Imperial College London

Neutral Theory and Fractals

High Performance Computing Programming Exercises

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Question 8

Perform a neutral theory simulation without speciation and plot species richness against time.

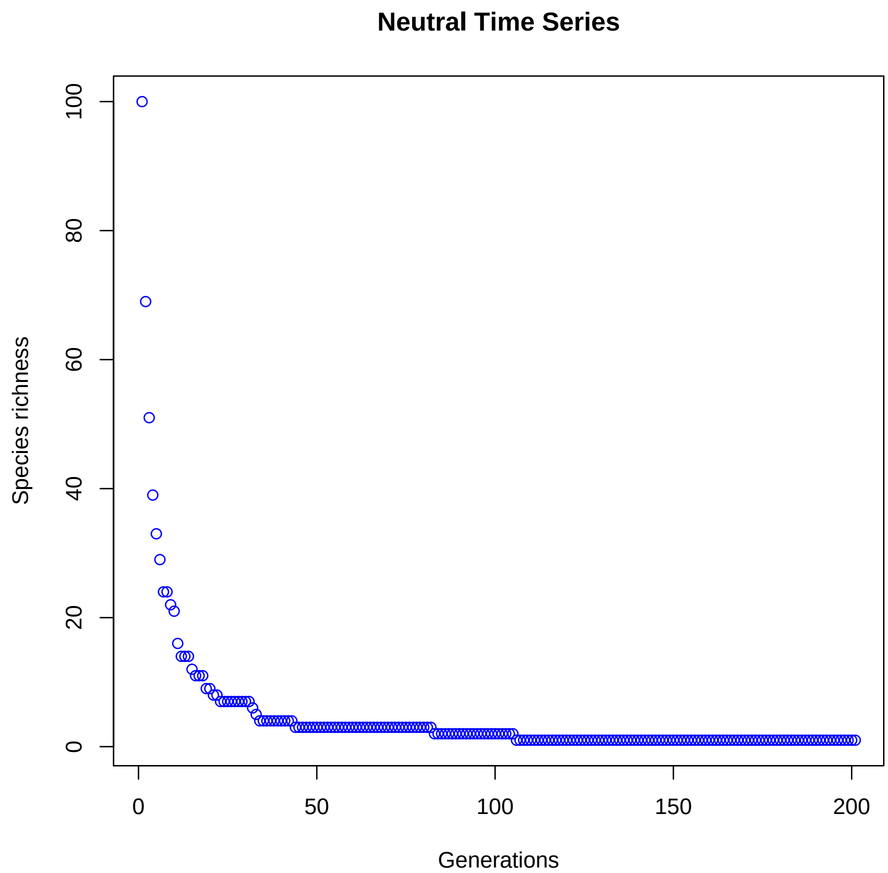

Figure 1: Time series of species richness over 200 generations. Derived from an initial condition of maximal diversity in a system with 100 individuals. The simulation is a zero sum assumption and does not feature speciation.

The above figure demonstrates that after many generations, this system will always converge to monodominance. This is because as a given individual dies, it is immediately replaced by the offspring of a different species. Over time the density of one particular species will increase, whilst the probablity of remaining in the community decreases for other species. Eventually, without speciation, the system will regress to one remaining species.

Question 12

Perform a neutral theory simulation with speciation and plot species richness against time. Do the initial conditions matter? Why?

