

Advanced Card Probability

Consider a deck of seven cards with the following values

A, A, 10, 10, J, J, Q.

A can be 11 or 1. 10, J, Q all have values of 10.

Two players want to play Blackjack, where the goal of the game is to draw two cards that add up to a value of 21. For example, a hand of A,Q would have a value of 21. If two A's are drawn, the value of the hand would be 12, so they don't go over 21.

Player one gets to have two independent hands, while player two only gets one hand. Player one draws first, adding to their second hand. Then player two draws a card. Then player one draws a card for their second hand. This repeats until each hand has two cards.

The question is, what is the probability that player one will have a better hand than player two? What about player one having a worse hand? What is the probability of a tie?

Run the code block below to run a simulation. The output is player 1 wins, player 2 wins, ties. Change `Number_Of_Games` to change the amount of trials.

In [10]:

```
import random
import matplotlib.pyplot as plt
# Definitions

Number_Of_Games = 100

def game():
    deck = [0, 0, 1, 1, 2, 2, 3]
    random.shuffle(deck)
    p1 = []
    p2 = []
    p3 = []
    for i in range(0, 6):
        card = deck.pop(random.randint(0, len(deck) - 1))
        if i % 3 == 0:
            p1.append(card)
        elif i % 3 == 1:
            p3.append(card)
        else:
            p2.append(card)
    p1_score = 0
    p2_score = 0
    p3_score = 0
    for card in p1:
        if card == 0:
            if p1_score == 11:
                p1_score += 1
            else:
                p1_score += 11
        else:
            p1_score += 10
```

```

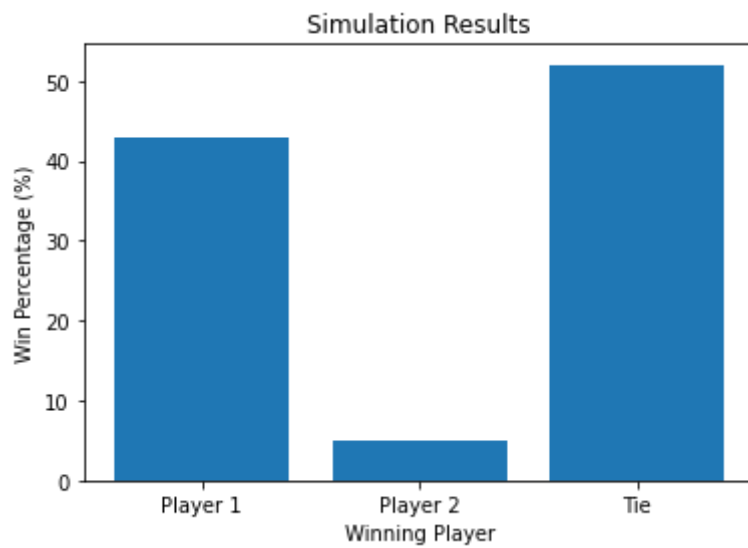
        p1_score += 10
    for card in p2:
        if card == 0:
            if p2_score == 11:
                p2_score += 1
            else:
                p2_score += 11
        else:
            p2_score += 10
    for card in p3:
        if card == 0:
            if p3_score == 11:
                p3_score += 1
            else:
                p3_score += 11
        else:
            p3_score += 10
    if (p2_score == p1_score and p2_score >= p3_score) or (p2_score == p3_score
        return 2
    else:
        return int((p2_score > p1_score) and (p2_score > p3_score))

def experiment():
    p1 = 0
    p2 = 0
    ties = 0
    for i in range(0, Number_Of_Games):
        res = game()
        if res == 0:
            p1 += 1
        elif res == 1:
            p2 += 1
        else:
            ties += 1
    l = ['Player 1', 'Player 2', 'Tie']
    print(str(p1) + ", " + str(p2) + ", " + str(ties))
    plt.bar(l,[p1, p2, ties])
    plt.xlabel("Winning Player")
    plt.ylabel("Win Percentage (%)")
    plt.title("Simulation Results")

experiment()

```

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Results

As you can see from the above simulation, the chance of player 1 winning is $\approx 45\%$. Player 2 wins $\approx 5\%$ of the time. Ties occur $\approx 50\%$ of the time.

Were these the results you calculated or expected?