classmate

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$w_s = \frac{1}{3} \times 0.99 + 30 \times 0.01 = 0.63\ min = \underline{38\ sec}$

$\lambda = 1/min \quad \mu = \frac{1}{0.63} \quad \rho = \underline{0.63}$

$\sigma_{ser}^2 = \sqrt{(0.33 - 0.38)^2 \times 0.99 + (30 - 0.38)^2 \times 0.01}$

$= \underline{8.78}$

$L = 0.63 + \frac{0.63^2 + 1^2 . 8.78^2}{2(1-0.63)}$

$= \underline{13}$

$L = \lambda w \quad w = \frac{L}{\lambda} = \underline{13\ min.}$

$\rho\left(1 + \frac{\rho}{2(1-\rho)}\right)$

$L_D = \rho + \frac{\rho}{2} L_1$

$0.99 \times \frac{1}{3} + 0.01 \times 30 = 0.63\ min$

$\frac{0.99 \times \frac{1}{3}}{0.63} \times \frac{1}{3} + \frac{0.01 \times 30}{0.63} \times 30$

$= 0.175 + 14 = 14.175$

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SO WHAT DID WE SEE LAST CLASS

RECAP

M/M/1/N queue $\longrightarrow$ $P_n = \dfrac{1-\rho}{1-\rho^{N+1}}\rho^n \quad \rho \neq 1$

$L = \dfrac{\rho}{1-\rho}\left[1-(N+1)P_N\right]$

P_N : blocking probability

$\rho = 1$, $P_n = \dfrac{1}{N+1}$

$L = N/2$

M/M/∞

$P_n = \dfrac{\rho^n e^{-\rho}}{n!}$

$L = \rho$

$L_q = 0$

M/M/m

$P_n = \dfrac{\rho^n}{n!}P_0 \quad n \le m$

$= \dfrac{\rho^n}{m!\,m^{n-m}}P_0$

$L_q = \dfrac{\rho}{1-\rho} \times \boxed{\dfrac{\rho^m}{m!\,(1-\rho)} \times P_0}$ — prob. of queuing

$\underbrace{P_m + P_{m+1} + P_{m+2} + \dots}$

$\rho = \dfrac{\lambda}{m\mu}$

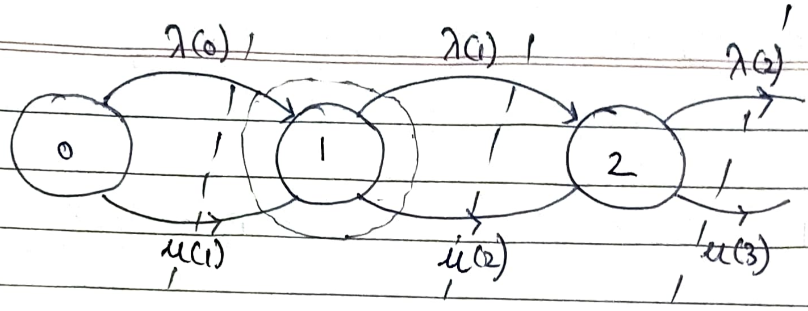
$L_q = \sum_{n=m+1}^{\infty}(n-m)P_n = \dfrac{1}{m!}\sum_{n=m+1}^{\infty}(n-m)m^m\rho^m P_0$

M/M/m/m $\longrightarrow$ max. m packets in system.

