

# The **gap-closing** estimand

Slides at  
[ianlundberg.org](https://ianlundberg.org)

A causal approach  
to study **interventions**  
that **close disparities**  
across social categories

Software at  
[ilundberg.github.io/gapclosing](https://ilundberg.github.io/gapclosing)

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Paper in *[Sociological Methods and Research](#)*. Replication code on [Dataverse](#). R package [gapclosing](#) on [CRAN](#). Research reported in this publication was supported by The Eunice Kennedy Shriver National Institute of Child Health & Human Development of the National Institutes of Health under Award Number P2CHD047879 and by the National Science Foundation under Award Number 2104607.



**The Gap-Closing Estimand:**  
A Causal Approach to Study Interventions  
That Close Disparities Across Social Categories

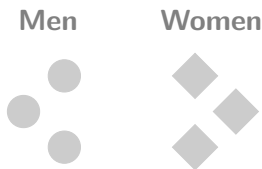
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Class**



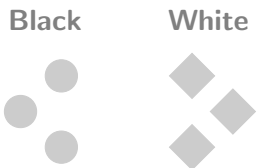
**Professional  
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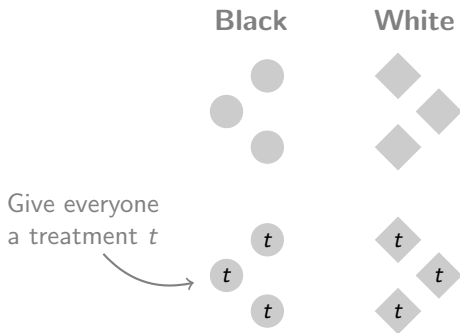
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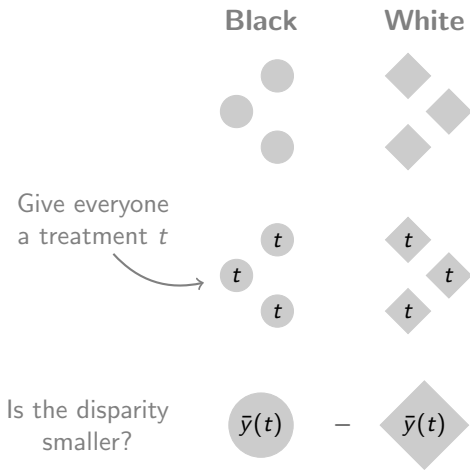
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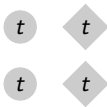
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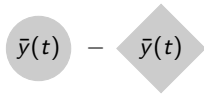
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Categories



Treatment

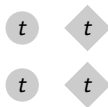


Counterfactual Disparity

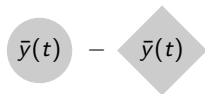




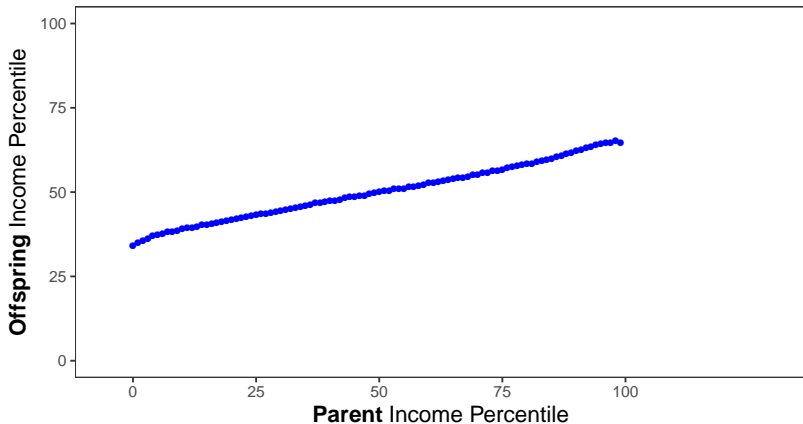
Categories



Treatment



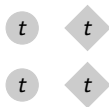
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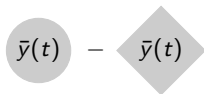
Chetty et al. 2017



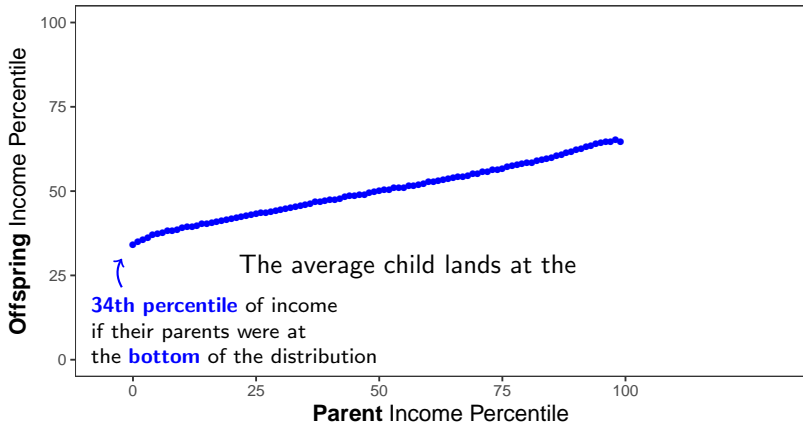
Categories



Treatment



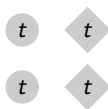
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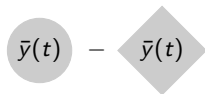
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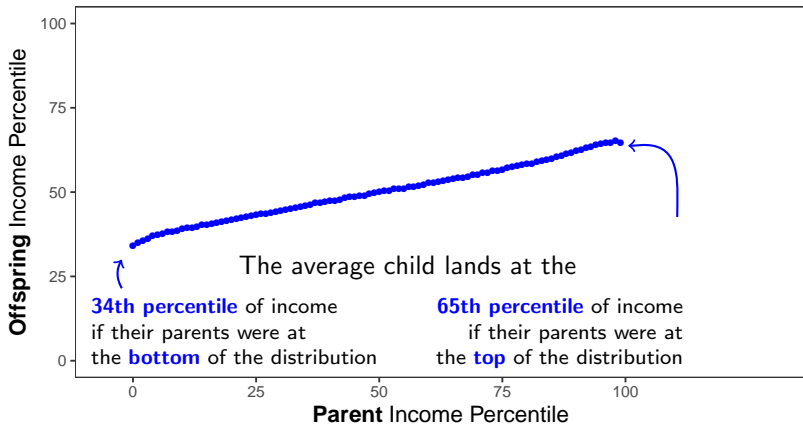
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Treatment



