Questions Sets For Practice

MODEL SET 1

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20

Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8

Time: 3 hours.

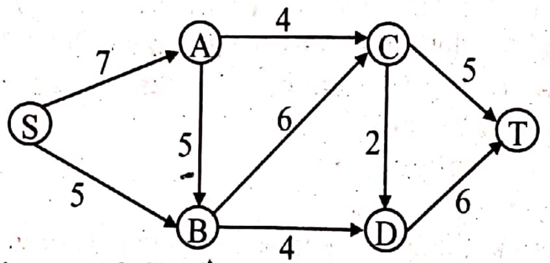
Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Long answer questions (any two) **[10*2=20]**

Section A

1. What is the major difference between mathematical induction and strong induction? Using both induction methods prove that $1 + 2 + 2^2 + 2^3 \ldots\ldots\ldots + 2^n = 2^{n+1} - 1$. (3 + 7)
2. Define network flow problem, residual graph and augmented path. Find the maximum flow in the given flow network graph from S to T.



3. What is spanning tree? Explain minimum spanning tree with suitable example.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. Find the set specified by the bit strings: 1110011100 and 1011011011 with reference to universal set U= {a, b, c, d, e, f, g, h, i, j}. Also find their union and intersection.
5. Explain Inclusion Exclusion principle. Describe in brief about computer representation of set?
6. Discuss adjacency and incidence matrix representation of graph with suitable example.
7. Solve the recurrence relation an = 2an-1 + an-2 -2an-2 for n≥3, a0 = 3, a1 = 6 and a2 = 9.
8. What is partial order set? Show that the set {Z+, |) is a partial order set.

9. What is fuzzy set? Explain with suitable example. Briefly explain membership function. What is Euler graph and Hamiltonian graph? Draw graph with 8 vertices which is Eulerian but not Hamiltonian and vice versa.

10. What is predicate? Translate following sentences into propositions using quantifiers.

    a. Every living thing is a plant or an animal.

    b. Every student of CSIT takes discrete class.

    c. Everything that glitters is not gold.

    d. Everyone loves someone

11. Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

12. Prove the statement: "The integer $3n + 2$ is odd if and only if the integer $9n + 5$ is even, where n is an integer."

MODEL SET 2

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20
Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8
Time: 3 hours.
Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. [2x10=20]

1. Define tautology, contradiction and contingency. Show that the hypothesis "if you send me an email then I will finish writing the program. If you don't send me an email then I will go to sleep early. If I will go to sleep early then I will wake up feeling refresh". Leads to conclusion "if I do not finish writing the program then I will wake up feeling refreshed".

2. Define linear homogeneous recurrence relation. Solve the given recurrence relation,

$a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, $a_0 = 5$ and $a_1 = -1$.

3. Define mathematical Induction. Use mathematical induction to show that $1^2+2^2+3^2+\text{---------}+n^2 = \frac{n(n+1)(2n+1)}{6}$, where $n \geq 1$.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. Define weakly connected and strongly connected directed graph with example.

5. State binomial theorem for positive index n. find the middle term in the expansion of $(3x+5y)^7$.

6. Define adjacency matrix and incidence matrix representation of graph with example.

7. What do you mean by partial order relation? Prove that the relation "is subset of ($\subseteq$)" is partial order on set power set P(S) of set S.

8. State Chinese remainder theorem and steps to be followed to solve system of linear congruencies and use it Solve the following system of congruences:

   $X \equiv 2 \pmod 3$

   $X \equiv 4 \pmod 5$

   $X \equiv 5 \pmod 7$

9. Determine whether each of these functions is a Bijection from R to R.

   a) f (x) = 2x + 1

   b) f (x) = x2 + 1

   c) f (x) = x3

   d) f (x) = (x2 + 1)/(x2 + 2)

10. Suppose that a person deposits $10,000 in a savings account at a bankyielding 11% per year with interest compounded annually. How much will be in the accountafter 30 years?

11. Prove that these four statements about the integer n areequivalent:

    a. n^2 is odd,

    b. $(1 - n)$ is even,

    c. n^3 is odd,

    d. $n^2 + 1$ is even.

12. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

MODEL SET 3

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20

Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

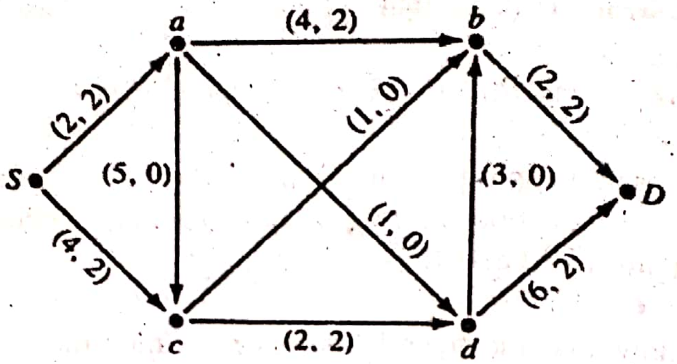
1. Show that the hypothesis "it's not sunny this afternoon and it's colder than yesterday," "We will go to swim only if it's sunny," "If we don't go to swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" leads to the conclusion "We will be home by sunset".
2. What is recurrence relation? Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with $a_0=3$, $a_1=7$ and $a_2=13$.
3. What is shortest path problem? Discuss Dijkstra's algorithm for finding the shortest path in a weighted graph between two vertices. Draw a weighted graph with at least 10 vertices and 15 edges and show each step using Dijkstra's algorithm to find shortest path between any two vertices of your choice.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. What are tautology, contradiction and contingency? Explain with example of each.
5. State the Pigeonhole principle. Find the minimum number of students in a class to be sure that three of them are born in the same month.
6. A man has six friends. How many ways he may invite one or more of them to a dinner?
7. Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many reasons does a representation of this planar graph split the plane?
8. Differentiate between existential and universal quantifier with suitable examples.
9. Prove that a connected multi graph has an Euler circuit if and only if each of its vertices has even degree.
10. Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

11. How can you relate domain and co-domain of functions with function in programming language? Discuss inverse and composite function with suitable example.
12. Find maximal flow for the network shown in the figure below.



MODEL SET 4

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20

Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

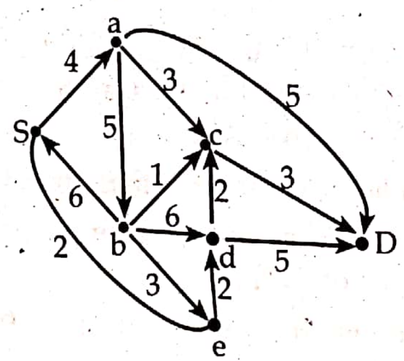
1. What do you mean by recurrence relation? Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.
2. Find sum of numbers 123,684 and 413,456 by representing the numbers as 4-tuple by using remainders modulo of pair- wise relatively prime numbers less than 100.
3. Define equivalence relation and partial order relation. How can you represent reflexive, symmetric and anti- symmetric relations using matrices? Explain with example.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. Let A(x) denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1), A(CS2), and A(MATH1)?

5. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.
   a) Someone in your school has visited Uzbekistan.
   b) Everyone in your class has studied calculus and C++.
   c) No one in your school owns both a bicycle and a motorcycle.
   d) There is a person in your school who is not happy.
   e) Everyone in your school was born in the twentieth century
6. The real number r is rational if there exist integers p and q with q = 0 such that r = p/q. A real number that is not rational is called irrational.
7. Show that the sequence{an} is a solution of the recurrence relation $a_n = -3a_n-1 + 4a_n-2$ if
   a) $a_n = 0$.
   b) $a_n = 1$.
   c) $a_n = (-4)^n$.
   d) $a_n = 2(-4)^n + 3$
8. State pigeonhole principle? What do you mean by Pascal's identity? Find the expansion of $(x+y)^4$.
9. Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable.
10. Show that a simple graph is connected if and only if it has a spanning tree.
11. What is shortest path problem? Discuss Dijkstra's algorithm for finding the shortest path in a weighted graph between two vertices. Draw a weighted graph with at least 12 vertices and 18 edges and show each step using Dijkstra's algorithm to find shortest path between any two vertices of your choice.
12. Find maximal flow for the network shown in the figure below.



MODEL SET 5

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20

Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

1. What is Eular's formula for planar graphs? How can Eular's formula for planar graphs be used to show that a simple graph is non-planar?

2. What do you mean by recurrence relation? Solve $a_n=a_{n-1}+2a_{n-2}$ with initial condition $a_0 = 2$ and $a_1 = 7$.

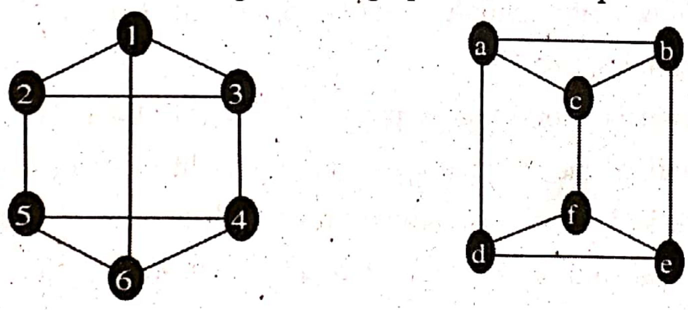
3. Use mathematical induction to prove that Prove that $1^2 + 3^2 + 5^2 + \dots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$ whenever n is a nonnegative integer.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. Prove that a connected multi graph has an Euler circuit if and only if each of its vertices has even degree.

