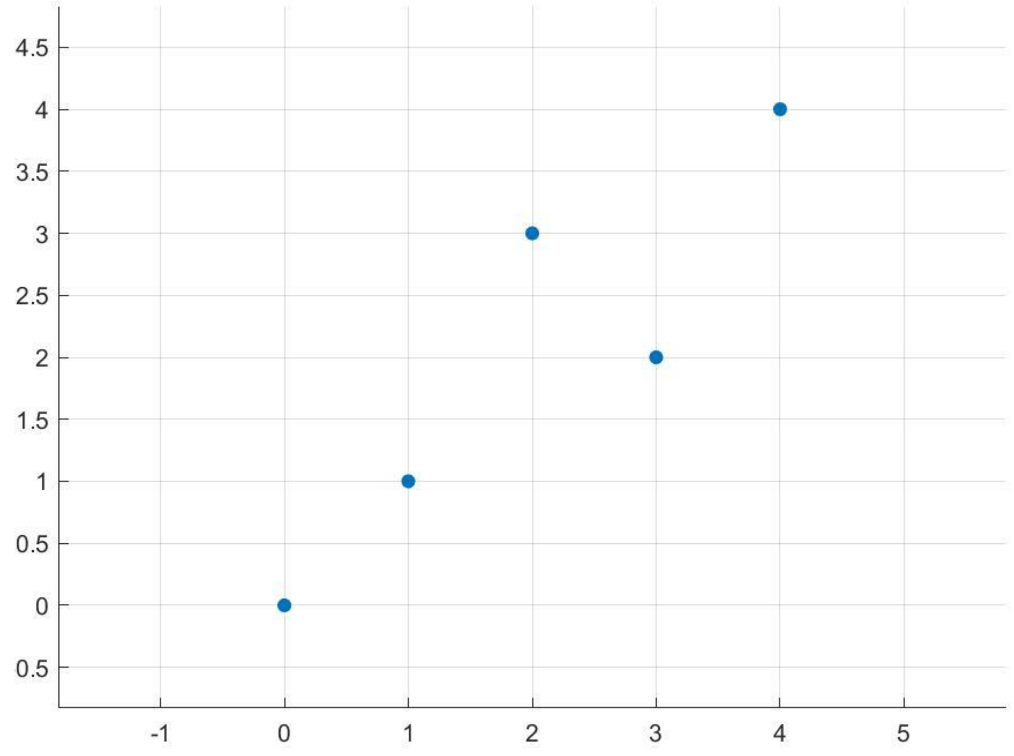
EXERCICE 1

Suppose that we have a set of 2D samples: $\left\{\boldsymbol{x}^{(1)}=\begin{pmatrix}0\\0\end{pmatrix}, \boldsymbol{x}^{(2)}=\begin{pmatrix}1\\1\end{pmatrix}, \boldsymbol{x}^{(3)}=\begin{pmatrix}2\\3\end{pmatrix}, \boldsymbol{x}^{(4)}=\begin{pmatrix}3\\2\end{pmatrix}, \boldsymbol{x}^{(5)}=\begin{pmatrix}4\\4\end{pmatrix}\right\} \subset \mathbb{R}^2$.
Answer the following questions doing all the computations manually.

(a) Draw the data and the eigenvectors that you think that the covariance matrix of the data will have.



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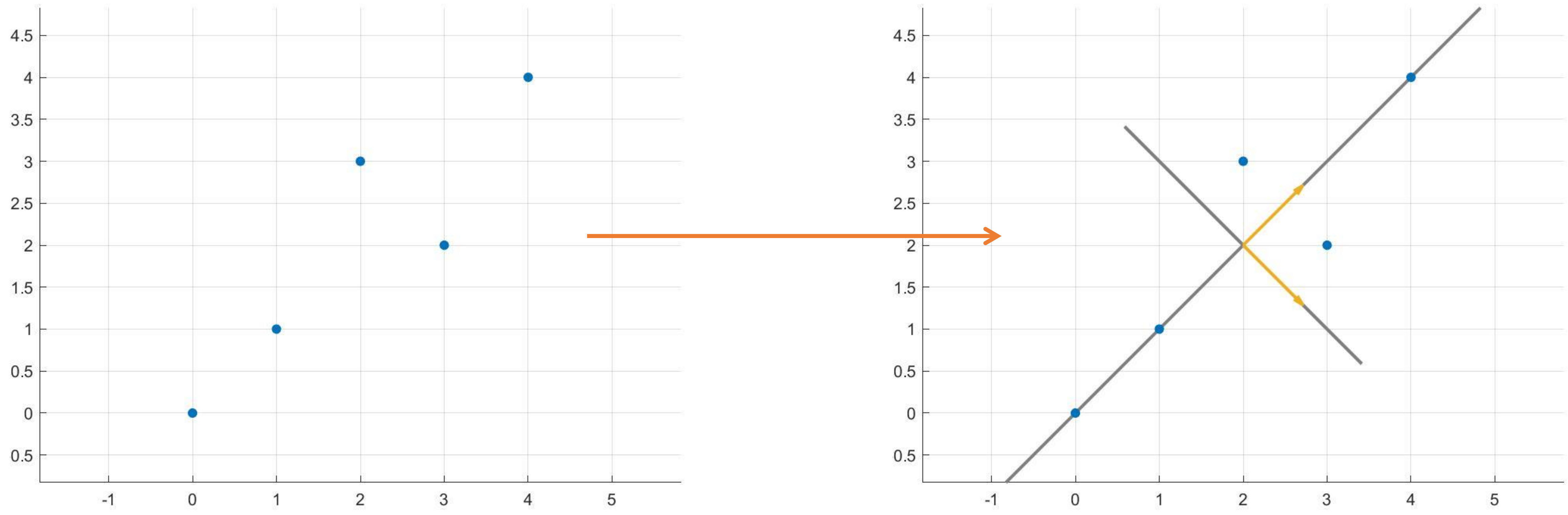

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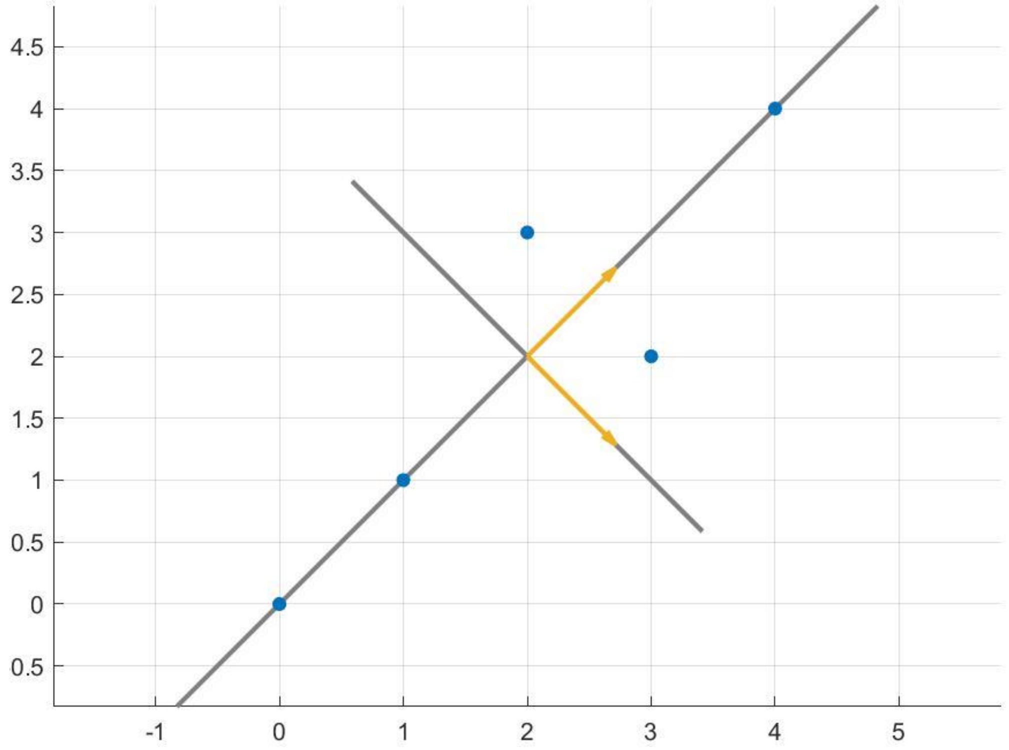
(b) Without doing any computation, draw in the figure from the previous exercise the projection of the points in the component that explains more variance. Show also for every point the information loss after the projection.



1) Which component explains more variance?

2) What will be the projection of all the points into this component?

3) What information have we lost?