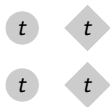
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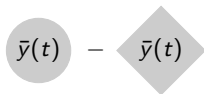
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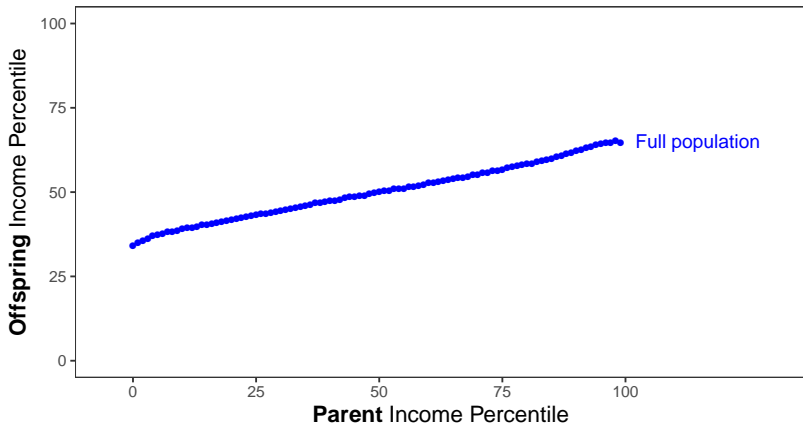
Categories



Treatment



Counterfactual Disparity

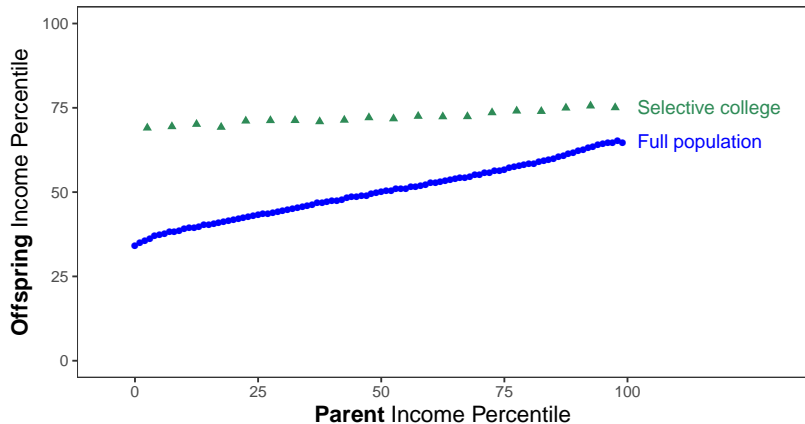


Chetty et al. 2017

● ◆  
● ◆  
Categories

● ◆  
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● ◆  
 $t$   $t$   
Treatment

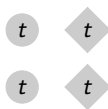
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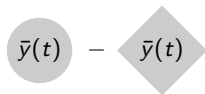
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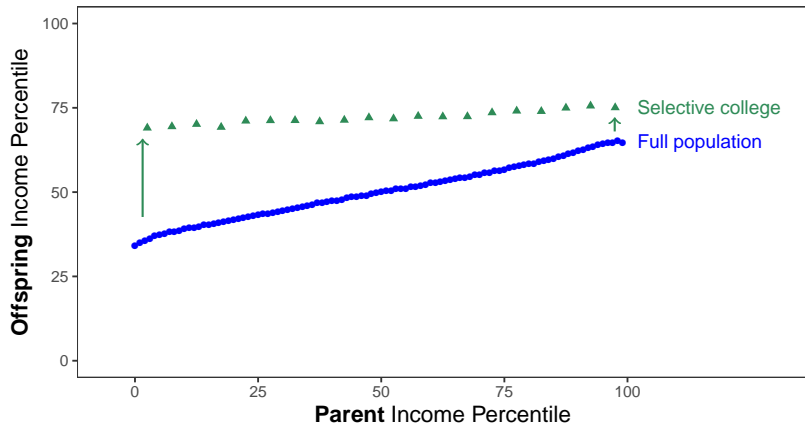
Categories



Treatment



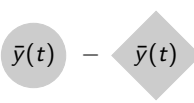
Counterfactual Disparity

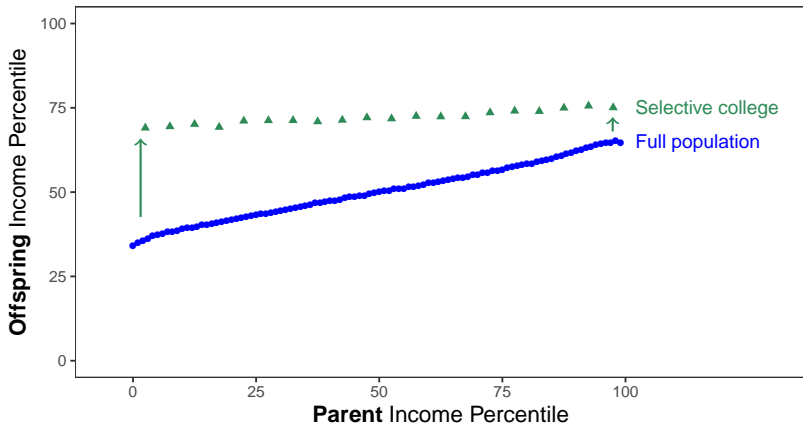


Chetty et al. 2017

  
**Categories**  
 Parent Income

  
**Treatment**  
 Selective College

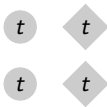
  
**Counterfactual Disparity**  
 Offspring Income



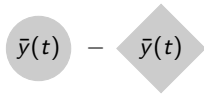
Chetty et al. 2017



Categories



Treatment



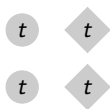
Counterfactual Disparity



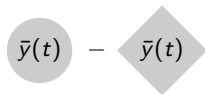


Categories

Race



Treatment



Counterfactual Disparity

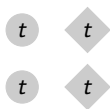


The difference in earnings  
between blacks and whites

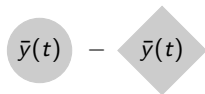
— Western 2006:12



Categories  
Race



Treatment



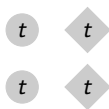
Counterfactual Disparity  
Earnings

The difference in earnings  
between blacks and whites  
would be reduced only by about 3 percent  
if the incarceration rate were zero

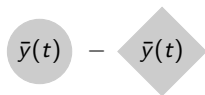
— Western 2006:12



Categories  
Race



Treatment  
Incarceration



Counterfactual Disparity  
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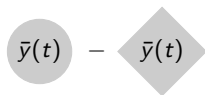
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Categories  
Race



Treatment  
Incarceration



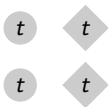
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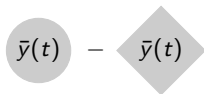
— Western 2006:12



Categories



Treatment



Counterfactual Disparity

We often want to know  
if **intervening** on a treatment variable  
would close gaps

# Causal decomposition analysis

# Causal decomposition analysis

$$\begin{array}{c} \text{Unadjusted} \\ Y = \beta(\text{Black}) + \epsilon \end{array}$$

$$\begin{array}{c} \text{Adjusted} \\ Y = \gamma(\text{Black}) + \vec{X}'\vec{\eta} + \delta \end{array}$$

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Effect of race



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✗ Effect of race

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Vanderweele &  
Robinson 2014

# Causal decomposition analysis

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- ✗ Effect of race
- ✓ Disparity after intervention on  $\vec{X}$

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Choice of intervention targets

