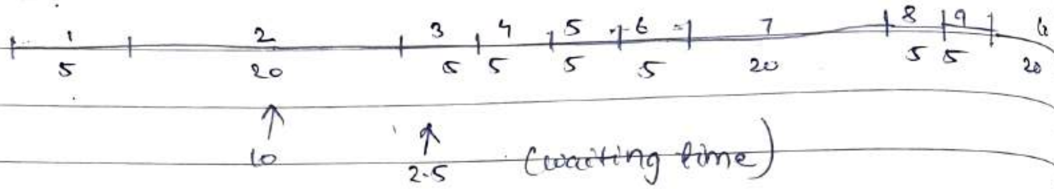
INTRODUCTION: QUEUING THEORY

70% → 5mins $0.7 \times 5 + 0.3 \times 20 = 9.5$
30% → 20mins how much you expect to wait on average = 4.75



total time = $7 \times 5 + 20 \times 3$
$= 35 + 60 = 95$ mins

∴ avg. waiting time = $\frac{35}{95} \times 5 + \frac{60}{95} \times 20 / 2$
$= \underline{7.23}$

0.8 → 5 mins $2 \times 30 + 8 \times 5$
0.2 → 30 mins $= 60 + 40 = 100$

∴ avg. waiting time = $\frac{40}{100} \times 5 + \frac{60}{100} \times 30 / 2$
$= \underline{10}$

* AK erlang (1909)

studied and formalized queing theory for telephone networks. traffic engg. airport take-off, landing.

PROBABILITY THEORY

events, probability (f^n), sample space
F   P Ω

(Ω, F, p) $\Omega = [0, 1]$ $\Omega = \{0, 1\}$

$F \subseteq 2^{\Omega}$ $F \notin 2^{\Omega}$ $F = \{\phi, \{1\}, \{0\}, \{1, 0\}\}$

if Ω is discrete $F = 2^{\Omega}$
if Ω is continuous $F \notin 2^{\Omega}$

AXIOMS OF PROBABILITY

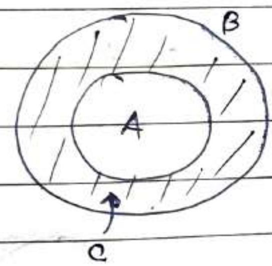
(i) $0 \le p(E) \le 1 \quad \forall \; E \in F$

(ii) $p(\Omega) = 1$

(iii) if $E_1 \cap E_2 = \phi$ $E_1, E_2 \in F$
then $p(E_1) + p(E_2) = p(E_1 \cup E_2)$

COROLLARIES

(i) if $A \subseteq B$
$p(A) \le p(B)$



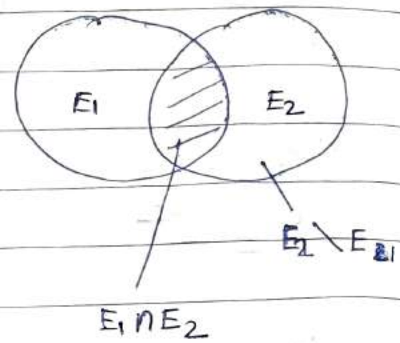
$A \cup (B \cap A')$

$p(A) + p(C) = p(B)$ ___ iii

since $p(C) \geq 0$ ___ i $A \cap C = \phi$

$\Rightarrow p(A) \le p(B)$

2. $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$



$p(E_1 \cup E_2)$

$= p(E_1 \cup (E_2 \setminus E_1))$

$= p(E_1) + p(E_2 \setminus E_1)$ — iii

$p(E_2) = p((E_1 \cap E_2) \cup (E_2 \setminus E_1))$

$= p(E_1 \cap E_2) + p(E_2 \setminus E_1)$ — iii

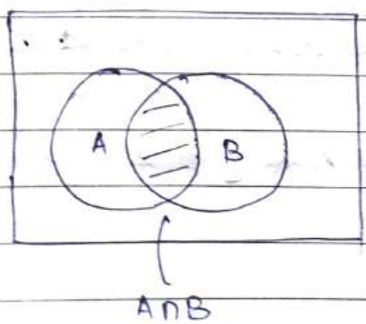
$\Rightarrow p(E_2 \setminus E_1) = p(E_2) - p(E_1 \cap E_2)$

$\therefore p(E_1 \cap E_2) = p(E_1) + p(E_2 \setminus E_1)$

$= p(E_1) + p(E_2) - p(E_1 \cap E_2)$

CONDITIONAL PROBABILITY

$p(A|B)$ probability of event A given B has ocurred.
$A, B \in F$



Ω is discrete,
all outcomes equally likely } say

$\therefore p(A|B) = \dfrac{N(A \cap B)}{N(B)}$

PROBABILITY MASS FUNCTION (PMF)

$p(X=x) = f_X(x)$ $\sum_{x\in\Omega} f_X(x) = 1$

HW roll a dice twice

$X(n_1, n_2) = n_1 + n_2$

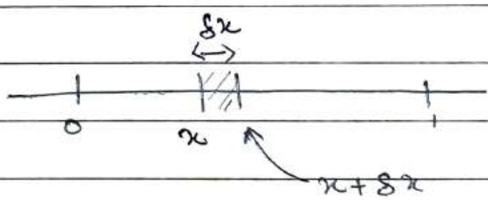
$p(X \in \{11, 12\}) = p\{(5,6), (6,5), (6,6)\}$

$= \frac{3}{36} = 1/12$

continuous random variable

$p(X = x)$ no X

$p(X \in [x, x+\delta x])$



$$f_X(x) = \lim_{\delta x \to 0} \frac{p(X \in [x, x+\delta x])}{\delta x}$$

probability density function (PDF)

cumulative distribution function (CDF)

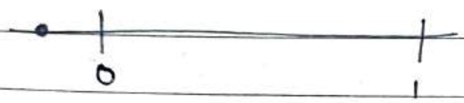
$F_X(x) = p(X \le x)$ $F_X(\infty) = 1$
$F_X(-\infty) = 0$

if $x_1 \le x_2$

$F_X(x_1) \le F_X(x_2)$

$p(A) \qquad p(B) \qquad A \subseteq B$

$f_X(x) = 0$



$\{x \in \mathbb{R} \mid f_X(x) \neq 0\}$ as support of X.

- gauss distribution
- poission distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad x \in (-\infty, \infty)$$

μ = mean, σ = standard deviation.

$\mu = 0$, $\sigma = 1$ is called normal distribution.

EXPECTATION

H.W. find out. binomial distribution geometric distribution

prove that. $E(ax+b) = aEx + b$.

$Y = aX + b$

$Y(\omega) = aX(\omega) + b$

RECAP

- probability axioms
- derived few important results
- conditional probability
- random variables (discrete & continuous)
- pmf, pdf, cdf

(i) $0 \leq p(E) \leq 1$

(ii) $p(\Omega) = 1$

(iii) if $E_1 \cap E_2 = \phi$ $\quad E_1, E_2 \in F$

$p(E_1 \cup E_2) = p(E_1) + p(E_2)$

$p(A|B) = \dfrac{p(A \cap B)}{p(B)}$ if $p(B) > 0$

gaussian distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad x \in (-\infty, \infty)$$

where, μ = mean, σ = standard deviation.

~~if~~ $\mu = 0 \quad \sigma = 1$ is called normal distribution

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EXPECTATION OF RV

X

e.g. i toss a coin n times, how many heads we expect?
$N/2$.

$$\Omega = \{\underbrace{HHH\ldots H}_{n}, \underbrace{HTH\ldots H}_{n}, \ldots, \underbrace{TTT\ldots T}_{n}\} \quad 2^n \text{ outcomes}$$

$X : \Omega \to \mathbb{R} \qquad X(\Omega) = \{0, 1, 2, \ldots n\}$

$X(\omega) = \#$ heads in 'ω' $\forall\, \Omega$
↳ sample

K experiments   1 = experiment is tossing coin n times

k_1 times   0 H   ~~E(X)~~ $\omega_0 \in \Omega$
k_2 times   1 H ω_1
⋮   ⋮ $k_1 + k_2 + \ldots k_{n+1} = K$.

k_{n+1} times nH ω_n

$$E(x) = \frac{0.k_1 + 1.k_2 + \dots + n.k_{n+1}}{k}$$

sample avg.

$$\lim_{k\to\infty} = 0 . \lim_{k\to\infty} \frac{k_1}{k} + 1 . \lim_{k\to\infty} \frac{k_2}{k} + \dots \, n . \lim_{k\to\infty} \frac{k_{n+1}}{k}$$

$$\lim_{k\to\infty} \frac{k_1}{k} = p(X=0)$$

$$\lim_{k\to\infty} \frac{k_i}{k} = p(X = i-1) \qquad i = \{1 \dots n+1\}$$

$$EX = \quad = \sum_{x \in X(\omega\Omega)} x \, p(X=x)$$ expectation for DRV.

$$X(\omega) = (x_1, \dots x_k)$$

$$EX = \int_{x : p(x) > 0} x . f_X(x) \, dx$$ expectation of CRV

$p: x \to x+dx$

$= f_X(x) \, dx$

HW. find out binomial distribution
geometric distribution.

prove that $E(aX+b) = aEx + b$

$$Y = aX + b$$
$$Y(\omega) = aX(\omega) + b$$

$$p(Y=y) = p(\{\omega \mid Y(\omega) = y\})$$
$$= p(\{\omega \mid aX(\omega) + b = y\})$$

bernoulli RV

coin toss $p(H) = p$

$X = 1 \quad p$

$\quad = 0 \quad \underbrace{1-p}_{\text{prob.}}$

if H, $X = 1$
if T, $X = 0$.

$E = 1 \cdot p + 0 \cdot (1-p) = p$.

if H $Y = 5 \cdot 1 + 3 = 8$

$Y = 5x + 3$

if T $Y = 5 \cdot 0 + 3 = 3$.

$EY = 8p + 3(1-p)$

E is linear operator.

$= 5p + 3$.

binomial RV $(n.p.)$

X = # successes in n independent bernolli trials.

$X_i = 1$ if ith bernoulli trial is success,
$\quad = 0$ otherwise.

$X = \sum_i X_i$

$EY = \sum_i EX_i = \sum_i p = n \cdot p$.

HW $E(g(x)) = \sum_x g(x)\, p(X = x)$

poisson RV

$X \in \{0, 1, 2, \ldots, \infty\}$

$$p(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} \qquad \sum_k p(X=k) = 1$$

$$EX = \sum_{k=0}^{\infty} k \cdot \frac{e^{-\lambda}\lambda^k}{k!} = \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^k}{(k-1)!}$$

$$= \sum_{k'=0}^{\infty} \frac{e^{-\lambda}\lambda^{k'+1}}{k'!}$$

$$= \lambda e^{-\lambda} \underbrace{\sum_{k'=0}^{\infty} \frac{\lambda^{k'}}{k'!}}_{e^{\lambda}} = \lambda$$

VARIANCE

how much spread the distribution values are.

$$E(X - EX)^2 \xrightarrow{\text{prove}} = EX^2 - [\underbrace{EX}_{\mu}]^2$$

M_1 – fist moment.

mean (μ) is called the first moment of RV.

EX^2 ~~variance~~ is called as the second moment.

EX^i is called the ith moment.