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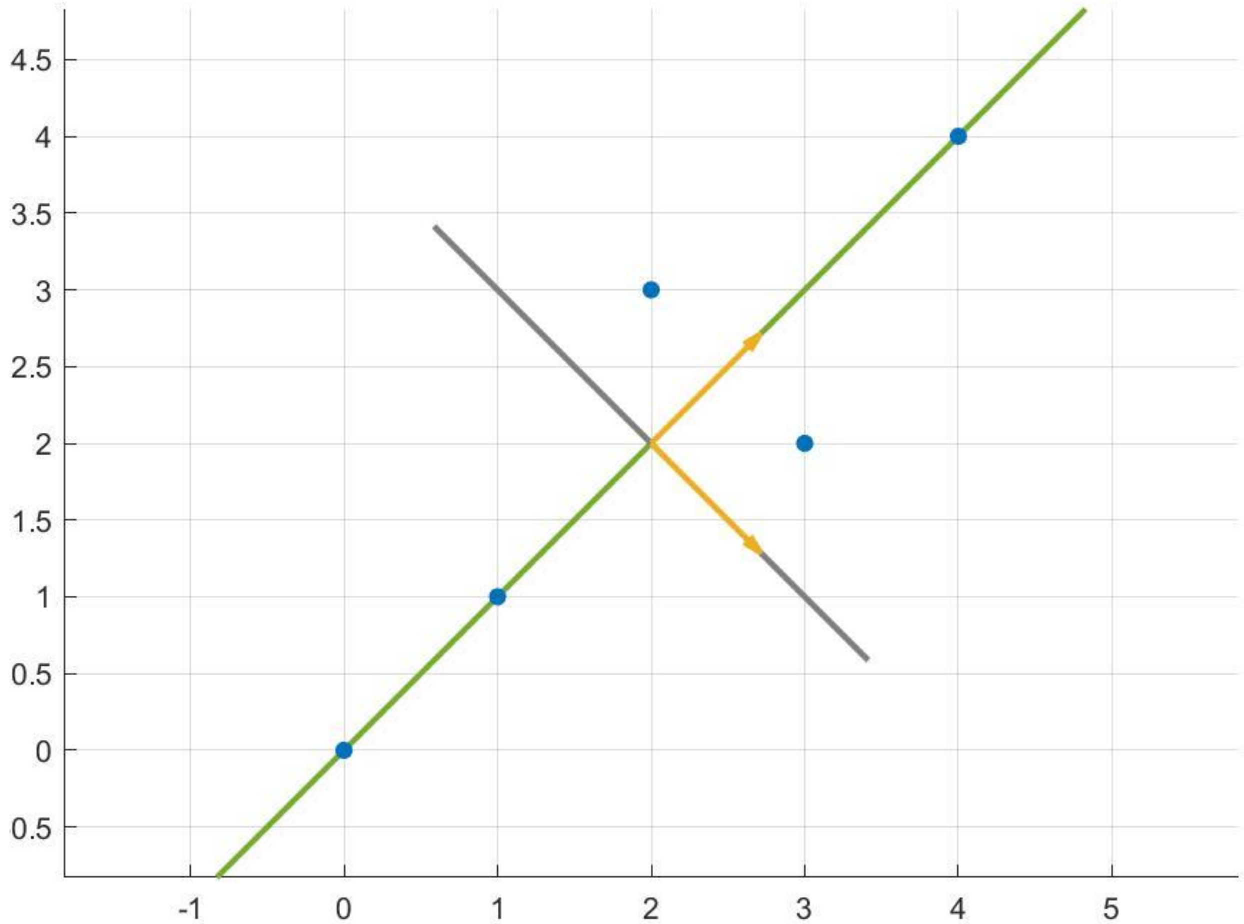

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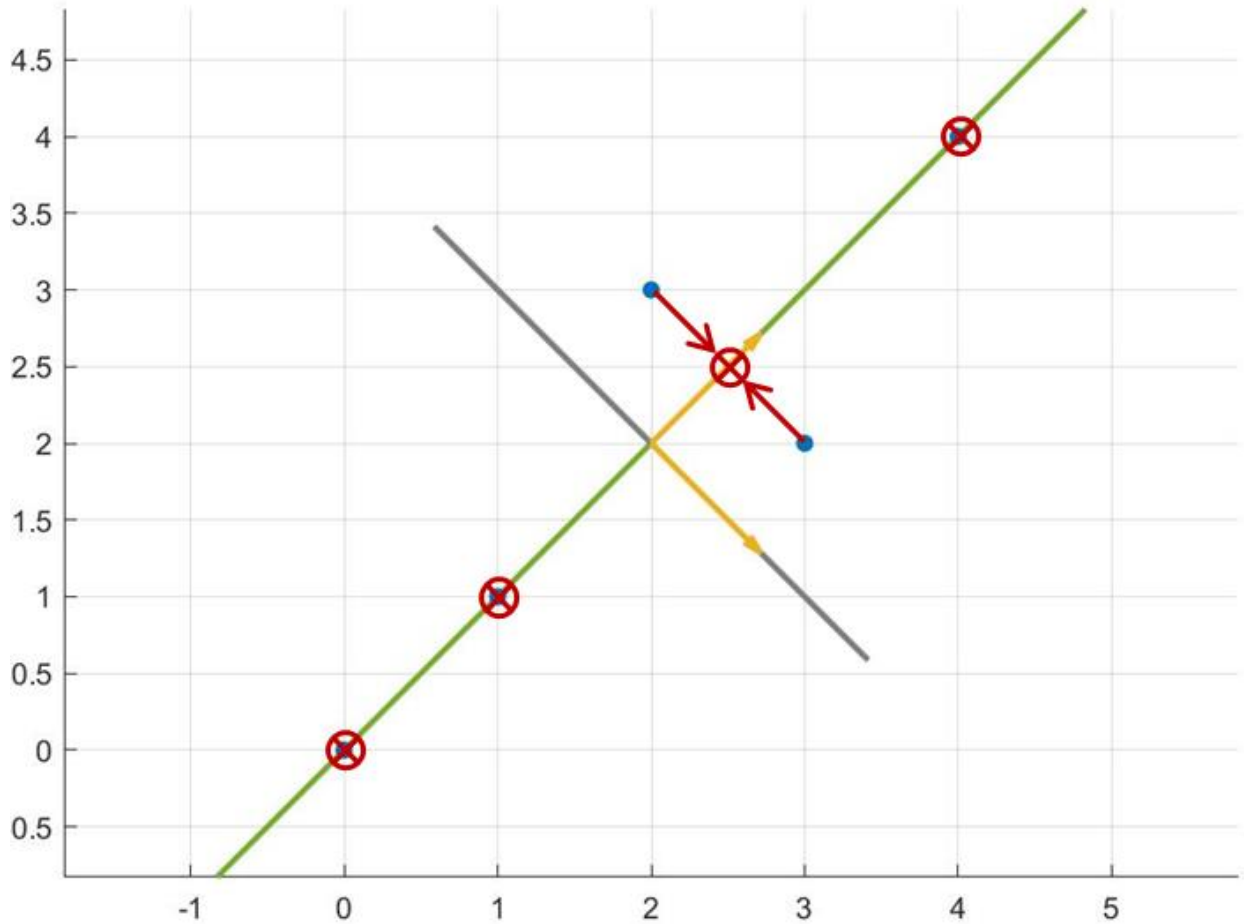