Jackson &  
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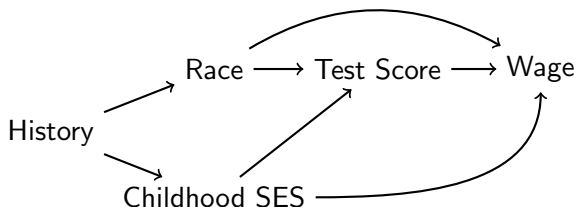
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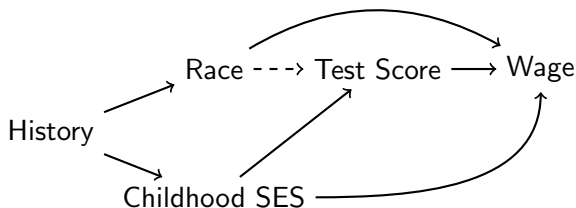
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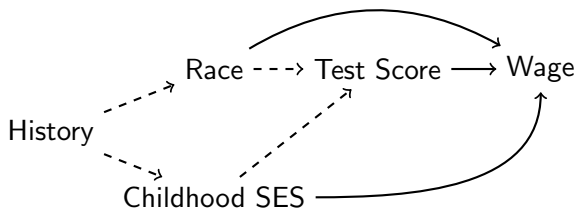
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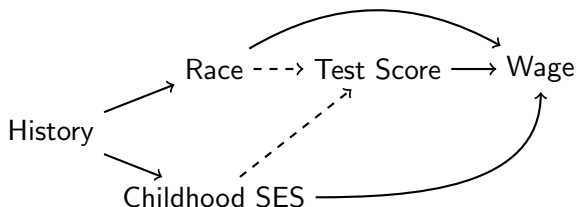
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Vanderweele &  
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Choice of intervention targets

Jackson &  
Vanderweele 2018

Equity: What should we equalize?

Jackson 2021



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Vanderweele &  
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Choice of intervention targets

Jackson &  
Vanderweele 2018

Equity: What should we equalize?

Jackson 2021

Systems may adapt to maintain inequity

Jackson & Arah 2020

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Vanderweele &  
Robinson 2014

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Jackson &  
Vanderweele 2018

Equity: What should we equalize?

Jackson 2021

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Jackson & Arah 2020

Present paper

Lundberg 2022

- Local intervention interpretation
- Doubly robust estimator
- Software

# How this works

1. Define an intervention
2. Make causal assumptions
3. Estimate

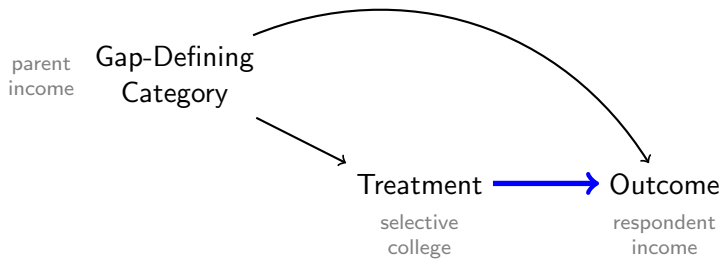
# Define an intervention

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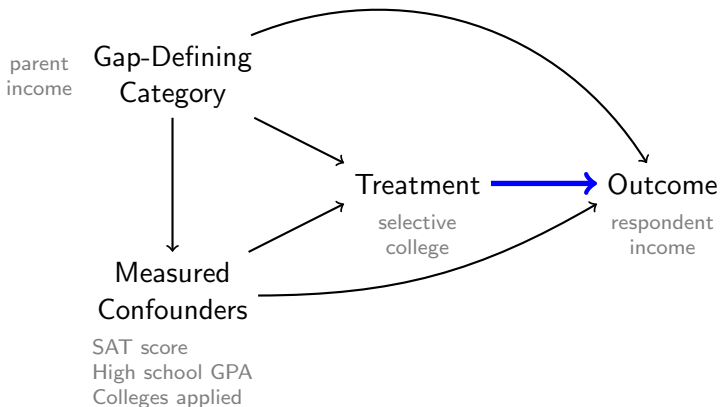
Using the Chetty et al. 2017 example,

What gap in respondent incomes would remain  
across categories of parent income  
if we intervened to send people to selective colleges?

# Make causal assumptions

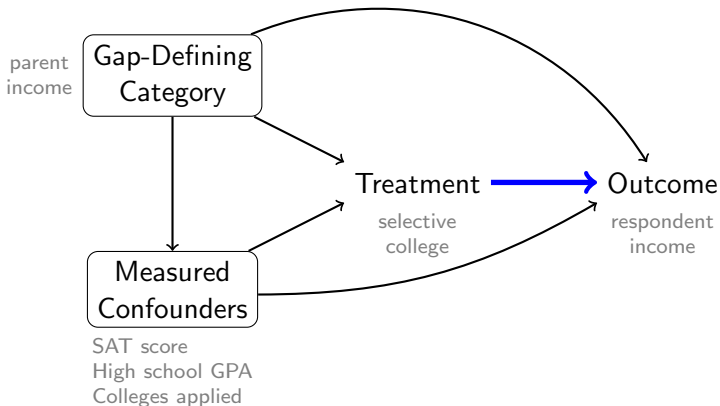


Pearl 2009

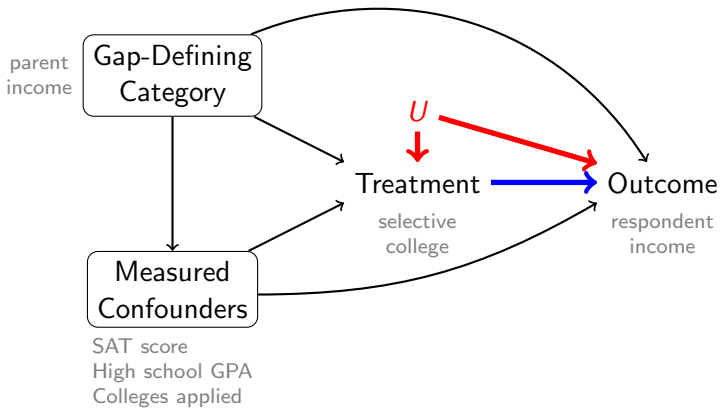


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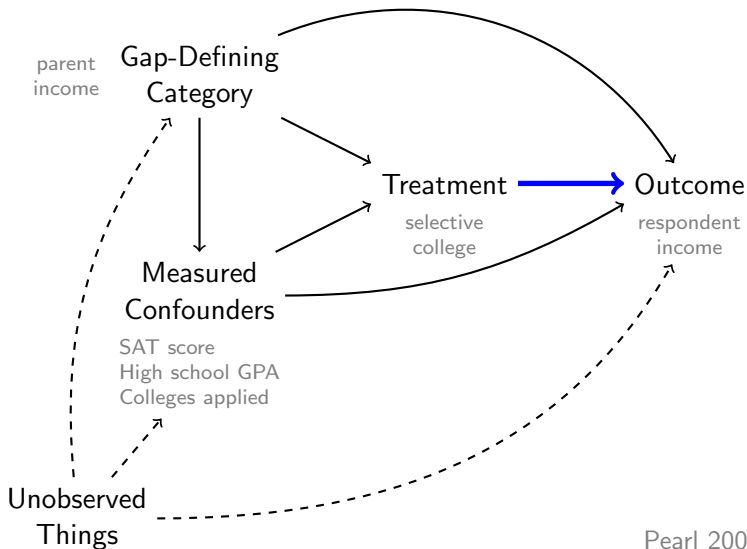