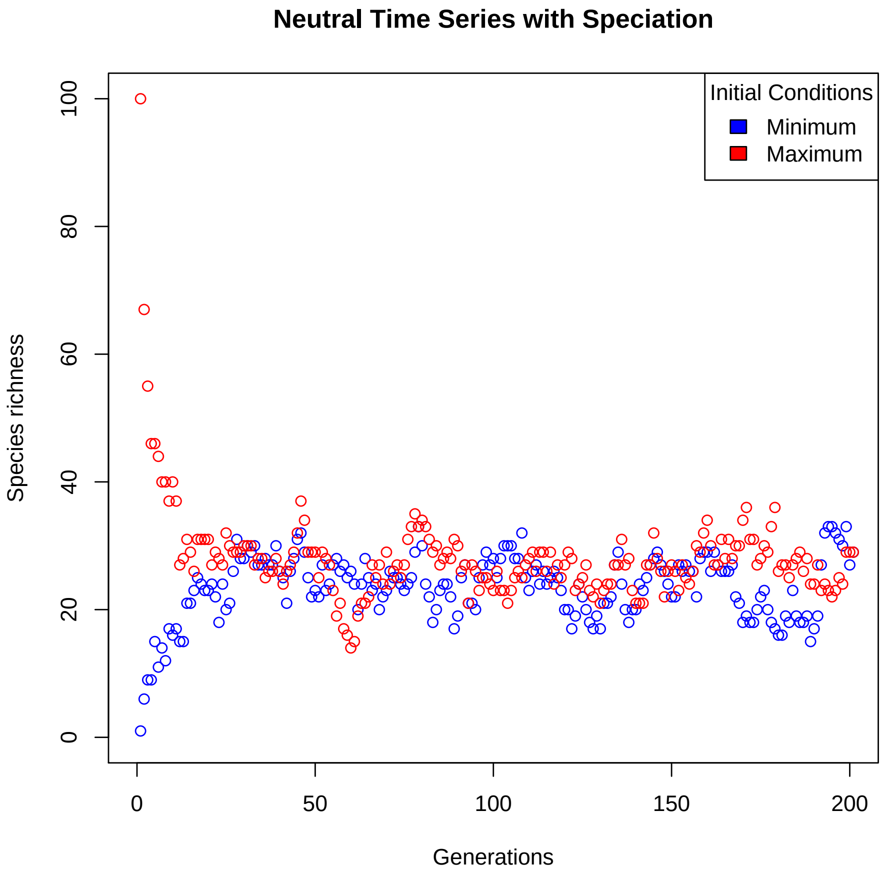

Figure 2: Time series of species richness over 200 generations. Derived from both minimum and maximal initial conditions of diversity in a system with speciation rate (ν= 0.1) and a community size of (J = 100).

This figure illustrates that the initial conditions of a community does not effect the species richness over time. Rather, the community dynamics are governed entirely by the speciation rate. Unlike figure 1, the species richness does not regress to a single species, but instead fluctuates between 15 and 35. This is due to the stochasticity introduced by ν, as new species arise in the population over time.

Question 16

Perform a neutral theory simulation with speciation for a 'burn in' period of 200 generations. Next record the species abundace octaves and repeatedly continue the simulation after the 'burn in' for another 2000 generations. Plot the average species abundance distribution. Do the initial conditions matter? Why?

