

What is Your Estimand?

Defining the Target Quantity
Connects Statistical Evidence
to Theory



Ian Lundberg

UCLA
Sociology
ianlundberg.org

Rebecca Johnson

Dartmouth Quantitative
Social Science
rebeccajohnson.io

Brandon M. Stewart

Princeton
Sociology
brandonstewart.org

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There is a question every quantitative study must answer:

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The **purpose** of the
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— We took [data source]



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There is a question every quantitative study must answer:

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$$Y = \beta_0 + X_1\beta_1 + X_2\beta_2 + \epsilon$$



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**Epistemological
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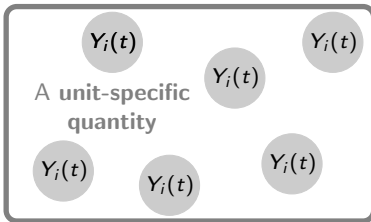
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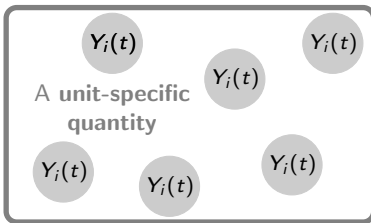
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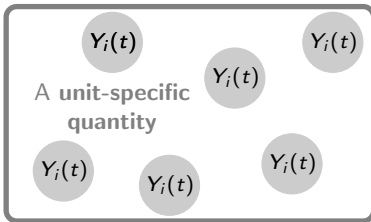


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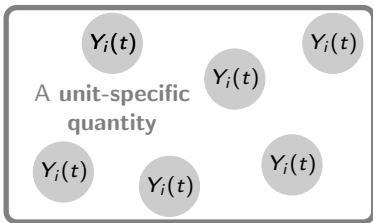


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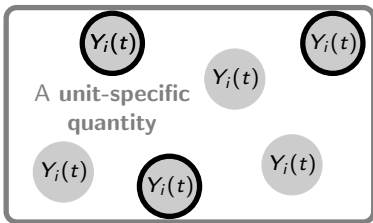


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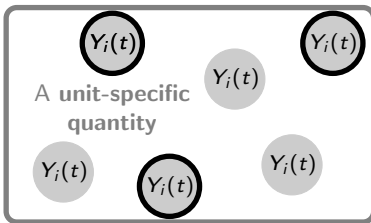


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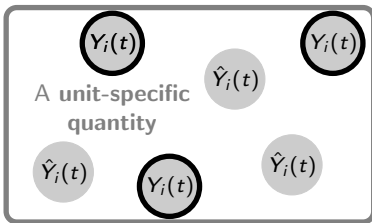


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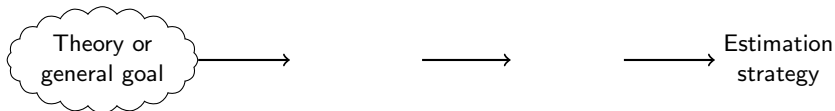
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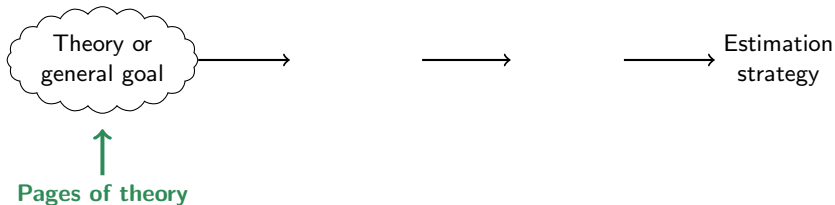
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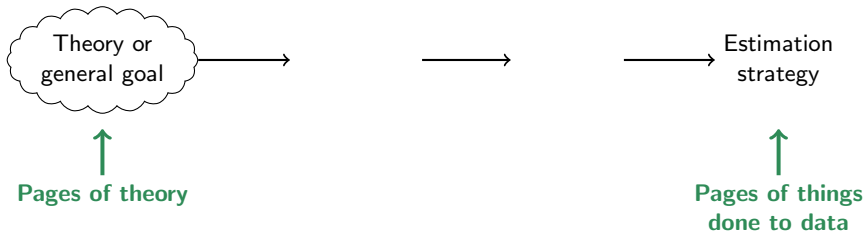
Research framework: Estimands connect theory to evidence