∴ moment generating functions

$$Ee^{tX} = 1 + tM_1 + \frac{t^2}{2!}M_2 + \ldots$$

MEMORYLESS RV

exponential RV $f_X(x) = \lambda e^{-\lambda x}$ if $x \geq 0$

$= 0$ o.w.

cdf $F_X(x) = 1 - e^{-\lambda x}$ $x \geq 0$

$F_X(x) = p(X \leq x)$

$p(X > t \mid X > t_0)$

$p(A \mid B) = \dfrac{p(A \cap B)}{p(B)}$ $\qquad \dfrac{P(X > t)}{P(X > t_0)}$

$p(A^c) = 1 - p(A)$ $\qquad = \dfrac{e^{-\lambda t}}{e^{-\lambda t_0}}$

$= \dfrac{1 - (1 - e^{-\lambda t})}{1 - (1 - e^{-\lambda t_0})}$

$p(X > t - t_0)$

$= e^{\lambda(t_0 - t)}$

$= e^{-\lambda(t - t_0)}$

$\therefore\ p(X > t \mid X > t_0) = p(X > t - t_0)$

markov property

game play learning.

physics engine

X: time at which 1st packet arrives

$t_0 = 100 \quad t = 105 \qquad t_0' = 5\text{sec}$

$t' = 10\text{sec}.$

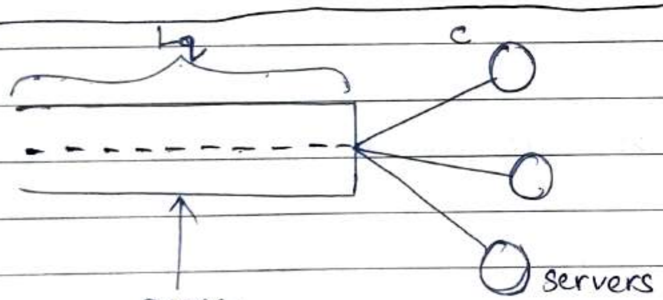
doesn't matter.

$$p(X > t \mid X > t_0) = p(X > t_0' \mid X > t_0')$$

memory less.

$$p(X_{t_{n+1}} = X_{t_n} = X_{t_n}, \ldots X_{t_0} = x_{t_0})$$

$$= p(X_{t_{n+1}} = x_{t_n} \mid X_{t_n} = x_{t_n}) \leftarrow \text{markov}$$



Ls: average or expected packet / customers / requests at servers.

Lq: -||- in the queue.

time spents in the system.

L = avg. or expected in the system.

$\boxed{L = L_q + L_s}$ $\qquad w = w_s + w_q$

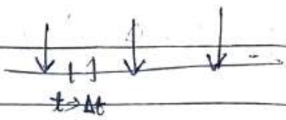
average rate of arrival λ

$$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} \to 0$$

(i) from $t \to t + \Delta t$

$$p(\text{one arrival}) = \lambda \Delta t + o(\Delta t)$$

(ii) $p(\text{more than one arrival})$



(iii) # arrivals in non-overlapping intervals are indepent of each other

$$0 \to 1 \to 2 \to 3 \to \cdots \; n \to$$

state n: till time t, n packets arrived.

$q_k(t)$ = prob. that there are k arrivals till time t.

$$k \geq 1 \qquad q_k(t+\Delta t) = q_{k-1}(t) \times \lambda \Delta t + q_k(t)(1 - \lambda \Delta t)$$

$$q_k(t+\Delta t) - q_k(t) = \lambda \Delta t \left(q_{k-1} - q_k\right)$$

$$\frac{q_k(t+\Delta t) - q_k(t)}{\Delta t} = \lambda q_{k-1} - \lambda q_k \quad \text{(1)}$$

for $k = 0$.

$$q_0(t+\Delta t) = q_0(1 - \lambda \Delta t)$$

$$\frac{q_0(t+\Delta t) - q_0(t)}{\Delta t} = -\lambda q_0(t) \quad \text{--- (2)}$$

$\lim_{\Delta t \to 0}$ in (2). $\frac{dq_0(t)}{dt} = -\lambda q_0(t)$

$$q_0(t) = e^{-\lambda t} \times c$$

$q_0(0) = 1 \Rightarrow c = 1$

$$q_0(t) = e^{-\lambda t}$$

$\lim_{\Delta t \to 0}$ in eq. (1).

$$\frac{dq_k(t)}{dt} = \lambda q_{k-1}(t) - \lambda q_k(t)$$

for $k=1$

$$\frac{dq_1(t)}{dt} = \lambda q_0(t) - \lambda q_1(t)$$

$$\frac{dq_1(t)}{dt} = -\lambda q_1(t) + \lambda e^{-\lambda t}$$

$$q_1(t) = (\lambda t) e^{-\lambda t}$$

$\vdots$

$$q_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

poisson distribution.

(memory less)

QUEUING THEORY

RECAP

- memoryless random variables
- exponential (λ) | poisson (λt) | geometric

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{oth.} \end{cases} \qquad p(X=n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

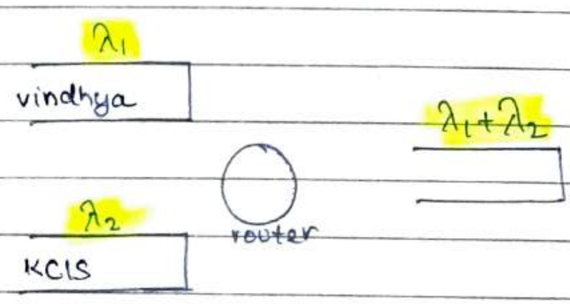
(i) two successive arrivals independent $\qquad \frac{o(\Delta t)}{\Delta t} \to 0$ as $\Delta t \to 0$

(ii) $p(\text{exactly one arrival in } \Delta t) = \lambda \Delta t + o(\Delta t)$

(iii) $p(\text{more than one arrival in } \Delta t) = o(\Delta t)$

then the process is a poisson process with parameter λt.

$$p(n \text{ arrivals in } t \text{ time units}) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$



$$p(K \text{ arrivals in } t \text{ time units}) = e^{-(\lambda_1+\lambda_2)t} \frac{((\lambda_1+\lambda_2)t)^K}{K!}$$

$= E_1$ K in first queue + 0 in second queue

E_2 K-1 in first queue + 1 in second queue

$\vdots$

E_{K+1} 0 in first queue + K in second queue.

$$= \sum_{K_1=0}^{K} p(\overbrace{K_1 \text{ packets in queue1}}^{A} \,\&\, \overbrace{K-K_1 \text{ packets in queue2}}^{B})$$

independent

$$= \sum_{K_1=0}^{K} e^{-\lambda_1 t} \frac{(\lambda_1 t)^{K_1}}{K_1!} \times e^{-\lambda_2 t} \frac{(\lambda_2 t)^{K-K_1}}{(K-K_1)!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)t}}{K!} \left\{ \sum_{K_1=0}^{K} \frac{K!}{K_1!\,(K-K_1)!} \times (\lambda_1 t)^{K_1} (\lambda_2 t)^{K-K_1} \right\}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)t}}{K!} ((\lambda_1+\lambda_2)t)^K$$

$=$ poisson $(\lambda_1+\lambda_2)$

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poisson RV $X(\lambda)$ $\quad E_X = \lambda$

exponential RV $X(\mu)$ $\quad E_X = 1/\mu$

$p(T \geq t) = p(\text{no packets in } t) = e^{-\lambda t}$

next packet arrival time.

$= 1 - p(T \leq t) \quad \therefore \quad p(T \leq t) = 1 - e^{-\lambda t}$

exponential.

$\lambda = 5$ packets / second

$p(5 \text{ packets in 2 seconds}) = e^{-10} \times \frac{10^5}{5!} = 0.03$

$p(5 \text{ packets in 1 second}) = e^{-5} \times \frac{5^5}{5!} = 0.17$

$p(10 \text{ packets in 1 second}) = e^{-5} \times \frac{5^{10}}{10!} = 0.0183$

A / B / C / D / E or A / B / C

A - arrival process (mean λ)

B - service rate (μ)

C - # servers

D - max. no of customers, or packets in the system.

E - population, max. no of customer will ever need the service

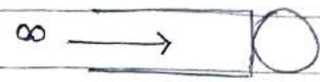
M/M/1

M - memoryless (poisson) λ

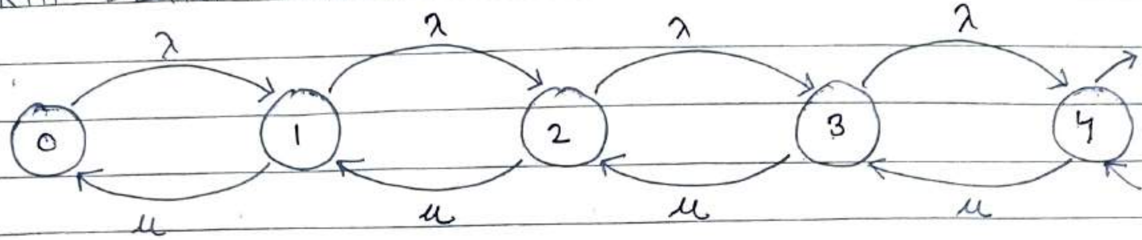
M - service process memoryless (μ) exponential.

1 - one server

∞ - buffer, ∞ - population size.

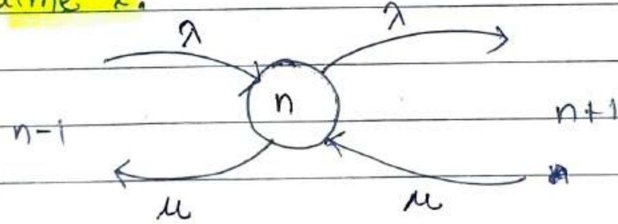


BIRTH - DEATH PROCESS



load factor $\rho = \frac{\lambda}{\mu} < 1$

$P_n(t)$ = probability of n coustomers in the system at time t.



$P_n(t + \Delta t)$ = no new arrival & no new departure & you were in state n at time t.