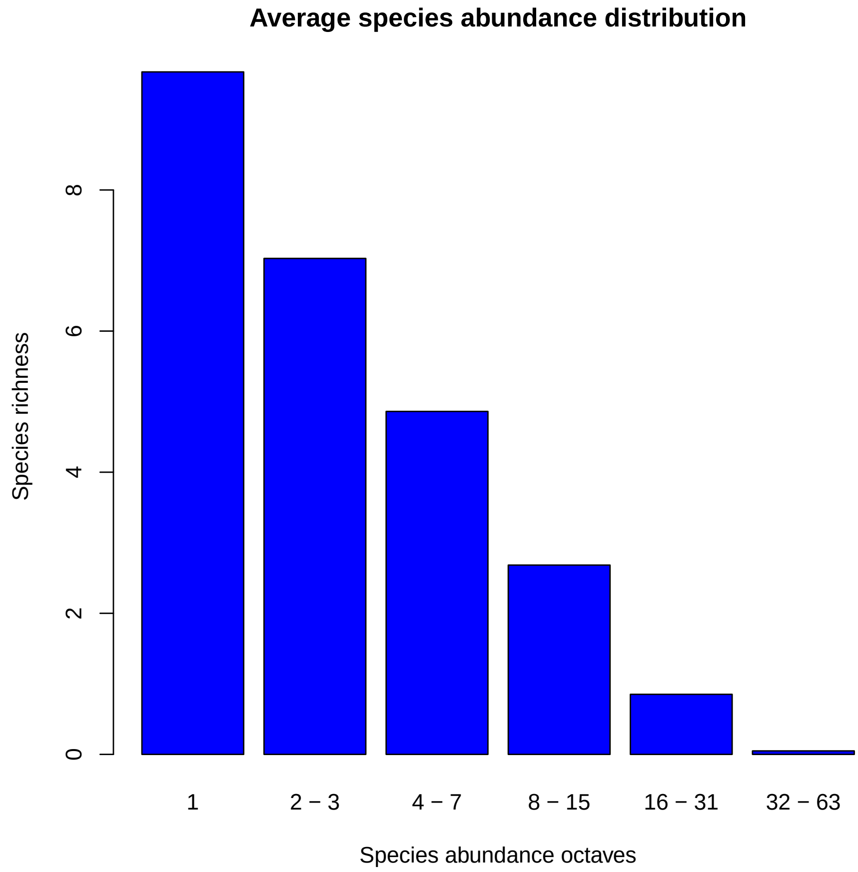

Figure 3: Bar chart of average species abundance distribution (octaves) after 2000 generations. Speciation rate (ν= 0.1) and a community size of (J = 100).

The initial conditions of the system do not impact the species richness of the community overall. During the 'burn in' period the community attains a state of equilibrium, therefore the true initial condition is the state of the community after 200 generations.

Question 20

Produce four bar charts of mean species abundance octaves for J = 500, J = 1000, J = 2500 and J = 5000 with a speciation rate of 0.00585.

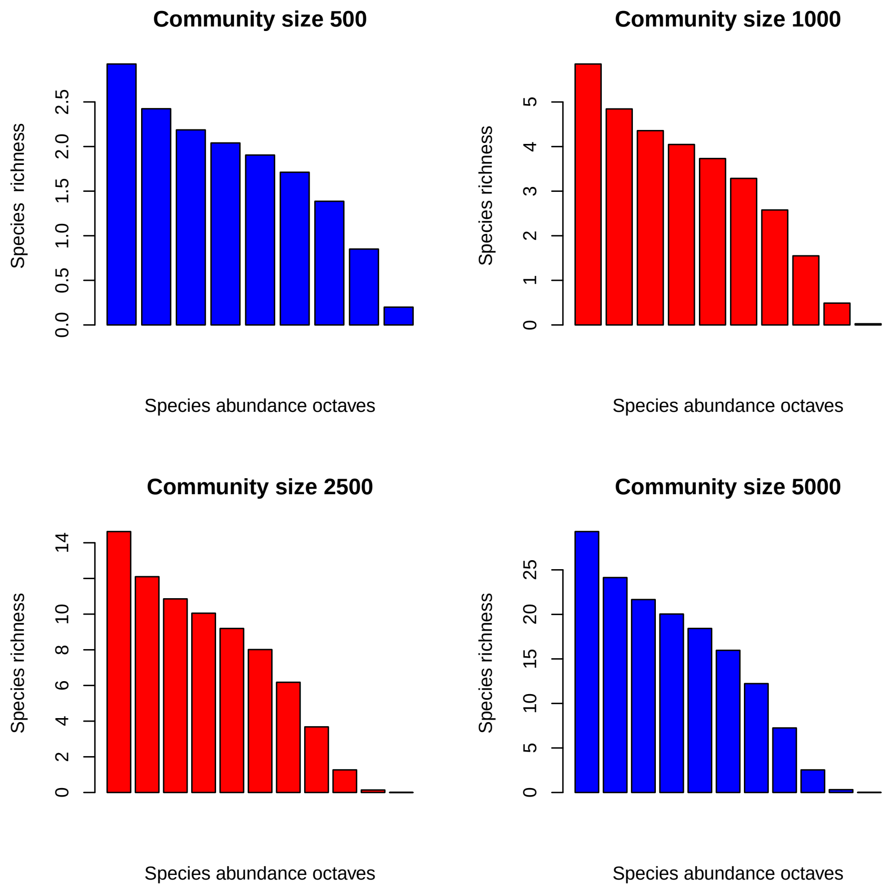

Figure 4: Average species abundance octaves for different community size generated from a cluster run of 100 times in parallel.