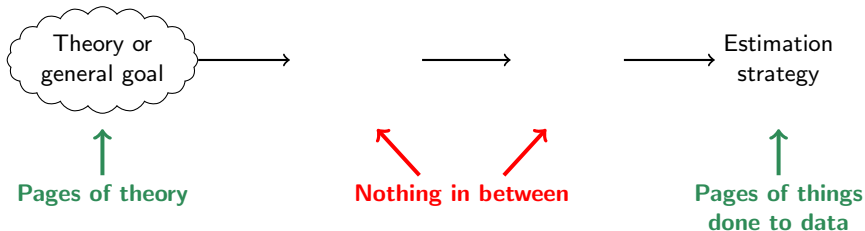
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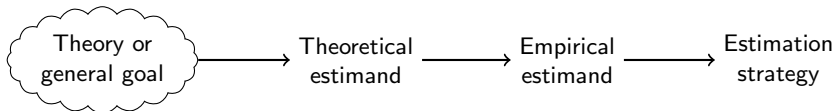
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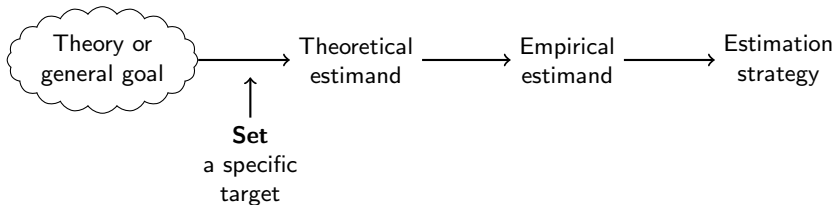
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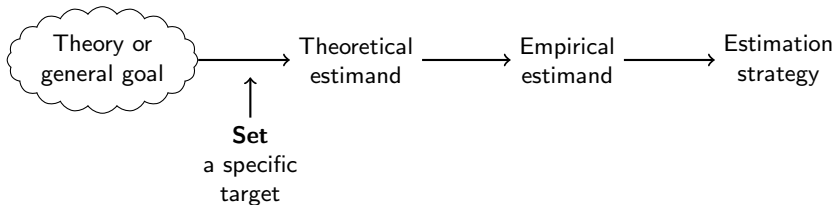
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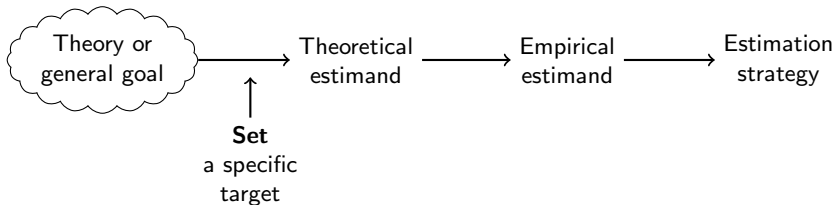
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Definition

A **unit-specific quantity**
aggregated over a
target population

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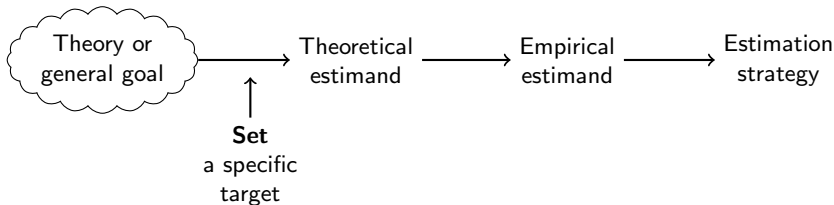
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$$\frac{1}{\text{Size of U.S. adult population}} \sum_{i \text{ in U.S. adult population}} \left(\text{Employed}_i \right)$$

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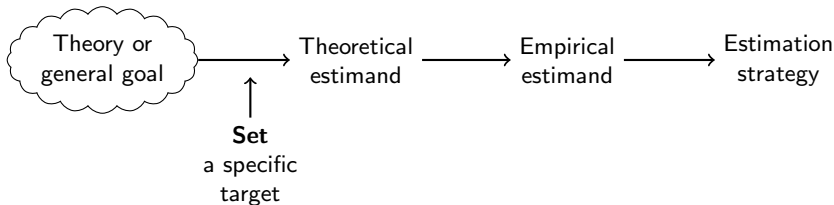
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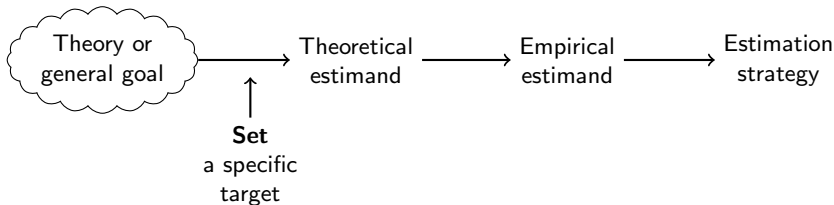
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Liebersohn 1987, Abbott 1988, Freedman 1991, Xie 2013, Hernán 2018

