel on a specified Publication Tab
Delete Ravel	

Calipers

A caliper is a range selector for an axis. Its customary use will be to select a range of dates, but a caliper can be applied to any axis to select a subset of data which is contiguous. In the next figure, calipers have been applied to extract the last 15 days' of data from this Ravel.

When a caliper is initially created, it spans the entire range of a given dimension. It is applied to sections of that data by moving the vertical left or right side of the parenthesis; the entire parenthesis is moved by clicking and dragging on the peak of the parenthesis—the length of the selection is thus maintained while the segment selected is altered.

Ravel currently allows one caliper per axis. Future releases will enable multiple calipers, which will allow non-contiguous segments of data to be displayed on the one plot or sheet.

Sort axis

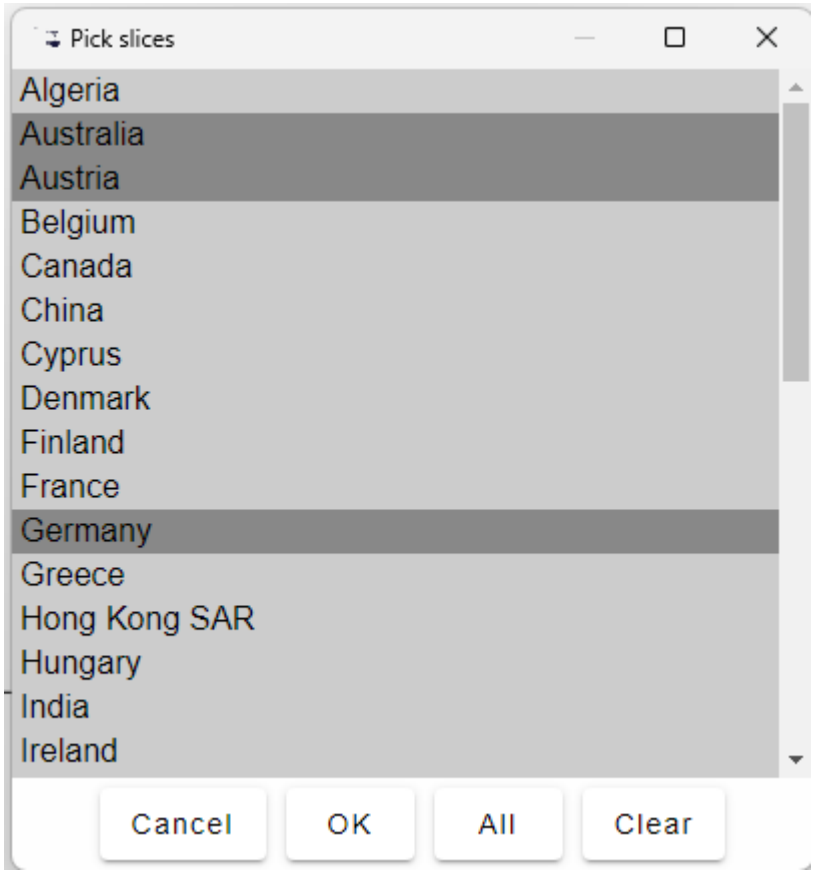
The default order of an axis is the same as the data input into it. Once loaded into a Ravel, the data can be sorted by the names of axis entries—for example, countries can be sorted alphabetically. This can be sorted forward (from A to Z) or in reverse.

Sort by value

The order of an axis can also be determined by the data values for specific axis selections. The command is only available if the axis selections return a single column of data for ranking—for example, in the inflation data, a single date is chosen so that the value of the inflation for all Reference Areas (countries) can be compared. There are four options generated by the combination of dynamic or static sorting, and forward or backward direction. Static sorting sorts the single axis by the value of the data, and does not alter that data as (for example) different dates are chosen using the selector dot. Dynamic sorting alters the order so that the axis is resorted when (for example) a different date is selected. In the next figure, dynamic sorting is applied to the Reference Area axis for the rate of inflation. The output is also put through a Slice §(4.2.13) operator, so that only the top 5 countries are output.

Pick axis slices

Pick axis slices lets you select an arbitrary number of slices—as was done above to choose just the USA and Japan to compare their rates of inflation. Individual slices can be chosen by left-clicking on the slice; to select more than one slice, hold down the control key as you click.



Flip Ravel

This is a convenience command, which is convenient when it would be preferable to have the output from a Ravel be on its left side rather than its right–for example, when the Ravel is linked to the Y2 axis of a plot.

4.14.5 Lock

Locks can only be attached to the output of Ravels. The Lock widget is used to fix the output of a Ravel to a particular configuration of data. After the Lock is closed (either by double-clicking on the Lock, or choosing Lock from the context menu), alterations can be made to the source Ravel without altering the output of the Lock. This makes it possible to take multiple slices of the data from one Ravel, and to work with them by assigning those slices to variables, manipulating the subset of data via additional Ravels, etc. It has a number of useful context menu commands.

Apply Lock state to Ravel

This menu item imposes the lock's state on the source Ravel. Since many locks can be applied to one Ravel, this makes it easy to edit the data as required to fine-tune individual locks. For example, the data for $Inflation_{Average}{}^{Normal}$ initially began in 1950. Since this included a very brief period of hyperinflation for Austria associated with Russia ceasing its posr-WWII occupation, it was advisable to alter the selection to start the caliper in 1955. This was easily done by applying the state of the lock to the Ravel, then editing the caliper, and opening and re-closing the lock.

4.14.6 Linking Ravels

When Ravels share dimensions, it is possible to link them together so that what is done to one Ravel—plotting, selecting data, rotating axes, etc.—is done to the other Ravel. This is very useful when you wish to examine relations between different sets of data.

Ravels are linked by dragging the mouse to select the desired Ravels, then choosing "Link selected Ravels" from the context menu when the mouse is hovering over the canvas.

In the next figure, the two Ravels whose borders are highlighted in brown have been linked. When the country "United States" is chosen for the Ravel containing house price data, the same selection is made for the Ravel containing household debt data. The plot and the regression analysis therefore compare United States house price changes to the change in the change in household debt.

When the selector dot for Country is moved to another country, the graph and the correlation result for household debt and house prices for that country replaces the information shown for the USA.

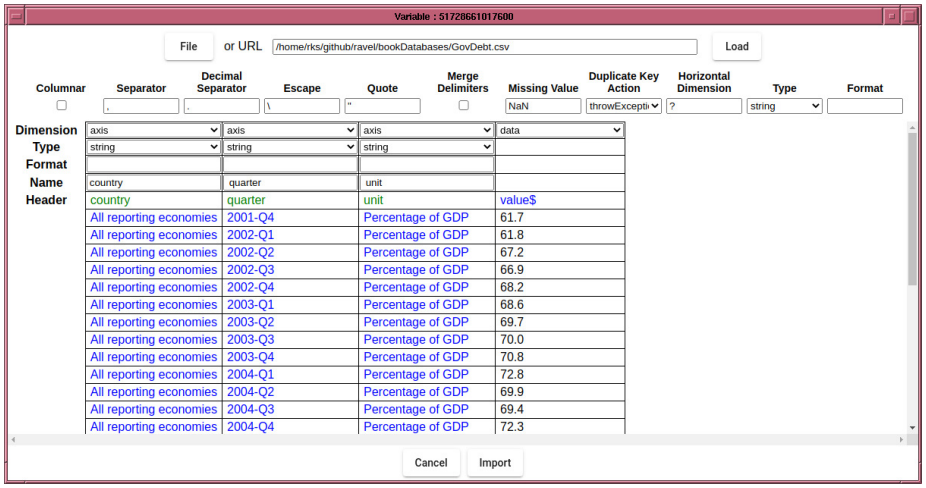
Linking can be refined using the context menu for Ravels—rather than for the canvas. If your mouse is over one of the selected Ravels rather than the canvas, the context menu option is "Lock specific handles". That brings up the detailed lock menu:

It is thus possible to turn on or off selecting of the same slices, arrow orientation, calipers and sort order if desired, while still linking the Ravels on other characteristics.