5. Determine whether the given two graphs are isomorphic or not?



6. Prove the theorem "If n is an integer, then n is odd if and only if n2 is odd."

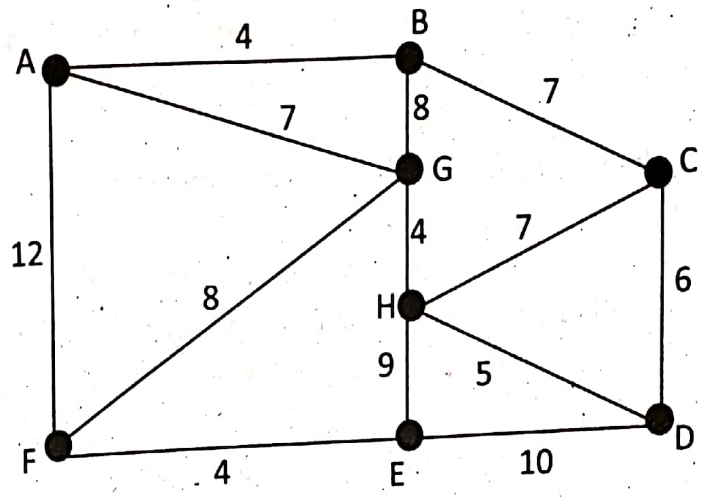
7. Prove$(p\rightarrow q)\wedge(q\rightarrow r)\rightarrow(p\rightarrow r)$ is tautologyby symbolic derivation.

8. Define BST. Draw the binary search tree for the following words Aask, Aart, Ante, Able, Alto, Also, Avid, Ant, Box, Book.

9. Find the domain and range of these functions.
   a. The function that assigns to each pair of positive integers the first integer of the pair
   b. The function that assigns to each positive integer its largest decimal digit

c. The function that assigns to a bit string the number of one's minus the number of zeros in the string
d. The function that assigns to each positive integer the largest integer not exceeding the square root of the integer
e. The function that assigns to a bit string the longest string of ones in the string

10. Write the Euclidean algorithm. Express gcd (252, 198) = 18 as a linear combination of 252 and 198.
11. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 3$ for n = 1, 2, 3, ... and suppose that a0 = 2. What are a1, a2, and a3?
12. Find two distinct minimal spanning tree of the weighted graph given below.



MODEL SET 6

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20
Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8
Time: 3 hours.
Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

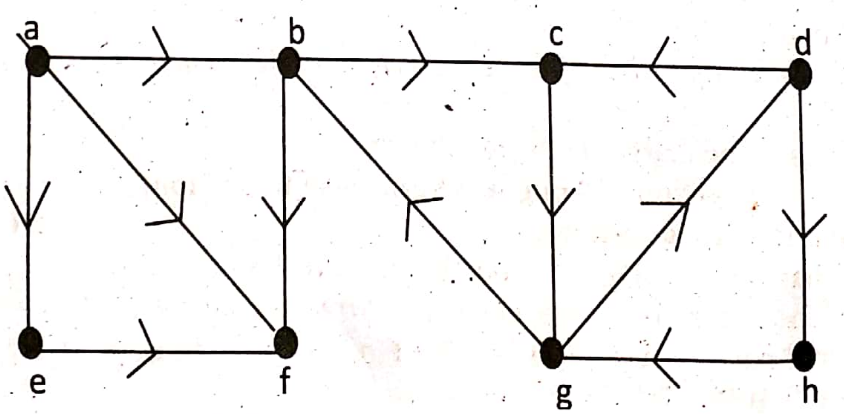
1. What is particular equation? What form does a particular solution of the linear non-homogeneous recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} +$ F(n) have when F(n) = 3^n, F(n) = $n3^n$, F(n) = $n^2 2^n$, and F(n) = $(n^2 + 1)3^n$?
2. Define composition of two functions? Let $f: R \to R$ and $g: R \to R$ are any two functions defined f(x) = 2x+1, g(x) = x^2-2. Find formulas which define the functions gof and fog. Also define countable and uncountable set. Show that set of integers are denumerable.

