Machine Learning - 2020-2021
Universitat Pompeu Fabra

Problems 6A: Support Vector Machine

Goal

The goal of this practice is to understand how Support Vector Machine is defined and work in supervised classification problems.

Exercises

1. Consider two classes: class $C_1 = \{\mathbf{x}_1 = \binom{1}{2}\}$ with a label 1 and class $C_2 = \{\mathbf{x}_2 = \binom{-1}{-2}, \mathbf{x}_3 = \binom{-2}{-1}\}$ with label -1.

   (a) Plot the points and write in parametric form the Support Vector Machine (SVM) classifier $g(\mathbf{x})$.

   (b) Write the primal problem, i.e., the optimisation problem that provides the SVM classifier.

   (c) Write the dual Lagrangian problem. Solve it and compute the values of α_i, $i = 1, 2, 3$.

   (d) Write the final classifier $g(\mathbf{x})$ and draw the decision hyperplane.

   (e) What is the margin value of the obtained classifier?

2. Consider two classes: the class $C_1 = \{\mathbf{x}_1 = \binom{0}{-1}\}$ with a label 1 and the class $C_2 = \{\mathbf{x}_2 = \binom{0}{1}, \mathbf{x}_3 = \binom{1}{1}\}$ with label -1.

   (a) Plot the points and write in parametric form the Support Vector Machine (SVM) classifier $g(\mathbf{x})$.

   (b) Write the primal problem, i.e., the optimisation problem that provides the SVM classifier.

   (c) Write the dual Lagrangian $\mathcal{L}_D$ and derive the values of $\alpha_i, i = 1, 2, 3$.

   (d) Write the final classifier $g(\mathbf{x})$ and draw the decision hyperplane.

   (e) What is the margin value of the obtained classifier?