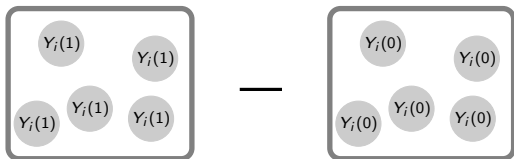
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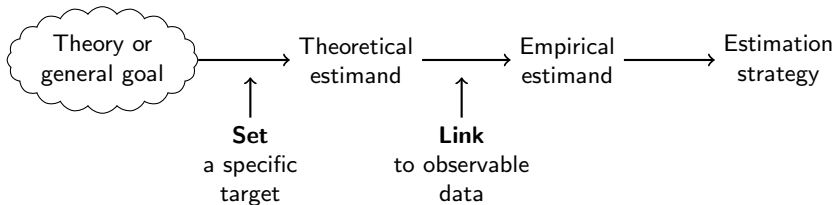
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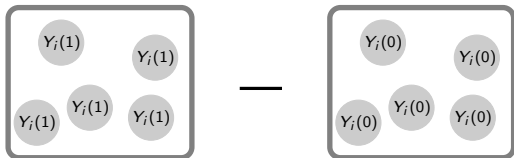
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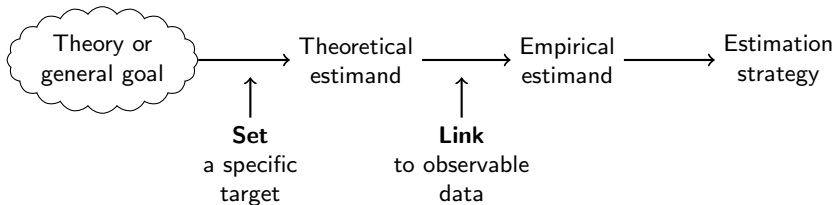
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A quantity involving
observable data

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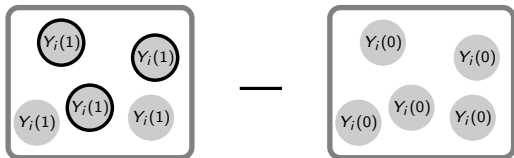
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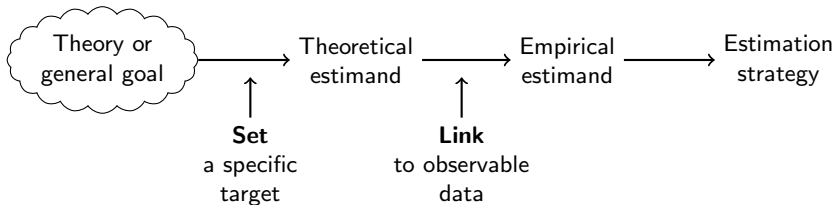
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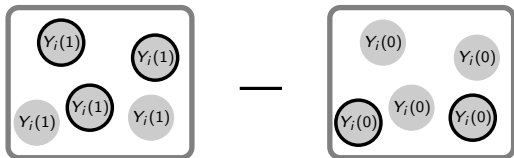
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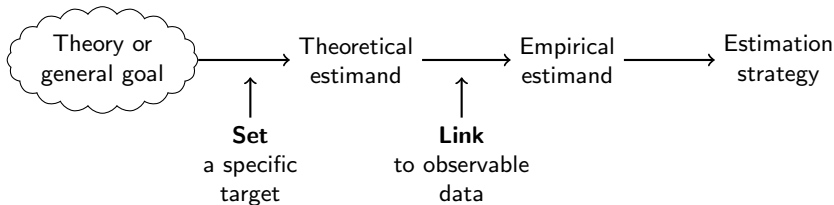
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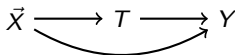
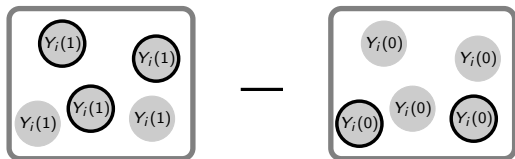
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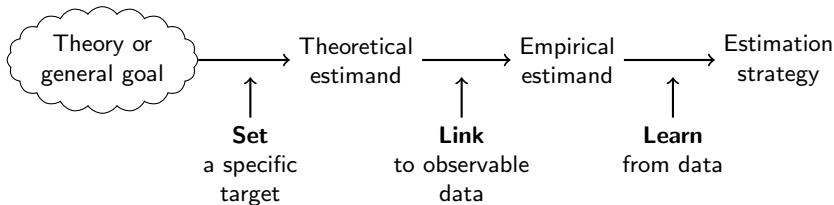
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Pearl 2009, Imbens and Rubin 2015,
Morgan and Winship 2015, Elwert and Winship 2014

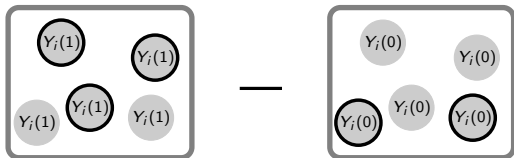
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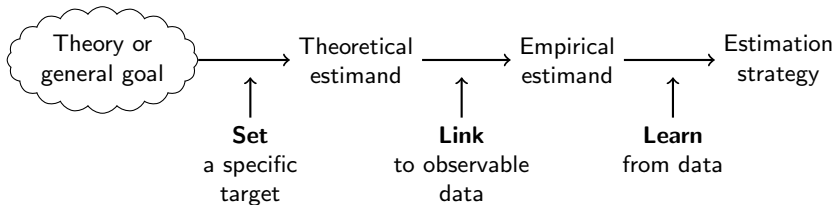
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An algorithm applied to data