$\longrightarrow$ m servers $\qquad L_q = 0$

memoryless arrival

$\longrightarrow$ memoryless service



$\lambda(0) P_0 = \mu(1) P_1$

$\lambda(1) P_1 = \mu(2) P_2$

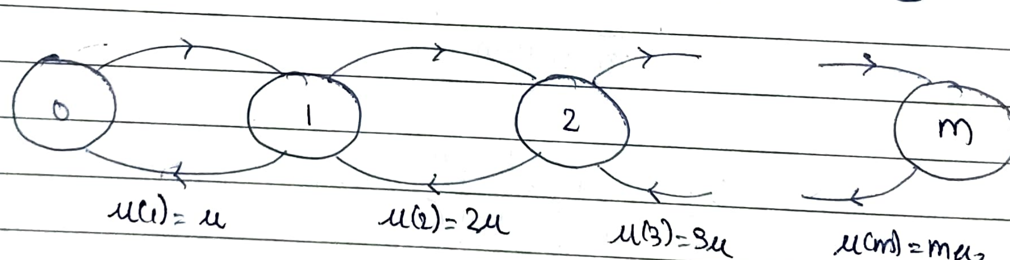
$\vdots$

$$P_n = \prod_{i=1}^{n} \frac{\lambda(i-1)}{\mu(i-1)} P_0$$

$$P_0 + P_1 + \dots = 1$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\lambda(i-1)}{\mu(i-1)}}$$

or using, $\lambda(0) P_0 + \mu(2) P_2 = \mu(1) P_1 + \lambda(1) P_1$



$$P_0 = \frac{1}{1 + \sum_{i=1}^{m} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n}$$

$$P_m = \frac{1}{m!} \left(\frac{\lambda}{\mu}\right)^m \Bigg/ 1 + \sum_{n=1}^{m} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)$$

M/G/1

M - memoryless arrival
G - general service process
1 - one server
∞ - buffer

t_{ser} : time taken to provide service to the customer
(If G = M, t_{ser} is exponentially distributed)

HW :→ σ^2_{ser} : var(t_{ser}) $\mu_{ser} = \frac{1}{E(t_{ser})}$

$$L = \rho + \frac{\rho^2 + \lambda^2 \sigma^2_{ser}}{2(1-\rho)}$$

HW. exercise
Show that L matches with $L = \frac{\rho}{1-\rho}$ when t_{ser} is exponentially distributed.

BAD POST OFFICE

t_{ser} = 20 sec 0.99 prob.
30 min 0.01 prob. W_s

one customer arrives per minute ($\lambda = 1$)

① what will be the avg. time spent by the customer in the post office? ← W

② on average how many customers will be there in the post office? ← L

L, W, W_s = $E[t_{ser}]$

$E_x = \sum_i x_i P_i$

$W_s = 20 \times 0.99 + 1800 \times 0.01 = 37.8$ sec

$\mu = \frac{1}{37.8}$ $\lambda = 1/60$

$\sigma^2_{ser} = Ex^2 - (Ex)^2$

$= 400 \times 0.99 + 1800^2 \times 0.01 - 37.8^2$

$= 31367.16$

$\rho = \frac{\lambda}{\mu} = \frac{1}{60} \times 37.8 = 0.626$ (~ 0.63)

$L = 12.94$

$W = \frac{L}{\lambda} = 12.94$ min.

$\lambda = 1$ customer / min $W_{ser} = 37.8$ sec

$W = 12.94$ min $\rho = 0.63$

M/D/1

t_{ser} is fixed/constant.

M - memoryless arrival
D - deterministic service

$\sigma^2_{ser} = 0$

$L_D = \rho + \frac{\rho^2 + 0}{2(1-\rho)}$

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ρ	L (M/M/1)	L_D (M/D/1)
0.1	0.111	0.106
0.2	0.250	0.225
0.5	1	0.75
0.8	4	2.4
0.9	9	4.95
0.99	99	50

$$\frac{L_D}{L} = \left[\rho + \frac{\rho^2}{2(1-\rho)}\right]\left[\frac{1-\rho}{\rho}\right] = \frac{2\rho(1-\rho)+\rho^2}{2(1-\rho)}$$

$$= \frac{\rho}{1-\rho}\left[\frac{2(1-\rho)+\rho}{2}\right] \times \left[\frac{1-\rho}{\rho}\right]$$