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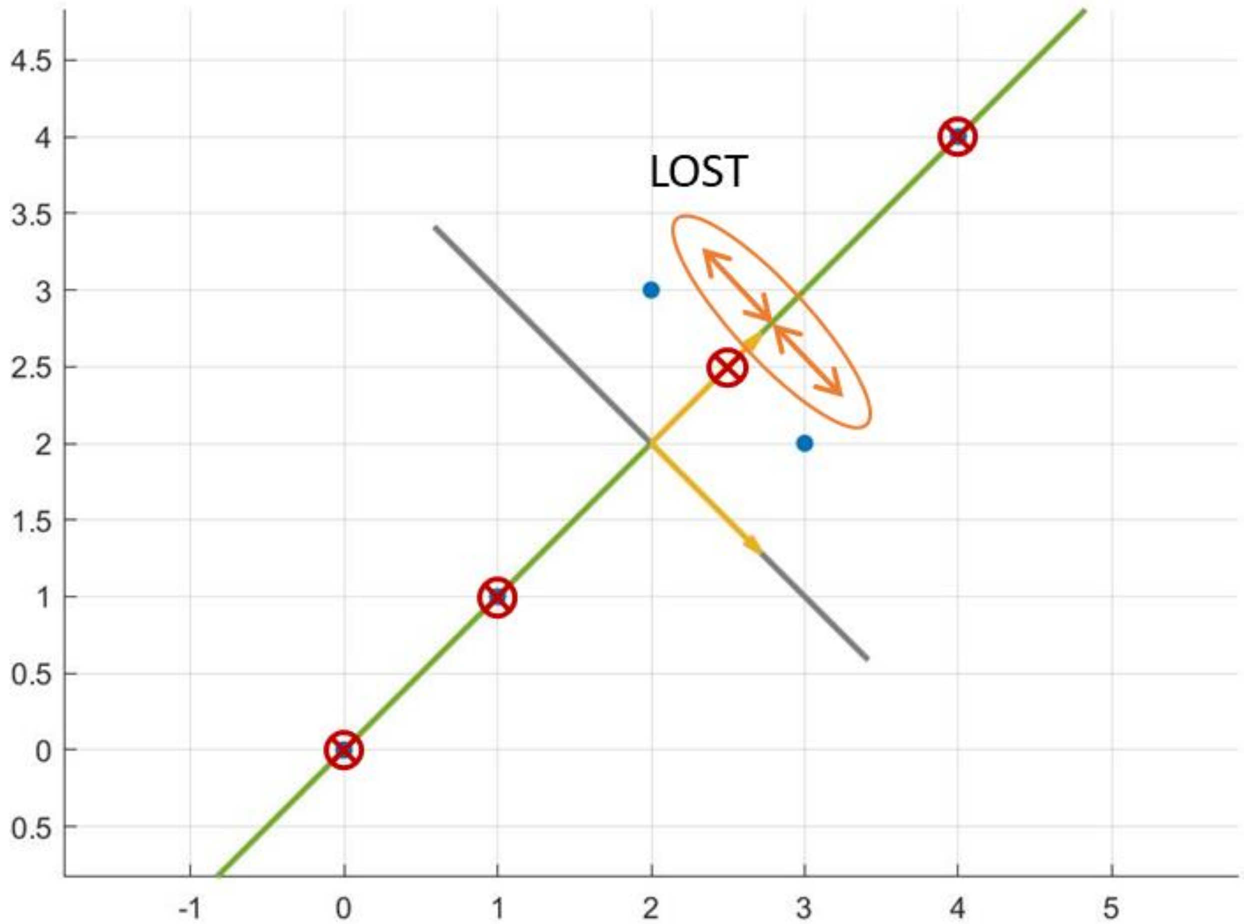

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(c) Compute the covariance matrix of the data and find the eigenvalues and eigenvectors.

COVARIANCE

$$\boldsymbol{\Sigma} = \frac{1}{N}(\boldsymbol{X} - \boldsymbol{\mu})(\boldsymbol{X} - \boldsymbol{\mu})^{\mathrm{T}} = ?$$

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$$\boldsymbol{\mu} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}^{(n)} = \begin{pmatrix}2\\2\end{pmatrix} \Longrightarrow \boldsymbol{X} - \boldsymbol{\mu} = \begin{pmatrix}-2 & -1 & 0 & 1 & 2\\-2 & -1 & 1 & 0 & 2\end{pmatrix} \Longrightarrow \boldsymbol{\Sigma} = \frac{1}{N}(\boldsymbol{X} - \boldsymbol{\mu})(\boldsymbol{X} - \boldsymbol{\mu})^{\mathrm{T}} = \frac{1}{5}\begin{pmatrix}10 & 9\\9 & 10\end{pmatrix}$$

$$\boldsymbol{\Sigma}_1 = \frac{1}{5}\begin{pmatrix}10 & 9\\9 & 10\end{pmatrix} \xrightarrow{\text{SVD}} \begin{cases}\lambda_1 = 3.8 & \boldsymbol{v}^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\\[2ex]\lambda_2 = 0.2 & \boldsymbol{v}^{(2)} = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}\end{cases}$$

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(d) Project the data over the basis obtained with PCA. Discuss the relation existing between the covariance of the projected data and the eigenvalues computed in the previous exercise.

New coordinate system $\left\{\boldsymbol{o} = \boldsymbol{\mu} = \binom{2}{2};\ \boldsymbol{v}^{(1)} = \frac{1}{\sqrt{2}}\binom{1}{1}, \boldsymbol{v}^{(2)} = \frac{1}{\sqrt{2}}\binom{-1}{1}\right\}$

$\downarrow$

Project the data into $\{\boldsymbol{\mu};\ \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$ = change of coordinate system

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Project the data into $\left\{\boldsymbol{\mu};\ \boldsymbol{v}^{(1)},\boldsymbol{v}^{(2)}\right\}$ = change of coordinate system

$$\boldsymbol{x}^{(n)}_{<\mathcal{V}>}={}_{<\mathcal{E}>}\mathrm{M}_{<\mathcal{V}>}\left(\boldsymbol{x}^{(n)}_{<\mathcal{E}>}-\boldsymbol{\mu}_{<\mathcal{E}>}\right)\xrightarrow[\text{the same time}]{\text{All the points at}}\boldsymbol{X}_{<\mathcal{V}>}={}_{<\mathcal{E}>}\mathrm{M}_{<\mathcal{V}>}\left(\boldsymbol{X}_{<\mathcal{E}>}-\boldsymbol{\mu}_{<\mathcal{E}>}\right)$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}-2&-1&0&1&2\\-2&-1&1&0&2\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}-4&-2&1&1&4\\0&0&1&-1&0\end{pmatrix}$$

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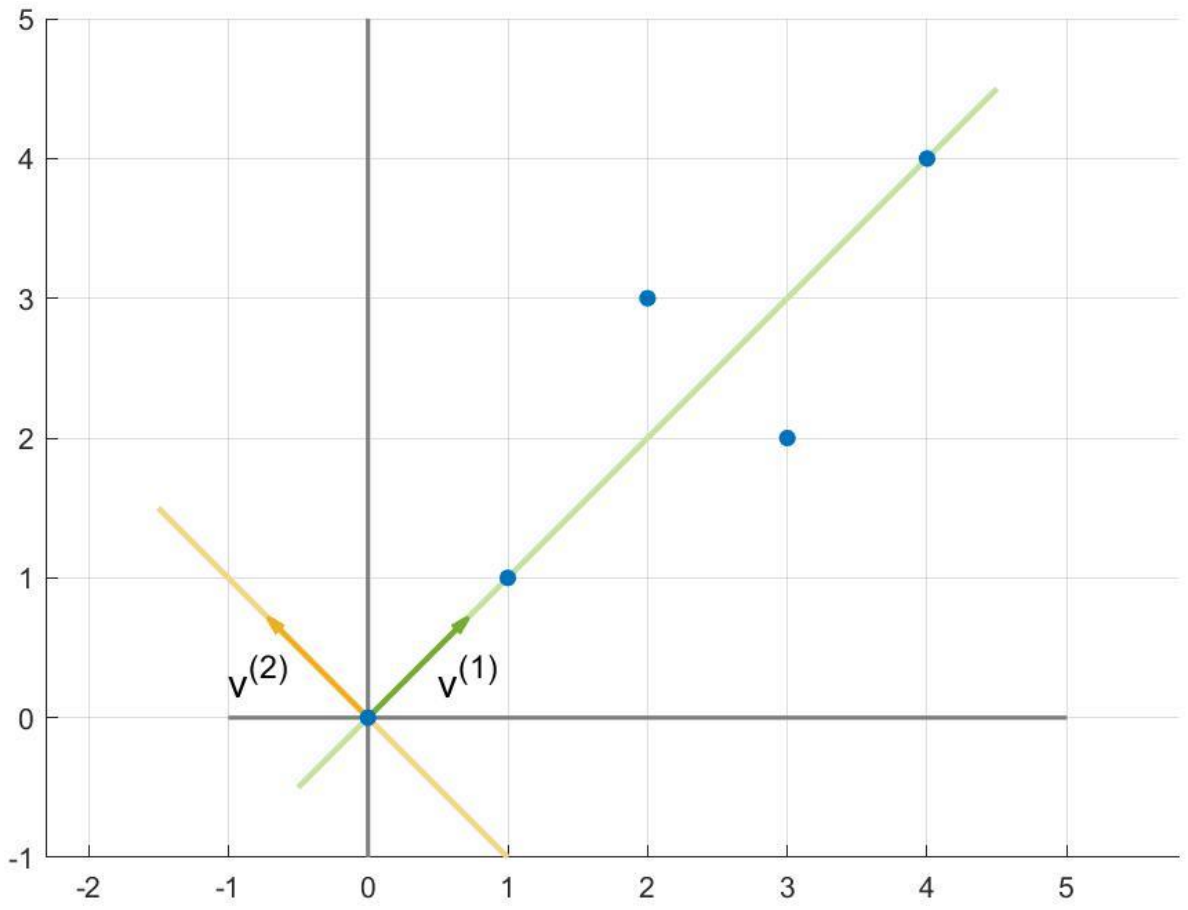
$$\boldsymbol{X}_{<v>} = \frac{1}{\sqrt{2}} \begin{pmatrix} \boxed{-4 \quad -2 \quad 1 \quad 1 \quad 4} \\ \boxed{0 \quad 0 \quad 1 \quad -1 \quad 0} \end{pmatrix}$$