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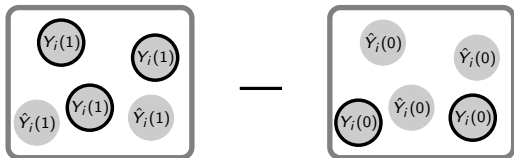
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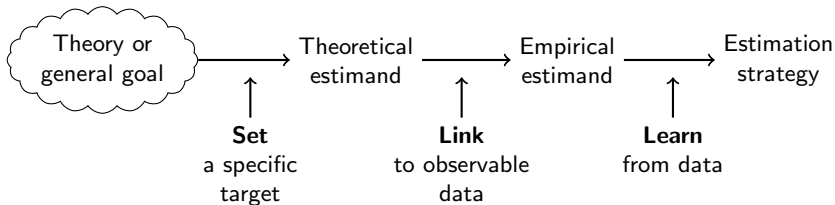
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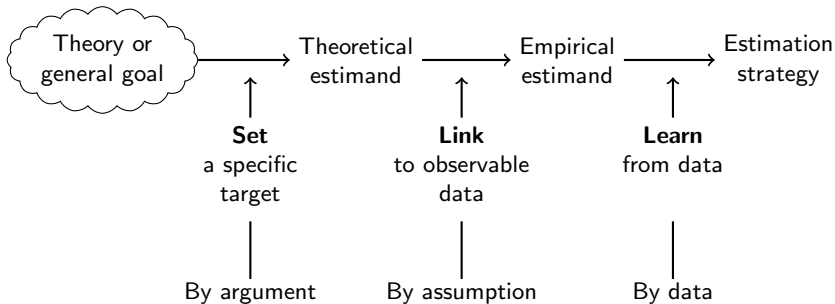


Young 2009, Watts 2014, Berk et al. 2019, Molina and Garip 2019

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An Empirical Analysis of Racial Differences in Police Use of Force

Roland G. Fryer Jr.

Harvard University and National Bureau of Economic Research

This paper explores racial differences in police use of force. On non-lethal uses of force, blacks and Hispanics are more than 50 percent more likely to experience some form of force in interactions with police. Adding controls that account for important context and civilian behavior reduces, but cannot fully explain, these disparities. On the most extreme use of force—officer-involved shootings—we find no racial differences either in the raw data or when contextual factors are taken into account. We argue that the patterns in the data are consistent with a model in which police officers are utility maximizers, a fraction of whom have a preference for discrimination, who incur relatively high expected costs of officer-involved shootings.

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TheUpshot
DATA DIVE

Surprising New Evidence Shows Bias in Police Use of Force but Not in Shootings



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OPINION | COMMENTARY

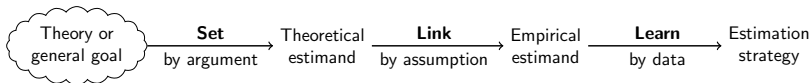
The Myth of Systemic Police Racism

WSJ OPINION

By Heather Mac Donald
June 2, 2020 1:44 pm ET

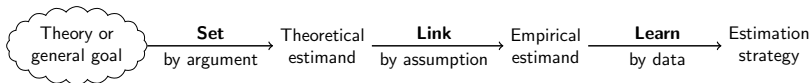
Reality check: study finds no racial bias in police shootings

The Guardian



Evidence: Police use lethal force at the same rate against black and white civilians who are stopped.