Question 21

What are the fractal dimensions of the Sierpinski carpet and the Menger sponge?

The Sierpinski Carpet

The Sierpinski Carpet is made up of 8 self-similar versions of itself on a scale of by $\frac{1}{3}$ of the original copy. Therefore, the dimensions of the Sierpinski Carpet can be calculated as follows:

Take the usual equation for calculating dimensions where n is the number of segments and s is the scale.

$$n = \frac{1}{s^D} \tag{1}$$

Replace n with 8 and s with $\frac{1}{3}$. This can be simplified to:

$$8 = \frac{1}{(\frac{1}{3})s^D} \tag{2}$$

Or more simply:

$$8 = 3^D \tag{3}$$

To find D, log both sides of the equation. This gives the dimensions of the Sierpinski Carpet.

$$D = \frac{log(8)}{log(3)} \approx 1.893 \tag{4}$$

The Menger Sponge

The Menger Sponge is made up of 27 self-similar versions of itself on a scale of $\frac{1}{3}$ of the original copy. However, the object is hollow in the centre, leaving 20 self-similar versions. The object can be calculated using the same method as the Sierpinski Carpet.The dimensions for the Menger sponge are:

Take the usual equation for calculating dimensions where n is the number of segments and s is the scale factor.

$$n = \frac{1}{s^D} \tag{5}$$

Replace n with 20 and s with $\frac{1}{3}$. This can be simplified to:

$$20 = \frac{1}{(\frac{1}{3})s^D} \tag{6}$$

Or more simply:

$$20 = 3^D \tag{7}$$

To find D, log both sides of the equation. This gives the dimensions of the Menger Sponge.

$$D = \frac{log(20)}{log(3)} \approx 2.727 \tag{8}$$

Question 22

The Chaos game

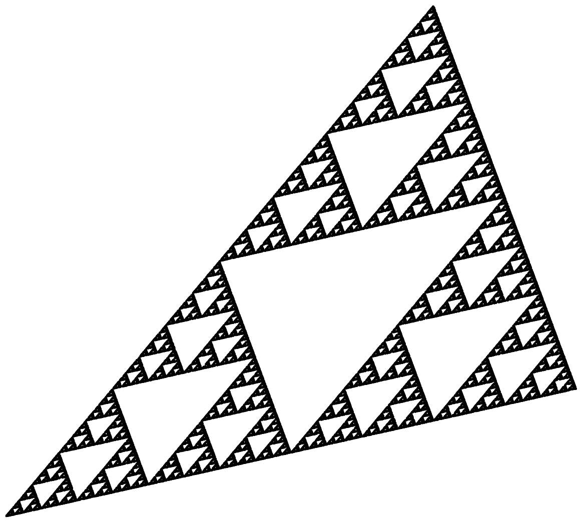

Figure 5: A fractal object created with three points and an initial point selected within at random. The process is repeated over 10,000 iterations.

Challenge Question E

Try starting the chaos game from a completely different initial position for X. What happens now and why?

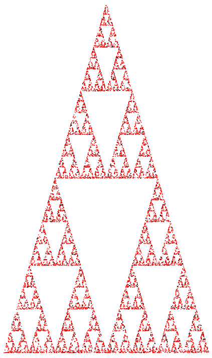

Figure 6: A Sierpinski triangle created with three points and a different initial point selected within at random. The process is repeated over 10,000 iterations. Points in black represent the first 1000 iterations of the chaos game.

