Discrete mathematics course
Chapter 5: Modeling Computation

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Formal language

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Example: The rabbit eats quickly.

Phrase-structure grammars

Definition 1.1.

A vocabulary (or alphabet) V is a finite, nonempty set of elements called symbols. A word (or sentence) over V is a string of finite length of elements of V. The empty string or null string, denoted by λ, is the string containing no symbols. The set of all words over V is denoted by V^*. A language over V is a subset of V^*.

Phrase-structure grammars

Definition 1.2.

A phrase-structure grammar $G = (V, T, S, P)$ consists of a vocabulary V, a subset T of V consisting of terminal symbols, a start symbol S from V, and a finite set of productions P. The set $V - T$ is denoted by N. Elements of N are called nonterminal symbols. Every production in P must contain at least one nonterminal on its left side.

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Example: let $G = (V, T, S, P)$, where $V = \{a, b, A, B, S\}$, $T = \{a, b\}$, S is the start symbol, and $P = \{S \to ABa, A \to BB, B \to ab, AB \to b\}$. G is an example of a phrase-structure grammar.

Phrase-structure grammars

Definition 1.3.

Let $G = (V, T, S, P)$ be a phrase-structure grammar. Let $w_0 = lz_0r$ (that is, the concatenation of l, z_0, and r) and $w_1 = lz_1r$ be strings over V. If $z_0 \to z_1$ is a production of G, we say that w_1 is directly derivable from w_0 and we write $w_0 \Rightarrow w_1$. If $w_0, w_1, ..., w_n$ are strings over V such that $w_0 \Rightarrow w_1, w_1 \Rightarrow w_2, ..., w_{n1} \Rightarrow w_n$,then we say that w_n is derivable from w_0, and we write $w_0 \overset{*}{\Rightarrow} w_n$. The sequence of steps used to obtain w_n from w_0 is called a derivation.

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Example: the string $Aaba$ is directly derivable from ABa in the grammar in the previous example, because $B \to ab$ is a production in the grammar. The string $abababa$ is derivable from ABa because $ABa \Rightarrow Aaba \Rightarrow BBaba \Rightarrow Bababa \Rightarrow abababa$, using the productions $B \to ab, A \to BB, B \to ab, andB \to ab$ in succession.

Phrase-structure grammars

Definition 1.4.

Let $G = (V, T, S, P)$ be a phrase-structure grammar. The language generated by G (or the language of G), denoted by $L(G)$, is the set of all strings of terminals that are derivable from the starting state S. In other words,

$$L(G) = \{w \in T^* | S \overset{*}{\Rightarrow} w\}.$$

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Example: let G be the grammar with vocabulary $V = S, A, a, b$, set of terminals $T = \{a, b\}$, starting symbol S, and productions $P = \{S \to aA, S \to b, A \to aa\}$. What is $L(G)$?

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Example: let G be the grammar with vocabulary $V = \{S, 0, 1\}$, set of terminals $T = \{0, 1\}$, starting symbol S, and productions $P = \{S \to 11S, S \to 0\}$. What is $L(G)$?

Exercises

- Exercise: give a phrase-structure grammar that generates the set $\{0^n 1^n | n = 0, 1, 2, ...\}$.

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- Exercise: find a phrase-structure grammar to generate the set $\{0^m1^n|m \quad and \quad n \quad are \quad nonnegative \quad integers\}$.

Types of Phrase-structure grammars

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- A **type 2** grammar can have productions only of the form $w_1 \to w_2$, where w_1 is a single symbol that is not a terminal symbol. (context-free grammars)
- A **type 3** grammar can have productions only of the form $w_1 \to w_2$ with $w_1 = A$ and either $w_2 = aB$ or $w_2 = a$,where A and B are nonterminal symbols and a is a terminal symbol, or with $w_1 = S$ and $w_2 = \lambda$. (regular grammars)

Derivation trees

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- The leaves of the tree represent the terminal symbols that arise.