Pearl 2009



Pearl 2009



Pearl 2009

# Estimate

		Outcome under treatment	Outcome under control
People in category 1	Person 1	?	$Y_1$
	Person 2	$Y_2$	?
	Person 3	$Y_3$	?
People in category 2	Person 4	?	$Y_4$
	Person 5	$Y_5$	?
	Person 6	?	$Y_6$

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## Learn a prediction function

---

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## Predict the whole table

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	Person 3	$\hat{Y}_3(1)$	$\hat{Y}_3(0)$
People in category 2	Person 4	$\hat{Y}_4(1)$	$\hat{Y}_4(0)$
	Person 5	$\hat{Y}_5(1)$	$\hat{Y}_5(0)$
	Person 6	$\hat{Y}_6(1)$	$\hat{Y}_6(0)$

Robins 1986  
Hahn 1998



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Problem: Optimization for the wrong task

## Learn a prediction function

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## Problem: Optimization for the wrong task

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Prediction error over  
**observed**  
cases

## Learn a prediction function

---

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## Problem: Optimization for the wrong task

---

Prediction error over  
**observed**  
cases

vs

Prediction error over  
**all**  
cases

Solution: Reweight errors to approximate the correct task

## Solution: Reweight errors to approximate the correct task

---

		Prediction under treatment
People in category 1	Person 1	$\hat{Y}_1(1)$
	Person 2	$\hat{Y}_2(1)$
	Person 3	$\hat{Y}_3(1)$
People in category 2	Person 4	$\hat{Y}_4(1)$
	Person 5	$\hat{Y}_5(1)$
	Person 6	$\hat{Y}_6(1)$

## Solution: Reweight errors to approximate the correct task

---

		Prediction under treatment	Outcome under treatment
People in category 1	Person 1	$\hat{Y}_1(1)$	?
	Person 2	$\hat{Y}_2(1)$	$Y_2$
	Person 3	$\hat{Y}_3(1)$	$Y_3$
People in category 2	Person 4	$\hat{Y}_4(1)$	?
	Person 5	$\hat{Y}_5(1)$	$Y_5$
	Person 6	$\hat{Y}_6(1)$	?

## Solution: Reweight errors to approximate the correct task

---

		Prediction under treatment	Outcome under treatment	Error
People in category 1	Person 1	$\hat{Y}_1(1)$	?	?
	Person 2	$\hat{Y}_2(1)$	$Y_2$	$\hat{Y}_2(1) - Y_2$
	Person 3	$\hat{Y}_3(1)$	$Y_3$	$\hat{Y}_3(1) - Y_3$
People in category 2	Person 4	$\hat{Y}_4(1)$	?	?
	Person 5	$\hat{Y}_5(1)$	$Y_5$	$\hat{Y}_5(1) - Y_5$
	Person 6	$\hat{Y}_6(1)$	?	?



## Solution: Reweight errors to approximate the correct task

---

		Prediction under treatment	Outcome under treatment	Error	Weight on error
People in category 1	Person 1	$\hat{Y}_1(1)$	?	?	
	Person 2	$\hat{Y}_2(1)$	$Y_2$	$\hat{Y}_2(1) - Y_2$	3 / 2
	Person 3	$\hat{Y}_3(1)$	$Y_3$	$\hat{Y}_3(1) - Y_3$	3 / 2
People in category 2	Person 4	$\hat{Y}_4(1)$	?	?	
	Person 5	$\hat{Y}_5(1)$	$Y_5$	$\hat{Y}_5(1) - Y_5$	3
	Person 6	$\hat{Y}_6(1)$	?	?	

## Solution: Reweight errors to approximate the correct task

---

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

		Prediction under treatment	Outcome under treatment	Error	Weight on error
People in category 1	Person 1	$\hat{Y}_1(1)$	?	?	
	Person 2	$\hat{Y}_2(1)$	$Y_2$	$\hat{Y}_2(1) - Y_2$	3 / 2
	Person 3	$\hat{Y}_3(1)$	$Y_3$	$\hat{Y}_3(1) - Y_3$	3 / 2
People in category 2	Person 4	$\hat{Y}_4(1)$	?	?	
	Person 5	$\hat{Y}_5(1)$	$Y_5$	$\hat{Y}_5(1) - Y_5$	3
	Person 6	$\hat{Y}_6(1)$	?	?	

## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate:  $(\text{Original Estimate}) - (\text{Estimated Bias})$

## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate:  $(\text{Original Estimate}) - (\text{Estimated Bias})$       Doubly  
Robust  
Estimation

Robins, Rotnitzky, & Zhao 1994  
Bang & Robins 2005

## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate: (Original Estimate) – (Estimated Bias)      Doubly  
Robust  
Estimation

Estimation Setting		Outcome Modeling	Treatment Modeling	Doubly Robust
Both Models Correct				
Outcome Model Incorrect				
Treatment Model Incorrect				

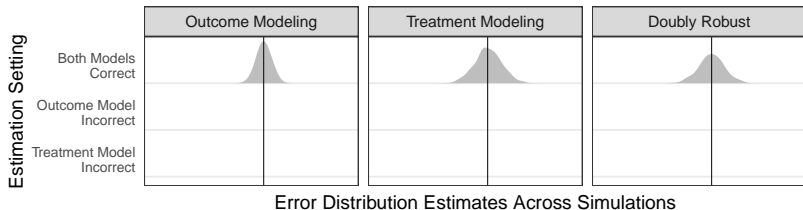
Error Distribution Estimates Across Simulations

Robins, Rotnitzky, & Zhao 1994  
Bang & Robins 2005

## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate: (Original Estimate) – (Estimated Bias)      Doubly  
Robust  
Estimation

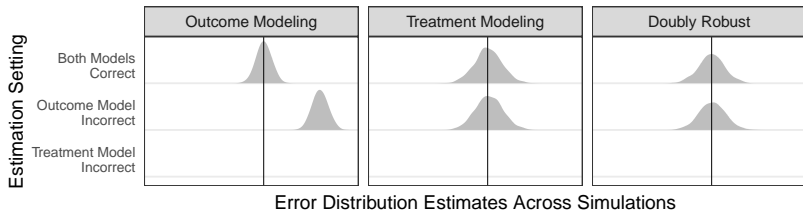


Robins, Rotnitzky, & Zhao 1994  
Bang & Robins 2005

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Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
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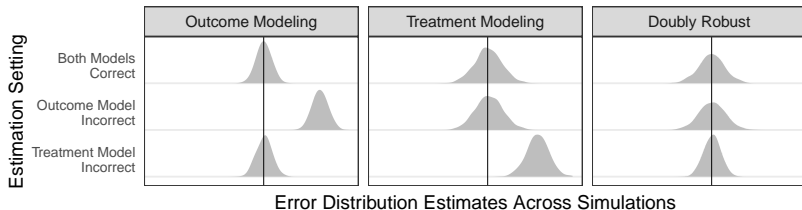


