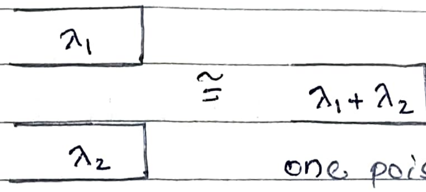
QUEUING THEORY

RECAP

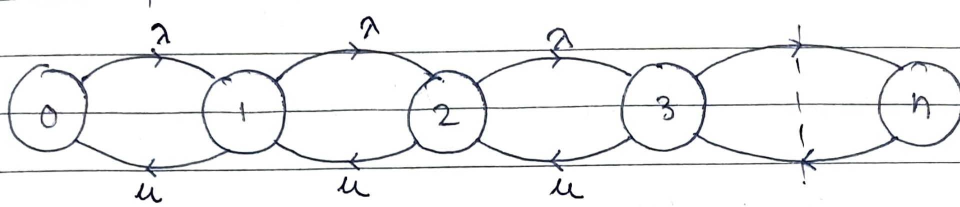
poisson process



2 poisson processes

one poisson process

birth-death process



State n : # of customers / packets in the system

A / B / C / D / E

A - arrival process (λ) mean
B - service process (mean μ)
C - # servers
D - # max of customers / packets (buffersize)
E - population size that will need service

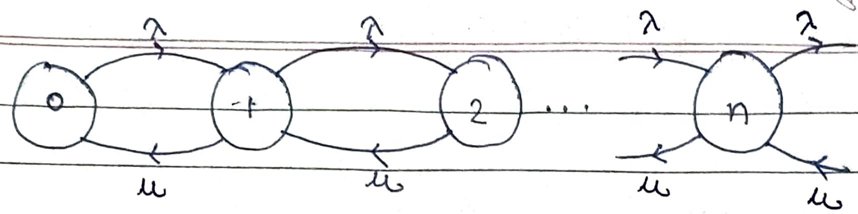
A / B / C ← D = ∞

A / B / C / D ← E = ∞

M / M / 1 ← one server

M (first): arrival process is memoryless (poisson arrival with arrival rate λ)

M (second): the service process is memory less (exponential with service rate μ)



$$\frac{dp_n'(t)}{dt} = -(\lambda+\mu)\, p_n(t) + \lambda p_{n-1}(t) + \mu p_{n+1}(t)$$