OR one new arrival & you were in state n-1 at time t

OR one new departure & you were in state n+1 at time t.

$$P_n(t+\Delta t) = (1-\lambda\Delta t)(1-\mu\Delta t)\,P_n(t) + \lambda\Delta t\,P_{n-1}(t) + \mu\Delta t\,P_{n+1}(t)$$

$$\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = -(\lambda+\mu)P_n(t) + \lambda\mu(\Delta t)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t)$$

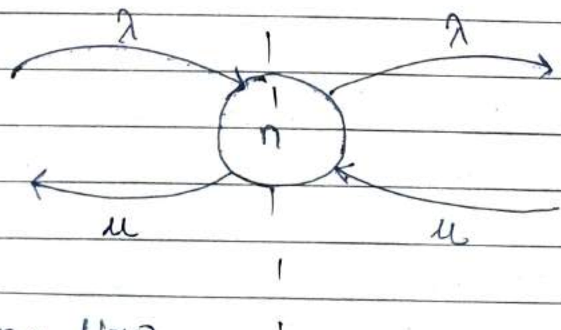
taking $\lim \Delta t \to 0$,

$$\frac{dP_n(t)}{dt} = -(\lambda+\mu)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t)$$

for steady state $\frac{dP_n(t)}{dt} = 0$.

$$\Rightarrow (\lambda+\mu)P_n(t) = \lambda P_{n-1}(t) + \mu P_{n+1}(t)$$

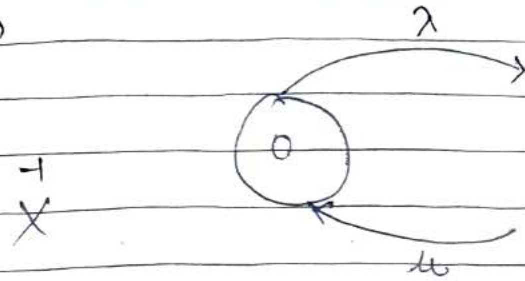
$$\Rightarrow (\lambda+\mu)P_n = \lambda P_{n-1} + \mu P_{n+1}$$



calculating flow

$$\underbrace{\lambda P_{n-1} + \mu P_{n+1}}_{\text{entry}} = \underbrace{(\lambda \cancel{p} + \mu)P_n}_{\text{exit}}$$

for $n=0$



$$\lambda P_0 = \mu P_1$$ M/M/1

$$\Rightarrow P_1 = \frac{\lambda}{\mu} P_0$$

$$\Rightarrow P_1 = \rho P_0$$

$$\cancel{\lambda P_0} + \mu P_2 = (\lambda + \cancel{\mu}) P_1$$

$$P_2 = \frac{\lambda P_1}{\mu} = \rho P_1 = \rho^2 P_0$$

$$P_n = \rho^n P_0$$

$$\sum_{n=0}^{\infty} P_n = 1$$

$$P_0 + \rho P_1 + \rho^2 P_2 + \dots = 1 \qquad P_0\left(\frac{1}{1-\rho}\right) = 1$$

$$P_0 = 1 - \rho$$

Utilization $U = 1 - P_0 = \rho$ M/M/1

	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$
$p(0 \le \ \le 3)$	0.9999	0.9375	0.34
$p(4 \le \ )$			
$p(12 \ge \ )$	10^{-12}		0.2835

$P_0 + P_1 + P_2 + P_3$ $1 - (P_0 + P_1 + \ldots P_{11})$

$= 0.9 + 0.09 + 0.009 + 0.0009$ $= 1 - \underbrace{0.999\ldots}_{12 \text{ times}}$

$\rho = 0.9 \quad \frac{0.9}{1-0.9} = 9 = L$

$W = \frac{9}{18} = 0.5 \quad W_{ser} = \frac{1}{20} = 0.05$

$W_q = 0.45$

$\rho = \frac{\lambda}{\mu} = \frac{19}{20} = 0.95$

$L = \frac{0.95}{0.05} = 19$

$W = 1 \quad W_q = \frac{19}{20} \times 60 = 57 sec$

$\rho = 0.2 \quad L = \frac{0.2}{0.8} = 0.25$

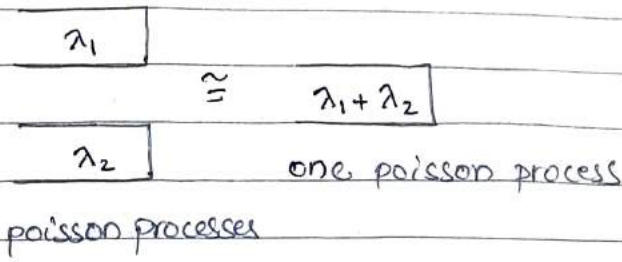
$W = \frac{0.25}{2} = 0.125$

$W_q = \frac{1}{8} - \frac{1}{10} \quad 0.025$

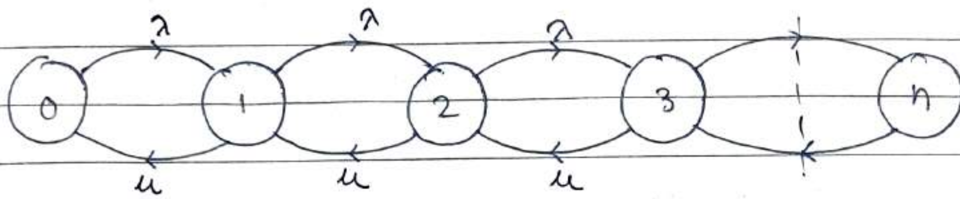
QUEUING THEORY

RECAP

poisson process



birth-death process



state n : # of customers / packets in the system

A/B/C/D/E

A - arrival process (λ) mean
B - service process (mean μ)
C - # servers
D - # max of customers / packets (buffersize)
E - population size that will need service

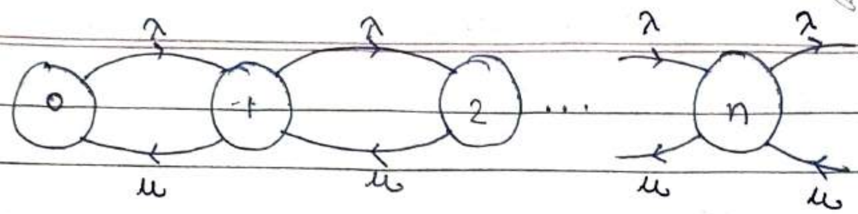
A/B/C ← $D = \infty$

A/B/C/D ← $E = \infty$

M/M/1 ← one server

M (first): arrival process is memoryless (poisson arrival with arrival rate λ)

M (second): the service process is memory less (exponential with service rate μ)



$$\frac{dP_n'(t)}{dt} = -(\lambda+\mu)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t)$$

$$P_n(0) = 0 \quad \forall\, n \neq 0$$

$t \to \infty$ we will assume $\frac{dP_n'}{dt} = 0$

$$\lambda P_{n-1} + \mu P_{n+1} = (\lambda+\mu)P_n$$

$$\lambda P_0 = \mu P_1$$

$$P_1 = \left(\frac{\lambda}{\mu}\right)P_0$$

$$= \rho P_0$$ load factor $(\rho < 1)$

$$\underline{\underline{P_n = \rho^n P_0}} \qquad P_0 = 1-\rho$$

$$P_n = \rho^n(1-\rho)$$

$$\underline{\underline{U = 1 - P_0}} \qquad U = \rho$$

$L = E(n)$

$$= 0 \times P_0 + 1 \times P_1 + \dots$$

$$= (1-\rho)\rho + 2 \times \rho^2(1-\rho) + \dots$$

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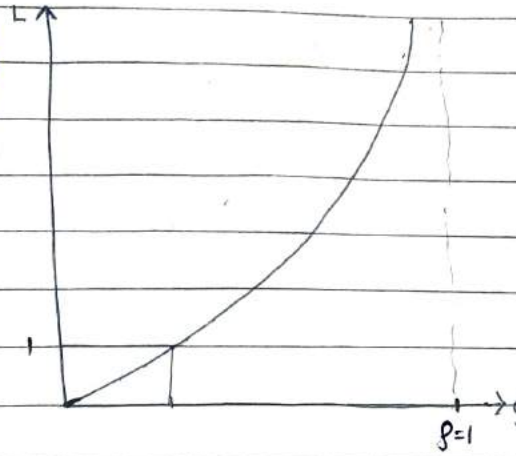
$$L - \rho L = (1-\rho)\rho(1+\rho+\rho^2+\ldots)$$

$$\underline{\underline{L = \rho \frac{1}{1-\rho}}} = \frac{\lambda}{\mu - \lambda}$$



① $\lambda = 0.9\mu \Rightarrow \rho = 0.9$

$$L = \frac{0.9}{0.1} = 9$$

② $\lambda = 0.95\mu$ or 0.99μ

$\downarrow$ $\downarrow$

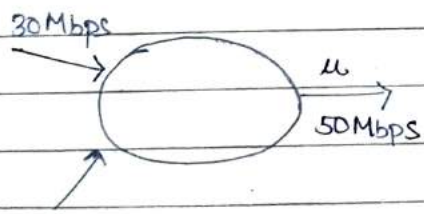
$L = 19$ 99

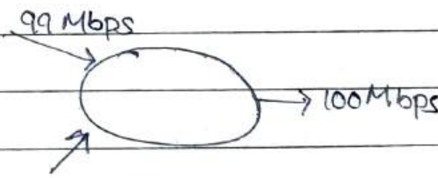
③ $\rho = 0.1$ $\rho = 0.5$

50%

$L = \frac{1}{9} = 0.111$ $L = 1$

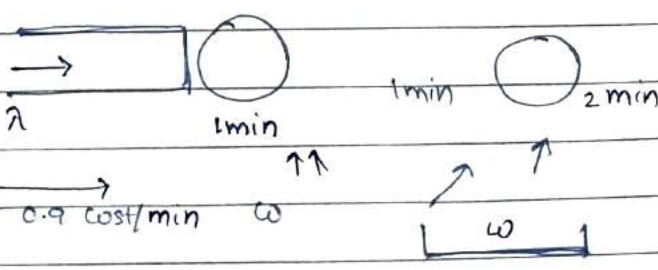
④





⑤ $L = L_q + L_s$

$\omega = \omega_q + \omega_s$



QoS : LITTLE'S LAW

$\boxed{L = \lambda w}$

w → avg. time spent by each packet

λ → avg. # packets / time

L → avg. # packets / customers.

$t \to 0$

E(# customers arrived in $[0, t]$) $= \lambda t$

$X = YZ$

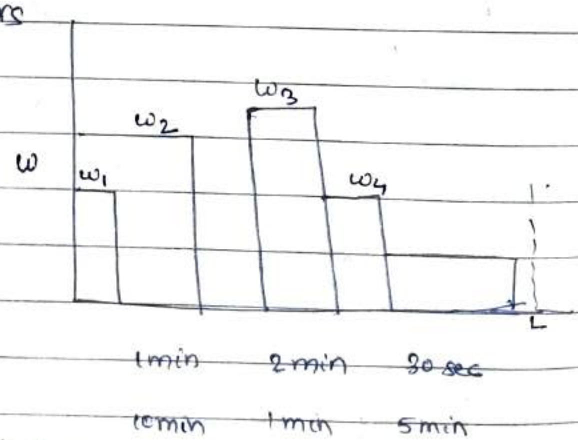
$EX = EY \cdot EZ$ ✗ not always true

on avg. each customer spends $\underline{w}$ unit of time in the system.

total time spent by all the customers those who arrived in $[0, t]$. $\lambda t \times w$

what is expected # customers L.

total time spent by these customers $= Lt$



$t \to \infty$ $\quad Lt = \lambda t w$

$$\underline{\underline{L = \lambda w}}$$ little's law.

(1) $\rho = 0.1$ $\lambda = 9$ customers / min

$$\omega = \frac{L}{\lambda} = \frac{\rho}{1-\rho} \times \frac{1}{\lambda} = \frac{0.1}{0.9} \times \frac{1}{9} = \frac{1}{81} \text{ mins.}$$

(2) $\lambda = 9$ customers / min $\mu = 10$ customers / min.

$$L = \frac{0.9}{0.1} = 9 \qquad \omega = \frac{L}{\lambda} = \frac{9}{9} = 1 \text{ min}$$

$$\boxed{\begin{aligned} \omega &= \omega_q + \omega_{ser} \\ L &= L_q + L_{ser} \end{aligned}}$$

$$\omega_{ser} = \frac{1}{\mu} \qquad \omega = \frac{L}{\lambda} = \frac{\lambda}{\mu-\lambda} \frac{1}{\lambda} = \frac{1}{\mu-\lambda}$$

(3) $\lambda = 9$ customers / min $\mu = 10$ customer / min

how much time will you spend in the queue on average?