4.15 Importing data into a parameter from a CSV file

After creating a parameter from the "Variable" drop-down in the "Insert" menu, right-clicking the parameter and selecting the option to "Import CSV", will open a dialogue box that allows you to select a CSV file. Upon selecting the file, a dialog is opened, allowing you to specify assorted encoding parameters.

An alternative is to click on the ImportData icon , which will create a new parameter for you to import the data into.
The dialog looks somewhat like this (developments in the import routine may change some layout details):



4.15.1 Quick instructions

- Data is typically (but not always) stored in the rightmost columns of a CSV file. Shift click to set the top left cell of the data. Columns to the left will be marked as axes.
- Select “axis” in the Dimension dropdown to include a column as an axis. Column data turns blue.
- Select “ignore” in the Dimension dropdown to exclude a column. Column data turns red.
- Select “data” in the Dimension dropdown to treat a column as a data column. Column data turns black.
- Select Type for included axis columns. Select one of three types:

  string most general type, data treated as is.

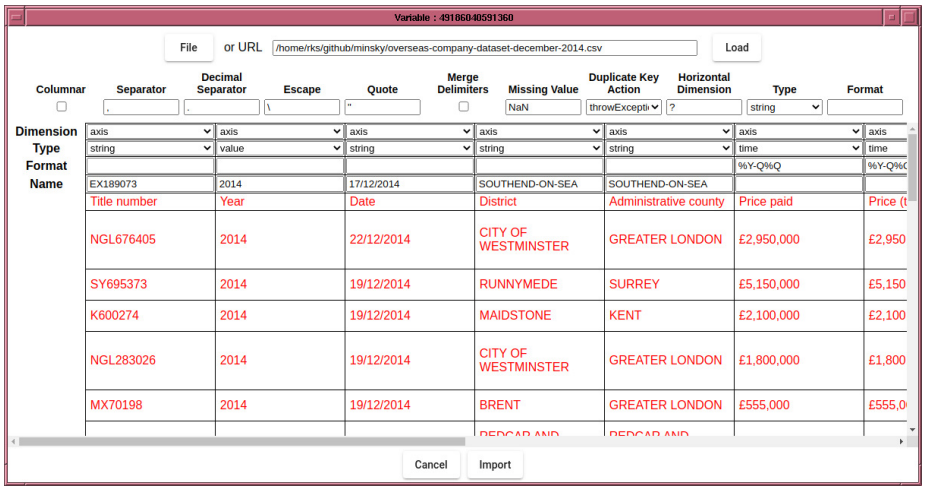
  value value data must be numerical, e.g. 1, 2, and not quoted (as in “1”, “2”, etc.)

  time data must refer to date-time data. The format field may be used to control interpretation of this data, though if this field is left blank, Ravel will interpret the data format itself. Blank format assumes data contains year/month/day/hour/minute/second separated by some non-numerical character. If fewer than 6 numerical fields present (from year to second), smallest units are set to minimum value (0 or 1 respectively).

- Click the OK button.
- Data is imported into the parameter.
- You may need to set units for the imported data field, which can be done via the Edit → Dimensions menu item.

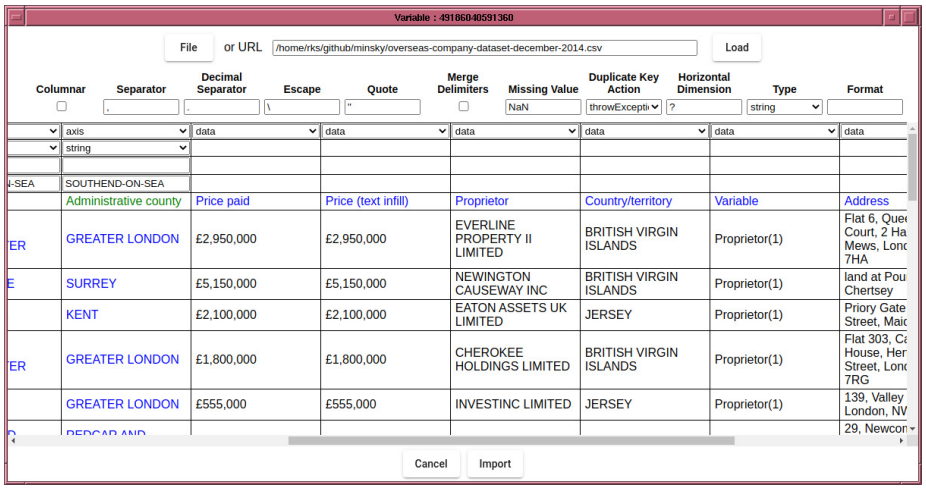
In the case shown above, the system has automatically guessed that the data is 3 dimensional, and that the first 3 columns give the axis labels for each dimension (shown in blue), and the 4th column contains the data. The first row has been automatically determined to be the first row of the file — with the dimension names are shown in green.

In this case, the automatic parsing system has worked things out correctly, but often times it needs help from the computer user. An example is as follows:

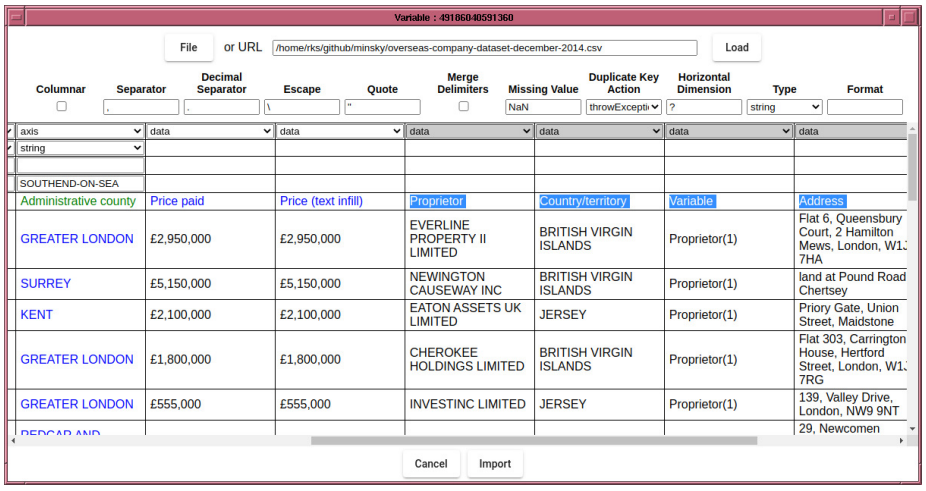


In this example, *Ravel* has failed to determine where the data starts, probably because of the columns to the right of the "Price" columns. So the first thing to do is tell it where the data is located. This is one function of the context menu for data importing. The options are:

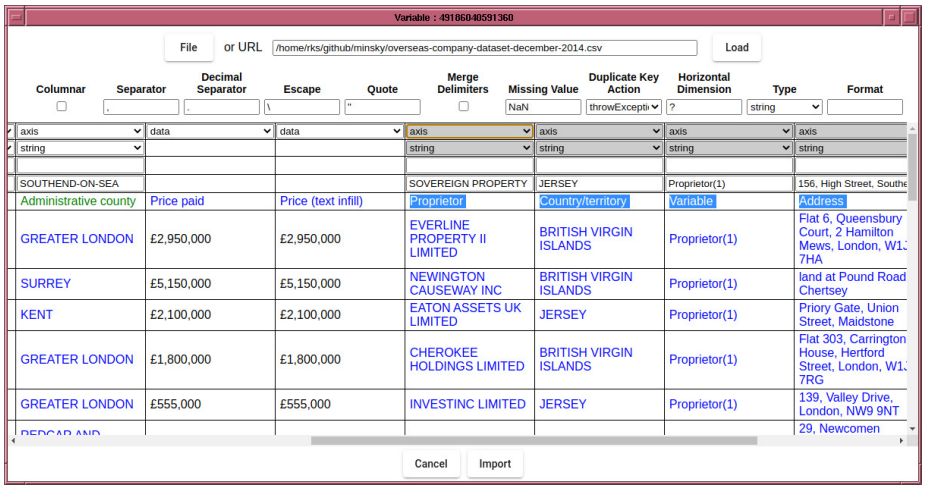
Command	Effect
Set as header row	Tell Ravel that the selected row contains the header information for the import
Auto-classify columns as axis/data	Invoke *Ravel*'s automated column classification routine
Populate column labels	Transfer the information from the header row to the Name row
Populate current column label	Transfer the information from the header to the Name for one column only
Set start of data row, and column	Tell *Ravel* where data begins in the file
Set start of data row	Tell *Ravel* where data begins in the file
Set start of data column	Tell *Ravel* where data begins in the file



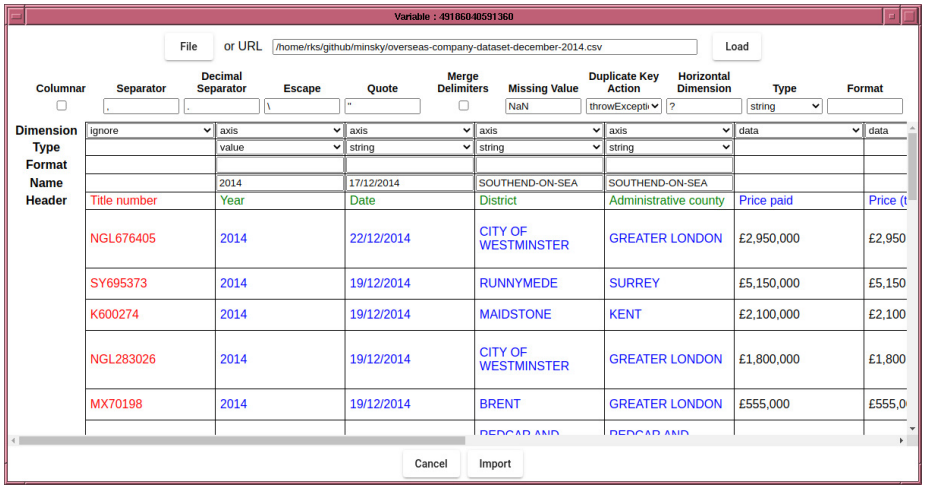
Note that this causes all columns to the right of "Price paid" to be treated as data, which is not right, since the columns to the right of "Price (text infill)" are text based columns, not data. So we need to mark those columns as either "axis" or "ignore". To do that, click and drag your mouse on the header field, which will cause those columns to be selected, like so:



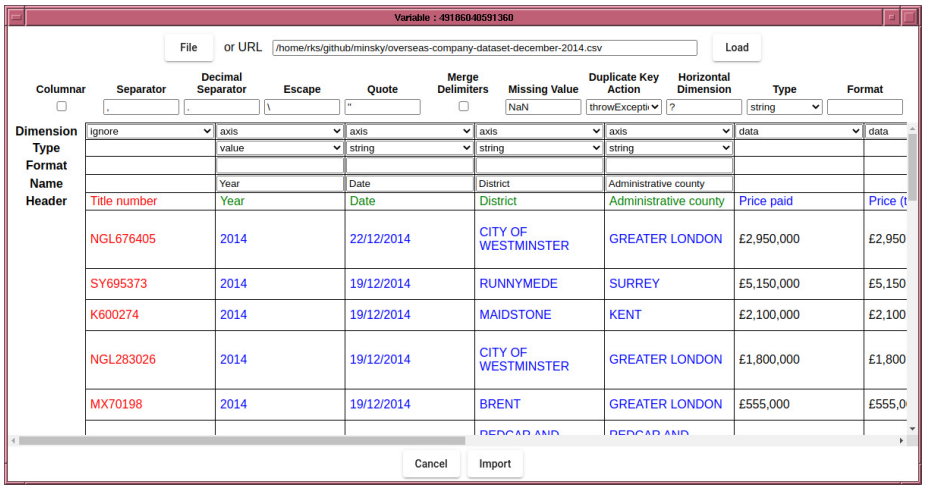
Then in the dimensions row, select "axis", which flips the selected columns:



Now the axes index labels are rendered in blue, the axes names in green and the data is in black. In this example, some axes have unique values, which are not particularly useful to scan over. Other examples might have columns that duplicate others, for example when the data has been entered via a database program that allows the user to type country codes (e.g., LX) rather than the full country name (Luxembourg). Only one of these axes is needed to uniquely specify the country, so you can exclude the unwanted dimension by choosing "ignore" in the "Dimension" row. The deselected columns are rendered in red, indicating that its data will not be imported into *Ravel*:

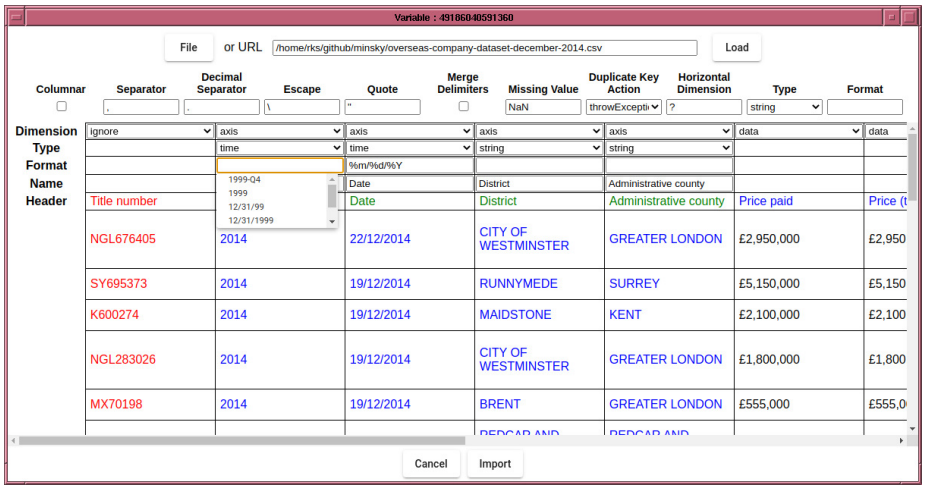


In this example, the axis names has not been correctly inferred. Whilst, one can manually edit the axis names in the "Name" line, a quick shortcut is to select "Populate column labels" from the context menu.



4.15.2 Temporal data

The Date column is current parsed as strings, which not only will be sorted incorrectly, but even if the data were in a YYYYMMDD format which is sorted correctly, will not have a uniform temporal spacing. It is therefore important to parse the Date column as temporal data, which is achieved by changing the column type to "time", and specifying a format string, which follows strftime conventions with the addition of a quarter specifier (`%Q`). If your temporal data is in the form Y*M*D*H*M*S, where * signifies any sequence of non-digit characters, and the year, month, day, hour minutes, second fields are regular integers in that order, then you can leave the format string blank . If some of the fields are missing, eg minutes and seconds, then they will be filled in with sensible defaults.