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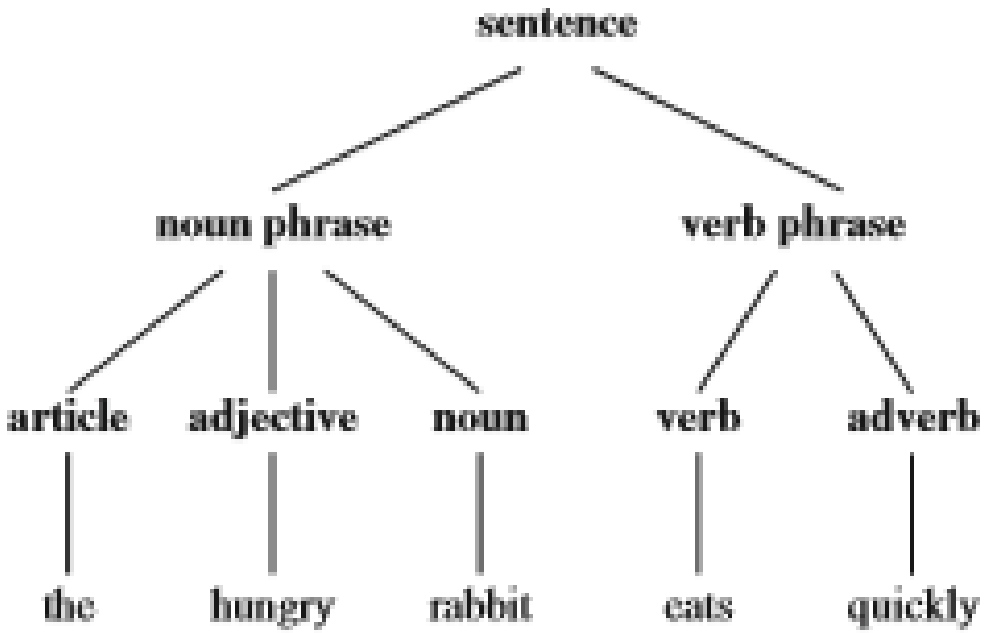

FIGURE 1 **A Derivation Tree.**

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- Use in the specification of the programming language ALGOL.
- The BackusNaur form is used to specify the syntactic rules of many computer languages, including Java.

Definitions

Definition 2.1.

A finite-state machine $M = (S, I, O, f, g, s_0)$ consists of a finite set S of states, a finite input alphabet I, a finite output alphabet O, a transition function f that assigns to each state and input pair a new state, an output function g that assigns to each state and input pair an output, and an initial state s_0.

Examples

- Construct the state diagram for the finite-state machine with the state table shown in Table on the left.

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TABLE 2

	f		g	
	Input		*Input*	
State	**0**	**1**	**0**	**1**
s_0	s_1	s_0	1	0
s_1	s_3	s_0	1	1
s_2	s_1	s_2	0	1
s_3	s_2	s_1	0	0

Examples

- Construct the state diagram for the finite-state machine with the state table shown in Table on the left.

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State	f Input 0	f Input 1	g Input 0	g Input 1
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s_1	s_3	s_0	1	1
s_2	s_1	s_2	0	1
s_3	s_2	s_1	0	0

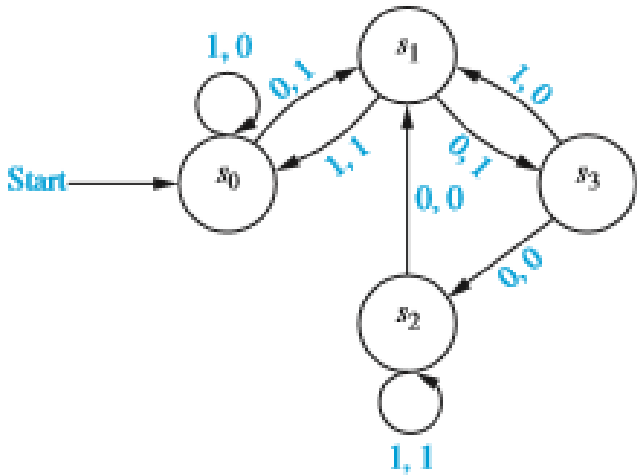

FIGURE 2 The State Diagram for the Finite-State Machine Shown in Table 2.