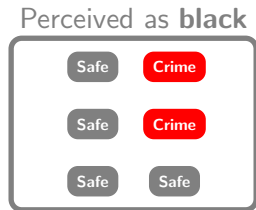
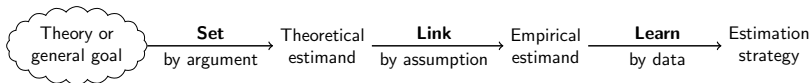
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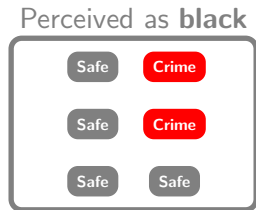
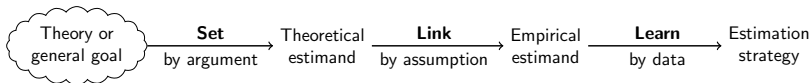
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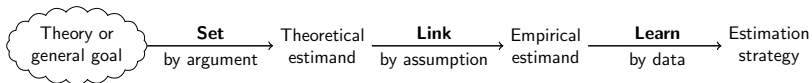
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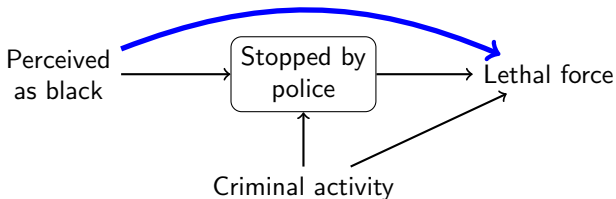
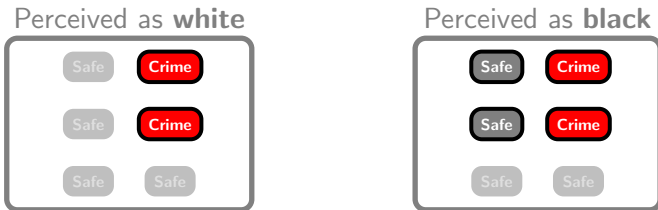
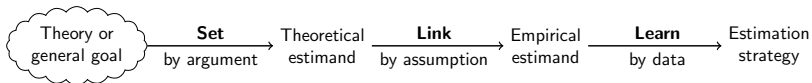
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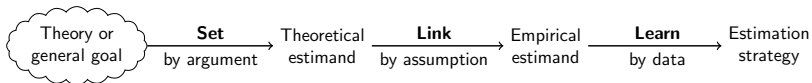
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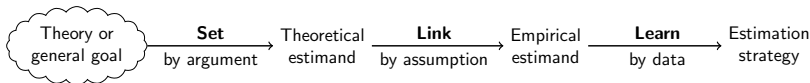
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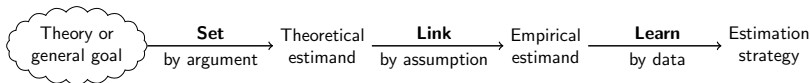


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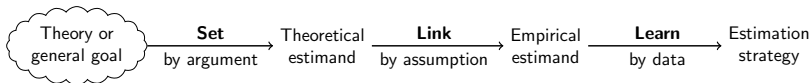


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Fryer responds:

“We use the term ‘racial differences’ 114 times in lieu of the more prescriptive wording—‘racial discrimination.’ We use the phrase ‘conditional on an interaction’ 20 times...I am not sure how many more ways we would have needed to caveat our results to satisfy [the critics].”

Fryer 2019. Fuller critique by Knox et al. 2020 and Durlauf and Heckman 2020.



Pal and Waldfogel (2016) estimate the family gap in pay.



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Is the theoretical estimand descriptive?



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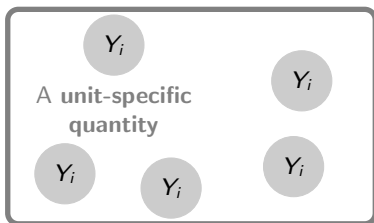
“the differential in hourly wages
between women with children
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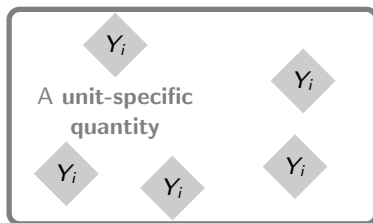
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Averaged over a
target population
of **mothers**



Averaged over a
target population
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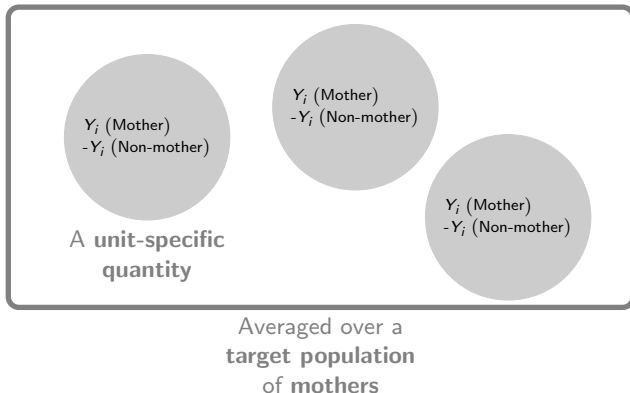
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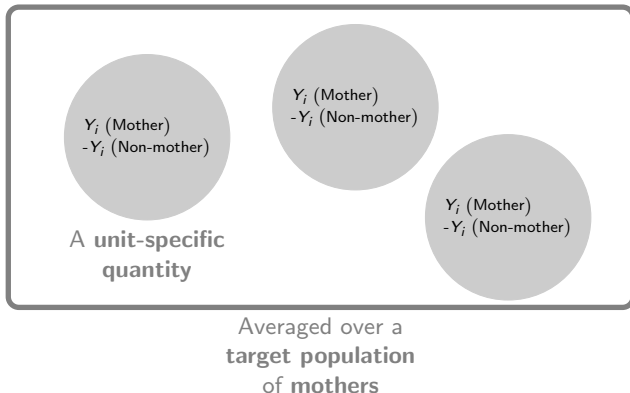




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Added complexity: Wages are undefined for the non-employed.