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Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
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Robust  
Estimation

Even better:

Chernozhukov et al. 2018  
Bickel 1982

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New Estimate:  $(\text{Original Estimate}) - (\text{Estimated Bias})$       Doubly  
Robust  
Estimation

Even better:      — Learn  $\hat{Y}_i$  in sample A  
                     — Estimate bias in sample B

Chernozhukov et al. 2018  
Bickel 1982

## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
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Robust  
Estimation

Even better:

- Learn  $\hat{Y}_i$  in sample A
- Estimate bias in sample B
- Cross fit: Swap roles and average

Chernozhukov et al. 2018  
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Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate:  $(\text{Original Estimate}) - (\text{Estimated Bias})$       Doubly  
Robust  
Estimation

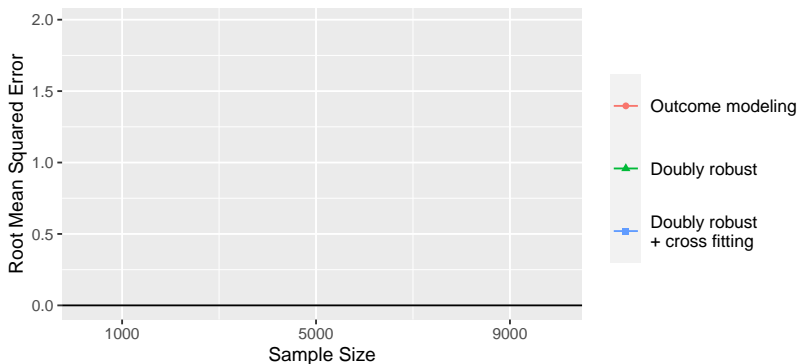
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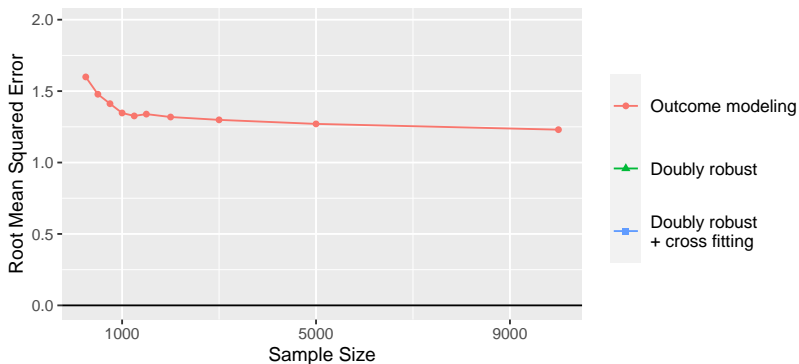
## Solution: Reweight errors to approximate the correct task

---



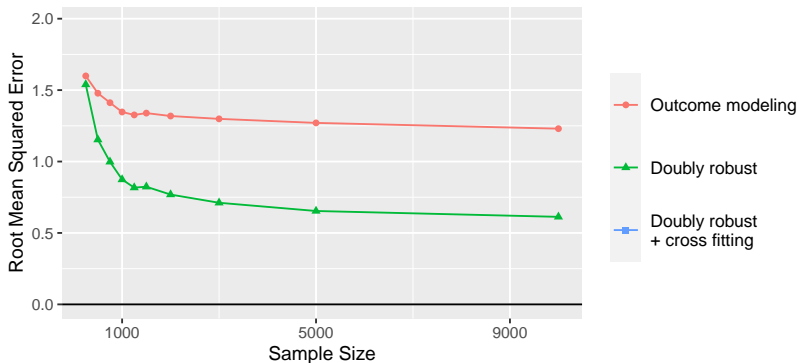
## Solution: Reweight errors to approximate the correct task

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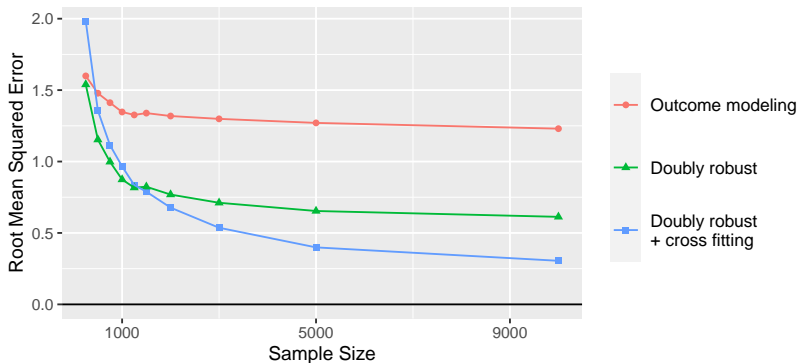
## Solution: Reweight errors to approximate the correct task

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---





## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
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New Estimate:  $(\text{Original Estimate}) - (\text{Estimated Bias})$       Doubly  
Robust  
Estimation

Even better:      — Learn  $\hat{Y}_i$  in sample A      Double  
Machine  
Learning  
— Estimate bias in sample B  
— Cross fit: Swap roles and average

Chernozhukov et al. 2018  
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## Solution: Reweight errors to approximate the correct task

Estimated bias:  $\text{Mean}(\hat{Y}_i - Y_i)$  with  
inverse probability of treatment weights

New Estimate: (Original Estimate) – (Estimated Bias)      Doubly  
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Estimation

Even better:      — Learn  $\hat{Y}_i$  in sample A      Double  
Machine  
Learning  
— Estimate bias in sample B  
— Cross fit: Swap roles and average

**So complicated!**

# gapclosing

An R package to estimate gap closing estimands. Install this package with the command

```
devtools::install_github("ilundberg/gapclosing").
```

```
estimate <- gapclosing(  
  data = simulated_data,  
  outcome_formula = formula(outcome ~ category + confounder),  
  treatment_formula = formula(treatment ~ category + confounder),  
  category_name = "category",  
  counterfactual_assignments = 1,  
  outcome_algorithm = "ranger",  
  treatment_algorithm = "ranger",  
  sample_split = "cross_fit",  
  se = T  
)
```

	description	estimate	se	ci.min	ci.max
	Factual gap	2.14	0.40	1.36	2.9
	Counterfactual gap	0.67	0.44	-0.19	1.5

How should one interpret results?