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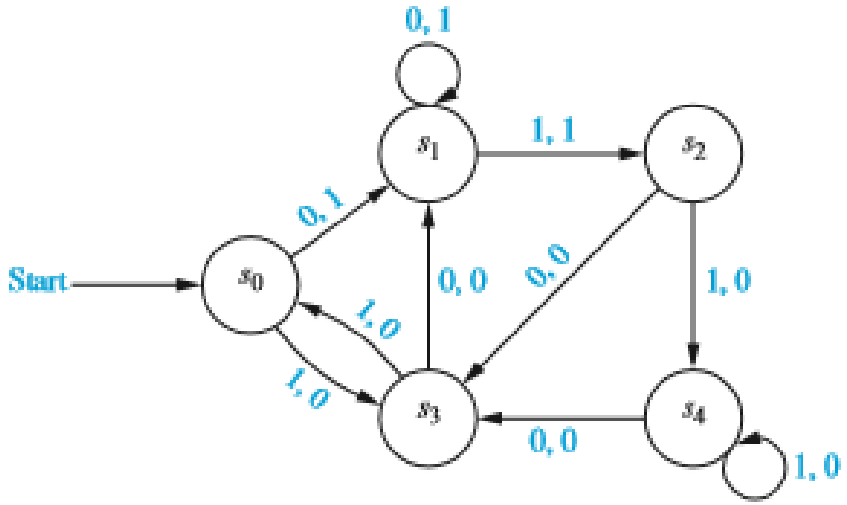

FIGURE 3 **A Finite-State Machine.**

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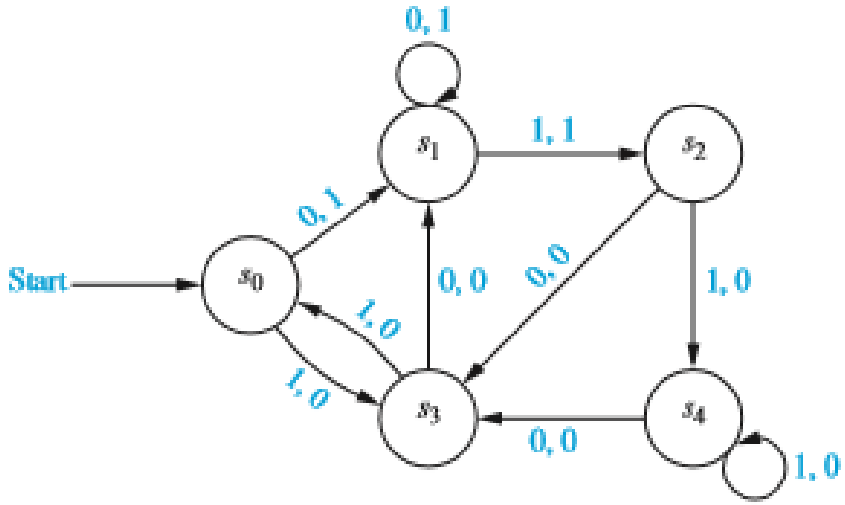

FIGURE 3 A Finite-State Machine.

TABLE 3

State	f Input 0	f Input 1	g Input 0	g Input 1
s_0	s_1	s_3	1	0
s_1	s_1	s_2	1	1
s_2	s_3	s_4	0	0
s_3	s_1	s_0	0	0
s_4	s_3	s_4	0	0

Examples

- Find the output string generated by the finite-state machine in previous Figure if the input string is 101011.