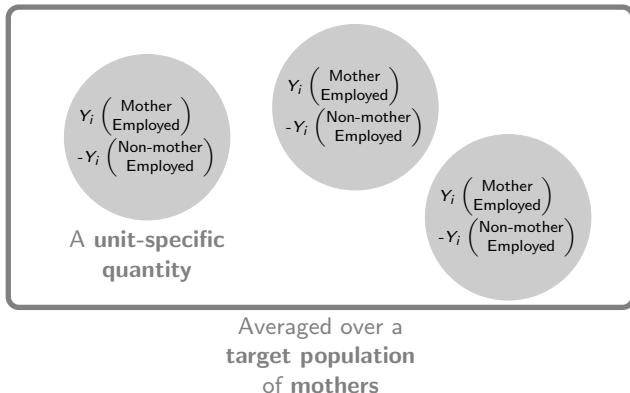


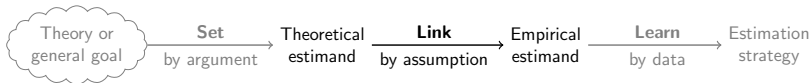


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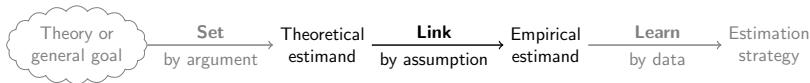
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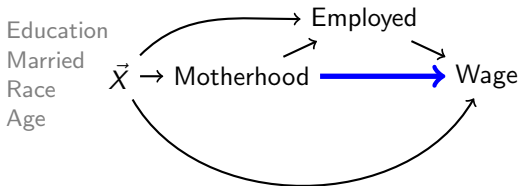


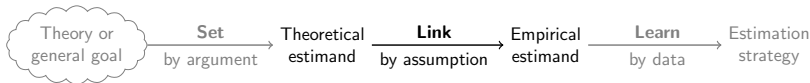


Unit-specific quantity: $Y_i \left(\begin{array}{c} \text{Mother,} \\ \text{Employed} \end{array} \right) - Y_i \left(\begin{array}{c} \text{Non-mother,} \\ \text{Employed} \end{array} \right)$

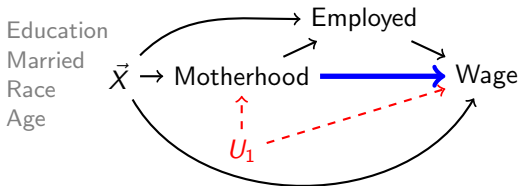


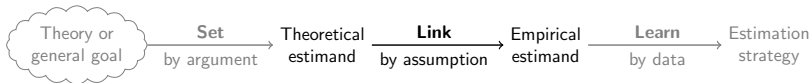
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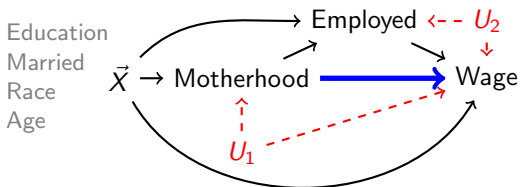


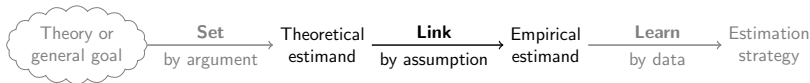
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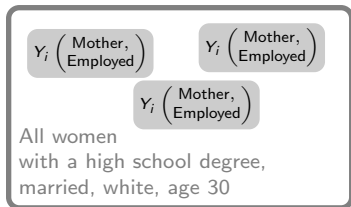
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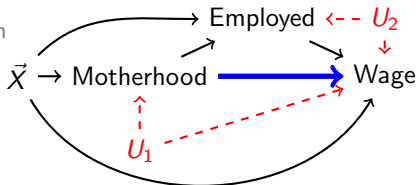
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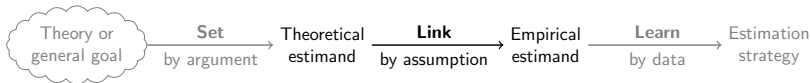
Potential outcomes



Focus on one $\vec{X} = \vec{x}_i$

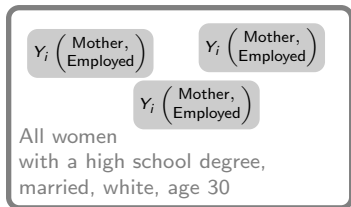
Education
Married
Race
Age





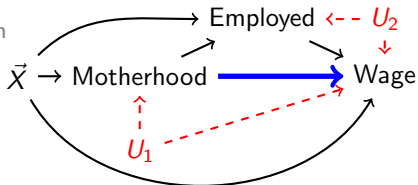
$$E \left(Y_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) \mid \vec{X} = \vec{x}_i \right)$$