Interpret with respect to a **target trial** (Hernán & Robins 2016)

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1. Sample  $\mathcal{S}$  from the population

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2. Assign treatment  $T = 1$  to  $\mathcal{S}$

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1. Sample  $\mathcal{S}$  from the population
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3. Observe the disparity  
across categories  $X$



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Goal: Expected result over  
hypothetical samples  $\mathcal{S}$

Interpret with respect to a **target trial** (Hernán & Robins 2016)

## Local intervention

1. Sample  $\mathcal{S}$  from the population
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Interpret with respect to a **target trial** (Hernán & Robins 2016)

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### Global intervention

1. Take the entire population  $\mathcal{P}$

Interpret with respect to a **target trial** (Hernán & Robins 2016)

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1. Take the entire population  $\mathcal{P}$
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Goal: Result of this procedure

Interpret with respect to a **target trial** (Hernán & Robins 2016)

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Difficulty: Causal inference

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1. Sample  $\mathcal{S}$  from the population
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Difficulty: Causal inference

### Global intervention

1. Take the entire population  $\mathcal{P}$
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Difficulty: Causal inference  
Equilibrium dynamics

Interpret with respect to a **target trial** (Hernán & Robins 2016)



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## Interpret with respect to a **target trial** (Hernán & Robins 2016)

Policy-relevant.



### Local intervention

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Goal: Expected result over hypothetical samples  $\mathcal{S}$

Difficulty: Causal inference

### Global intervention

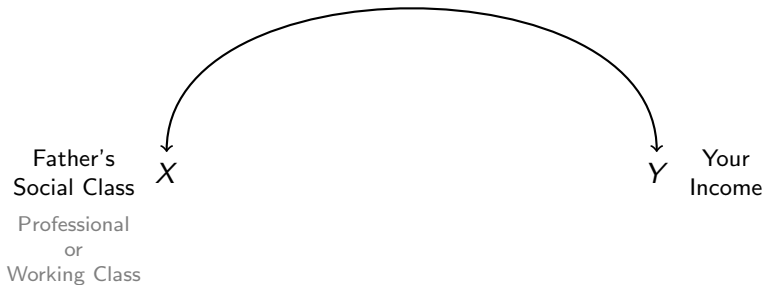
1. Take the entire population  $\mathcal{P}$
2. Assign treatment  $T = 1$  to  $\mathcal{P}$
3. Observe the disparity across categories  $X$

Goal: Result of this procedure

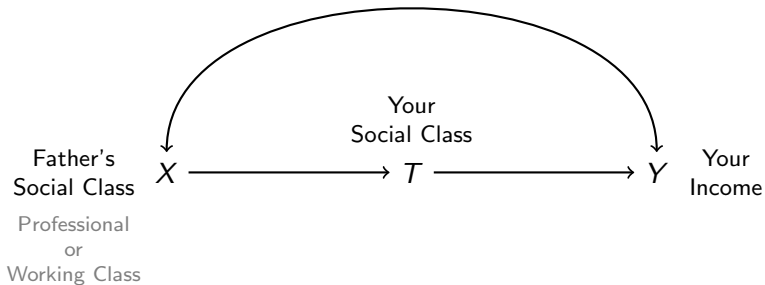
Difficulty: Causal inference  
Equilibrium dynamics

# Empirical Examples

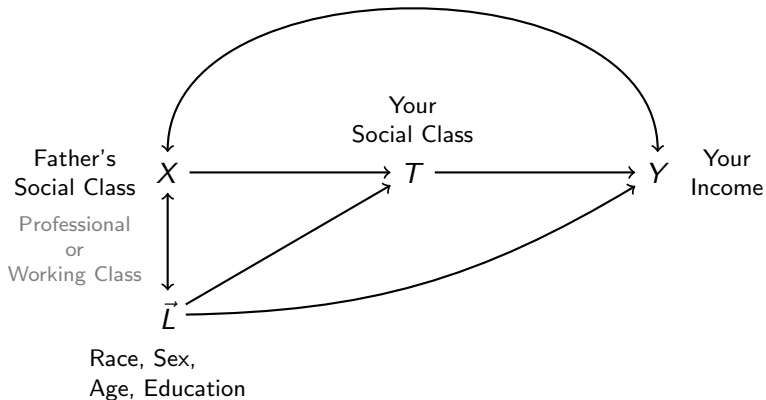
## Empirical Example 1: Economic Mobility



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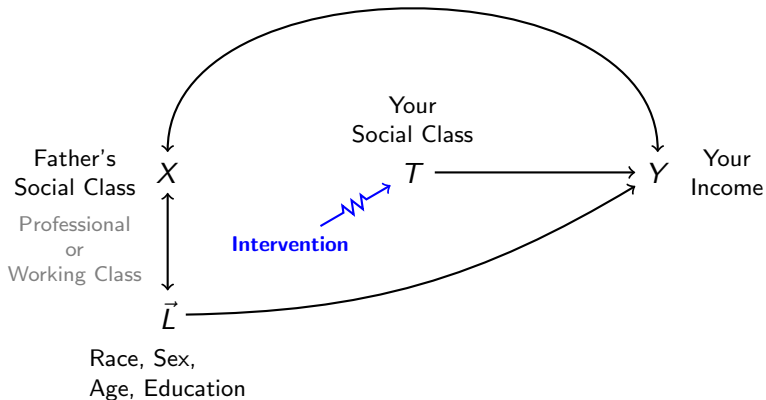


## Empirical Example 1: Economic Mobility

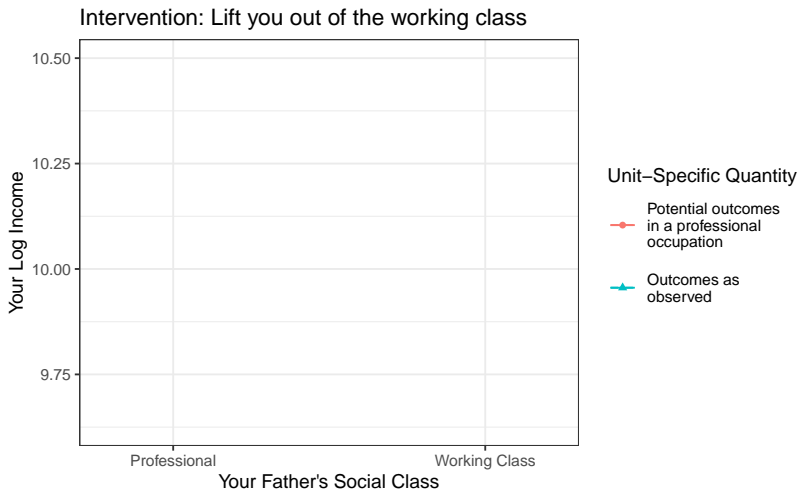




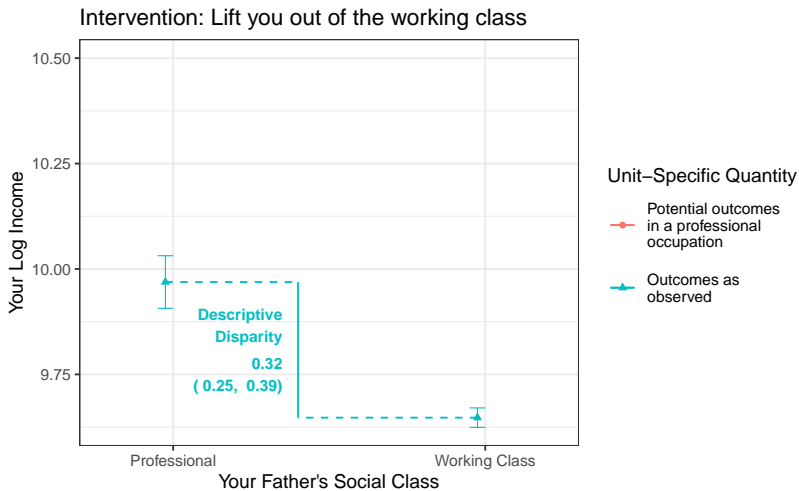
## Empirical Example 1: Economic Mobility