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TABLE 4

Input	1	0	1	0	1	1	—
State	s_0	s_3	s_1	s_2	s_3	s_0	s_3
Output	0	0	1	0	0	0	—

Examples

- In a certain coding scheme, when three consecutive 1s appear in a message, the receiver of the message knows that there has been a transmission error. Construct a finite-state machine that gives a 1 as its current output bit if and only if the last three bits received are all 1s.

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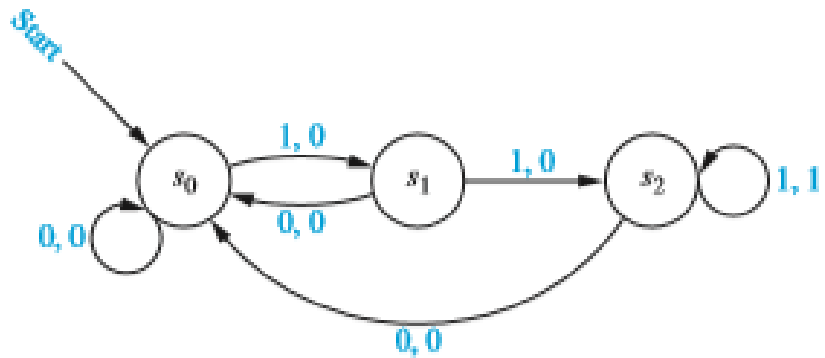

FIGURE 6 **A Finite-State Machine That Gives an Output of 1 If and Only If the Input String Read So Far Ends with 111.**

Definitions

Definition 2.2.

Let $M = (S, I, O, f, g, s_0)$ be a finite-state machine and $L \subseteq I^*$. We say that M recognizes (or accepts) L if an input string x belongs to L if and only if the last output bit produced by M when given x as input is a 1.

Set of Strings

Definition 3.1.

Suppose that A and B are subsets of V^*, where V is a vocabulary. The *concatenation* of A and B, denoted by AB, is the set of all strings of the form xy, where x is a string in A and y is a string in B.

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- Example: let $A = \{1, 00\}$. Find A^n for n=0,1,2, and 3.

Set of Strings

Definition 3.2.

Suppose that A is a subset of V^*. Then the *Kleene closure* of A, denoted by A^*, is the set consisting of concatenations of arbitrarily many strings from A. That is, $A^* = \cup_{k=0}^{\infty} A^k$

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- Example: what are the Kleene closures of the sets $A = \{0\}, B = \{0, 1\}$, and $C = \{11\}$?

Finite-State Automata

Definition 3.3.

A finite-state automaton $M = (S, I, f, s_0, F)$ consists of a finite set S of states, a finite input alphabet I, a transition function f that assigns a next state to every pair of state and input (so that $f : S \times I \to S$), an initial or start state s_0, and a subset F of S consisting of final (or accepting states).

Finite-State Automata

- Construct the state diagram for the finite-state automaton $M = (S, I, f, s_0, F)$, where $S = \{s_0, s_1, s_2, s_3\}$, $I = \{0, 1\}$, $F = \{s_0, s_3\}$, and the transition function f is given in the Table below.

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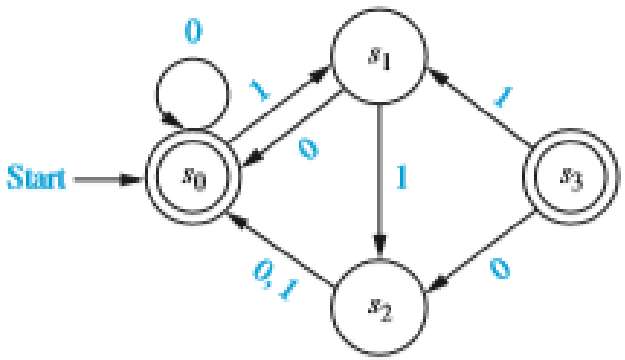

FIGURE 1 The State Diagram for a Finite-State Automaton.

Language Recognition by Finite-State Machines

Definition 3.4.

