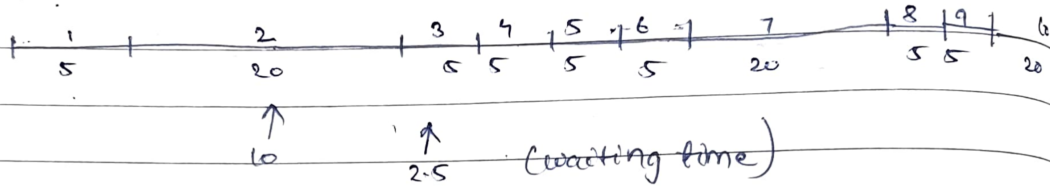
INTRODUCTION: QUEUING THEORY

70% → 5mins $\quad 0.7\times5 + 0.3\times20 = 9.5$

30% → 20mins $\quad$ how much you expect to wait on average = 4.75



total time $= 7\times5 + 20\times3$

$= 35 + 60 = 95$ mins

$\therefore$ avg. waiting time $= \frac{35}{95}\times5 + \frac{60}{95}\times20 \,/\, 2$

$= \underline{\underline{7.23}}$

0.8 → 5 mins $\quad 2\times30 + 8\times5$

0.2 → 30 mins $\quad = 60 + 40 = 100$

$\therefore$ avg. waiting time $= \frac{40}{100}\times5 + \frac{60}{100}\times30 \,/\, 2$

$= \underline{\underline{10}}$

* AK erlang (1909)

studied and formalized queing theory for telephone networks. traffic engg. airport take-off, landing.

PROBABILITY THEORY

events, probability (f^n), sample space.

F $\quad$ P $\quad$ Ω

(Ω, F, p) $\quad$ $\Omega = [0, 1]$ $\quad$ $\Omega = \{0, 1\}$

$F \subseteq 2^{\Omega}$ $\quad$ $F \not\subset 2^{\Omega}$ $\quad$ $F = \{\phi, \{1\}, \{0\}, \{1, 0\}\}$.

if Ω is discrete $F = 2^{\Omega}$

if Ω is continuous $F \not\subset 2^{\Omega}$

AXIOMS OF PROBABILITY

(i) $0 \le p(E) \le 1 \quad \forall \; E \in F$

(ii) $p(\Omega) = 1$

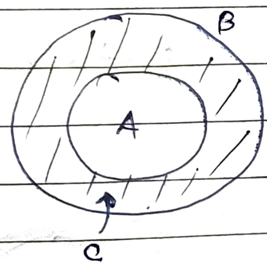
(iii) if $E_1 \cap E_2 = \phi$ $\quad E_1, E_2 \in F$

then $p(E_1) + p(E_2) = p(E_1 \cup E_2)$

COROLLARIES

(i) if $A \subset B$

$p(A) \le p(B)$



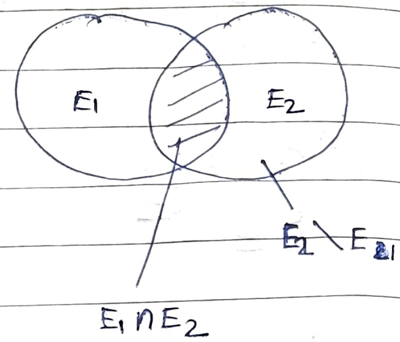
$A \cup (B \cap A')$

$p(A) + p(C) = p(B)$ ___ iii

since $p(C) \geq 0$ ___ i $\quad$ $A \cap C = \phi$

$\Rightarrow p(A) \le p(B)$

2. $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$



$p(E_1 \cup E_2)$

$= p(E_1 \cup (E_2 \setminus E_1))$

$= p(E_1) + p(E_2 \setminus E_1)$ — iii

$p(E_2) = p((E_1 \cap E_2) \cup (E_2 \setminus E_1))$

$= p(E_1 \cap E_2) + p(E_2 \setminus E_1)$ — iii

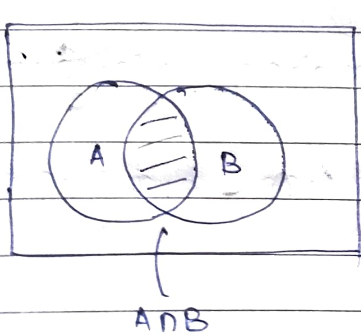
$\Rightarrow p(E_2 \setminus E_1) = p(E_2) - p(E_1 \cap E_2)$

$\therefore P(E_1 \cap E_2) = p(E_1) + p(E_2 \setminus E_1)$

$= p(E_1) + p(E_2) - p(E_1 \cap E_2)$

CONDITIONAL PROBABILITY

$p(A \mid B)$ probability of event A given B has ocurred.
$A, B \in F$



Ω is discrete,
all outcomes equally likely } say

$$\therefore p(A \mid B) = \frac{N(A \cap B)}{N(B)}$$

PROBABILITY MASS FUNCTION (PMF)

$p(X = x) = f_X(x)$

$$\sum_{x \in \Omega} f_X(x) = 1$$

HW roll a dice twice

$X(n_1, n_2) = n_1 + n_2$

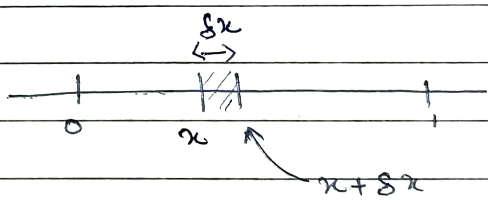
$p(X \in \{11, 12\}) = p\{(5,6), (6,5), (6,6)\}$

$= \frac{3}{36} = 1/12$

continuous random variable

$p(X = x)$ no X

$p(X \in [x, x+\delta x])$



$$f_X(x) = \lim_{\delta x \to 0} \frac{p(X \in [x, x+\delta x])}{\delta x}$$

probability density function (PDF)

cumulative distribution function (CDF)

$F_X(x) = p(X \leq x)$

$F_X(\infty) = 1$

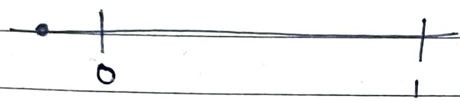
$F_X(-\infty) = 0$

if $x_1 \leq x_2$

$F_X(x_1) \leq F_X(x_2)$

$p(A)$ $p(B)$ $A \subseteq B$

$f_X(x) = 0$



$\{x \in \mathbb{R} \mid f_X(x) \neq 0\}$ as support of X.

- gauss distribution
- poission distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad x \in (-\infty, \infty)$$

μ = mean, σ = standard deviation.

$\mu = 0$, $\sigma = 1$ is called normal distribution.