$\boldsymbol{v}^{(1)}$

$\boldsymbol{v}^{(2)}$



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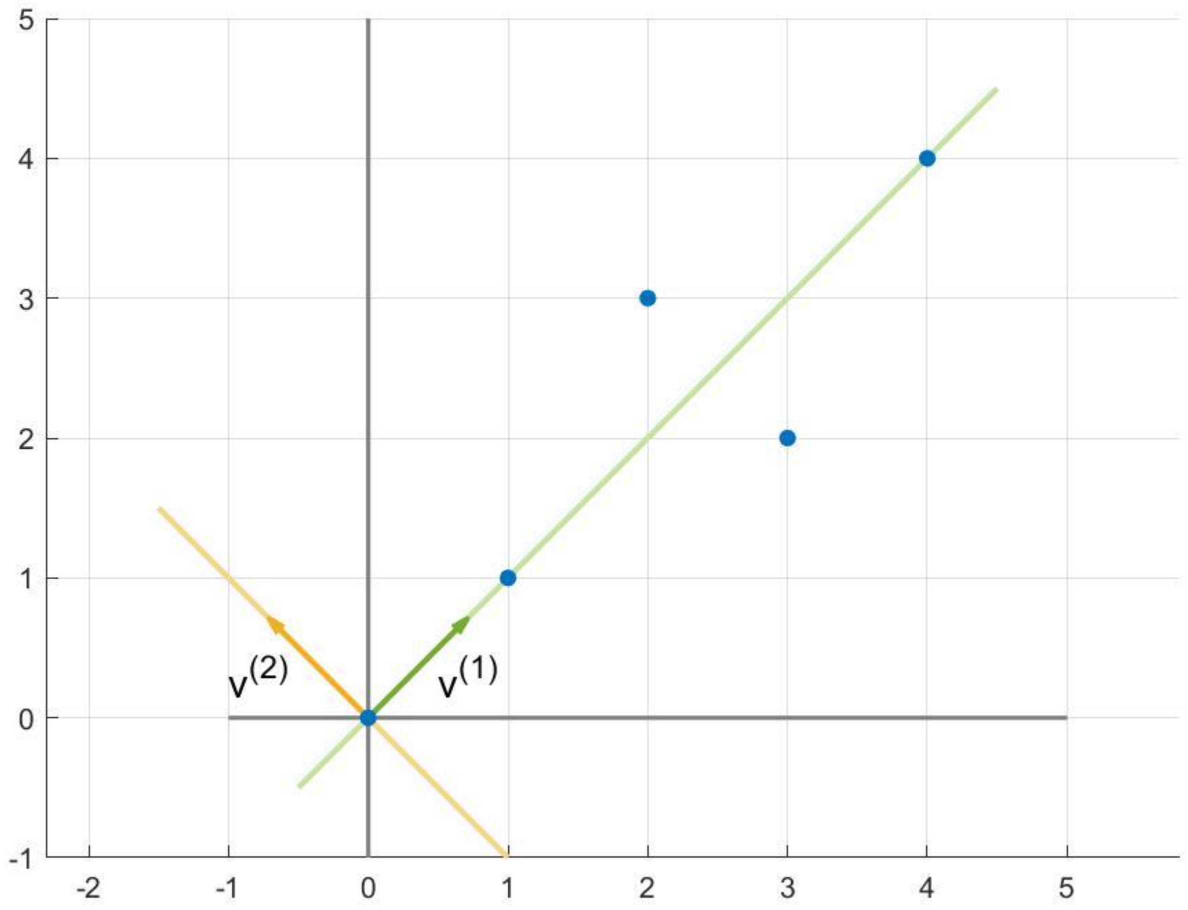
What if we computed the covariance of the data in the new basis?

$$\boldsymbol{\Sigma}_{<\mathcal{V}>} = \frac{1}{N}(\boldsymbol{X}_{<\mathcal{V}>} - \boldsymbol{\mu}_{<\mathcal{V}>})(\boldsymbol{X}_{<\mathcal{V}>} - \boldsymbol{\mu}_{<\mathcal{V}>})^{\mathrm{T}} = ?$$



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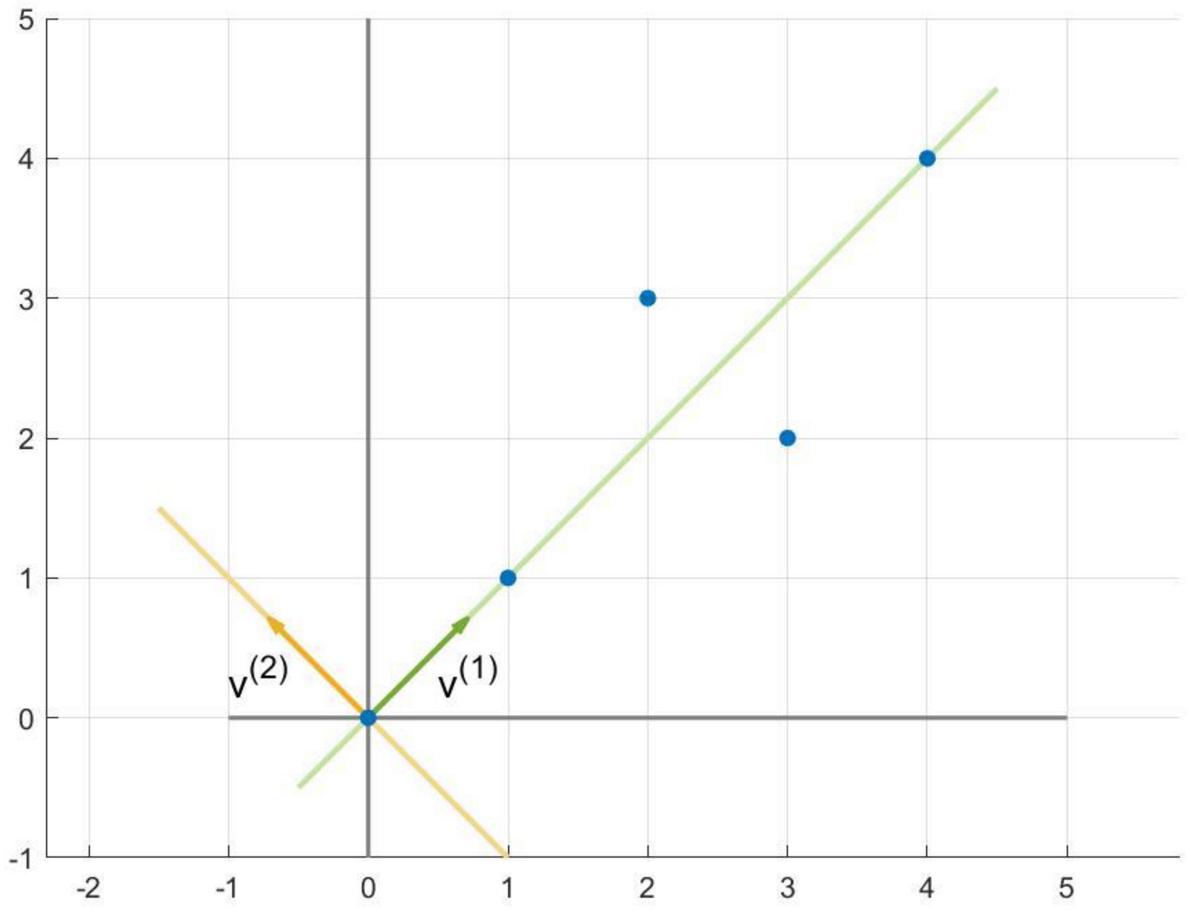
$$\boldsymbol{\Sigma}_{<v>} = \frac{1}{N}(\boldsymbol{X}_{<v>} - \boldsymbol{\mu}_{<v>})(\boldsymbol{X}_{<v>} - \boldsymbol{\mu}_{<v>})^{\mathrm{T}} = ?$$

$$\Longrightarrow \boldsymbol{\Sigma}_{<v>} = \begin{pmatrix} \overset{\lambda_1}{3.8} & 0 \\ 0 & \underset{\lambda_2}{0.2} \end{pmatrix}$$