3. Define any five inference rules. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book.

Section B (Short Answer Questions)

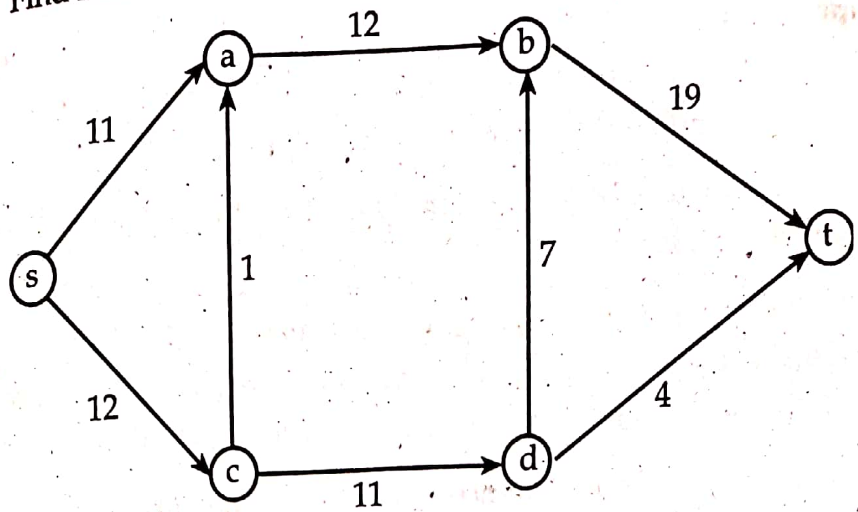
Attempt any EIGHT questions. [8 × 5=40]

4. What is meant by chromatic number? How can you color bi-partite graph k2,3 with minimum number of color.
5. Using direct and indirect proof method show that "Square of an even integer is an even".
6. How can you relate domain and co-domain of functions with function in programming language? Discuss inverse and composite function with suitable example.
7. Differentiate between existential and universal quantifier with suitable examples.
8. Show that matrix addition is associative; that is, show that if A, B, and C are all m × n matrices, then A + (B + C) = (A + B) + C
9. Give a proof by contradiction of the theorem "If 3n + 2 is odd, then n is odd."
10. What is induction recursion? Prove that $1^3+2^3+3^3+\ldots\ldots+n^3= n^2(n+1)^2/4$ by using induction.
11. What do you mean by directed graph? Consider the diagraph D given below.



a. Find the directed walk in D of length 8
b. Find a directed path in D of longest possible length
c. Is D strongly connected? If not why?
d. Identify the Source and Sink vertices.

12. Find max flow and min cut of following weighted graph



MODEL SET 7

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20

Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

1. What is binary tree? Proof that A full m-ary tree with
   a. n vertices has $i = (n - 1)/m$ internal vertices and $l = [(m - 1)n + 1]/m$ leaves,
   b. i internal vertices has $n = mi + 1$ vertices and $l = (m - 1)i + 1$ leaves,
   c. l leaves has $n = (ml - 1)/(m - 1)$ vertices and $i = (l - 1)/(m - 1)$ internal vertices.
2. Define mathematical induction. How it is differ from strong mathematical induction? Use mathematical induction to prove that $2^n < n!$ For every integer n with $n \geq 4$ (Note that this inequality is false for n = 1, 2, and 3.)
3. Define fallacies. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

Section B (Short Answer Questions)

Attempt any EIGHT questions. [8 × 5=40]

4. Prove that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

5. Differentiate between existential and universal quantifiers with suitable examples.

6. State the Pigeonhole principle. How many students must be in a class to guarantee that at least two students receive the same score on the final exam is graded on a scale from 0 to 100?

7. State which rule of inference is the basis of the following argument; "It is below freezing now. Therefore, it is either below freezing or raining now."

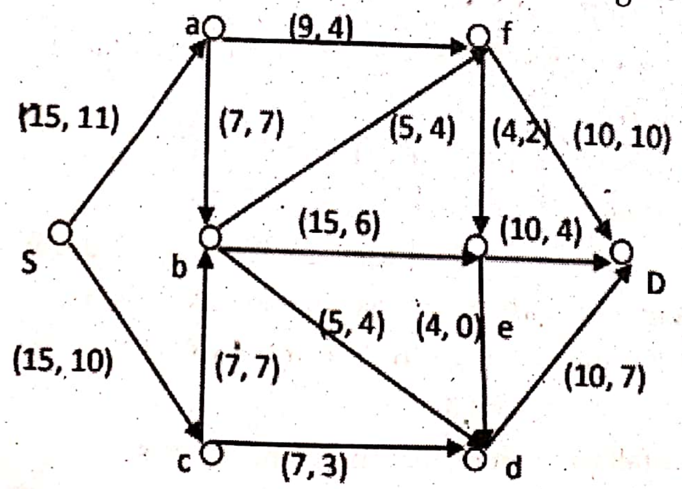
8. How can you show that two graphs are isomorphic? Discuss invariants that can be used to show that two graphs are not isomorphic with suitable example.