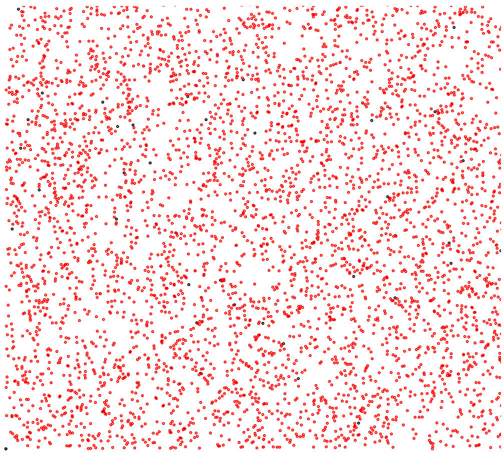

Figure 7: Using 4 coordinated and an initial point selected at random. The process is repeated over 10,000 iterations. Points in black represent the first 1000 iterations of the chaos game.

The fractal object produced is the same regardless of the initial position chosen, even if the initial position lies outside of the attractor. The (very tiny) black points of the first 1000 interations do not radiate out from any definitive starting position. This is because for each point in the sequence, a factor of $\frac{1}{2}$ is applied between the original point and a random target vertex. This method will result in randomly positioned points all across the attractor. When four points are given, the points completely fill the attractor at random. Sadly no fractal appears but we are instead left with a work of art Yayoi Kusama would be proud of.

Question 25

Spiral

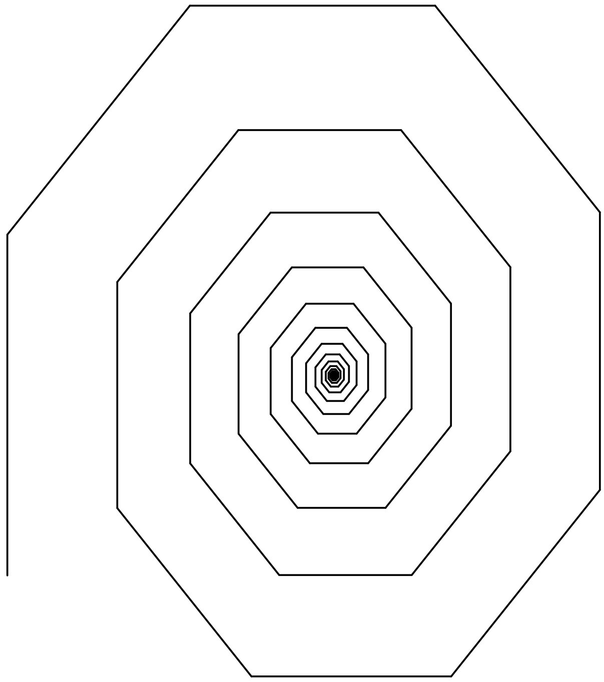

Figure 8: A spiral generated iteratively from a given start point, direction and length.

When generating the spiral in R, the following error message is given: **Error: C stack usage 7971072 is too close to the limit**. This is because in our aim to produce an infinitesimal fractal object, we have overlooked the fact that it is impossible for the computer to allocate enough memory to this calculation. This error can be overcome by defining a minimum length, in this case, 0.000000001.