Var of the data in the direction of $\boldsymbol{v}^{(1)}$

Var of the data in the direction of $\boldsymbol{v}^{(2)}$



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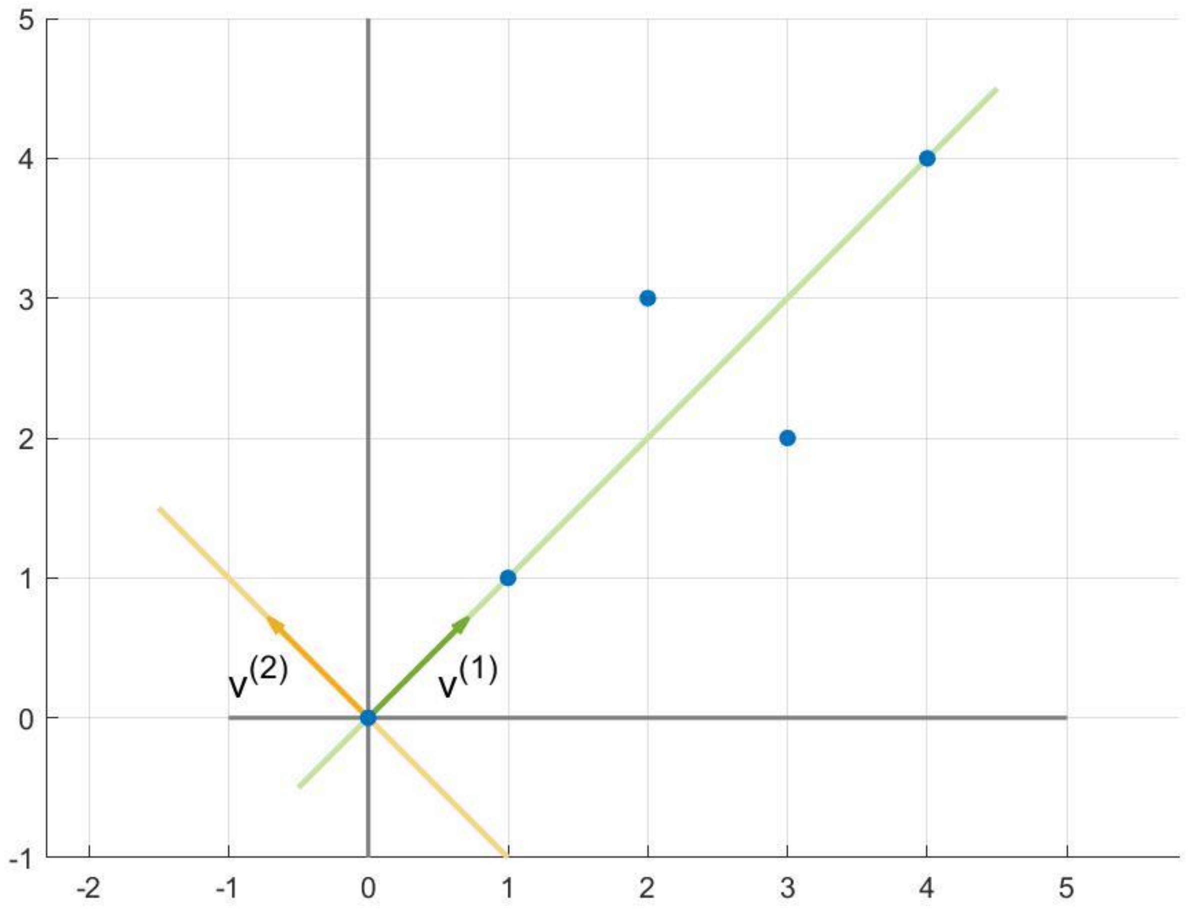
(e) Choose the component that maximizes the variance of the data. Project and re-project the data using the chosen component. Draw the results.

Which component maximizes the variance of the data?



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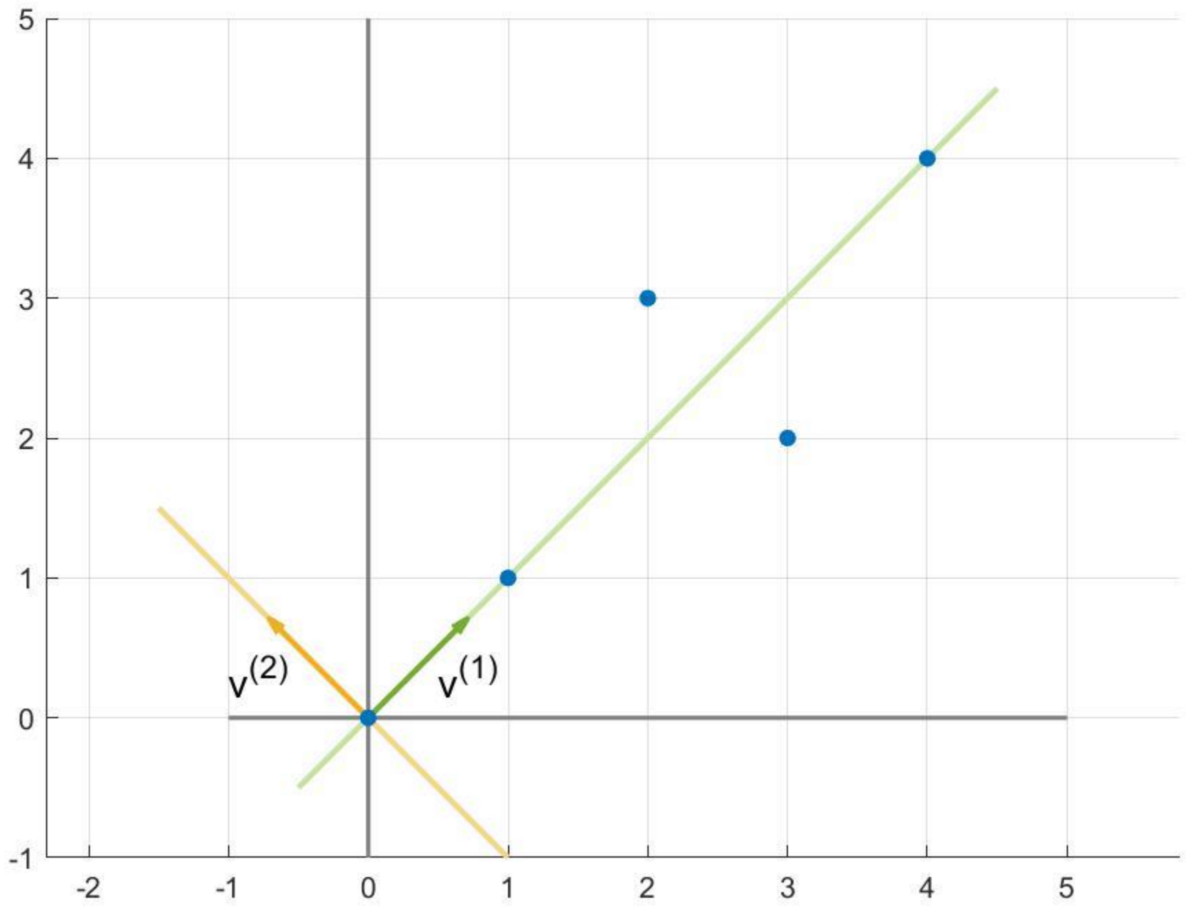
Which component maximizes the variance of the data?

λ_1 = variance in $\boldsymbol{v}^{(1)} \Longrightarrow \frac{\lambda_1}{\lambda_1+\lambda_2} = \frac{3.8}{3.8+0.2} = 95\%$ of var

λ_2 = variance in $\boldsymbol{v}^{(2)} \Longrightarrow \frac{\lambda_2}{\lambda_1+\lambda_2} = \frac{0.2}{3.8+0.2} = 5\%$ of var



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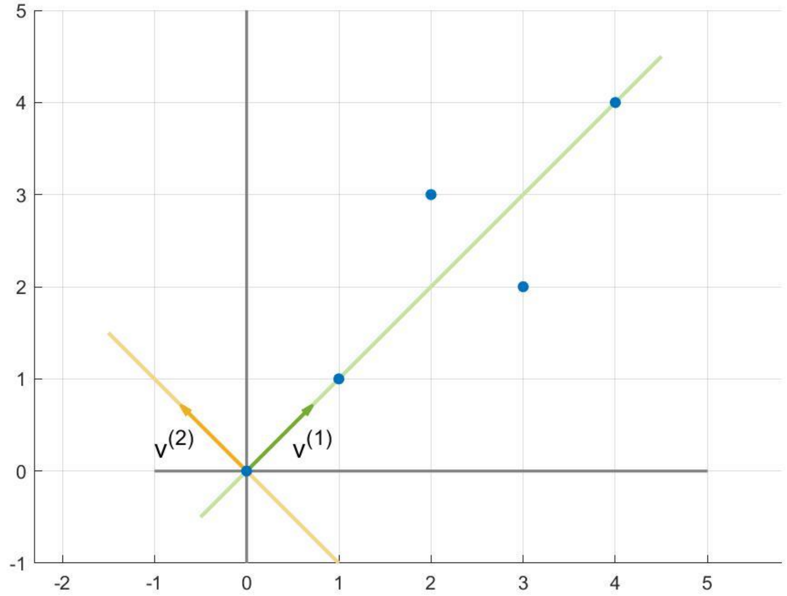
REMINDER