$\omega = 1$ min, $\omega_q = \omega - \omega_{ser} = 1\text{min} - 0.1 = 0.9 \text{ min} = 54 \text{ sec.}$

(4). $\lambda = 18$ customer / min $\mu = 20$ customer / min

what is the expected time to be spent in the queue?

$$\rho = 0.9 \qquad L = \frac{\rho}{1-\rho} = 9$$

$$\omega = \frac{L}{\lambda} = \frac{9}{18} = 0.5 \text{ min}$$

$$\omega_q = \omega - \frac{1}{20} = 0.5 - 0.05 = 0.45 \text{ min} = 27 \text{ sec}$$

(5) $\lambda = 19$ customers/min $\quad \mu = 20$ customer/min

what is w_q?

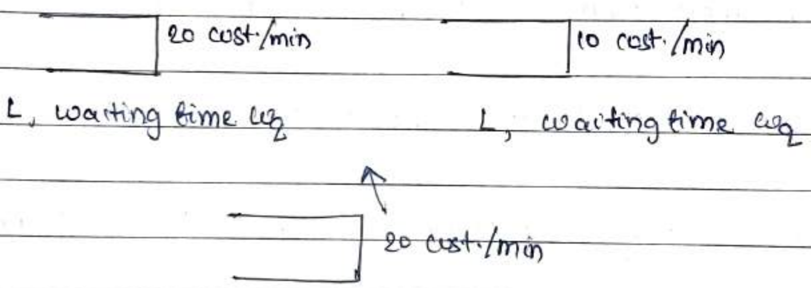
$L = 19 \quad w = 1 \quad w_q = 1 - \frac{1}{20} = \frac{19}{20} \times 60 = 57$ sec.

(6) $\lambda = 2$ customers/min $\quad \mu = 10$ customers/min

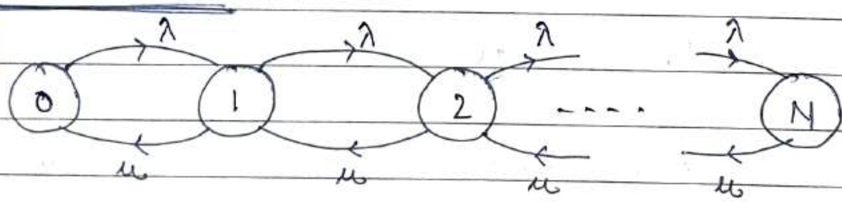
$\rho = 0.2 \quad L = \frac{\rho}{1-\rho} = \frac{1}{4} \quad w = \frac{1}{8}$

$w_q = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$ min $= \frac{60}{40} = 1.5$ sec.

HW (7)



M/M/1/N



$\lambda P_{n-1} = \mu P_{n}$

$0 = \frac{dP_n(t)}{dt} = -(\mu) P_N + \lambda P_{N-1}$

$P_N = \left(\frac{\lambda}{\mu}\right) P_{N-1}$

$$\frac{dP_{N-1}(t)}{dt} = 0$$

$$-(\lambda+\mu) P_{N-1} + \lambda P_{N-2} + \mu P_N$$

$$P_n = \rho^n P_0 \qquad n = 0 \dots N$$

$$\sum_{n=0}^{N} P_n = 1 \qquad P_0 + \rho P_0 + \rho^2 P_0 + \dots \rho^N P_0 = 1$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \qquad P_n = \rho^n \frac{1-\rho}{1-\rho^{N+1}}$$

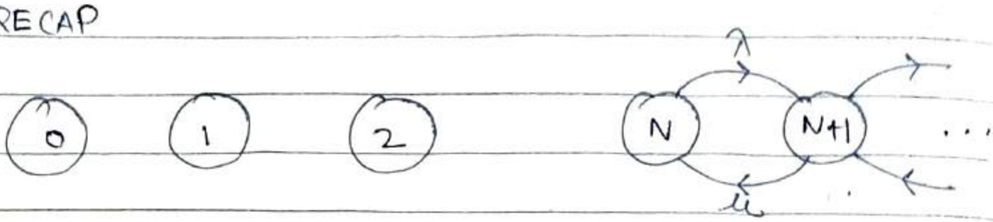
P_N blocking probability

M/M/∞ , M/M/m

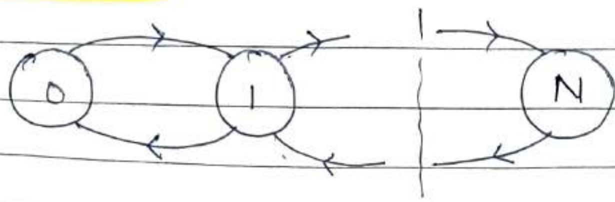
QUIZ
mon. / thu.
on moodle
syllabus: prob. theory,
m/m/1

QUEUING THEORY

RECAP



M/M/1/N



A/B/C/D/E

∞ ∞

max # customers in the system.

$$\lambda P_{N-1} = \mu P_N$$

$$\lambda P_{N-2} = \mu P_{N-2} \quad \cdots \quad \lambda P_0 = \mu P_1$$

$$P_n = \rho^n P_0 \qquad \rho = \frac{\lambda}{\mu} \qquad n = 0, 1, 2, \ldots N$$

$$\sum_{n=0}^{N} P_n = 1$$

$$P_0 \left(1 + \rho + \rho^2 + \ldots \rho^N \right) = 1$$

$$P_0 \left(\frac{1 - \rho^{N+1}}{1 - \rho} \right) = 1 \Rightarrow P_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

blocking probability $P_N = \frac{1 - \rho}{1 - \rho^{N+1}} \rho^N$ $(\rho > 1 ?)$

= Pr. (when queue is full)

for M/M/1 $\rho = 2$ $N = 2$

$U = 1 - P_0 = \rho$ $\qquad P_0 = \frac{1-2}{1-4} = \frac{1}{3}$ $\qquad U = 1 - \frac{1}{3} = \frac{2}{3}$

M/M/1/N

$\rho = 1 \Rightarrow P_0 = \frac{1}{N+1} \quad P_n = \frac{1}{N+1}$ } equiprobable

(1) $N = 5 \Rightarrow \quad P_0 = \frac{1-0.1}{1-0.1^6} = \frac{0.9}{0.999999} \approx \underline{\underline{0.9}} \quad P_N = \underline{\underline{9\times10^{-6}}}$

$\rho = 0.1$

$\rho = 0.5 \Rightarrow \quad P_0 = 0.508 \qquad P_N = \underline{\underline{0.016}}$

$\rho = 1 \Rightarrow \quad P_0 = 1/6 = \underline{\underline{0.167}} \qquad P_N = \underline{\underline{0.167}}$

$\rho = 2 \Rightarrow \quad P_0 = \underline{\underline{0.016}} \qquad P_N = \underline{\underline{0.508}}$

$\rho = 5 \Rightarrow \quad P_0 = \underline{\underline{0.00026}} \qquad P_N = \underline{\underline{0.8}}$

(2) $\rho = 0.9$

$N = 1 \quad P_N = 0.47$

$N = 2 \quad P_N = 0.3$

$N = 20 \quad P_N = 0.014$

$N = 100 \quad P_N = 2.7\times10^{-6}$

$L = E(n) = 0.P_0 + 1.P_1 + 2.P_2 + \dots N.P_N$

$= P_0\left(\rho + 2\rho^2 + 3\rho^3 + \dots N\rho^N\right)$

$\rho L = P_0\left(\rho^2 + 2\rho^3 + \dots (N-1)\rho^N + N\rho^{N+1}\right)$

$$L - \rho L = P_0\left(\rho + \rho^2 + \rho^3 + \dots \rho^N - N\rho^{N+1}\right)$$

$$L(1-\rho) = P_0\left\{\rho\frac{(1-\rho^{N+1})}{1-\rho} - N\rho^{N+1}\right\}$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$L(1-\rho) = \rho - N\rho^{N+1}\left(\frac{1-\rho}{1-\rho^{N+1}}\right)$$

$$= \rho - \frac{N\rho^{N+1}}{1-\rho^{N+1}} + \frac{N\rho^{N+2}}{1-\rho^{N+1}} \;??$$

$$\boxed{L = \frac{\rho}{1-\rho}\left(1-(N+1)P_N\right)}$$

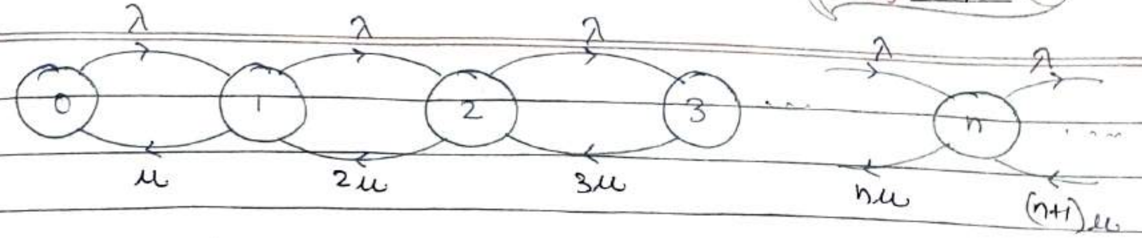
for $\rho = 1$, $L = \frac{1+2+\dots N}{N+1} = \frac{N}{2}$

M/M/∞ QUEUING SYSTEM — ∞ servers
∞ customers
∞ population

$L = L_q + L_s$ $\quad$ $L = \lambda\omega$

$$L = \frac{\rho}{1-\rho}\left(1-(N+1)P_N\right)$$

$$\omega = \frac{1}{\mu-\lambda}\left(1-(N+1)P_N\right)$$



o server1

o server2

⋮

o server n

$$\lambda P_{n-1} = n\mu P_n$$

$$P_n = \left(\frac{\rho^n}{n!}\right) P_0$$

$$\sum_{n=0}^{\infty} P_n = 1 \qquad P_0\left(1 + \sum_{n=1}^{\infty} \frac{\rho^n}{n!}\right) = 1$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\rho^n}{n!}} = e^{-\rho}$$

$$\underline{\underline{P_0 = e^{-\rho}}}$$

$$\underline{\underline{P_n = \left(\frac{\rho^n}{n!}\right) e^{-\rho}}}$$

$$L = \sum_{n=0}^{\infty} n P_n = \sum_{n=1}^{\infty} \frac{n}{n!} \rho^n e^{-\rho}$$

$$= \rho \sum_{n'=0}^{\infty} \frac{\rho^{n'}}{n'!} e^{-\rho}$$

$$= \rho\, e^{\rho} \cdot e^{-\rho} = \rho$$

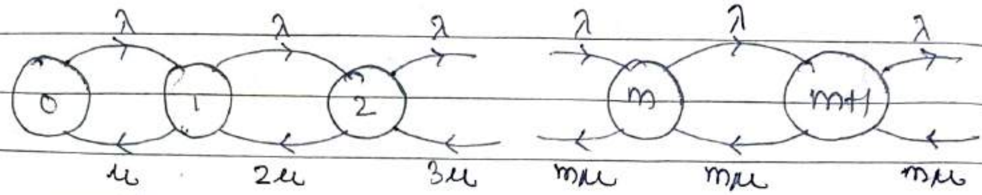
$$\therefore \underline{\underline{L = \rho}}$$

memoryless service

$M/M/m$ QUEUING SYSTEM

$M/M/m$ → m servers

memoryless arrival



$$\lambda P_{n-1} = n\mu P_n \qquad n \le m$$

$$m\mu P_n \qquad \text{if } n > m$$

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right) P_0 = \frac{\rho}{n!} P_0 \qquad \text{if } n \le m$$

$$= \frac{\rho}{m!(m)^{n-m}} P_0 \qquad \text{if } n > m$$

$$\sum_{n=0}^{\infty} P_n = 1$$

$$\Rightarrow P_0 = \frac{1}{1 + \sum_{n=1}^{m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \sum_{n=m}^{\infty} \frac{1}{m^{n-m}}\left(\frac{\lambda}{\mu}\right)^{n-m}}$$

$$\rho = \frac{\lambda}{m\mu} \text{ in } M/M/m$$

$$L_q = \sum_{n=m+1}^{\infty} (n-m) P_n \qquad P_m$$

$$= \sum_{n=m+1}^{\infty} (n-m) \cdot \frac{1}{m!} \cdot \frac{1}{m^{n-m}} \cdot \left(\frac{\rho}{m}\right)^n \cdot P_0$$

$$L_q = \frac{\rho}{1-\rho} \cdot \left(\frac{\rho^m}{m!(1-\rho)} \times P_0\right)$$

$= P_m + P_{m+1} + \dots$

prob. (queuing)

after queue starts filling in, the system starts behaving similar to M/M/1 with service rate mμ.