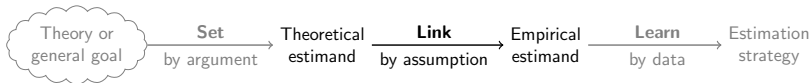
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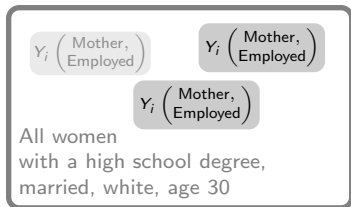
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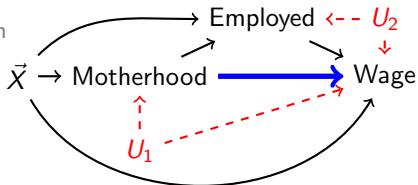
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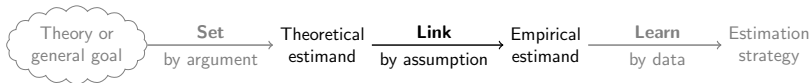
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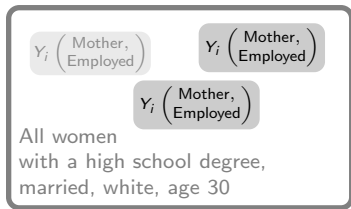
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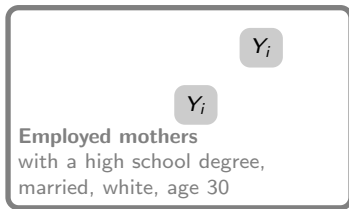


$$E \left(Y_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) \middle| \vec{X} = \vec{x}_i \right) \quad ? \quad E \left(Y_i \middle| \begin{matrix} \text{Motherhood} = \text{Mother,} \\ \text{Employment} = \text{Employed,} \\ \text{Covariates } \vec{X} = \text{Observed } \vec{x}_i \end{matrix} \right)$$

Potential outcomes

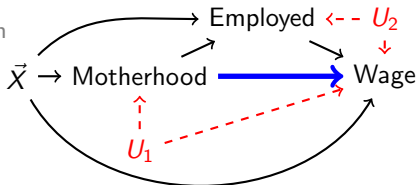


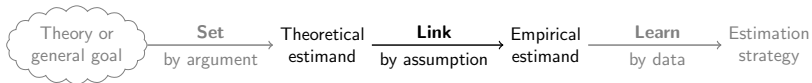
Realized outcomes



Focus on one $\vec{X} = \vec{x}_i$

Education
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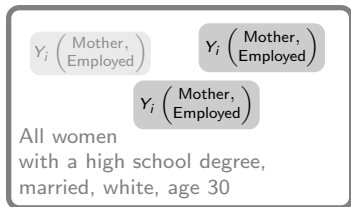




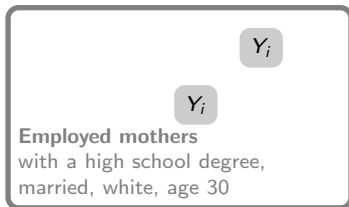
$$E \left(Y_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) \mid \vec{X} = \vec{x}_i \right) = E \left(Y_i \mid \begin{matrix} \text{Motherhood} = \text{Mother,} \\ \text{Employment} = \text{Employed,} \\ \text{Covariates } \vec{X} = \text{Observed } \vec{x}_i \end{matrix} \right)$$

By the DAG

Potential outcomes

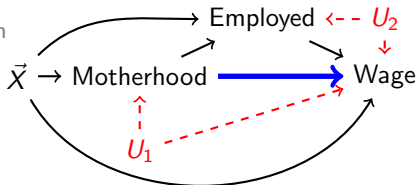


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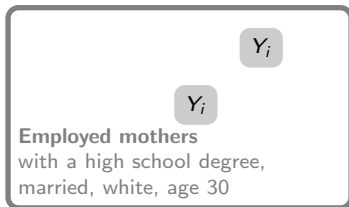
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Realized outcomes

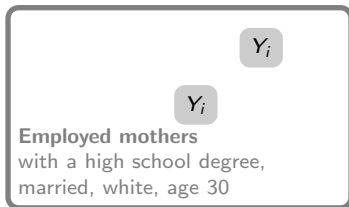




This can be estimated
by machine learning!

$$\longrightarrow E \left(Y_i \mid \begin{array}{l} \text{Motherhood} \\ \text{Employment} \\ \text{Covariates } \vec{X} \end{array} \right) = \begin{array}{l} \text{Mother,} \\ \text{Employed,} \\ \text{Observed } \vec{x}_i \end{array}$$

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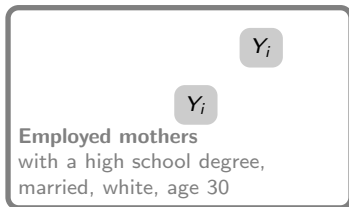


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↓
Any prediction algorithm
that minimizes squared errors

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Realized outcomes





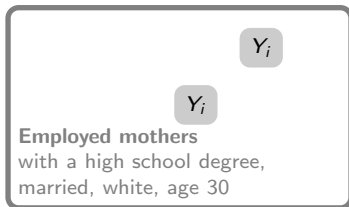
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Generalized
Additive
Model