Project the data into $\{\boldsymbol{\mu};\ \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$ = **change of coordinate system**

Project the data into $\{\boldsymbol{v}^{(1)}\}$ = taking only the component corresponding to $\boldsymbol{v}^{(1)}$



$$\boldsymbol{X}_{<\mathcal{V}>} = \frac{1}{\sqrt{2}}\begin{array}{c}\begin{pmatrix}\boxed{-4 & -2 & 1 & 1 & 4}\\ \boxed{0 & 0 & 1 & -1 & 0}\end{pmatrix}\end{array}\begin{array}{l}\boldsymbol{v}^{(1)}\\ \boldsymbol{v}^{(2)}\end{array} \longrightarrow \boldsymbol{X}_{<\boldsymbol{v}^{(1)}>} = \frac{1}{\sqrt{2}}\begin{pmatrix}-4 & -2 & 1 & 1 & 4\end{pmatrix}$$

$\boldsymbol{x}^{(5)}_{<\boldsymbol{v}^{(1)}>}$

$\boldsymbol{x}^{(1)}_{<\boldsymbol{v}^{(1)}>}$

How do we do this without changing of basis?

EXERCICE 1

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(e) Choose the component that maximizes the variance of the data. Project and re-project the data using the chosen component. Draw the results.

Project the data into $\{\boldsymbol{v}^{(1)}\}$ = taking only the component corresponding to $\boldsymbol{v}^{(1)}$

$$\boldsymbol{X}_{<\mathcal{V}>} = \frac{1}{\sqrt{2}} \begin{array}{cc} \begin{pmatrix} -4 & -2 & 1 & 1 & 4 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix} & \begin{array}{l} \boldsymbol{v}^{(1)} \\ \boldsymbol{v}^{(2)} \end{array} \end{array} \longrightarrow \boldsymbol{X}_{<\boldsymbol{v}^{(1)}>} = \frac{1}{\sqrt{2}} \begin{pmatrix} -4 & -2 & 1 & 1 & 4 \end{pmatrix}$$

$$\boldsymbol{X}_{<\mathcal{V}>} = {}_{<\mathcal{E}>}\mathrm{M}_{<\mathcal{V}>} \left(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right) = \begin{pmatrix} | & | \\ \boldsymbol{v}^{(1)} & \boldsymbol{v}^{(2)} \\ | & | \end{pmatrix}^{\mathrm{T}} \left(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right) = \begin{pmatrix} - & {\boldsymbol{v}^{(1)}}^{\mathrm{T}} & - \\ - & {\boldsymbol{v}^{(2)}}^{\mathrm{T}} & - \end{pmatrix} \left(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right)$$

$$= \begin{pmatrix} {\boldsymbol{v}^{(1)}}^{\mathrm{T}} \left(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right) \\ {\boldsymbol{v}^{(2)}}^{\mathrm{T}} \left(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right) \end{pmatrix} = \begin{pmatrix} \boldsymbol{X}_{<\boldsymbol{v}^{(1)}>} \\ \boldsymbol{X}_{<\boldsymbol{v}^{(2)}>} \end{pmatrix}$$

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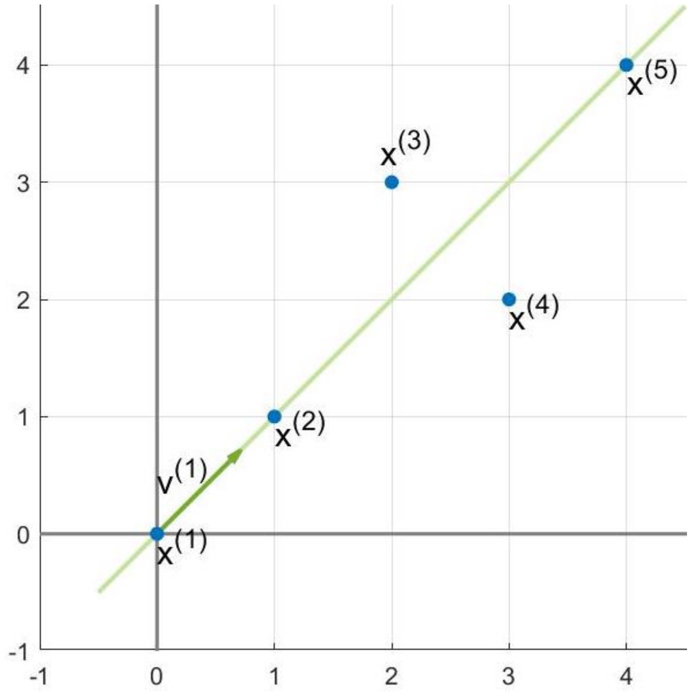
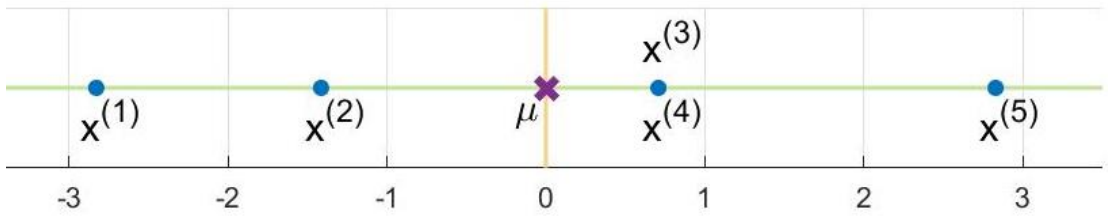
(e) Choose the component that maximizes the variance of the data. Project and re-project the data using the chosen component. Draw the results.

$$\boldsymbol{X}_{<\boldsymbol{v}^{(1)}>} = \boldsymbol{v}^{(1)^{\mathrm{T}}}(\boldsymbol{X}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}-2 & -1 & 0 & 1 & 2\\ -2 & -1 & 1 & 0 & 2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}-4 & -2 & 1 & 1 & 4\end{pmatrix}$$



Project the data into $\{\boldsymbol{v}^{(1)}\}$



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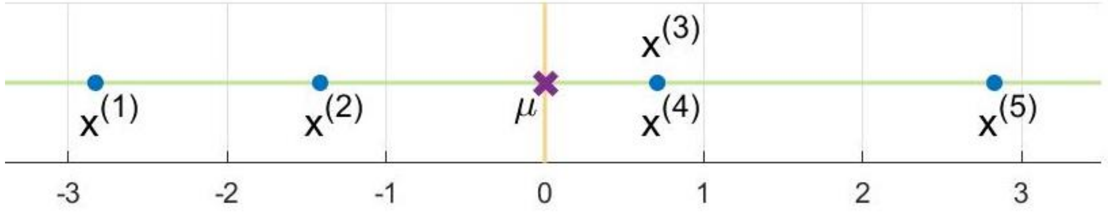
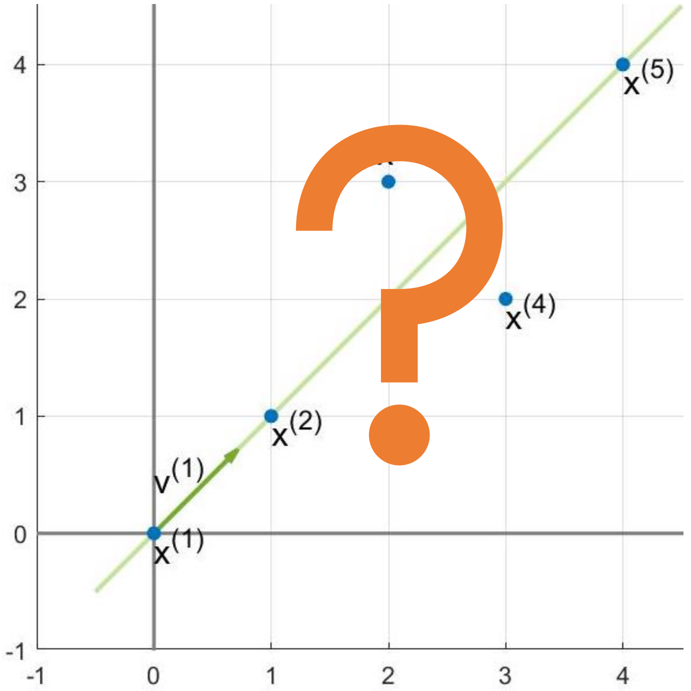
(e) Choose the component that maximizes the variance of the data. Project and re-project the data using the chosen component. Draw the results.