$$p_n(0) = 0 \quad \forall\, n \neq 0$$

$t \to \infty$ we will assume $\frac{dP_n'}{dt} = 0$

$$\lambda P_{n-1} + \mu P_{n+1} = (\lambda+\mu) P_n$$

$$\lambda P_0 = \mu P_1$$

$$P_1 = \left(\frac{\lambda}{\mu}\right) P_0$$

load factor $(\rho < 1)$

$$= \rho P_0$$

$$\underline{\underline{P_n = \rho^n P_0}} \qquad P_0 = 1 - \rho$$

$$P_n = \rho^n (1-\rho)$$

$$\underline{\underline{U = 1 - P_0}} \qquad U = \rho$$

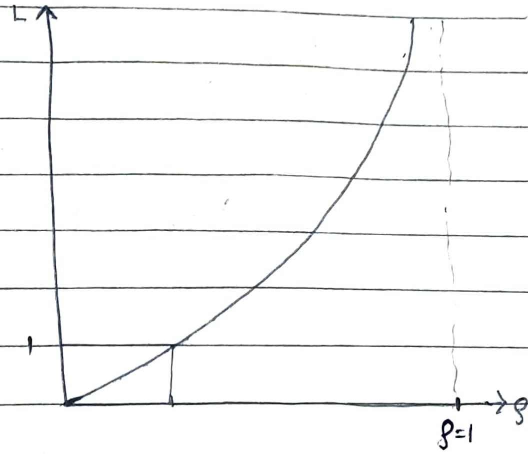
$$L = E(n)$$

$$= 0 \times P_0 + 1 \times P_1 + \dots$$

$$= (1-\rho)\rho + 2 \times \rho^2 (1-\rho) + \dots$$

$$L - \rho L = (1-\rho)\,\rho\,(1+\rho+\rho^2+\dots)$$

$$L = \rho\,\frac{1}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$



① $\lambda = 0.9\mu \Rightarrow \rho = 0.9$

$$L = \frac{0.9}{0.1} = 9$$

② $\lambda = 0.95\mu$ or 0.99μ

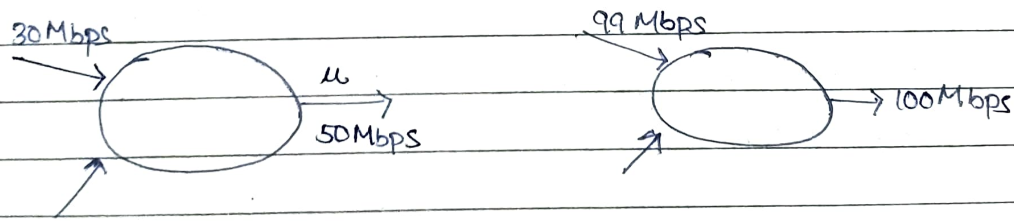
$\downarrow$ $\downarrow$

$L = 19$ $\quad$ 99

③ $\rho = 0.1$ $\quad$ $\rho = 0.5$ $\quad$ 50%

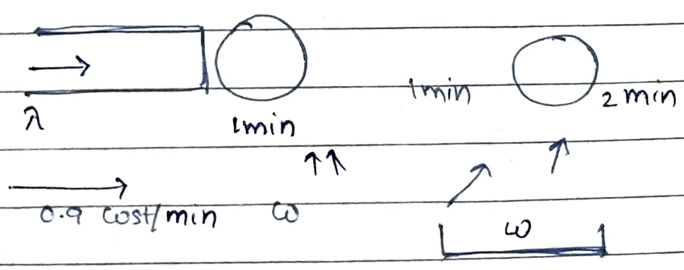
$L = \frac{1}{9} = 0.111$ $\quad$ $L = 1$ $\quad$ $\cancel{\rho \times 0.5}$

④



⑤ $L = L_q + L_s$

$W = W_q + W_s$



QoS : LITTLE'S LAW

$L = \lambda w$ → avg. time spent by each packet

λ → avg. # packets / time

L → avg. # packets / customers.

$t \to 0$

E(# customers arrived in $[0, t]$) = λt

$X = YZ$

$EX = EY \cdot EZ$ ✗ not always true

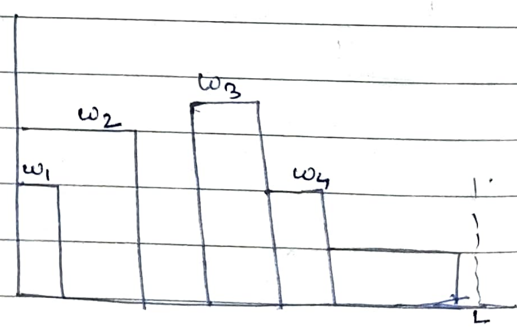
on avg. each customer spends $\underline{w}$ unit of time in the system.

total time spent by all the customers those who arrived in $[0, t]$. $\lambda t \times w$

what is expected # customers L.

total time spent by these customers $= Lt$

w



1min 2min 30 sec

10min 1min 5min

$t \to \infty$ $Lt = \lambda t w$

$\underline{L = \lambda w}$ little's law.

① $\rho = 0.1$ $\lambda = 9$ customers/min

$$W = \frac{L}{\lambda} = \frac{\rho}{1-\rho} \times \frac{1}{\lambda} = \frac{0.1}{0.9} \times \frac{1}{9} = \frac{1}{81} \text{ mins.}$$

② $\lambda = 9$ customers/min $\mu = 10$ customers/min.

$$L = \frac{0.9}{0.1} = 9 \qquad W = \frac{L}{\lambda} = \frac{9}{9} = 1 \text{ min}$$

$$\boxed{\begin{aligned} W &= W_q + W_{ser} \\ L &= L_q + L_{ser} \end{aligned}}$$

$$W_{ser} = \frac{1}{\mu} \qquad W = \frac{L}{\lambda} = \frac{\lambda}{\mu-\lambda} \frac{1}{\lambda} = \frac{1}{\mu-\lambda}$$

③ $\lambda = 9$ customers/min $\mu = 10$ customer/min

how much time will you spend in the queue on average?