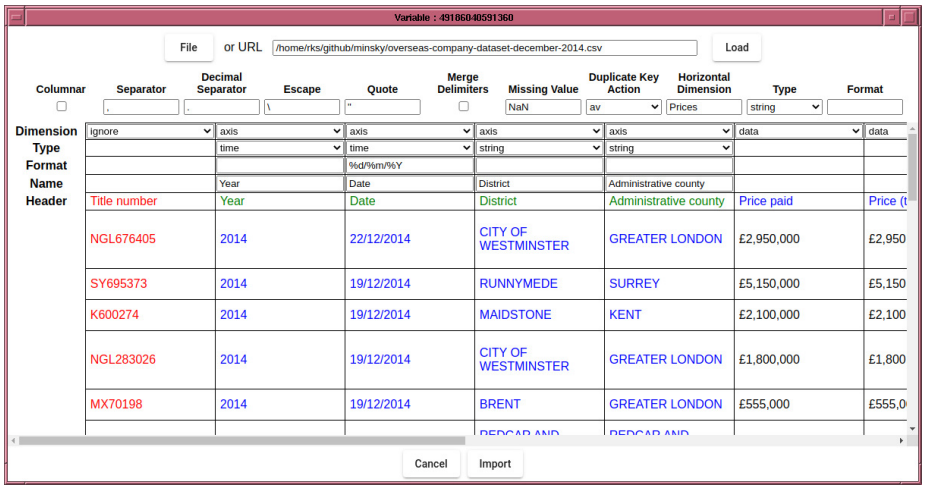
Code	Description
%a or %A	The name of the day of the week according to the current locale, in abbreviated form or the full name.
%b or %B	The month name according to the current locale, in abbreviated form or the full name.
%d	Day of month in range 01 to 31
%e	Day of month in range 1 to 31
%H	Hour in range 0 to 23
%I	Hour in range 1 to 12
%m	Month as a decimal number (01 to 12)
%M	Minute in range 00 to 59
%Q	Quarter (0=1st January, 1=1st March etc)
%p	AM or PM
%s	Number of seconds since epoch (1st January 1970)
%S	Seconds in range 00 to 59
%y	Two digit year YY
%Y	Four digit year YYYY
%z	numerical timezone offset
%Z	Timezone name
%%	Literal % character

Table 4.1: Table of strftime codes

A *Strftime* formatted string consists of escape codes (with leading % characters). All other characters are treated as matching literally the characters of the input. So to match a date string of the format YYYY-MM-DD HH:MM:SS+ZZ (ISO format), use a format string "`%Y-%m-%d %H:%M:%S+%Z`". Similarly, for quarterly data expressed like 1972-Q1, use "`%Y-Q%Q`". Note that only %Y and %y can be mixed with %Q (nothing else makes sense anyway).

Note that if your month data uses month names instead of numerical month representation, the it is important you select %d or %e depending on whether your day data has leading zeros or not. This is unfortunately a restriction of the underlying date import routine, which may change in the future.

Even in the current settings, you may still get a message "exhausted memory — try reducing the rank", or a similar message about hitting a 20% of physical memory threshold. In some cases, "titles" and "addresses" might be pretty much unique for each record, leading to a large, but very sparse hypercube. If you remove those columns, then you may encounter the "Duplicate key" message. In this case, you can aggregate over these records—which is desirable anyway to analyse trends in aggregate data. You do this by setting "Duplicate Key Action" to sum (or perhaps average for this example). After some additional playing around with dimensions to aggregate over, we can get the data imported.



4.15.3 Duplicate keys

In a hypercube, data is indexed by a list of indices, collectively known as a key. The indices may be strings, integers or date/time values. If more than one value exists in the CSV file for a given key, Minsky throws a "Duplicate key" exception. This exception gives you the option of writing a report, which is basically a sorted version of the original CSV file, with the errors listed at the beginning. You can open this report in a spreadsheet to see if data needs to be corrected or removed.

Duplicates will arise if you load data from a transactional database which stores the exact time of every transaction. Here the correct choice on "Duplicate Key Action" is to sum the entries so that all sales for a particular day (or month, quarter, or year) are aggregated for analysis of trends in the data.

4.15.4 Counter mode

As mentioned previously, Ravel is designed to work with numerical data. However, it is possible to work with purely symbolic data by using *counter mode*. This mode counts the number of times a key is present in the data file — either zero or one, or in the case of duplicate keys, a value greater than one. You can feed this into a Ravel and perform rollups to analyse the symbols statistically, for example as histograms.

To select counter mode, select the "counter" check box in the input dialog form. If no data columns are present at all, Ravel automatically selected counter mode.

4.16 Minsky-specific commands

4.16.1 File Menu Minsky commands

Library Opens a repository of models for the Minsky simulation system.

Recording Record the states of a model as it is being built for later replay. This is useful for demonstrating how to build a model, but bear in mind that recorded logs are not, in general, portable between versions of Minsky.

Replay recording Replay a recording of model states.

4.16.2 Options Menu Commands

The options menu allows you to customise aspects of Minsky.

Preferences

- Godley table show values. When ticked, the values of flow variables are displayed in the Godley table whilst a simulation is running. This will tend to slow down the simulation somewhat.
- Godley table output style — whether +/− or DR/CR (debit/credit) indicators are used.
- Enable multiple equity columns - whether Godley table have more than one equity columns.
- Number of recent files to display — affects the recent files §(2.3.1) menu.
- Wrap long equations in LaTeX export. If ticked, use the breqn package to produce nicer looking automatically line-wrapped formulae. Because not all LaTeX implementations are guaranteed to support breqn, untick this option if you find difficulty.
- select a font for variable names etc.

Background colour — select a colour from which a colour scheme is computed.

4.16.3 Simulation Menu Commands

The Simulation Menu invokes the Simulation Parameters form.

- Time unit allows one to specify units for the time dimension for dimensional analysis. eg “year”, “s” etc.
- Min and Max step size control aspect of the adaptive Runge-Kutta equation solver, which trade off performance and accuracy of the model. The algorithm is adaptive, so the step size will vary according to how stiff the system of equations is.

  - Specifying a minimum step size prevents the system from stalling, at the expense of accuracy when the step size reaches that minimum.
  - Specifying a maximum step size is useful to ensure one has sufficient data points for smooth plots.

- An iteration is the time between updates to plots. Increasing the number of solver steps per iteration decreases the overhead involved in updating the display, at the expense of smoothness of the plots.
- Start time is the value of the system t variable when the system is reset.
- Run until time can be used to pause the simulation once t reaches a certain value. Setting this to "Infinity" causes the simulation to run indefinitely, or until some arithmetic error occurs.
- The algorithm is adaptive, so the step size will vary according to how stiff the system of equations is.
- Solver order determines how high order a polynomial is used for approximating the actual system. A first order explicit solver is the classic Jacobi method, which is the fastest, but least accurate solver. 4th order is the adaptive Runge-Kutta equation solver.

4.16.4 Record/Replay Buttons

These buttons control the recording / replay mode of *Minsky*. You can record your interactions with *Minsky*, and replay those interactions for demonstration/presentation purposes.

1. Record a session of building/modifying a model. Note that replaying a recorded session always starts from a blank canvas, so if you're recording the modification of a model, ensure that the first thing recorded is to load the model being modified. This button is a toggle button, so clicking it again finishes the session, and closes the file.
2. Simulate/Replay button. Pressing this button puts *Minsky* into replay mode, and asks for a recording file. You may use the run buttons (run/pause,stop and step), as well as the speed slider, to control the replay. This button is a toggle button, so clicking it again returns *Minsky* back to the default simulation mode.