M/M/m/m QUEUING SYSTEM

memoryless arrival

memoryless service

m servers

max # customers / packets in system

P_0 P_n.

M/G/1

general service process

M/D/1

deterministic service process.

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$$w_s = \frac{1}{3} \times 0.99 + 30 \times 0.01 = 0.63 \text{ min} = \underline{\underline{38 \text{ sec}}}.$$

$$\lambda = 1/\text{min} \qquad \mu = \frac{1}{0.63} \qquad \rho = \underline{\underline{0.63}}.$$

$$\sigma_{ser}^2 = \sqrt{(0.33 - 0.38)^2 \times 0.99 + (30 - 0.38)^2 \times 0.01}$$

$$= \underline{\underline{8.78}}$$

$$L = 0.63 + \frac{0.63^2 + 1^2 \cdot 8.78^2}{2(1-0.63)}$$

$$= \underline{\underline{13}}$$

$$L = \lambda w \qquad w = \frac{L}{\lambda} = \underline{\underline{13}} \text{ min.}$$

$$\rho\left(1 + \frac{\rho}{2(1-\rho)}\right)$$

$$L_D = \rho + \frac{\rho}{2} L_1$$

$$0.99 \times \frac{1}{3} + 0.01 \times 30 = 0.63 \text{ min}$$

$$\frac{0.99 \times \frac{1}{3}}{0.63} \times \frac{1}{3} + \frac{0.01 \times 30}{0.63} \times 30$$

$$= 0.175 + 14 = 14.175$$

SO WHAT DID WE SEE LAST CLASS

RECAP

M/M/1/N queue $\longrightarrow$ $P_n = \frac{1-\rho}{1-\rho^{N+1}} \rho^n \quad \rho \neq 1$

$L = \frac{\rho}{1-\rho}\left[1-(N+1)P_N\right]$

P_N: blocking probability

$\rho = 1, \quad P_n = \frac{1}{N+1}$

$L = N/2$

M/M/∞

$P_n = \frac{\rho^n}{n!} e^{-\rho}$

$L = \rho$

$L_q = 0$

M/M/m $\quad P_n = \frac{\rho^n}{n!} P_0 \quad n \leq m$

$= \frac{\rho^n}{m!\, m^{n-m}} P_0$

$L_q = \frac{\rho}{1-\rho} \times \frac{\rho^m}{m!(1-\rho)} \times P_0$ — prob. of queuing

$\underbrace{P_m + P_{m+1} + P_{m+2} + \dots}$

$\rho = \frac{\lambda}{m\mu}$

$L_q = \sum_{n=m+1}^{\infty} (n-m) P_n = \frac{1}{m!} \sum_{n=m+1}^{\infty} (n-m) m^m \rho^m P_0$

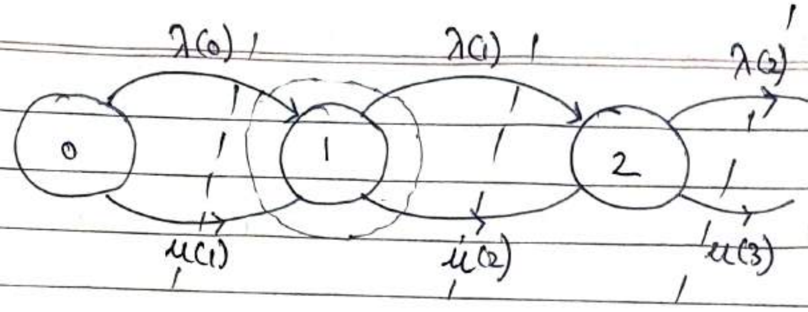
M/M/m/m $\longleftrightarrow$ max. m packets in system.

$\rightarrow$ m servers

$L_q = 0$

memoryless arrival

memoryless service



$\lambda(0)\, P_0 = \mu(1)\cdot P_1$

$$P_n = \prod_{i=1}^{n} \frac{\lambda(i-1)}{\mu(i-1)} P_0$$

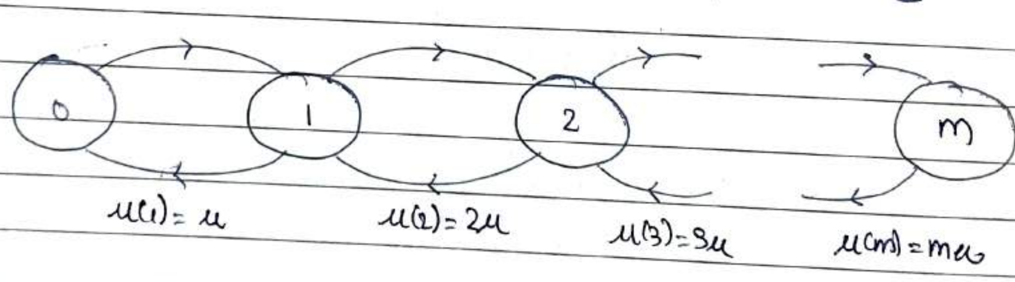
$\lambda(1)\, P_1 = \mu(2)\, P_2$

$\vdots$

$P_0 + P_1 + \dots = 1$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\lambda(i-1)}{\mu(i-1)}}$$

or using, $\lambda(0) P_0 + \mu(2) P_2 = \mu(1) P_1 + \lambda(1) P_1$



$$P_0 = \frac{1}{1 + \sum_{i=1}^{m} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n}$$

$$P_m = \frac{1}{m!} \left(\frac{\lambda}{\mu}\right)^m \Big/ 1 + \sum_{n=1}^{m} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)$$

M/G/1

M - memoryless arrival
G - general service process
1 - one server
∞ - buffer

t_{ser} : time taken to provide service to the customer
(if G = M, t_{ser} is exponentially distributed)

HW → σ^2_{ser} : var(t_{ser}) $\mu_{ser} = \frac{1}{E(t_{ser})}$

HW. exercise
show that L matches with $L = \frac{\rho}{1-\rho}$ when t_{ser} is exponentially distributed.

$$L = \rho + \frac{\rho^2 + \lambda^2 \sigma^2_{ser}}{2(1-\rho)}$$

BAD POST OFFICE

t_{ser} = 20 sec 0.99 prob.
  30 min 0.01 prob. W_s

one customer arrives per minute ($\lambda = 1$)

(1) what will be the avg. time spent by the customer in the post office? ← W

(2) on average how many customers will be there in the post office? ← L

$L, W, W_s = E[t_{ser}]$

$E_X = \sum_i x_i p_i$

$w_s = 20 \times 0.99 + 1800 \times 0.01 = 37.8$ sec

$\mu = \frac{1}{37.8}$ $\lambda = 1/60$

$\sigma^2_{ser} = Ex^2 - (Ex)^2$

$= 400 \times 0.99 + 1800^2 \times 0.01 - 37.8^2$

$= 31367.16$

$\rho = \frac{\lambda}{\mu} = \frac{1}{60} \times 37.8 = 0.626$ (~ 0.63)

$L = 12.94$

$w = \frac{L}{\lambda} = 12.94$ min.

$\lambda = 1$ customer/min $w_{ser} = 37.8$ sec

$w = 12.94$ min $\rho = 0.63$

M/D/1

M - memoryless arrival.

D - deterministic service

t_{ser} is fixed/constant.

$\sigma^2_{ser} = 0$

$L_D = \rho + \frac{\rho^2 + 0}{2(1-\rho)}$

ρ	L (M/M/1)	L_D (M/D/1)
0.1	0.111	0.106
0.2	0.250	0.225
0.5	1	0.75
0.8	4	2.4
0.9	9	4.95
0.99	99	50

$$\frac{L_D}{L} = \left[\rho + \frac{\rho^2}{2(1-\rho)}\right]\left[\frac{1-\rho}{\rho}\right] = \frac{2\rho(1-\rho)+\rho^2}{2(1-\rho)}$$

$$= \frac{\rho}{1-\rho}\left[\frac{2(1-\rho)+\rho}{2}\right] \times \left[\frac{1-\rho}{\rho}\right]$$

$$\frac{L_D}{L} = 1 - \frac{\rho}{2}$$

$U = \rho \quad (1 - P_0)$

M/M/1

$\rho \to 1 \qquad \frac{L_D}{L} \to 1/2$

will D/D/1 $\quad \lambda < \mu \quad \rho < 1 \quad L_q = 0$

M/G/1, M/D/1

M/M/1 approximates M/M/1

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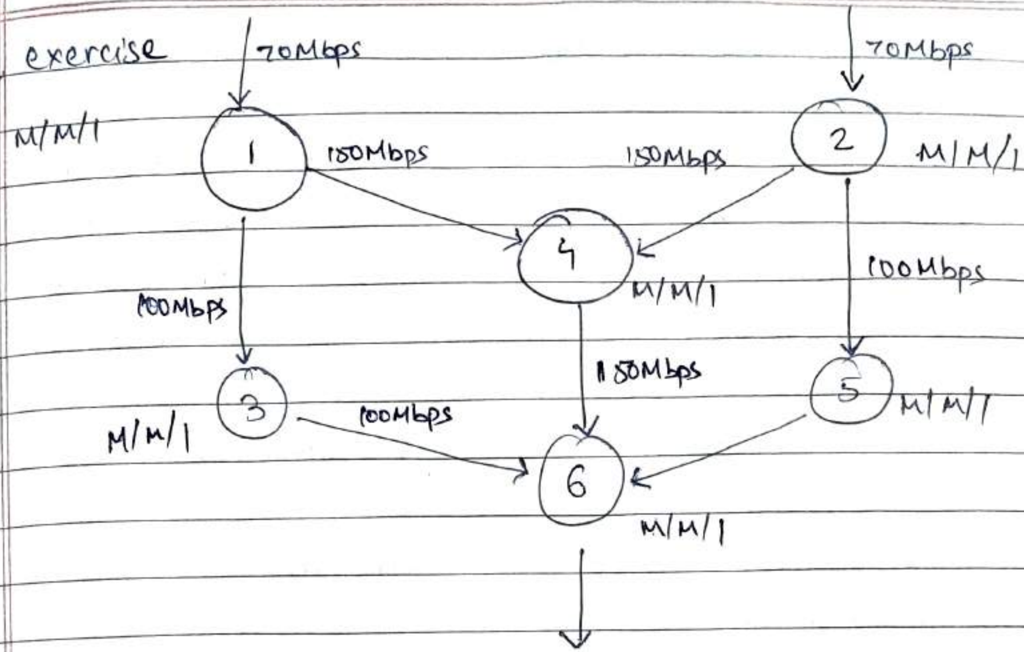
$L = \lambda w$ ← little's law