9. Prove $(p \wedge \neg q) \rightarrow \neg(p \leftrightarrow r) \Leftrightarrow \neg p \vee q \vee \neg r$ by symbolic derivation.

10. Use mathematical induction to prove that $3 + 3 * 5 + 3 * 5^2 + \ldots + 3 * 5^n = 3\,(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.

11. Define the binomial coefficient and give the general term of the binomial coefficient. Show that the sum of the binomial coefficient is 2^n.

12. Find maximal flow for the network shown in the figure below.



MODEL SET 8

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/Science Full Marks: 60 + 20 + 20
Discrete Structure (CSC 160) Pass Marks: 24 + 8 + 8
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **[2x10=20]**

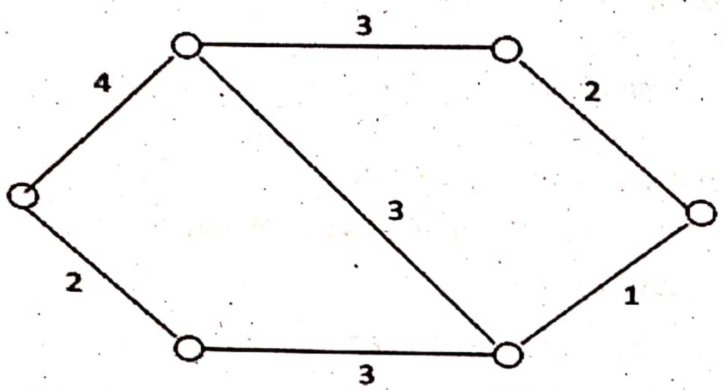
1. Define Euler and Hamiltonian circuits and paths with examples illustrating the existence and nonexistence of them.
2. Define Rules of Inference for quantified statements. Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."
3. Define recurrence relation. Define recurrence relation for Fibonacci series. Find the solution to the recurrence relation $a_n=-3a_{n-1}-3a_{n-2}-a_{n-3}$ with initial conditions $a_0=1$, $a1=-2$, and $a2=-1$.

Section B (Short Answer Questions)

Attempt any EIGHT questions. **[8 × 5=40]**

4. In how many ways can the letter of the word MONDAY be arranged? How many of these arrangements do notbegin with M? How many begin with M and do not end with Y.
5. Find an explicit formula for the Fibonacci numbers, with recursion relation $f_{n-1}+f_{n-2}$ and $f_0=0$, $f_1=1$
6. Explain the method of proving theorems by direct, indirect, contradiction and by cases.
7. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if
   a) $a_n = 0$.
   b) $a_n = 1$.
   c) $a_n = (-4)^n$.
   d) $a_n = 2(-4)^n + 3$

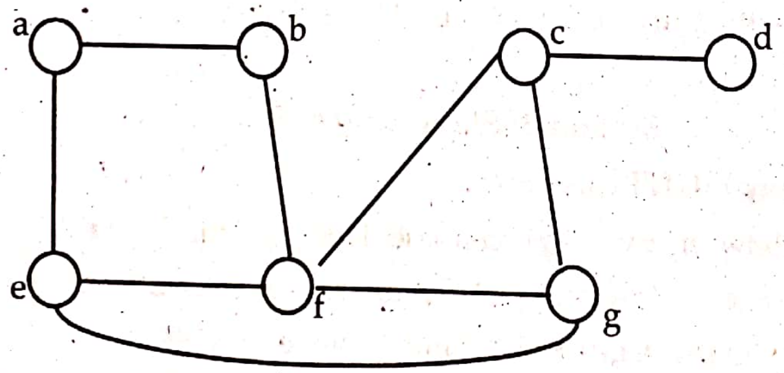
8. Discuss the shortest path algorithm of Dijkstra for finding the shortest path between two vertices. Use this algorithm to find the length of the shortest path between a and z in the following weighted graph?



9. Explain the concept of network flows and max-flow min-cut theorem with suitable examples.

10. What do you mean by combination with an example? Also show that $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

11. Find a spanning tree of the simple graph in the following graph, if it exists.



Can there be more possibilities?

12. State the converse and contra-positive of each of the following implications.

a. If it snows today, I will stay home.

b. We play the game if it is sunny.

c. If a positive integer is a prime then it has no divisors other than 1 and itself.