4.16.5 Run Buttons

The Run buttons respectively:

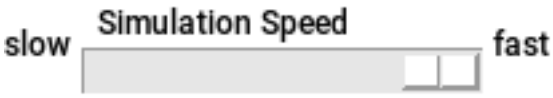
1. Start a simulation–when started the button changes to a pause icon, allowing you to pause the simulation ⏸.

2. Stop a simulation and reset the simulation time to zero

3. Step through the simulation one iteration at a time.

4. Reverse checkbox changes the simulation time direction. Bear in mind that a simulation will eventually diverge from its original trajectory due to chaotic effects.

4.16.6 Speed slider



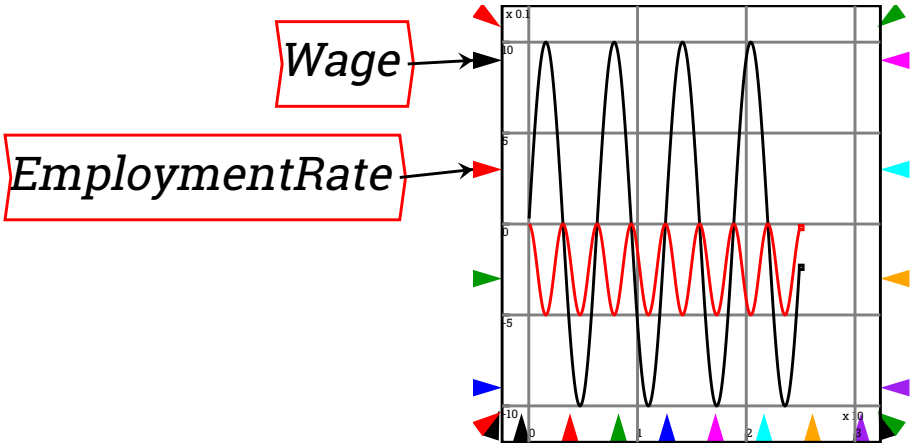
The speed slider controls the rate at which a model is simulated. The default speed is the maximum speed your system can support, but you can slow this down to more closely observe dynamics at crucial points in a simulation.

4.16.7 Simulation time

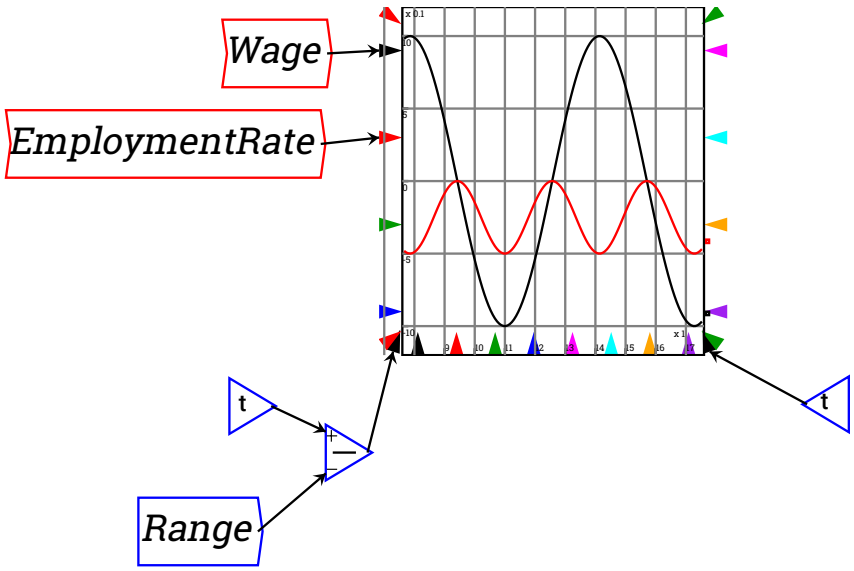
In the right hand top corner is a textual display of the current simulation time t, and the current (adaptive) difference between iterations Δt.

Time t embeds a reference to the simulation time on the Canvas. This is not necessary in most simulations, but can be useful if you want to make a time-dependent process explicit, or control the appearance of a graph.

For example, by default a graph displays the simulation time on the horizontal axis, so that cycles get compressed as a simulation runs for a substantial period:

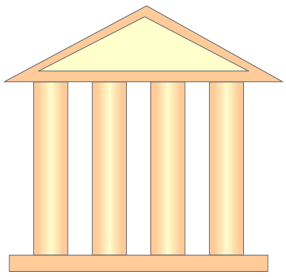


If a Time block is added to the marker for the x-axis range, you can control the number of years that are displayed. This graph is set up to show a ten year range of the model only:

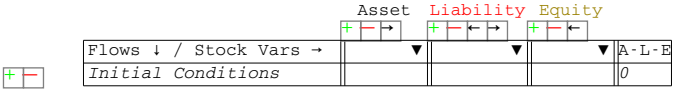


Godley Table 🏛 . This is the fundamental element of *Minsky* that is not found (yet) in any other system dynamics program.

Clicking on it and placing the resulting Bank Icon on the Canvas enters a Godley table into your model:

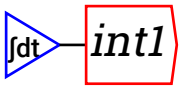


Double-click on the Bank Icon (or right-click and choose "Open Godley Table" from the context menu) and you get a double-entry bookkeeping table we call a Godley Table, which looks like the following onscreen:

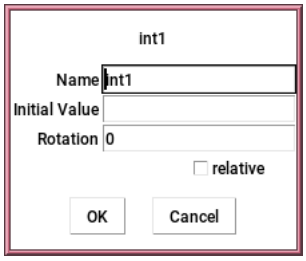


Use this table to enter the bank accounts and financial flows in your model. We discuss this later §(1.2.1).

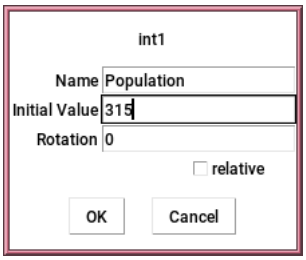
Integration . This inserts a variable whose value depends on the integral of other variables in the system. This is the essential element for defining a dynamic model. Click on it and the following entity will appear at the top left hand side of the canvas (and move with your mouse until you click to place it somewhere:



"int1" is just a placeholder for the integration variable, and the first thing you should do after creating one is give it a name. Double-click on the "int1", or right click and choose Edit. This will bring up the following menu:



Change the name to something appropriate, and give it an initial value. For example, if you were building a model that included America's population, you would enter the following:



The integrated variable block would now look like this:

To model population, you need to include a growth rate. According to Wikipedia, the current US population growth rate is 0.97 percent per annum. Expressed as an equation, this says that the annual change in population, divided by its current level, equals 0.0097:

$$\frac{1}{\text{Population}(t)} \cdot \left(\frac{d}{dt} \text{Population}(t) \right) = 0.0097$$