queuing System	L	L_s	L_q	W	W_q	W_s	P_0, P_n
M/M/1 $\rho = \frac{\lambda}{\mu} < 1$	$\frac{\rho}{1-\rho}$	ρ	$\frac{\rho^2}{1-\rho}$	$\frac{L}{\lambda}$	$\frac{L_q}{\lambda}$	$\frac{1}{\mu}$	$P_0 = 1-\rho$, $P_n = \rho^n P_0$
M/M/∞ $\rho = \frac{\lambda}{\mu} < \infty$	ρ	ρ	0	$\frac{1}{\mu}$	0		$P_0 = e^{-\rho}$, $P_n = \frac{\rho^n}{n!} P_0$
M/M/1/N $\rho = \frac{\lambda}{\mu} < \infty$	$\rho \neq 1$: $\frac{\rho}{1-\rho}[1-(N+1)P_N]$; $\rho = 1$: $N/2$					$\frac{1}{\mu}$	$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$, $P_n = \rho^n P_0$ ($\rho \neq 1$); $\rho = 1$: $P_n = \frac{1}{N+1}$
M/M/m $\rho = \frac{\lambda}{m\mu} < 1$	λw		$\frac{\rho}{1-\rho} \times Pr(q)$ $(P_m + P_{m+1} + \cdots)$	$\frac{L_q}{\lambda} + \frac{1}{\mu}$	$\frac{L_q}{\lambda}$	$\frac{1}{\mu}$	
M/D/1 $\rho = \frac{\lambda}{\mu} < 1$	$\rho + \frac{\rho^2}{2(1-\rho)}$					$\frac{1}{\mu}$	

HW exercise



1-4-6 & 2-4-6 Strategy 1

1-3-6 & 2-5-6 Strategy 2

which one is better.

ROUTNG

make evil go away.

verse 1

there is a world
that is virtual and different
it can be so cold
makes us stand up for what's right
our hope through our life
if we reset it to the start

chorus

here we are, going far
to save all that we love
if we give, all we've got
we will make it through
here we are, like a star
shining bright on your world
today (make evil go away)

mini-chorus

code lyoko we'll, reset it all
code lyoko be, there when you call
code lyoko we, will stand real tall
code lyoko stronger after all

verse 2

a world of machines
it can shadow human nature
all that we need
is the way to find the answer
and one thing is for sure
you can count on us for good.

chorus ...

verse 3

we'll do our best
to never let you down
we're up to the test
to turn this world around

mini-chorus ...

mini-chorus ...

X

by Noam Kaniel (vocals)

written by
Ygal Amar
Franck Keller

A WORLD WITHOUT DANGER
CODE LYOKO (June 6, 2004)

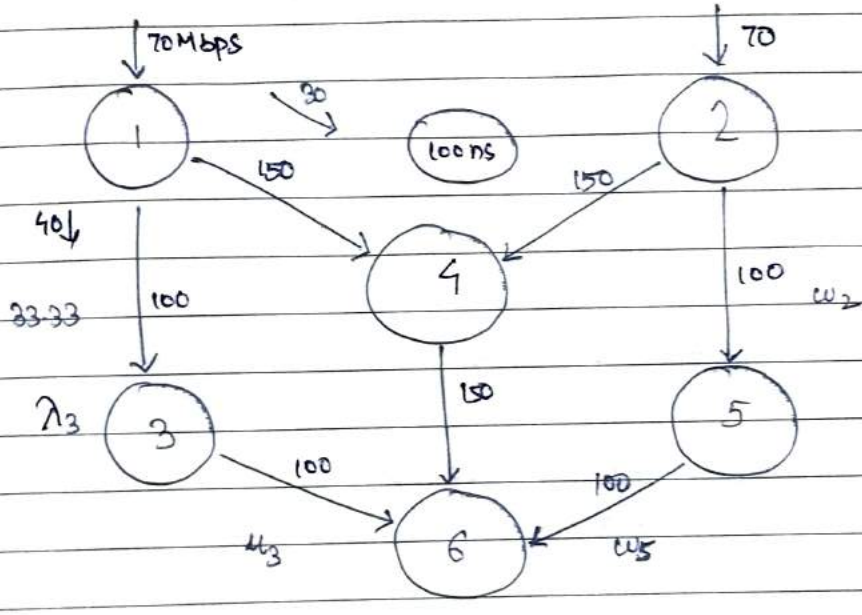
RECAP

M/M/m
M/M/m/m
M/G/1
M/D/1

queuing system

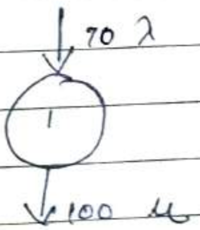
M/M/1

L, W, W_s, L_q, W_q, L_s tabular summary



routing 1 1-3-6 & 2-5-6
2 1-4-6 & 2-4-6

routing 1



$\rho = \frac{70}{100} = 0.7$

w_1: delay from 1 → 3

$w_1 = \frac{L_1}{\lambda_1} = \frac{\rho_1}{\lambda_1(1-\rho_1)} = \frac{7}{3 \times 70 \times 10^6} = \frac{10^{-6}}{30} = \frac{100}{3}$ ns $= 33.3$ ns

$\lambda_3 = 70 \times 10^6$ $\rho_3 =$ $w_3 = 33.33$ ns.

$w_2 = w_5 = 33.33$ ns

<u>routing 2</u>

1 - 4 - 6 2 - 4 - 6

$\lambda_1 = 70$ $\mu_1 = 150$ $\rho_1 = \frac{70}{150}$

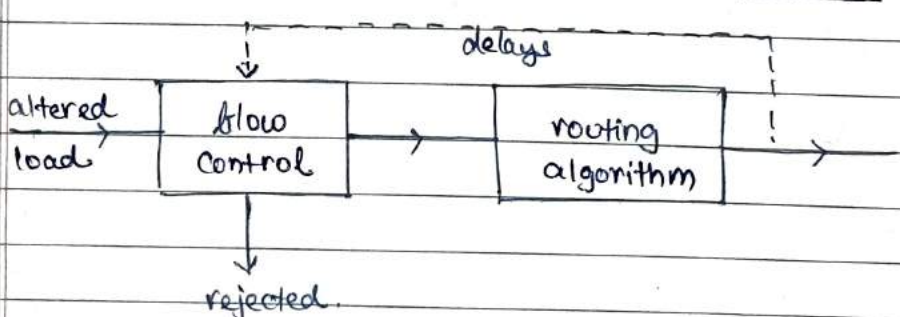
1 - 4 $w_1 = \frac{L_1}{\lambda_1} = \frac{70}{70\,(80) \times 10^6} = 12.5$ ns

2 - 4 $w_2 = 12.5$ ns

4 - 6 $\lambda_4 = 140$ $\mu_4 = 150$ $\rho_4 = \frac{140}{150}$

$w_4 = \frac{L_4}{\lambda_4} = 100$ ns

net delay $w_1 + w_4$ or $w_2 + w_4 = \underline{112.5 \text{ ns}}$

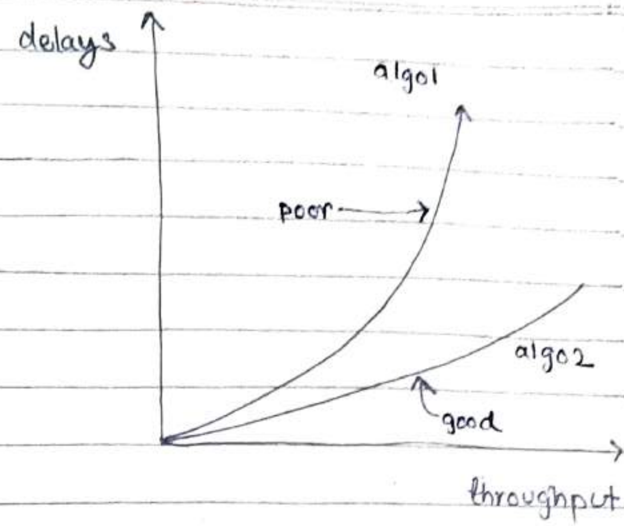


<u>throughput = offered load - rejected load</u>

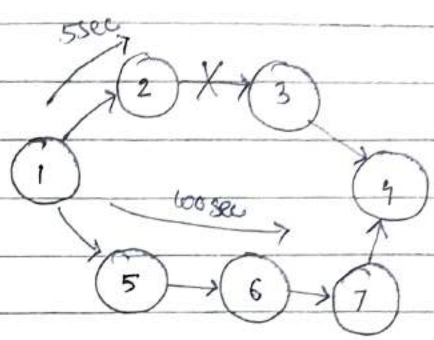
ROUTING

ensure delays are minimized
maximize throughputs

routing → protocols
routing → algorithms



(i) static vs. dynamic
(ii) flat vs. hierarchical
(iii) ~~flat~~ intra-domain vs. inter-domain
(iv) distributed vs. centralized
(v) single vs. multipath
(vi) link state vs. distance vector



need to know whole network (dijkstra)
(hierarchical routing)
bellman-ford

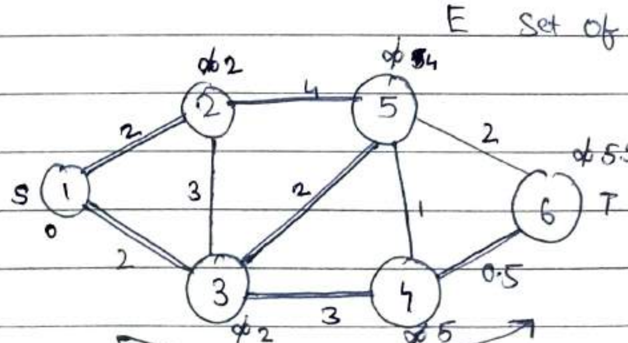
ROUTNG ALGORITHMS (shortest-path algorithms)

(i) dijkstra
(ii) bellman-ford
(iii) floyd-warshall

DIJKSTRA

$G = (V, E)$ V set of vertices
E set of edges

$s = \{1\}$



every step we add 1 node to our shortest path tree

$S = \{1\}$

initialization $D_1 = 0$ $D_j = d_{ji}$ $\forall j \neq 1$

$P = \{\}$

ALGORITHM

(1) $D_i = \min_{j \notin P} D_j$ $i \notin P$

$P = P \cup \{i\}$

(2) $D_j = \min\{D_j, \underset{2 + 3}{D_i + d_{ij}}\}$ $\forall j \notin P$

goto (1) until $P = G$

initialization $D_1 = 0$ $D_2 = 2$ $D_3 = 2$ $D_4 = \infty$ $D_5 = \infty$ $D_6 = \infty$
$P = \{1\}$

(1). $P = \{1, 2\}$
$D_2 = 2$, $D_3 = 2$ $D_4 = \infty$ $D_5 = 6$ $D_6 = \infty$

(2). $P = \{1, 2, 3\}$
$D_2 = 2$ $D_3 = 2$ $\underline{D_4 = 5}$ $\underline{D_5 = 4}$ $D_6 = \infty$

(3) $P = \{1, 2, 3, 5\}$
$D_2 = 2$ $D_3 = 2$ $D_5 = 4$ $D_4 = 5$ $D_6 = 5.5$

(4) $P = \{1, 2, 3, 4, 5, 6\}$
done.

running time $|V| = N$ # iterations = $N-1$

$O(N)$ in each iteration

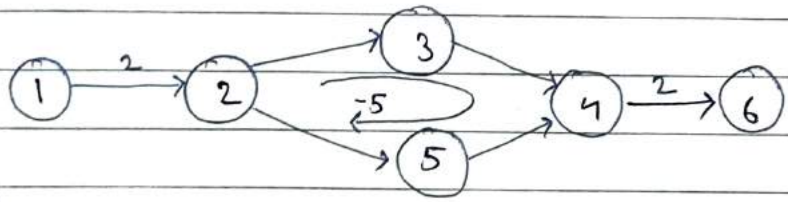
$O(N^2) \rightarrow O(N \log N)$ $O(|E| + |V| \log |V|)$

dijkstra disadvantage

(i) -ve edges are not allowed.