Realized outcomes





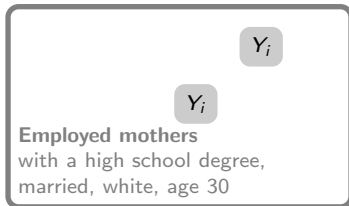
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Any prediction algorithm
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Generalized Additive Model Random Forest

Realized outcomes



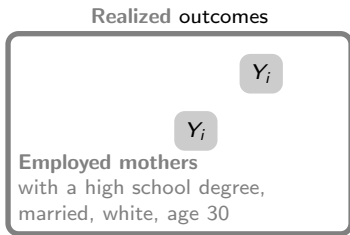


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Any prediction algorithm
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Ordinary Least Squares Generalized Additive Model Random Forest





Mechanics: How **predictive algorithms** estimate the **estimand**



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1) Learn an algorithm to predict the outcome



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- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

$$\hat{E} \left(Y_i \left| \begin{array}{l} \text{Motherhood} \\ \text{Employment} \\ \text{Covariates } \vec{X} \end{array} \right. \right) = \begin{array}{l} \text{Mother,} \\ \text{Employed,} \\ \text{Observed } \vec{x}_i \end{array}$$



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

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Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
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$$\hat{Y}_i \begin{pmatrix} \text{Non-mother,} \\ \text{Employed} \end{pmatrix} = \hat{E} \left(Y_i \left| \begin{array}{ll} \text{Motherhood} & = \text{Non-mother,} \\ \text{Employment} & = \text{Employed,} \\ \text{Covariates } \vec{X} & = \text{Observed } \vec{x}_i \end{array} \right. \right)$$



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$$\hat{Y}_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) - \hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

$$\hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right) = \hat{E} \left(Y_i \left| \begin{matrix} \text{Motherhood} & = & \text{Non-mother,} \\ \text{Employment} & = & \text{Employed,} \\ \text{Covariates } \vec{X} & = & \text{Observed } \vec{x}_i \end{matrix} \right. \right)$$

- 3) Average over the target population

$$\frac{1}{n} \sum_{i=1}^n \left(\hat{Y}_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) - \hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right) \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

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This is called an **imputation estimator**

Hahn, 1998

Abadie & Imbens 2006

Also called the parametric *g*-formula in biostatistics, Hernán & Robins 2020



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age
+ Interaction between age and motherhood

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age
+ Interaction between age and motherhood
+ Allow a smooth curve for age rather than quadratic

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

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Choose an algorithm by **predictive performance**

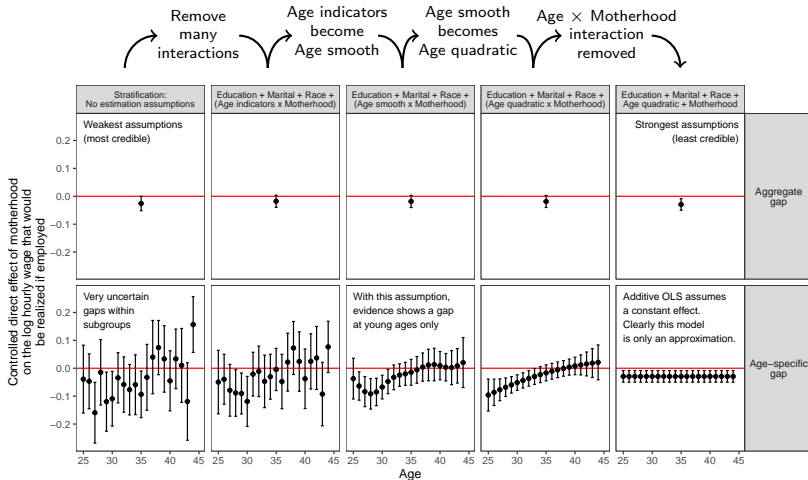
Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible	OLS with a quadratic for age
	+ Interaction between age and motherhood
	+ Allow a smooth curve for age rather than quadratic
	+ Include each age as a separate indicator variable
Most flexible	+ Include all interactions among all predictors

Increasingly strong estimation assumptions: Less credibility →





What is your estimand?

← Every quantitative study should answer this question

Defining the Target Quantity
Connects Statistical Evidence
to Theory

Ian Lundberg

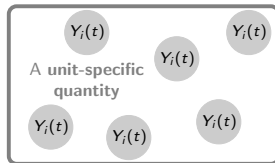
ilundberg@princeton.edu
ianlundberg.org

Rebecca Johnson

rebecca.ann.johnson@dartmouth.edu
rebeccajohnson.io

Brandon Stewart

bms4@princeton.edu
brandonstewart.org



Averaged over a
target population

Preprint on [SocArxiv](#)
Code on [Dataverse](#)

American Sociological Review