Exercise in the Camera Model

Henrik Aanæs, haa@imm.dtu.dk

February 4, 2015

This is the first exercise in view geometry, related to Chapter 1 of my lecture notes. First there will be a few questions on homogeneous coordinates, followed by tasks on computing with the pinhole camera.

1 Homogeneous Coordinates

Question 1. What are the inhomogeneous version of the following 2D homogeneous points

$$\begin{bmatrix}1\\2\\1\end{bmatrix} \quad , \quad \begin{bmatrix}4\\2\\2\end{bmatrix} \quad , \quad \begin{bmatrix}6\\4\\-1\end{bmatrix} \quad , \quad \begin{bmatrix}5\\3\\0.5\end{bmatrix} \quad ?$$

Question 2. What is the inhomogeneous version of the following 3D homogeneous points

$$\begin{bmatrix}1\\10\\-3\\1\end{bmatrix} \quad , \quad \begin{bmatrix}2\\-4\\1.1\\2\end{bmatrix} \quad , \quad \begin{bmatrix}0\\0\\-1\\10\end{bmatrix} \quad , \quad \begin{bmatrix}-15\\3\\6\\3\end{bmatrix} \quad ?$$

Question 3. A line is given by

$$x + 2y = 3 \quad .$$

If this were to be written in the homogeneous form, i.e. $\mathbf{l^T q} = \mathbf{0}$, how would l look?

Question 4. Given the line in question 3, which of the following 2D homogeneous coordinates are on this line?

1. $[3, 0, 1]^T$
2. $[6, 0, 2]^T$
3. $[1, 1, 2]^T$
4. $[1, 1, 1]^T$
5. $[110, -40, 10]^T$

6. $[11, 4, 1]^T$

Question 5. Using the theory of homogeneous coordinates, what is the intersection of the two lines given by

$$\mathbf{l_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \quad and \quad \mathbf{l_2} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \text{ ?}$$

Question 6. What is the effect of multiplying a homogeneous 2D point $\mathbf{x}$ by the following matrix $\mathbf{A}$, i.e. $\mathbf{Ax}$, where $\mathbf{A}$ is given by:

$$\mathbf{A} = \begin{bmatrix} 10 & 0 & 2 \\ 0 & 10 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 7. With the matrix $\mathbf{A}$ given in Question 6., compute $\tilde{\mathbf{x}} = \mathbf{Ax}$ and convert $\tilde{\mathbf{x}}$ to inhomogeneous coordinates, and validate your interpretation from Question 6. with $\mathbf{x}$ being equal to

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 3 \\ 0.5 \end{bmatrix},$$

which corresponds to the homogeneous coordinates from Question 1.

Question 8. A line is given by $\mathbf{l^T x} = \mathbf{0}$ for homogeneous points $\mathbf{x}$ and

$$\mathbf{l} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -1 \end{bmatrix}.$$

What is the distance of the homogeneous points $\mathbf{x}$ to this line, when the points are given by

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 1 \end{bmatrix}, \quad \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 4 \end{bmatrix} \text{ ?}$$

Question 9. What would be the answer in Question 8. if $\mathbf{l}$ was changed to

$$\mathbf{l} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \text{ ?}$$

2 Pinhole Camera – Outer Orientation

Here you should exercise in computing with the pinhole camera model, in particular the outer orientation corresponding to the cameras position relative to the object. For you to have something to 'photograph' a sample object is supplied in the accompanying file `Box3D`. To see this box try the following script:

```
clear
close all
Q=Box3D;
plot3(Q(1,:),Q(2,:),Q(3,:),'.'),
axis equal
axis([-1 1 -1 1 -1 5])
xlabel('x')
ylabel('y')
zlabel('z')
```

To keep it simple try to project this 3D object $\mathbf{Q}$ via the simplest projection matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} .$$

The result is obtained as follows, given you have defined `P`:

```
figure
q=P*[Q;ones(1,size(Q,2))];
q(1,:)=q(1,:)./q(3,:);
q(2,:)=q(2,:)./q(3,:);
q(3,:)=q(3,:)./q(3,:);
plot(q(1,:),q(2,:),'.')
axis([-0.3 0.3 -0.3 0.3])
```

Question 10. Try the above MatLab code, using:

$$\mathbf{P} = \mathbf{A}[\mathbf{R}\mathbf{t}] \quad , \quad \mathbf{A} = \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad \mathbf{t} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} .$$

Question 11. Try to change the outer orientation of the camera by setting the rotation to

$$\mathbf{R} = \begin{bmatrix} 0.9397 & 0.3420 & 0 \\ -0.3420 & 0.9397 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} .$$

Do the projection with the new $\mathbf{P}$. What is the effect? Explain this by the appearance of $\mathbf{R}$.

Question 12. Try to further change the translation to

$$\mathbf{t} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} .$$

Do the projection with the new $\mathbf{P}$. What is the effect? Explain this by the appearance of $\mathbf{t}$.

3 Pinhole Camera – Inner Orientation

Now we will also try experimenting with the inner orientation, which is a model of the cameras optics. Since this will give some other units of the resulting 'image' change your plotting commands from

```
plot(q(1,:),q(2,:),'.')
axis([-0.3 0.3 -0.3 0.3])
```

to

```
plot(q(1,:),q(2,:),'.')
axis equal
axis([0 640 0 480])
```

Question 13. Change the inner orientation $\mathbf{A}$ to

$$\mathbf{A} = \begin{bmatrix} f & 0 & \Delta x \\ 0 & f & \Delta y \\ 0 & 0 & 1 \end{bmatrix} ,$$

where $f = 1000$, $\Delta x = 200$, and $\Delta y = 200$. Try this projection of the Box and explain.

Question 14. In Question 13. change Δx to 300, project and explain.

Question 15. Change the focal length f to 1200, and keep $\Delta x = 300$, and $\Delta y = 200$. Project and explain.

Question 16. Change the focal length f to 2000, and keep $\Delta x = 300$, and $\Delta y = 200$. Also change the translation to

$$\mathbf{t} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} .$$

Project and compare to the result in Question 14. Explain.