$$\frac{L_D}{L} = 1 - \frac{\rho}{2}$$

$U = \rho \quad (1-P_0)$

M/M/1

$\rho \to 1 \qquad \frac{L_D}{L} \to 1/2$

will D/D/1 $\quad \lambda < \mu \quad \rho < 1 \quad L_q = 0$

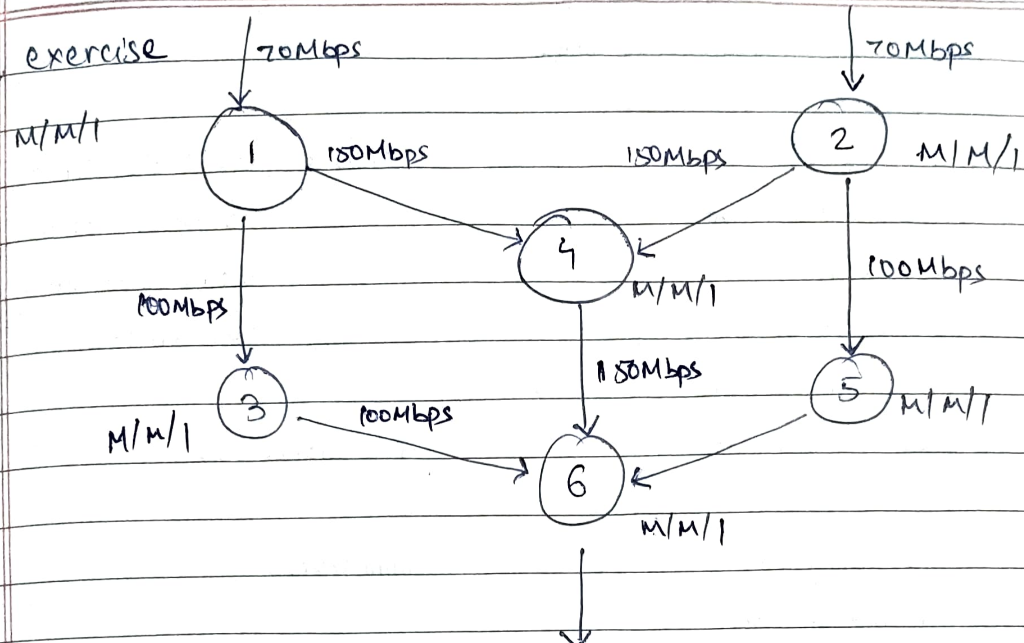
M/G/1, M/D/1

M/M/1 approximates M/M/1

$L = \lambda w$ ← little's law

queuing System	L	Ls	Lq	W	Wq	Ws	P_0, P_n
M/M/1 $\rho = \frac{\lambda}{\mu} < 1$	$\frac{\rho}{1-\rho}$	ρ	$\frac{\rho^2}{1-\rho}$	$\frac{L}{\lambda}$	$\frac{L_q}{\lambda}$	$\frac{1}{\mu}$	$P_0 = 1-\rho$ $P_n = \rho^n P_0$
M/M/∞ $\rho = \frac{\lambda}{\mu} < \infty$	ρ	ρ	0	$\frac{1}{\mu}$	0		$P_0 = e^{-\rho}$ $P_n = \frac{\rho^n}{n!} P_0$
M/M/1/N $\rho = \frac{\lambda}{\mu} < \infty$	$\rho \neq 1$: $\frac{\rho}{1-\rho}[1-(N+1)\rho^N]$ $\rho = 1$: $N/2$					$\frac{1}{\mu}$	$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ $P_n = \rho^n P_0$ ← $\rho \neq 1$ $\rho = 1$: $P_n = \frac{1}{N+1}$
M/M/m $\rho = \frac{\lambda}{m\mu} < 1$	λw		$\frac{\rho}{1-\rho} \times Pr(q)$ $(P_m + P_{m+1} + \dots)$	$\frac{L_q}{\lambda} + \frac{1}{\mu}$	$\frac{L_q}{\lambda}$	$\frac{1}{\mu}$	
M/D/1 $\rho = \frac{\lambda}{\mu} < 1$	$\rho + \frac{\rho^2}{2(1-\rho)}$					$\frac{1}{\mu}$	

HW exercise



1-4-6 & 2-4-6 Strategy 1

1-3-6 & 2-5-6 Strategy 2

which one is better.