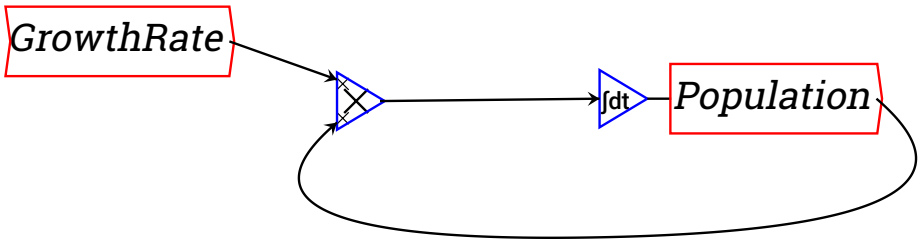
To express this as an integral equation, firstly we multiply both sides of this equation by Population to get:

$$\frac{d}{dt} \text{Population}(t) = 0.0097 \cdot \text{Population}(t)$$

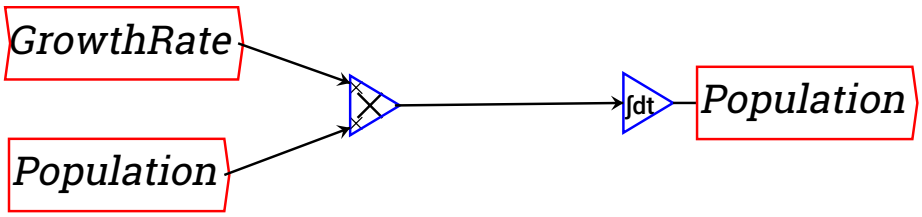
Then we integrate both sides to get an equation that estimates what the population will be T years into the future as:

$$\text{Population}(T) = 315 + \int_0^T 0.0097 \cdot \text{Population}(t) dt$$

Here, 315 (million) equals the current population of the USA, the year zero is today, and T is some number of years from today. The same equation done as a block diagram looks like this:



Or you can make it look more like the mathematical equation by right-clicking on "Population" and choosing "Copy item". This creates a copy of the Population variable, enabling you to wire up the equation this way:



Derivative Operator This operator symbolically differentiates its input, provided the input is differentiable. An error is generated if the input is not differentiable.

Chapter 5

User defined functions

Much of this chapter is exerpted from exprtk's read.txt file

5.1 Introduction

The C++ Mathematical Expression Toolkit Library (ExprTk) is a simple to use, easy to integrate and extremely efficient run-time mathematical expression parsing and evaluation engine. The parsing engine supports numerous forms of functional and logic processing semantics and is easily extensible.
With Minsky's user defined functions, expressions can refer to Minsky variables accessible from the current scope (ie local Minsky variables will hide global variables), and also parameters declared as part of the function name. One can also call other user defined functions, which is the only way a user defined function with more than 2 parameters can be used. For 0-2 parameters, user defined functions can be wired into a Minsky computation.
ExprTk identifiers (such as variable names and function names) consist of alphanumeric characters plus '_' and '.'. They must start with a letter. Minsky is reserving the underscore and full stop to act as an escape sequence, in order to refer to the full range of possible Minsky variable identifiers, including all unicode characters. This section will be updated once that feature is in place — for now, please avoid using those characters in identifiers.

5.2 Capabilities

The ExprTk expression evaluator supports the following fundamental arithmetic operations, functions and processes:

Types: Scalar, Vector, String

Basic operators: `+, -, *, /, %, ^`

Assignment: `:=, +=, -=, *=, /=, %=`

Equalities & Inequalities: `=, ==, <>, !=, <, <=, >, >=`

Logic operators: and, mand, mor, nand, nor, not, or, shl, shr, xnor, xor, true, false

Functions: abs, avg, ceil, clamp, equal, erf, erfc, exp, expm1, floor, frac, log, log10, log1p, log2, logn, max, min, mul, ncdf, nequal, root, round, roundn, sgn, sqrt, sum, swap, trunc

Trigonometry: acos, acosh, asin, asinh, atan, atanh, atan2, cos, cosh, cot, csc, sec, sin, sinc, sinh, tan, tanh, hypot, rad2deg, deg2grad, deg2rad, grad2deg

Control structures: if-then-else, ternary conditional, switch-case, return-statement

Loop statements: while, for, repeat-until, break, continue

String processing: in, like, ilike, concatenation

Optimisations: constant-folding, simple strength reduction and dead code elimination

Calculus: numerical integration and differentiation

5.3 Example expressions

The following is a short listing of infix format based mathematical expressions that can be parsed and evaluated using the ExprTk library.

- `sqrt(1 - (3 / x^2))`
- `clamp(-1, sin(2 * pi * x) + cos(y / 2 * pi), +1)`
- `sin(2.34e-3 * x)`
- `if(((x[2] + 2) == 3) and ((y + 5) <= 9),1 + w, 2 / z)`
- `inrange(-2,m,+2) == if(({-2 <= m} and [m <= +2]),1,0)`
- `({1/1}*[1/2]+(1/3))-{1/4}^[1/5]+(1/6)-({1/7}+[1/8]*(1/9))`
- `a * exp(2.2 / 3.3 * t) + c`
- `z := x + sin(2.567 * pi / y)`
- `u := 2.123 * {pi * z} / (w := x + cos(y / pi))`
- `2x + 3y + 4z + 5w == 2 * x + 3 * y + 4 * z + 5 * w`
- `3(x + y) / 2.9 + 1.234e+12 == 3 * (x + y) / 2.9 + 1.234e+12`
- `(x + y)3.3 + 1 / 4.5 == [x + y] * 3.3 + 1 / 4.5`

- `(x + y[i])z + 1.1 / 2.7 == (x + y[i]) * z + 1.1 / 2.7`
- `(sin(x / pi) cos(2y) + 1) == (sin(x / pi) * cos(2 * y) + 1)`
- `75x^17 + 25.1x^5 - 35x^4 - 15.2x^3 + 40x^2 - 15.3x + 1`
- `(avg(x,y) <= x + y ? x - y : x * y) + 2.345 * pi / x`
- `while (x <= 100) { x -= 1; }`
- `x <= 'abc123' and (y in 'AString') or ('1x2y3z' != z)`
- `((x + 'abc') like '*123*') or ('a123b' ilike y)`
- `sgn(+1.2^3.4z / -5.6y) <= {-7.8^9 / -10.11x }`

5.4 Copyright notice

5.5 Built-in operations & functions

5.5.1 Arithmetic & Assignment Operators

OPERATOR	DEFINITION
`+`	Addition between x and y. (eg: `x + y`)
`-`	Subtraction between x and y. (eg: `x - y`)
`*`	Multiplication between x and y. (eg: `x * y`)
`/`	Division between x and y. (eg: `x / y`)
`%`	Modulus of x with respect to y. (eg: `x % y`)
`^`	x^y. (eg: `x ^ y`)
`:=`	Assign the value of x to y. Where y is either a variable or vector type. (eg: `y := x`)
`+=`	Increment x by the value of the expression on the right hand side. Where x is either a variable or vector type. (eg: `x += abs(y - z)`)
`-=`	Decrement x by the value of the expression on the right hand side. Where x is either a variable or vector type. (eg: `x[i] -= abs(y + z)`)
`*=`	Assign the multiplication of x by the value of the expression on the righthand side to x. Where x is either a variable or vector type. (eg: `x *= abs(y / z)`)
`/=`	Assign the division of x by the value of the expression on the right-hand side to x. Where x is either a variable or vector type. (eg: `x[i + j] /= abs(y * z)`)
`%=`	Assign x modulo the value of the expression on the right hand side to x. Where x is either a variable or vector type. (eg: `x[2] %= y ^ 2`)

5.5.2 Equalities & Inequalities

OPERATOR	DEFINITION
`==` or `=`	True only if x is strictly equal to y. (eg: `x == y`)
`<>` or `!=`	True only if x does not equal y. (eg: `x <> y` or `x != y`)
`<`	True only if x is less than y. (eg: `x < y`)
`<=`	True only if x is less than or equal to y. (eg: `x <= y`)
`>`	True only if x is greater than y. (eg: `x > y`
`>=`	True only if x greater than or equal to y. (eg: `x >= y`)