Re-project the data



Re-project the data = **undo** change of coordinate system

From $\{\mathbf{0};\ \boldsymbol{e}^{(1)}, \boldsymbol{e}^{(2)}\}$ to $\{\boldsymbol{\mu};\ \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$ $\longrightarrow$ $\boxed{\boldsymbol{X}_{<\mathcal{V}>}} = {}_{<\mathcal{E}>}\mathrm{M}_{<\mathcal{V}>}\ (\boxed{\boldsymbol{X}_{<\mathcal{E}>}} - \boldsymbol{\mu}_{<\mathcal{E}>})$

From $\{\boldsymbol{\mu};\ \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\}$ to $\{\mathbf{0};\ \boldsymbol{e}^{(1)}, \boldsymbol{e}^{(2)}\}$ $\longrightarrow$ ${}_{<\mathcal{V}>}\mathrm{M}_{<\mathcal{E}>} \boxed{\boldsymbol{X}_{<\mathcal{V}>}} + \boldsymbol{\mu}_{<\mathcal{E}>} = \boxed{\boldsymbol{X}_{<\mathcal{E}>}}$

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(e) Choose the component that maximizes the variance of the data. Project and re-project the data using the chosen component. Draw the results.

Re-project the data

$$\boldsymbol{X}_{\text{reproj}} = \boldsymbol{v}^{(1)} \cdot \boldsymbol{X}_{<\boldsymbol{v}^{(1)}>} + \boldsymbol{\mu}_{<\mathcal{E}>} = ?$$

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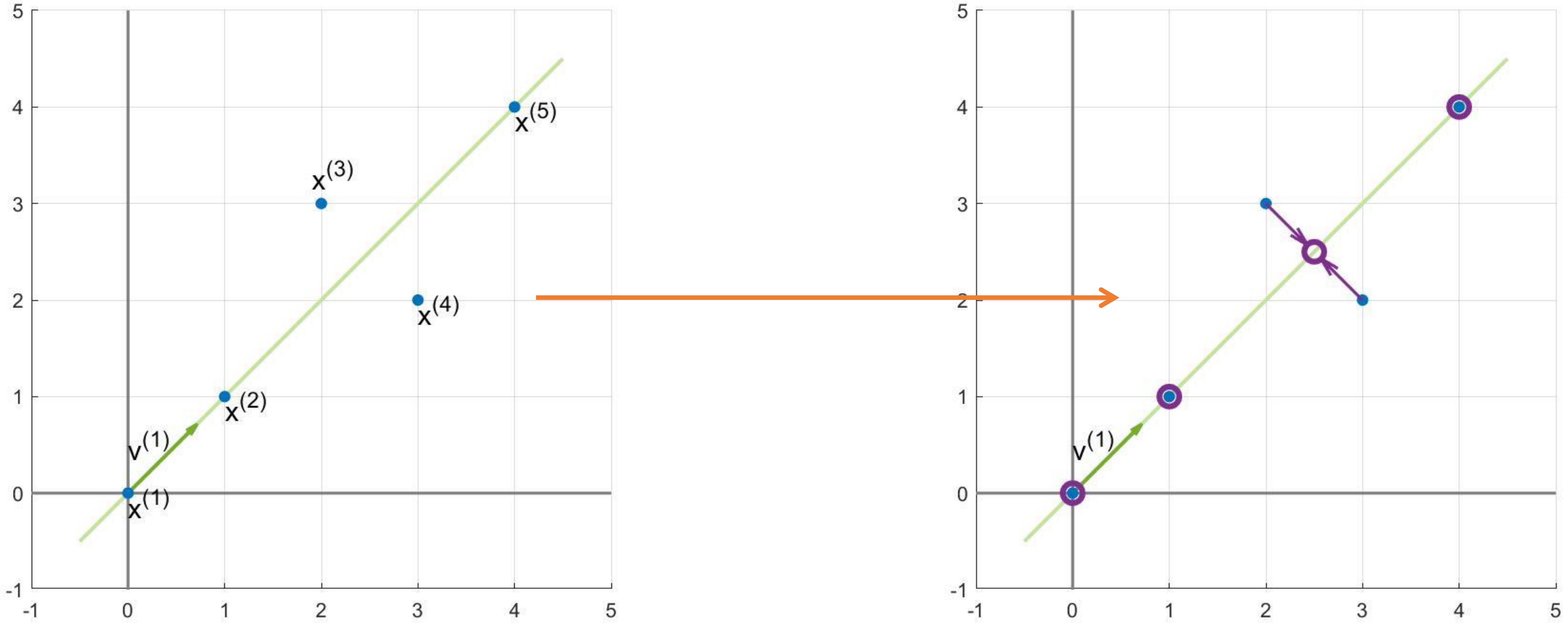
Re-project:

$$\boldsymbol{X}_{\text{reproj}}=\boldsymbol{v}^{(1)}\cdot\boldsymbol{X}_{<\boldsymbol{v}^{(1)}>}+\boldsymbol{\mu}_{<\mathcal{E}>}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}-4 & -2 & 1 & 1 & 4\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}=\begin{array}{c}\begin{array}{ccccc} & & \boldsymbol{x}^{(3)} & & \end{array}\\ \begin{pmatrix}0 & 1 & 2.5 & 2.5 & 4\\0 & 1 & 2.5 & 2.5 & 4\end{pmatrix}\\ \begin{array}{ccccc} & & & \boldsymbol{x}^{(4)} & \end{array}\end{array}$$



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(f) Project and re-project the point $\boldsymbol{p} = \begin{pmatrix}2 & 4\end{pmatrix}^{\mathrm{T}}$ using only the component with maximum variance. Draw the results and compute the reprojection error.

Project: $\boldsymbol{p}_{<\boldsymbol{v}^{(1)}>} = {\boldsymbol{v}^{(1)}}^{\mathrm{T}}(\boldsymbol{p}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}) =?$

Re-project: $\boldsymbol{p}_{\mathrm{reproj}} = \boldsymbol{v}^{(1)} \cdot \boldsymbol{p}_{<\boldsymbol{v}^{(1)}>} + \boldsymbol{\mu}_{<\mathcal{E}>} =?$

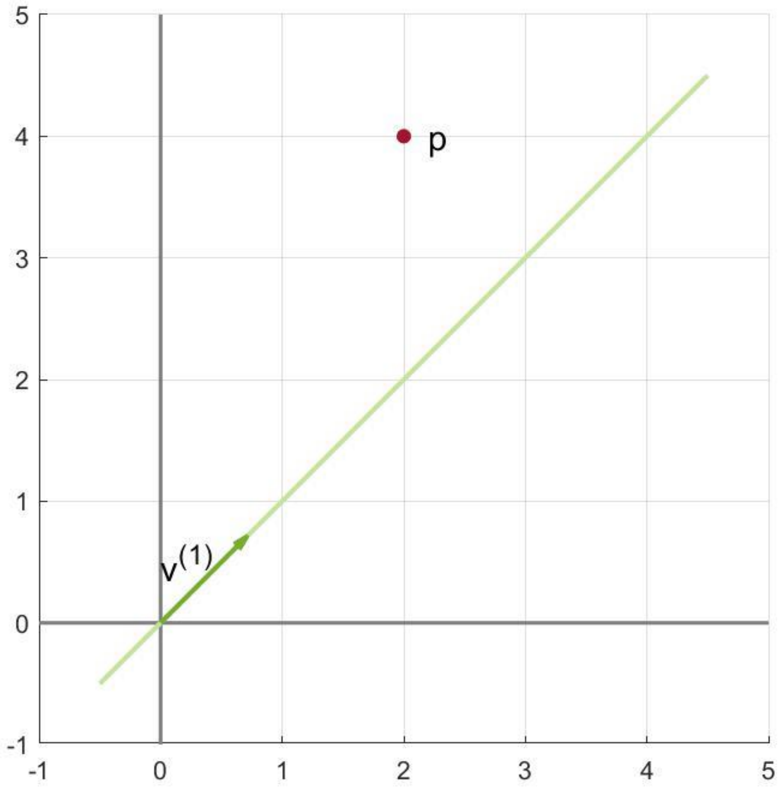
EXERCICE 1

$$\left\{\boldsymbol{x}^{(1)} = \begin{pmatrix}0\\0\end{pmatrix}, \boldsymbol{x}^{(2)} = \begin{pmatrix}1\\1\end{pmatrix}, \boldsymbol{x}^{(3)} = \begin{pmatrix}2\\3\end{pmatrix}, \boldsymbol{x}^{(4)} = \begin{pmatrix}3\\2\end{pmatrix}, \boldsymbol{x}^{(5)} = \begin{pmatrix}4\\4\end{pmatrix}\right\} \subset \mathbb{R}^2$$

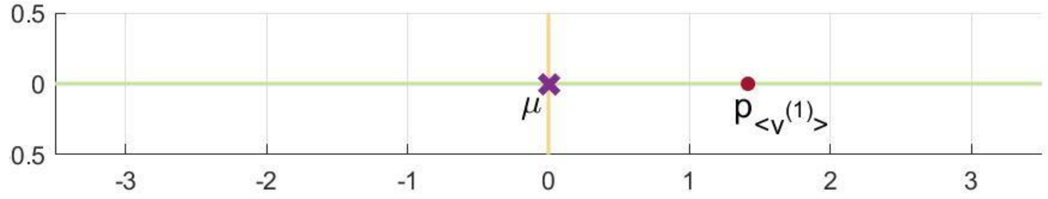
(f) Project and re-project the point $\boldsymbol{p} = \begin{pmatrix}2 & 4\end{pmatrix}^{\mathrm{T}}$ using only the component with maximum variance. Draw the results and compute the reprojection error.