A string x is said to be recognized or accepted by the machine $M = (S, I, f, s_0, F)$ if it takes the initial state s_0 to a final state, that is, $f(s_0, x)$ is a state in F. The language recognized or accepted by the machine M, denoted by $L(M)$, is the set of all strings that are recognized by M. Two finite-state automata are called equivalent if they recognize the same language.

Language Recognition by Finite-State Machines

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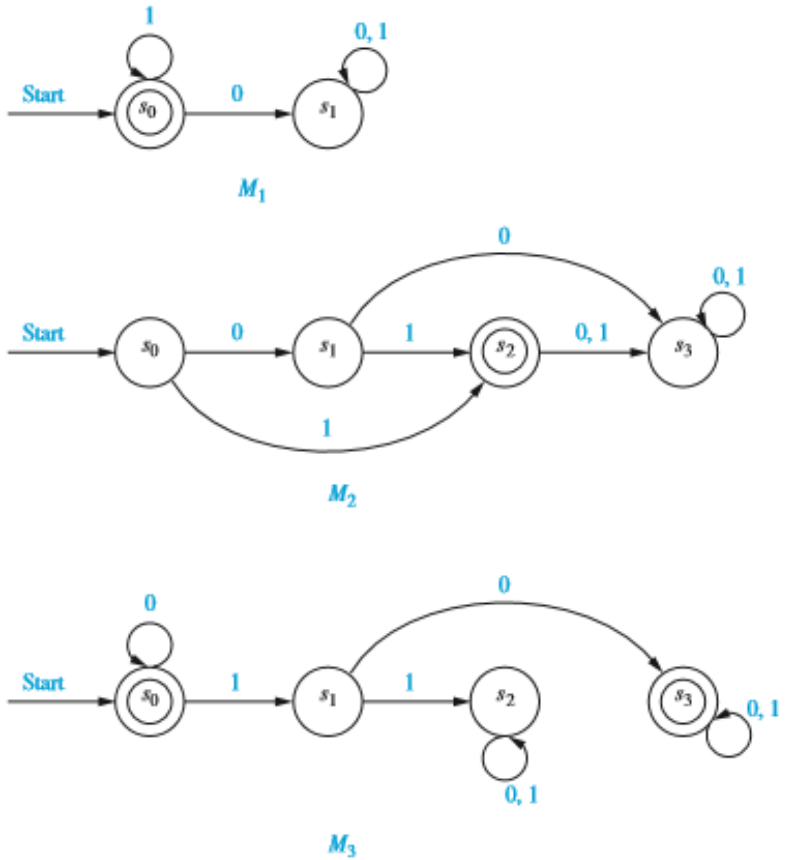

Non-deterministic Finite-State Automata

Definition 3.5.

A non-deterministic finite-state automaton $M = (S, I, f, s_0, F)$ consists of a set S of states, an input alphabet I, a transition function f that assigns a set of states to each pair of state and input (so that $f : S \times I \to P(S)$), a starting state s_0, and a subset F of S consisting of the final states.

Nondeterministic Finite-State Automata

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s_1	s_3	s_4
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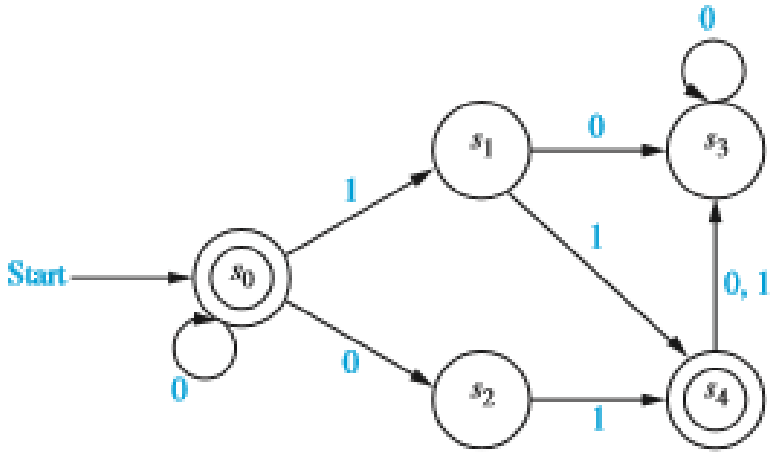

FIGURE 7 **A Nondeterministic Finite-State Automaton.**

Nondeterministic Finite-State Automata

Theorem 3.6.