5.5.3 Boolean Operations

OPERATOR	DEFINITION
`true`	True state or any value other than zero (typically 1).
`false`	False state, value of exactly zero.
`and`	Logical AND, True only if x and y are both true. (eg: `x and y`)
`mand`	Multi-input logical AND, True only if all inputs are true. Left to right short-circuiting of expressions. (eg: `mand(x > y, z < w, u or v, w and x)`)
`mor`	Multi-input logical OR, True if at least one of the inputs are true. Left to right short-circuiting of expressions. (eg: `mor(x > y, z < w, u or v, w and x)`)
`nand`	Logical NAND, True only if either x or y is false. (eg: `x nand y`)
`nor`	Logical NOR, True only if the result of x or y is false (eg: `x nor y`)
`not`	Logical NOT, Negate the logical sense of the input. (eg: `not(x and y) == x nand y`)
`or`	Logical OR, True if either x or y is true. (eg: `x or y`)
`xor`	Logical XOR, True only if the logical states of x and y differ. (eg: `x xor y`)
`xnor`	Logical XNOR, True iff the biconditional of x and y is satisfied. (eg: `x xnor y`)
`&`	Similar to AND but with left to right expression short circuiting optimisation. (eg: `(x & y) == (y and x)`)
`|`	Similar to OR but with left to right expression short circuiting optimisation. (eg: `(x | y) == (y or x)`)

5.5.4 General Purpose Functions

FUNCTION	DEFINITION
`abs`	Absolute value of x. (eg: `abs(x)`)
`avg`	Average of all the inputs. (eg: `avg(x,y,z,w,u,v) == (x + y + z + w + u + v) / 6`)
`ceil`	Smallest integer that is greater than or equal to x.
`clamp`	Clamp x in range between r0 and r1, where r0 ¡ r1. (eg: `clamp(r0,x,r1)`)
`equal`	Equality test between x and y using normalised epsilon
`erf`	Error function of x. (eg: `erf(x)`)
`erfc`	Complimentary error function of x. (eg: `erfc(x)`)
`exp`	e^x (eg: `exp(x)`)
`expm1`	e^{x-1} where x is very small. (eg: `expm1(x)`)
`floor`	Largest integer that is less than or equal to x. (eg: `floor(x)`)
`frac`	Fractional portion of x. (eg: `frac(x)`)
`hypot`	$\sqrt{x^2 + y^2}$ (eg: `hypot(x,y) = sqrt(x*x + y*y)`)
`iclamp`	Inverse-clamp x outside of the range r0 and r1. Where r0 ¡ r1. If x is within the range it will snap to the closest bound. (eg: `iclamp(r0,x,r1)` $= \begin{cases} r0 & \text{if} \quad x \leq r0 \\ x & \text{if} \quad r0 \leq x \leq r1 \\ r1 & \text{if} \quad x \geq r1 \end{cases}$)
`inrange`	In-range returns 'true' when x is within the range $[r_0, r_1]$. Where $r_0 < r_1$. (eg: `inrange(r0,x,r1)`)
`log`	Natural logarithm $\ln x$. (eg: `log(x)`)
`log10`	$\log_{10} x$. (eg: `log10(x)`)
`log1p`	$\ln(1 + x)$, where x is very small. (eg: `log1p(x)`)
`log2`	$\log_2 x$. (eg: `log2(x)`)
`logn`	$\log_n x$, where n is a positive integer. (eg: `logn(x,8)`)
`max`	Largest value of all the inputs. (eg: `max(x,y,z,w,u,v)`)
`min`	Smallest value of all the inputs. (eg: `min(x,y,z,w,u)`)
`mul`	Product of all the inputs. (eg: `mul(x,y,z,w,u,v,t) == (x * y * z * w * u * v * t)`)
`ncdf`	Normal cumulative distribution function. (eg: `ncdf(x)`)
`nequal`	Not-equal test between x and y using normalised epsilon
`pow`	x^y. (eg: `pow(x,y) == x ^ y`)
`root`	$\sqrt[n]{x}$, where n is a positive integer. (eg: `root(x,3) == x^(1/3)`)
`round`	Round x to the nearest integer. (eg: `round(x)`)
`roundn`	Round x to n decimal places (eg: `roundn(x,3)`) where $n > 0$ is an integer. (eg: `roundn(1.2345678,4) == 1.2346`)
`sgn`	Sign of x, -1 where $x < 0$, $+1$ where $x > 0$, else zero. (eg: `sgn(x)`)
`sqrt`	$\sqrt{x}$, where $x >= 0$. (eg: `sqrt(x)`)
`sum`	Sum of all the inputs. (eg: `sum(x,y,z,w,u,v,t) == (x + y + z + w + u + v + t)`)
`swap` `<=>`	Swap the values of the variables x and y and return the current value of y. (eg: `swap(x,y)` or `x <=> y`)
`trunc`	Integer portion of x. (eg: `trunc(x)`)

5.5.5 Trigonometry Functions

FUNCTION	DEFINITION
`acos`	Arc cosine of x expressed in radians. Interval $[-1, +1]$ (eg: `acos(x)`)
`acosh`	Inverse hyperbolic cosine of x expressed in radians. (eg: `acosh(x)`)
`asin`	Arc sine of x expressed in radians. Interval $[-1, +1]$ (eg: `asin(x)`)
`asinh`	Inverse hyperbolic sine of x expressed in radians. (eg: `asinh(x)`)
`atan`	Arc tangent of x expressed in radians. Interval $[-1, +1]$ (eg: `atan(x)`)
`atan2`	Arc tangent of (x/y) expressed in radians. $[-\pi, +\pi]$ (eg: `atan2(x,y)`)
`atanh`	Inverse hyperbolic tangent of x expressed in radians. (eg: `atanh(x)`)
`cos`	Cosine of x. (eg: `cos(x)`)
`cosh`	Hyperbolic cosine of x. (eg: `cosh(x)`)
`cot`	Cotangent of x. (eg: `cot(x)`)
`csc`	Cosecant of x. (eg: `csc(x)`)
`sec`	Secant of x. (eg: `sec(x)`)
`sin`	Sine of x. (eg: `sin(x)`)
`sinc`	Sine cardinal of x. (eg: `sinc(x)`)
`sinh`	Hyperbolic sine of x. (eg: `sinh(x)`)
`tan`	Tangent of x. (eg: `tan(x)`)
`tanh`	Hyperbolic tangent of x. (eg: `tanh(x)`)
`deg2rad`	Convert x from degrees to radians. (eg: `deg2rad(x)`)
`deg2grad`	Convert x from degrees to gradians. (eg: `deg2grad(x)`)
`rad2deg`	Convert x from radians to degrees. (eg: `rad2deg(x)`)
`grad2deg`	Convert x from gradians to degrees. (eg: `grad2deg(x)`)

5.5.6 String Processing

FUNCTION	DEFINITION
= , ==, !=, <>, <=, >=, < , >	All common equality/inequality operators are applicable to strings and are applied in a case sensitive manner. In the following example x, y and z are of type string. (eg: `not((x <= 'AbC') and ('1x2y3z' <> y)) or (z == x)`)
`in`	True only if x is a substring of y. (eg: `x in y` or `'abc' in 'abcdefgh'`)
`like`	True only if the string x matches the pattern y. Available wildcard characters are '*' and '?' denoting zero or more and zero or one matches respectively. (eg: `x like y` or `'abcdefgh' like 'a?d*h'`)
`ilike`	True only if the string x matches the pattern y in a case insensitive manner. Available wildcard characters are '*' and '?' denoting zero or more and zero or one matches respectively. (eg: `x ilike y` or `'a1B2c3D4e5F6g7H' ilike 'a?d*h'`)
`[r0:r1]`	The closed interval $[r0, r1]$ of the specified string. eg: Given a string x with a value of 'abcdefgh' then: 1. `x[1:4] == 'bcde'` 2. `x[ :5] == x[:10 / 2] == 'abcdef'` 3. `x[2 + 1: ] == x[3:] =='defgh'` 4. `x[ : ] == x[:] == 'abcdefgh'` 5. `x[4/2:3+2] == x[2:5] == 'cdef'` Note: Both r0 and r1 are assumed to be integers, where r0 ¡= r1. They may also be the result of an expression, in the event they have fractional components truncation will be performed. (eg: $1.67 \rightarrow 1$)