BELLMAN-FORD

-ve edges are allowed
but, no -ve wt. cycles. ← shortest path is not defined



init. ~~init~~ $D_1^h = 0$ $h = 1 \ldots n$

$D_i^0 = \infty$ $\forall i = 2 \ldots n$ $i \neq 1$

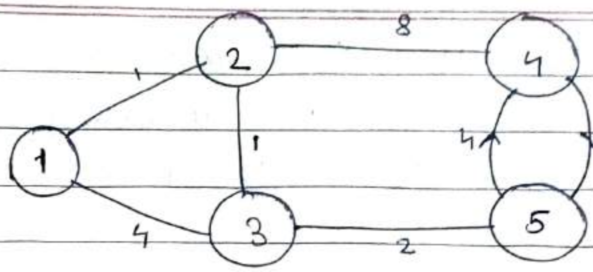
algorithm. $D_i^{h+1} = \min \{ d_{ij} + D_j^h \}$ $\forall i \neq 1$

terminate if $D_i^h = D_i^{h-1}$ $\forall i$

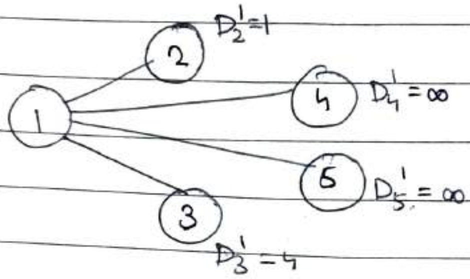
(D_j^h min. cost of reaching j using atmost h edges)

$D_i^{h+1} \leq \min_{j \neq 1} \{ d_j + D_i^h, D_i^h \}$

every step, shortest path with n edges to each node is calculated.



$D_1^1 = 0 \quad D_2^0 = D_3^0 = D_4^0 = D_5^0 = \infty$



running time

$O(|V||E|) \quad O(N^3)$

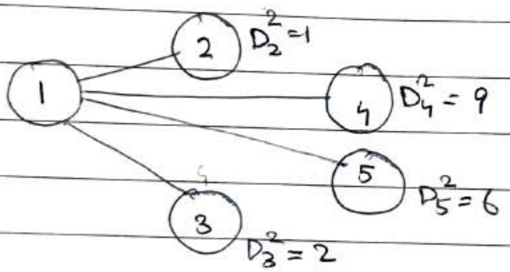
$|V| = N$

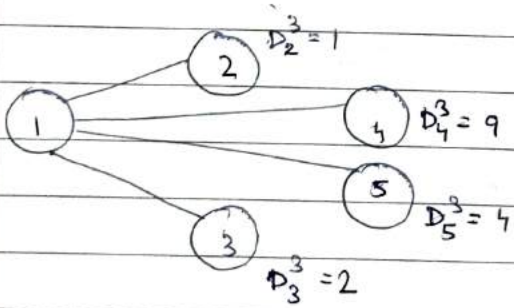
$O(m|E|)$

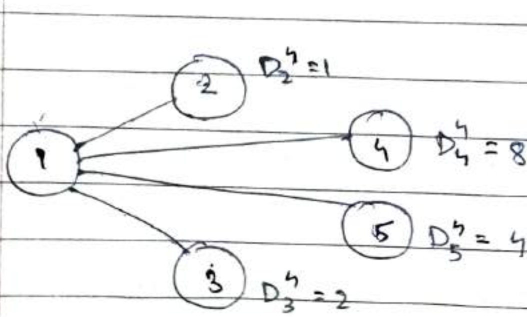
\# iteration required.

$m << N$

$|E| << N^2$







PROPOSITION

(a) D_i^h s ~~are~~ generated by the algorithm are equal to the shortest path from i to 1 of # edges $\leq h$.

(b) the algorithm terminates iff all cycles not containing node 1 are having non--ve costs. if algo terminates, it so in $h \leq N$ & at termination, D_i^h is the cost of shortest path from 1 to i (or i to 1).

HW $K \leq h$ prove $h+1$

$D_i^{K+1} \leq D_i^K \quad \forall K \leq h.$

FLOYD - WARSHALL
(all-pair shortest path)

$D_{ij}^0 = d_{ij} \quad \forall i,j \quad$ for $h = 0 \ldots N-1$

$$D_{ij}^{h+1} = \min_K \left[D_{ij}^h , D_{iK}^h + D_{Kj}^h \right]$$

iteration - h:

we have the shortest path cost from i to j using nodes $1, 2, \ldots h$ ($h = 0$, w/o any node)

$O(N^3) \quad O(|V|^3)$

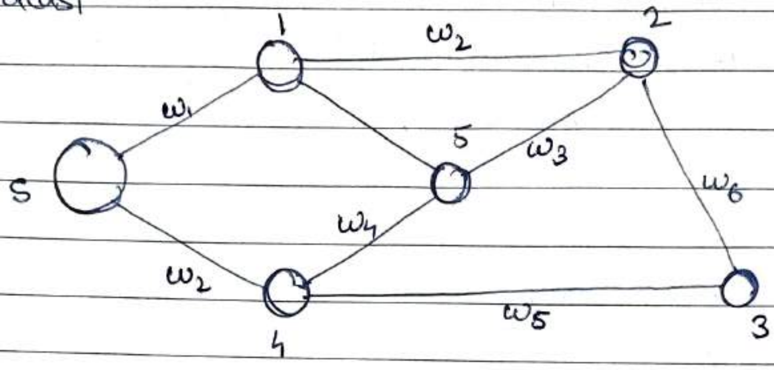
ROUTING AND VPN

RECAP

routing algorithms (shortest path) — dijkstra - greedy; bellman-ford, flgodyd-warshall } dynamic programming

unicast - routing

broadcast

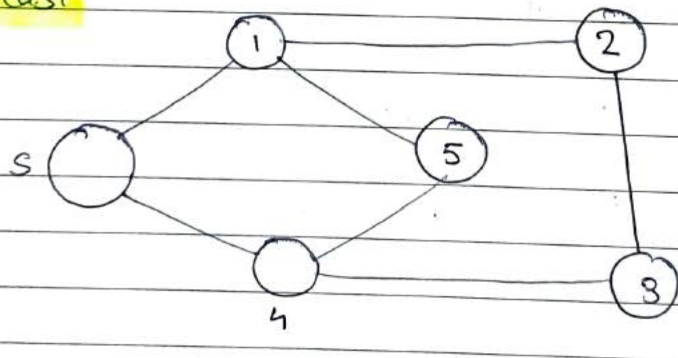


min (s, 2)

min. $\sum c(s,i)$

MST — Kruskal's, prim's

multicast



$s \to \{1, 3, 5\}$

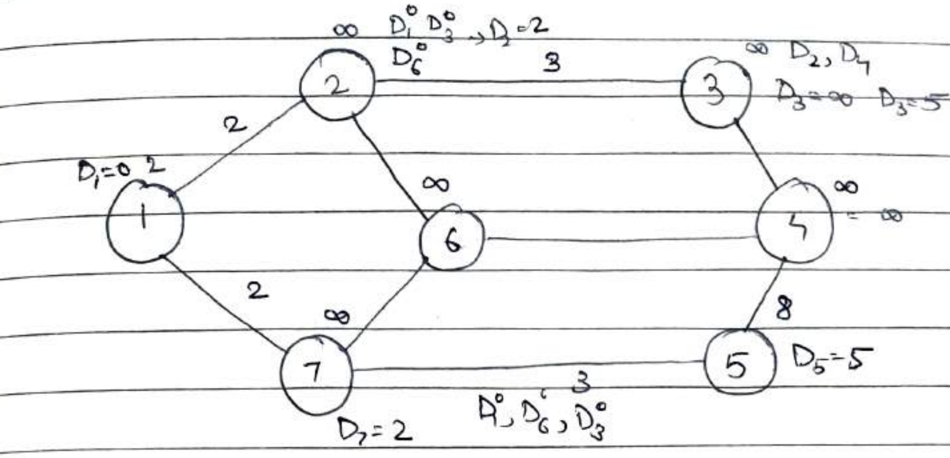
M st M spans $\{1, 3, 5\}$

Cost $(M) = \sum_{e \in M} w_e$ is minimized.

min cost Steiner tree $\longrightarrow$ NP-complete

PIM

ARPANET (1969)



BELLMAN-FORD

$\forall h \quad D_1^h=0, \quad D_i^0=\infty \quad \forall i$

$$D_i^h = \min_{j\in N(i)} \left[d_{ij} + D_j^{h-1} \right] \quad i\neq 1$$

$\underbrace{d_{i3} \quad d_{i5} \quad d_{i1}}_{?}$

what if each node synchronously start.

this is challenging

if node == 1
$D_1^h = 0 \quad \forall h$

if node == i
$D_i^0 = \infty$

- -

$$D_i^h = \underbrace{\min_{j\in N(i)} d_{ij} + D_j^{h-1}}$$

the algorithm will also converge

$$D_i^{i+1} = \min_{j\in N(i)} d_{ij} + D_j^{t-1}$$

825 ms will broadcast D_i's to its neighboors.

DYNAMIC ROUTING

distance vector routing	link state routing
routers should know only its neighboors	routers should know whole network.
(you keeps only dist. to neighbours)	(you keep whole network and all link states & delays)
bellman-ford (slower)	dijkstra (faster)
(if some link goes dow, knowledge passing is slow)	(more BW as more traffic is generated)
less BW & less traffic	

Static routing

typically routing is fixed
(smaller organizations)

ARPANET
distance vector routing (1969)

RIP (v1) ⟶ RIP (v2)
(1998) → RFC 2453
request for
comments.

hops allowed to
be 15

DYNAMIC ROUTING

interior gateway
(autonomous system AS
within its)

exterior gateway
(between diff. AS)

distance vector

- RIPv1
  - RIPv2
    (RFC 2453)
- IGRP
  - EIGRP
    (RFC 7868)

link state

- OSPF
- IS-IS

(1)

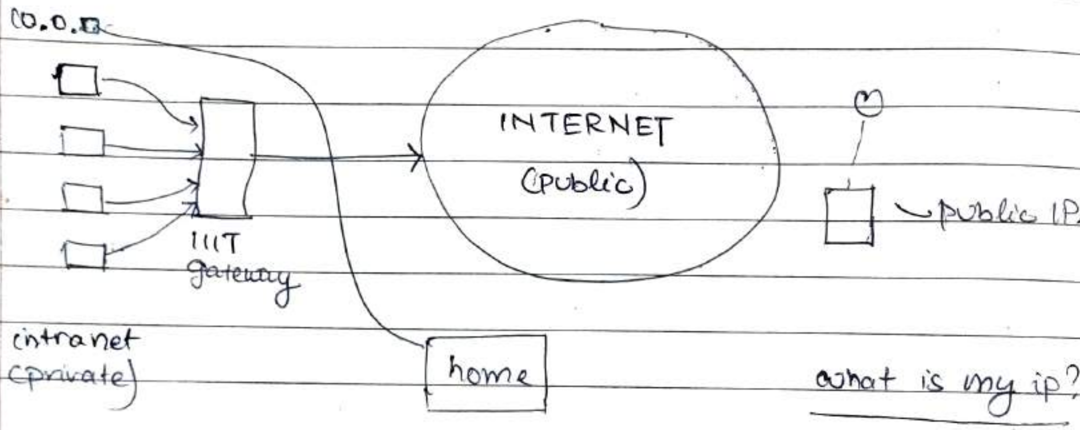
EGP
↓
BGP (port 179)
· border gateway protocols
current version 2006,
RFC 4271

VPN

What is VPN?