If the language L is recognized by a non-deterministic finite-state automaton M_0, then L is also recognized by a deterministic finite-state automaton M_1.

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- Example: what are the strings in the regular sets specified by the regular expressions 10^*,$(10)^*$, $0 \cup 01$, $0(0 \cup 1)^*$, and $(0^*1)^*$?

Kleene Theorem

Theorem 4.2 (Kleene theorem).

A set is regular if and only if it is recognized by a finite-state automaton.

Regular Sets and Regular Grammars

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- Example: construct a non-deterministic finite-state automaton that recognizes the language generated by the regular grammar $G = (V, T, S, P)$, where $V = \{0, 1, A, S\}$, $T = \{0, 1\}$, and the productions in P are $S \to 1A$, $S \to 0$, $S \to \lambda$, $A \to 0A$, $A \to 1A$, and $A \to 1$.

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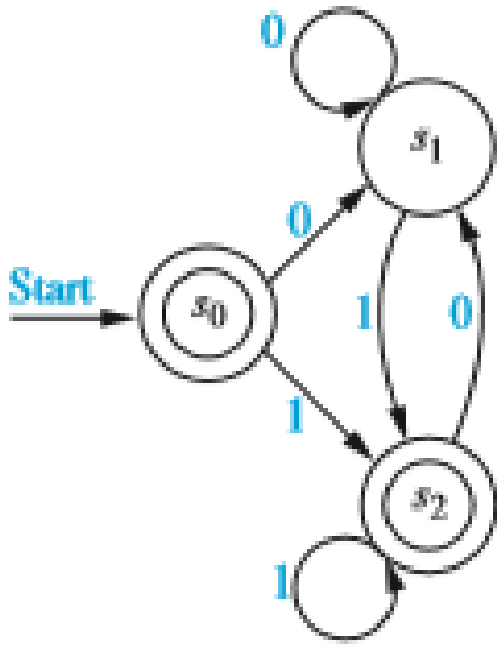

A Set Not Recognized by a Finite-State Automaton

- Example: Show that the set $\{0^n 1^n | n = 0, 1, 2, ...\}$, made up of all strings consisting of a block of 0s followed by a block of an equal number of 1s, is not regular.

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- Linear bounded automaton
- Turing machine

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- Turing machines have read and write capabilities on the tape as the control unit moves back and forth along this tape, changing states depending on the tape symbol read.

Definition

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A Turing machine $T = (S, I, f, s_0)$ consists of a finite set S of states, an alphabet I containing the blank symbol B, a partial function f from $S \times I$ to $S \times I \times \{R, L\}$, and a starting state s_0.