FUNCTION	DEFINITION
`:=`	Assign the value of x to y. Where y is a mutable string or string range and x is either a string or a string range. eg: 1. `y := x` 2. `y := 'abc'` 3. `y := x[:i + j]` 4. `y := '0123456789'[2:7]` 5. `y := '0123456789'[2i + 1:7]` 6. `y := (x := '0123456789'[2:7])` 7. `y[i:j] := x` 8. `y[i:j] := (x + 'abcdefg'[8 / 4:5])[m:n]` Note: For options 7 and 8 the shorter of the two ranges will denote the number characters that are to be copied.
`+`	Concatenation of x and y. Where x and y are strings or string ranges. eg 1. `x + y` 2. `x + 'abc'` 3. `x + y[:i + j]` 4. `x[i:j] + y[2:3] + '0123456789'[2:7]` 5. `'abc' + x + y` 6. `'abc' + '1234567'` 7. `(x + 'a1B2c3D4' + y)[i:2j]`
`+=`	Append to x the value of y. Where x is a mutable string and y is either a string or a string range. eg: 1. `x += y` 2. `x += 'abc'` 3. `x += y[:i + j] + 'abc'` 4. `x += '0123456789'[2:7]`
`<=>`	Swap the values of x and y. Where x and y are mutable strings. (eg: `x <=> y`)

FUNCTION	DEFINITION
[]	The string size operator returns the size of the string being actioned. eg: 1. `'abc'[] == 3` 2. `var max_str_length := max(s0[],s1[],s2[],s3[]` 3. `('abc' + 'xyz')[] == 6` 4. `(('abc' + 'xyz')[1:4])[] == 4`

5.5.7 Control Structures

STRUCTURE	DEFINITION
`if`	If x is true then return y else return z.eg: 1. `if (x, y, z)` 2. `if ((x + 1) > 2y, z + 1, w / v)` 3. `if (x > y) z;` 4. `if (x <= 2*y) { z + w };`

`if-else`	The if-else/else-if statement. Subject to the condition branch the statement will return either the value of the consequent or the alternative branch. eg:
	1. `if (x > y) z; else w;`
	2. `if (x > y) z; else if (w != u) v;`
	3. `if (x < y) { z; w + 1; } else u;`
	4. `if ((x != y) and (z > w))`
	`{`
	`y := sin(x) / u;`
	`z := w + 1;`
	`}`
	`else if (x > (z + 1))`
	`{`
	`w := abs (x - y) + z;`
	`u := (x + 1) > 2y ? 2u : 3u;`
	`}`

`switch`	The first true case condition that is encountered will determine the result of the switch. If none of the case conditions hold true, the default action is assumed as the final return value. This is sometimes also known as a multi-way branch mechanism. eg:
	`switch`
	`{`
	`case x > (y + z) : 2 * x / abs(y - z);`
	`case x < 3 : sin(x + y);`
	`default : 1 + x;`
	`}`

`while`	The structure will repeatedly evaluate the internal statement(s) 'while' the condition is true. The final statement in the final iteration will be used as the return value of the loop. eg:
	`while ((x -= 1) > 0)`
	`{`
	`y := x + z;`
	`w := u + y;`
	`}`

FUNCTION	DEFINITION

`repeat/until`	The structure will repeatedly evaluate the internal statement(s) 'until' the condition is true. The final statement in the final iteration will be used as the return value of the loop. eg:
	
	`repeat`
	`  y := x + z;`
	`  w := u + y;`
	`until ((x += 1) > 100)`

`for`	The structure will repeatedly evaluate the internal statement(s) while the condition is true. On each loop iteration, an 'incrementing' expression is evaluated. The conditional is mandatory whereas the initialiser and incrementing expressions are optional. eg:
	
	`for (var x := 0; (x < n) and (x != y); x += 1)`
	`{`
	`  y := y + x / 2 - z;`
	`  w := u + y;`
	`}`

`break/break[]`	Break terminates the execution of the nearest enclosed loop, allowing for the execution to continue on external to the loop. The default break statement will set the return value of the loop to NaN, where as the return based form will set the value to that of the break expression. eg:
	
	`while ((i += 1) < 10)`
	`{`
	`  if (i < 5)`
	`    j -= i + 2;`
	`  else if (i % 2 == 0)`
	`    break;`
	`  else`
	`    break[2i + 3];`
	`}`

`continue`	Continue results in the remaining portion of the nearest enclosing loop body to be skipped. eg:
	
	`for (var i := 0; i < 10; i += 1)`
	`{`
	`  if (i < 5)`
	`    continue;`
	`  j -= i + 2;`
	`}`

FUNCTION	DEFINITION

`return`	Return immediately from within the current expression. With the option of passing back a variable number of values (scalar, vector or string). eg:
	1. `return [1];`
	2. `return [x, 'abx'];`
	3. `return [x, x + y,'abx'];`
	4. `return [];`
	5. `if (x < y)`
	`return [x, x - y, 'result-set1', 123.456];`
	`else`
	`return [y, x + y, 'result-set2'];`

`?:`	Ternary conditional statement, similar to that of the above denoted if-statement. eg: 1. `x ? y : z` 2. `x + 1 > 2y ? z + 1 : (w / v)` 3. `min(x,y) > z ? (x < y + 1) ? x : y : (w * v)`

`~`	Evaluate each sub-expression, then return as the result the value of the last sub-expression. This is sometimes known as multiple sequence point evaluation. eg:
	`~(i := x + 1, j := y / z, k := sin(w/u)) == (sin(w/u)))`
	`~{i := x + 1; j := y / z; k := sin(w/u)} == (sin(w/u)))`

`[*]`	Evaluate any consequent for which its case statement is true. The return value will be either zero or the result of the last consequent to have been evaluated. eg:
	`[*]`
	`{`
	`case (x + 1) > (y - 2) : x := z / 2 + sin(y / pi);`
	`case (x + 2) < abs(y + 3) : w / 4 + min(5y,9);`
	`case (x + 3) == (y * 4) : y := abs(z / 6) + 7y;`
	`}`

`[]`	The vector size operator returns the size of the vector being actioned. eg: 1. `v[]` 2. `max_size := max(v0[],v1[],v2[],v3[])`

Note: In the tables above, the symbols x, y, z, w, u and v where appropriate may represent any of one the following:

1. Literal numeric/string value
2. A variable
3. A vector element
4. A vector
5. A string
6. An expression comprised of [1], [2] or [3] (eg: `2 + x /vec[3])`

5.6 Fundamental types

ExprTk supports three fundamental types which can be used freely in expressions. The types are as follows:

Scalar Type The scalar type is a singular numeric value. The underlying type is that used to specialise the ExprTk components (float, double, long double, MPFR et al).

Vector Type The vector type is a fixed size sequence of contiguous scalar values. A vector can be indexed resulting in a scalar value. Operations between a vector and scalar will result in a vector with a size equal to that of the original vector, whereas operations between vectors will result in a vector of size equal to that of the smaller of the two. In both mentioned cases, the operations will occur element-wise.

String Type The string type is a variable length sequence of 8-bit chars. Strings can be assigned and concatenated to one another, they can also be manipulated via sub-ranges using the range definition syntax. Strings however can not interact with scalar or vector types.