Project:

$$\boldsymbol{p}_{<\boldsymbol{v}^{(1)}>} = {\boldsymbol{v}^{(1)}}^{\mathrm{T}}\left(\boldsymbol{p}_{<\mathcal{E}>} - \boldsymbol{\mu}_{<\mathcal{E}>}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix} \cdot \left(\begin{pmatrix}2\\4\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}\right) = \frac{2}{\sqrt{2}}$$



Project $\boldsymbol{p}$ into $\left\{\boldsymbol{v}^{(1)}\right\}$



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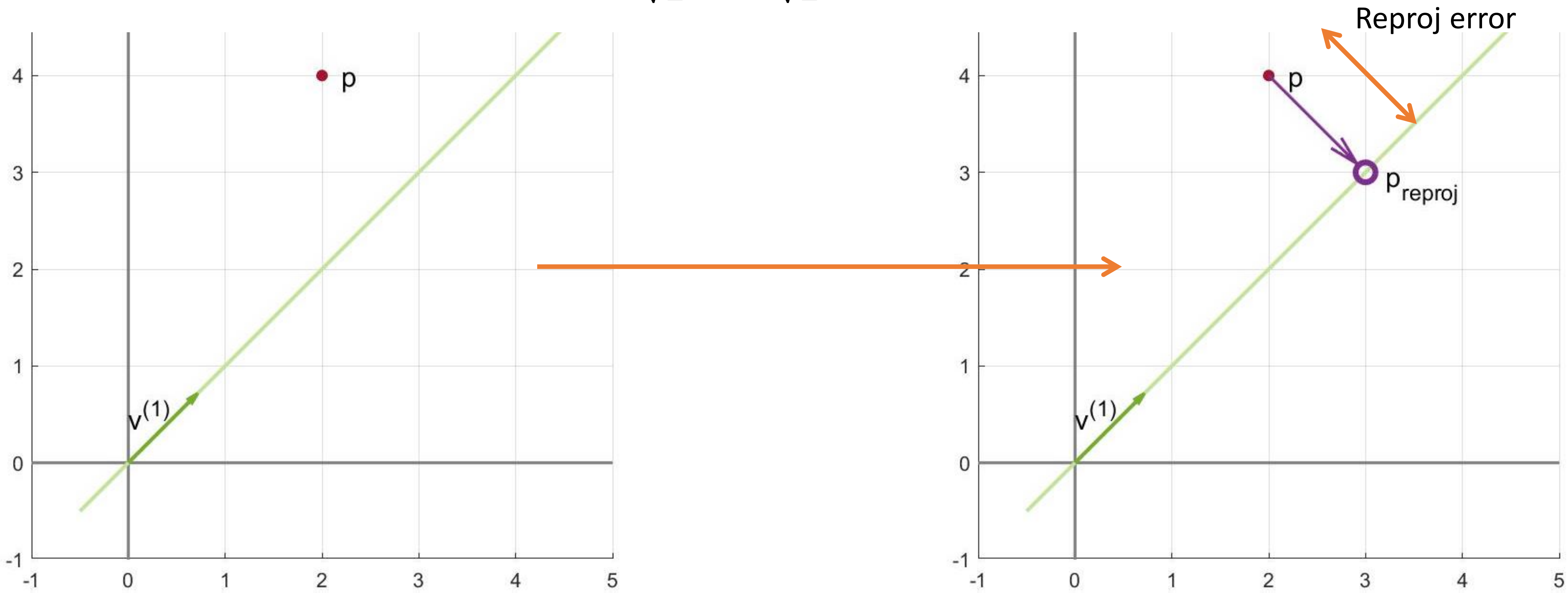
Re-project: $$\boldsymbol{p}_{\mathrm{reproj}} = \boldsymbol{v}^{(1)} \cdot \boldsymbol{p}_{<\boldsymbol{v}^{(1)}>} + \boldsymbol{\mu}_{<\mathcal{E}>} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \cdot \frac{2}{\sqrt{2}} + \begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$$



EXERCICE 1

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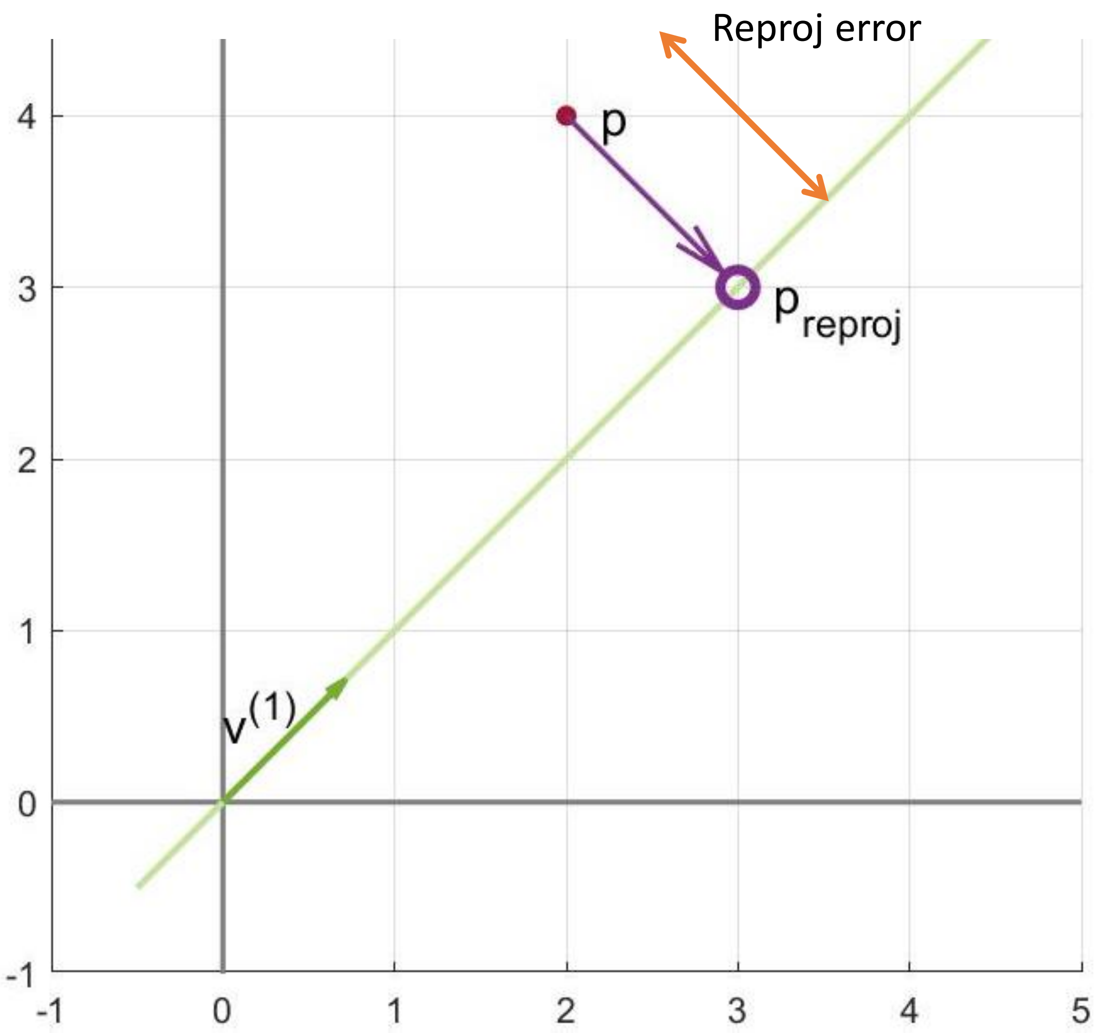

$$\textbf{Reproj error} = d(\boldsymbol{p},\boldsymbol{p}_{\text{reproj}}) = d\left(\begin{pmatrix}2\\4\end{pmatrix},\begin{pmatrix}3\\3\end{pmatrix}\right) = \sqrt{2}$$