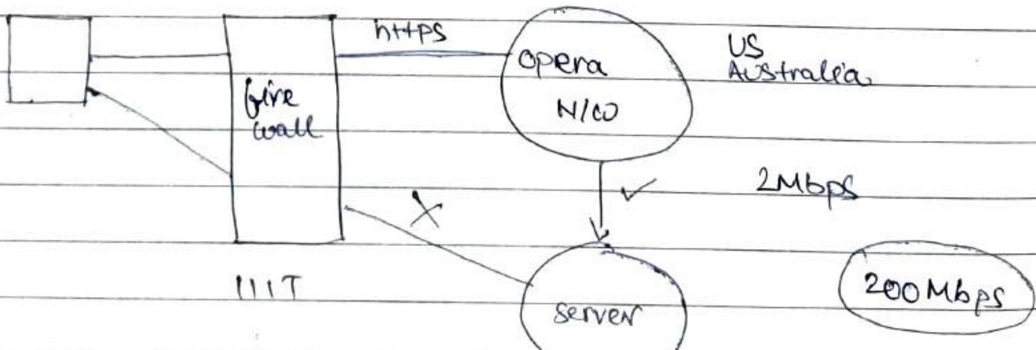
- virtual private network
- to access the mess portal.
- Connect to corporate network.
- to access computing resources within IIIT when outside campus.

secure communication.



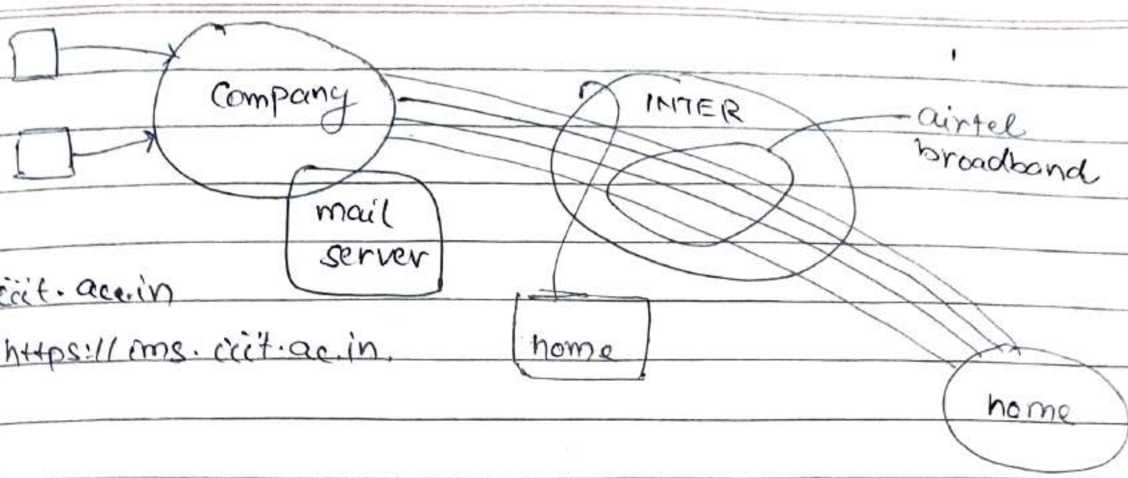
what is my ip?

opera VPN or some VPN clients to browse



10-12 Mbps

whatsApp call (port)



ciit.ac.in
https://ims.ciit.ac.in.

purpose of VPN

provide data confidentiality
data integrity

authentication
(use hash fns)

SHA SHA2
MD5
RIPEMD

encryption

① (public key) assymmetric key
② symmetric key

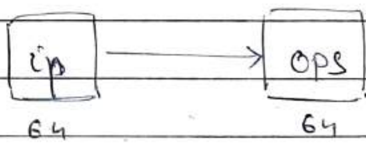
DES, 3DES, AES, blowfish.

RSA, Elgamael
EC

IPV4

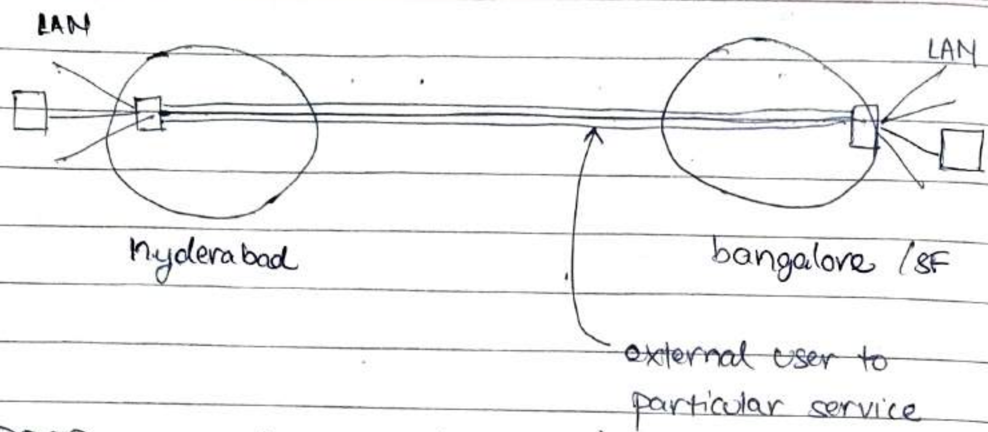
10.x.x.x
256 256 256

192.x.x.x



VPN

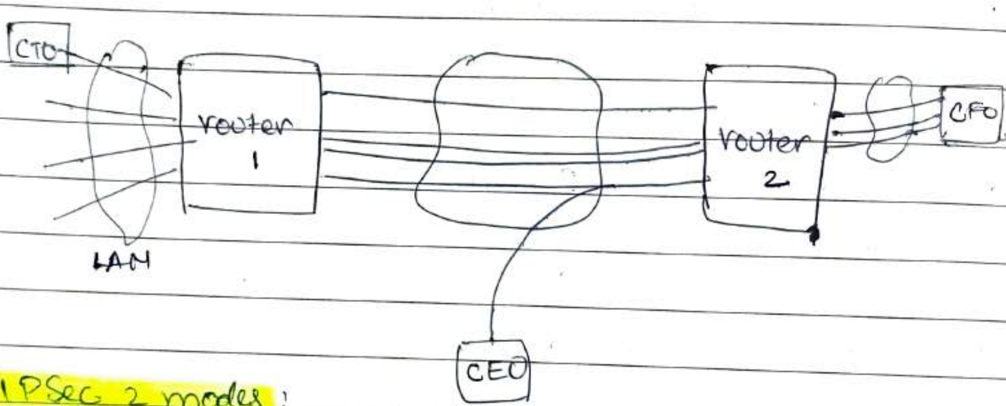
- user to service (SSL)
- user to LAN
- across two LAN.



PPTP - windows 95 (RFC 2637)

L2TP - RFC 2661, 3931

IPSec - used for 2 LANs. RFC 4301



IPSec 2 modes:

- transport —— device - device
- tunnel —— router - router, or device - router

ENC → ESP RFC 4303 IKE - RFC 4306

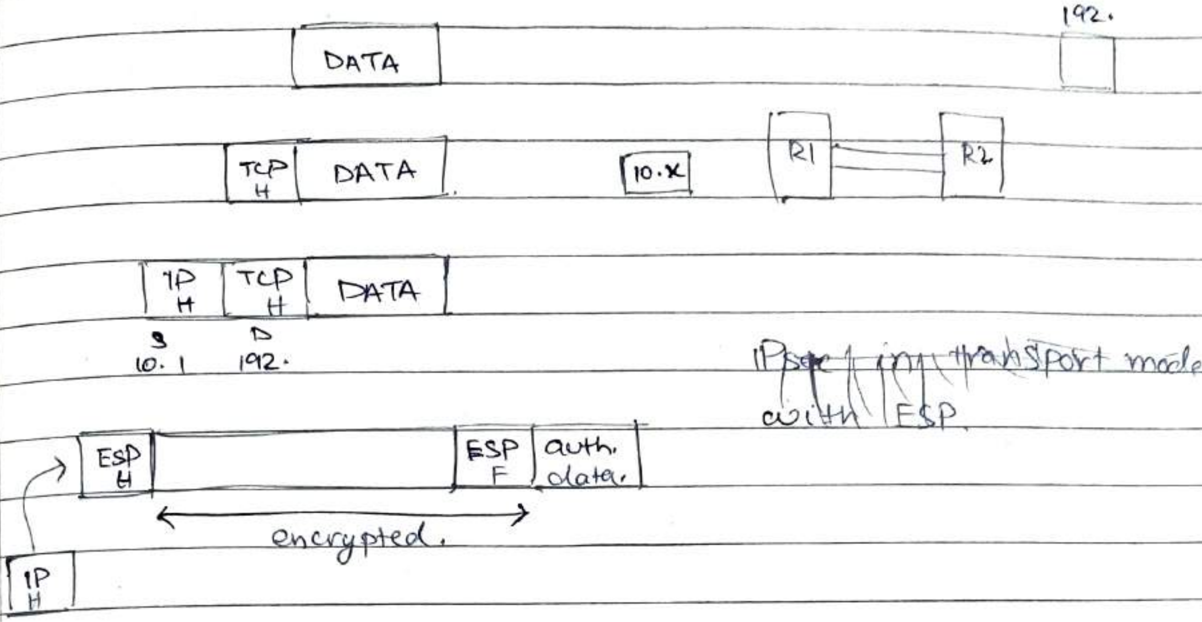
AH → AH RFC 4302

authenticate the data

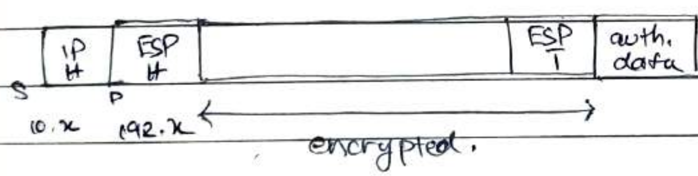
- encrypt the data
(authentication possible)

nullencryption

IPSec tunnel in ESP



IPSec in transport mode with ESP



VPN-A	VPN-B
3DES-CBC	AES
HMAC-SHA1	AES-XCBC
1024 bit	2048-bit

IKE (diffie hellman) RFC 4869 what all cryptographic operation supported.