$W = 1$ min, $W_q = W - W_{ser} = 1\text{min} - 0.1 = 0.9$ min $= 54$ sec.

④. $\lambda = 18$ customer/min $\mu = 20$ customer/min

What is the expected time to be spent in the queue?

$$\rho = 0.9 \qquad L = \frac{\rho}{1-\rho} = 9$$

$$W = \frac{L}{\lambda} = \frac{9}{18} = 0.5 \text{ min}$$

$$W_q = W - \frac{1}{20} = 0.5 - 0.05 = 0.45 \text{ min} = 27 \text{ sec}$$

(5) $\lambda = 19$ customers/min $\quad \mu = 20$ customer/min

what is w_q?

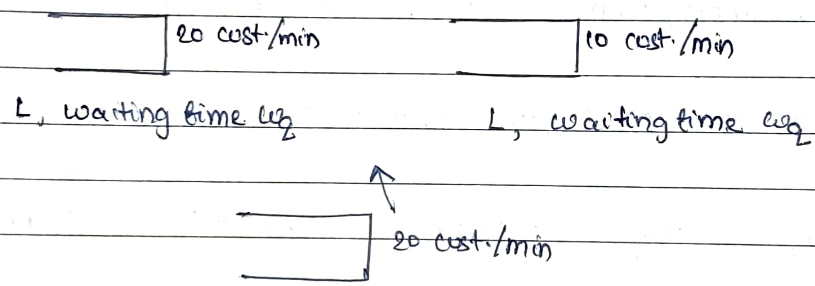
$L = 19 \quad w = 1 \quad w_q = 1 - \frac{1}{20} = \frac{19}{20} \times 60 = 57$ sec.

(6) $\lambda = 2$ customers / min $\quad \mu = 10$ customers /min

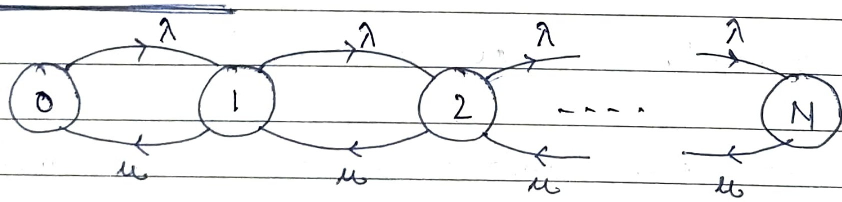
$\rho = 0.2 \quad L = \frac{\rho}{1-\rho} = \frac{1}{4} \quad w = \frac{1}{8}$

$w_q = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$ min $= \frac{60}{40} = 1.5$ sec.

HW (7)



M/M/1/N



$\lambda P_{n-1} = \mu P_n$

$0 = \frac{dP_n(t)}{dt} = -(\mu) P_N + \lambda P_{N-1}$

$P_N = \left(\frac{\lambda}{\mu}\right) P_{N-1}$

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$$\frac{dP_{N+1}(t)}{dt} = 0$$

$$-(\lambda+\mu) P_{N-1} + \lambda P_{N-2} + \mu P_N$$

$$P_n = \rho^n P_0 \qquad n = 0 \ldots N$$

$$\sum_{n=0}^{N} P_n = 1 \qquad P_0 + \rho P_0 + \rho^2 P_0 + \ldots \rho^N P_0 = 1$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \qquad P_n = \rho^n \frac{1-\rho}{1-\rho^{N+1}}$$

P_N blocking probability

M/M/∞ , M/M/m

QUIZ
mon. / thu.
on moodle
syllabus: prob. theory, M/M/1