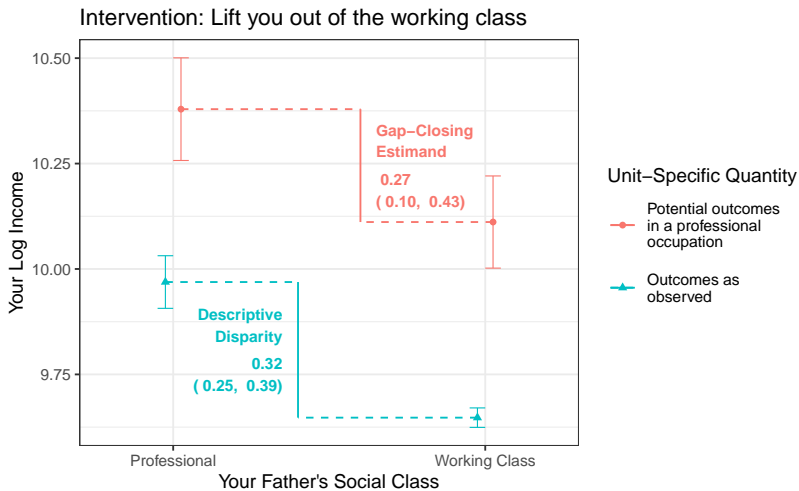
## Empirical Example 1: Economic Mobility



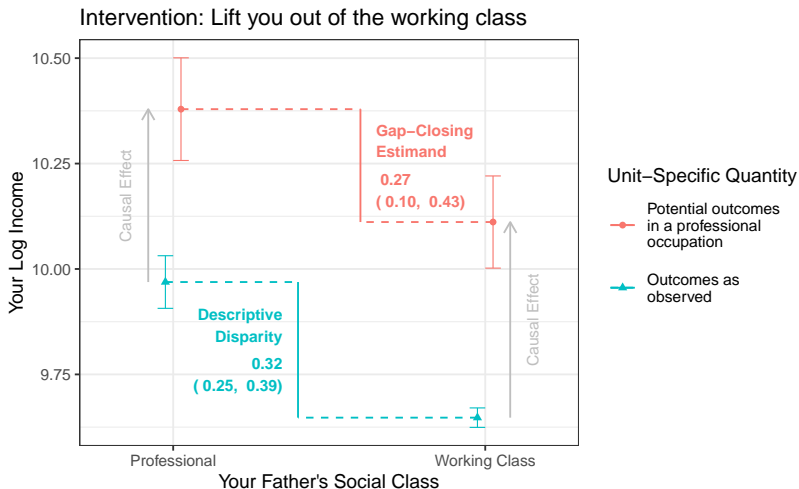
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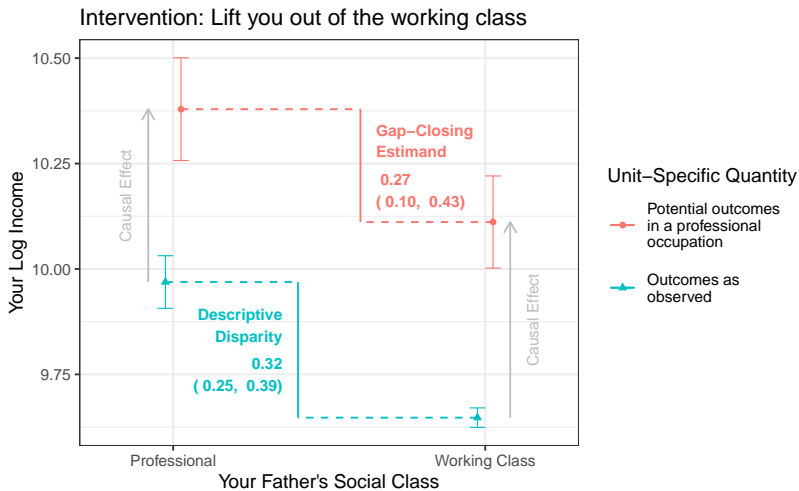


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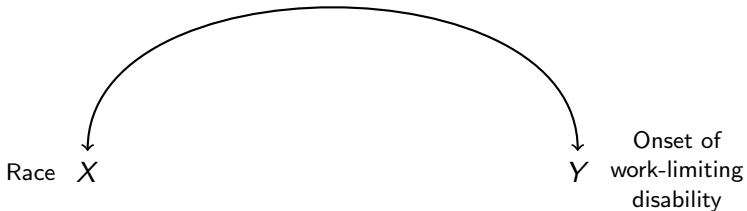