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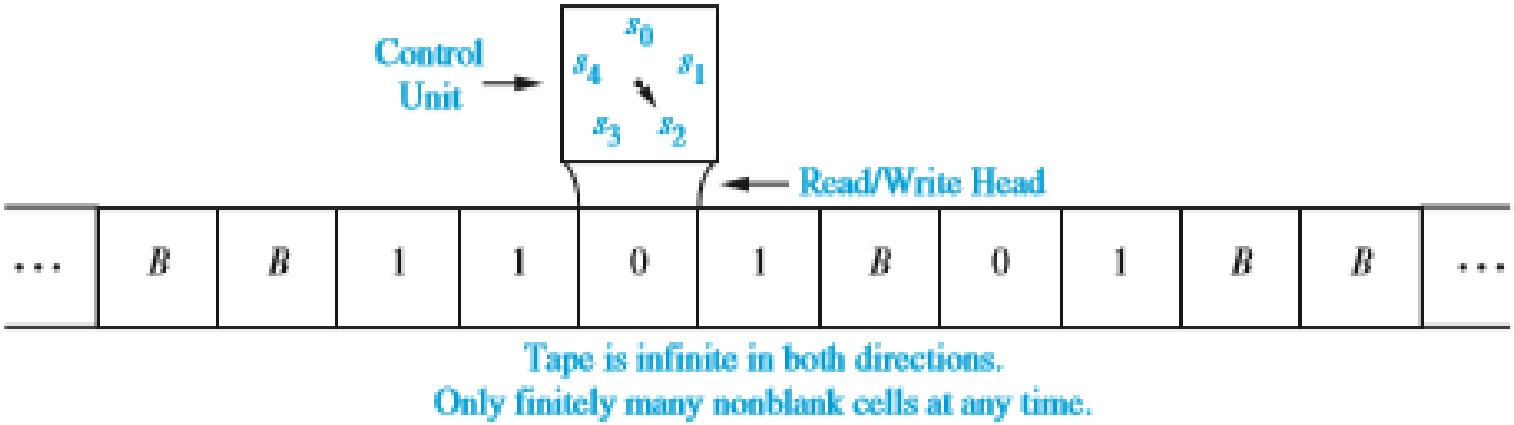

FIGURE 1 A Representation of a Turing Machine.

Example

- What is the final tape when the Turing machine T defined by the seven five-tuples $(s_0, 0, s_0, 0, R), (s_0, 1, s_1, 1, R), (s_0, B, s_3, B, R), (s_1, 0, s_0, 0, R), (s_1, 1, s_2, 0$ and $(s_2, 1, s_3, 0, R)$ is run on the tape shown in Figure (a) below?

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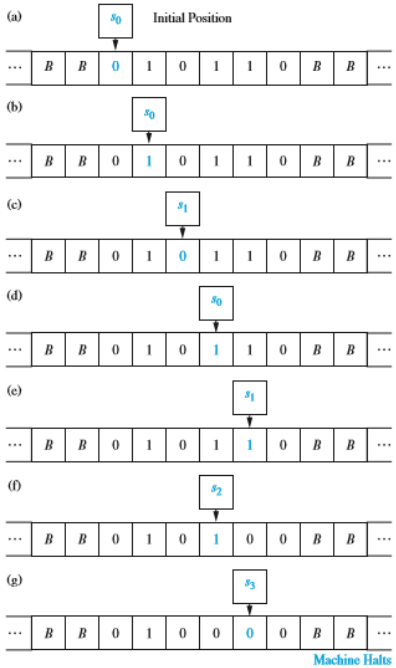

Turing Machines to Recognize Sets

Definition 5.2.

Let V be a subset of an alphabet I. A Turing machine $T = (S, I, f, s_0)$ recognizes a string x in V^* if and only if T, starting in the initial position when x is written on the tape, halts in a final state. T is said to recognize a subset A of V^* if x is recognized by T if and only if x belongs to A.

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Some definitions

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Theorem 5.5.

The halting problem is an unsolvable decision problem. That is, no Turing machine exists that, when given an encoding of a Turing machine T and its input string x as input, can determine whether T eventually halts when started with x written on its tape.

Some definitions

Definition 5.6.

A decision problem is in P, the *class of polynomial-time problems*, if it can be solved by a deterministic Turing machine in polynomial time in terms of the size of its input. That is, a decision problem is in P if there is a deterministic Turing machine T that solves the decision problem and a polynomial $p(n)$ such that for all integers n, T halts in a final state after no more than $p(n)$ transitions whenever the input to T is a string of length n. A decision problem is in NP, the *class of non-deterministic polynomial-time problems*, if it can be solved by a non-deterministic Turing machine in polynomial time in terms of the size of its input. That is, a decision problem is in NP if there is a non-deterministic Turing machine T that solves the problem and a polynomial $p(n)$ such that for all integers n, T halts for every choice of transitions after no more than $p(n)$ transitions whenever the input to T is a string of length n.