Question 27

Tree

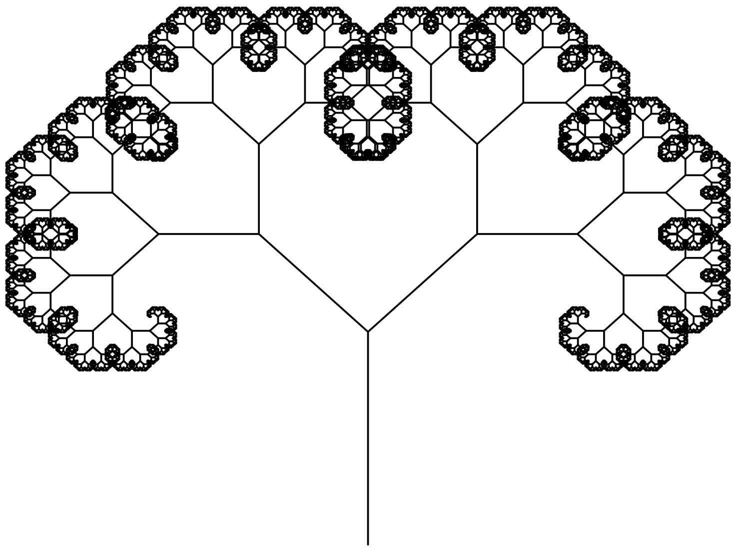

Figure 9: A fractal tree.

Question 29

Fern

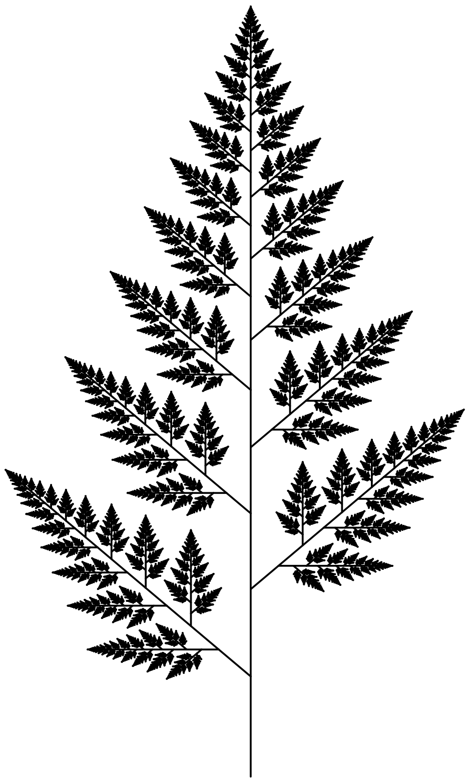

Figure 10: Arguably the jewel in the crown of the HPC exercises. A fern fractal object.

Challenge F

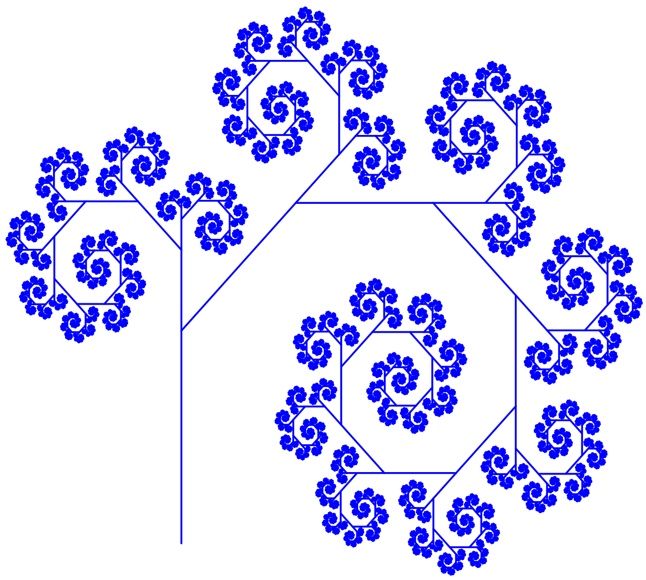

Figure 11: The physical impossibility of infinity in the mind of someone finite.

The image produced by smaller line size thresholds takes longer to run and results in a 'fluffier' fractal. This is because as the line threshold decreases, more self-similar units are produced at smaller and smaller self-similar scales.

Artist statement

My work centres on the existential dread brought on by the fullest feeling of the sublime, i.e. the immensity of the universe's extent and duration. This is symbolised here in the infinite self-similarity of the above fractal. Although the observer may be under the illusion that they are viewing the whole object, it is impossible and in fact absurd to attempt to grasp the full extent of this infinite object. Thus, this reflects the absurdity of human kind to search for meaning in the world beyond that which we give it.