## Empirical Example 1: Economic Mobility

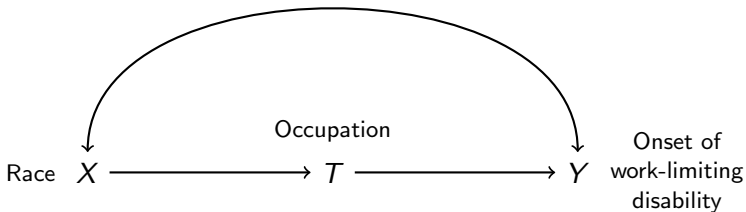
`plot_two_categories()`



## Empirical Example 2: Racial Disparities in Health

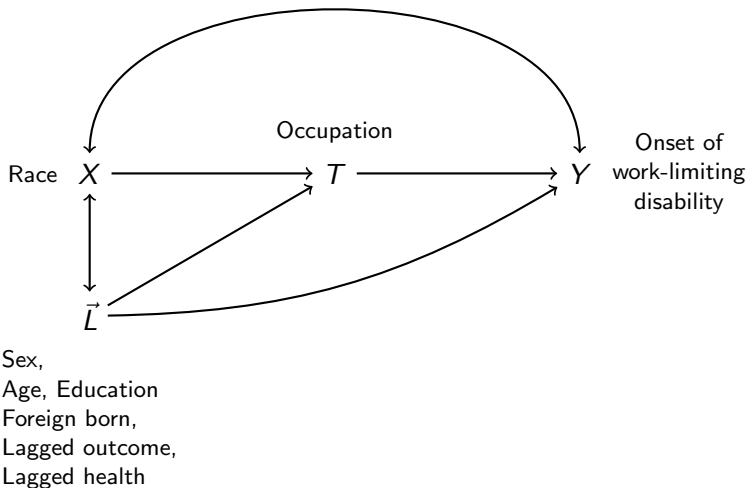


## Empirical Example 2: Racial Disparities in Health

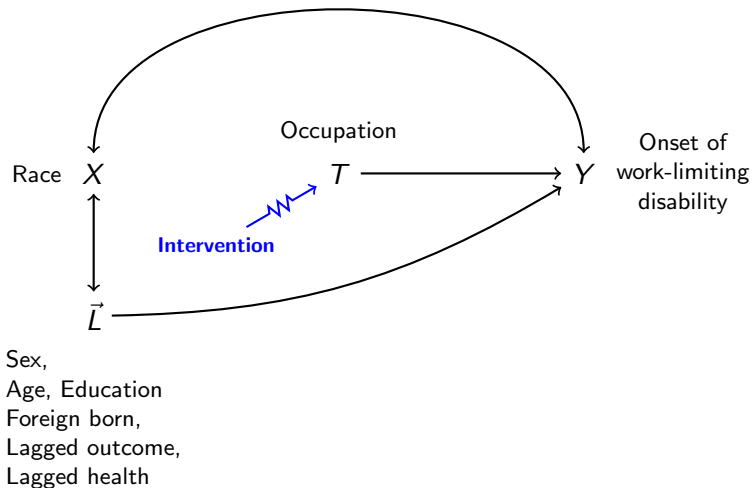




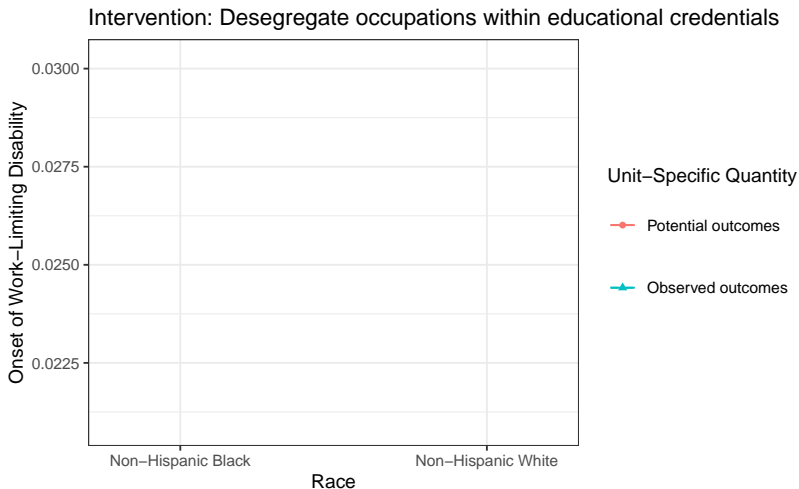
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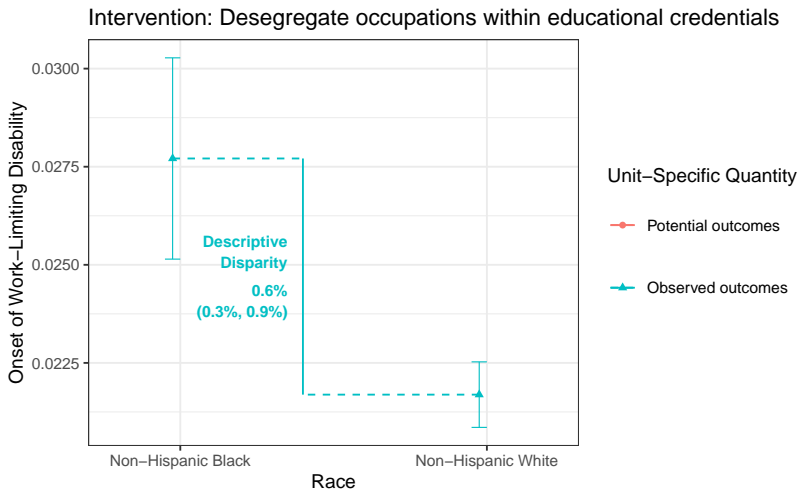
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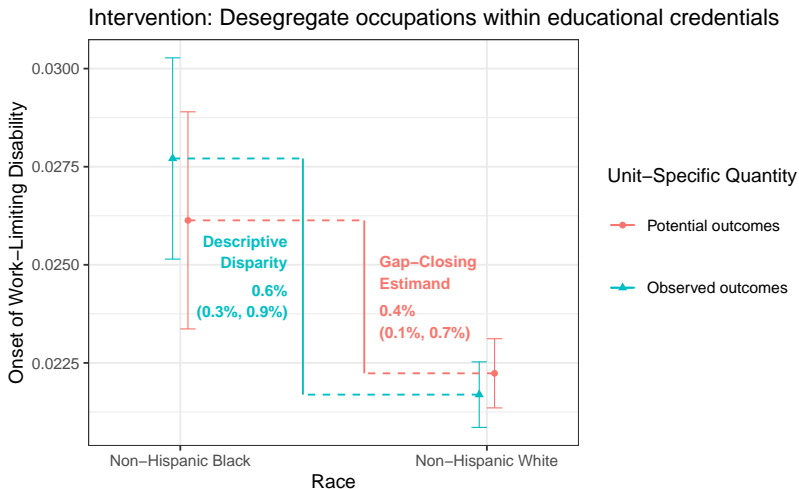
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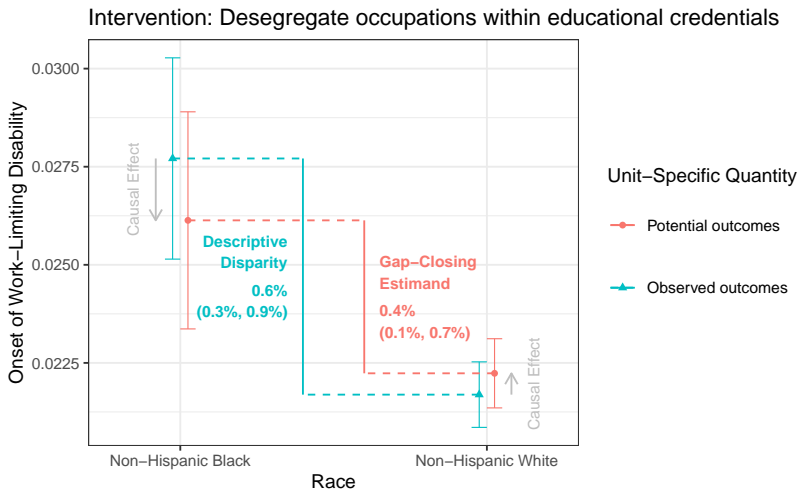
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# Discussion

# Gap closing estimands

- ▶ Define the goal
  - ▶ two target populations
  - ▶ hypothetical intervention
- ▶ Make causal assumptions
  - ▶ draw a DAG
- ▶ Estimate
  - ▶ fit an outcome model
  - ▶ change the treatment
  - ▶ predict for everyone
  - ▶ average within the populations



Now try it!

[ilundberg.github.io/gapclosing](https://ilundberg.github.io/gapclosing)

Ian Lundberg

[ianlundberg.org](https://ianlundberg.org)

[ilundberg@cornell.edu](mailto:ilundberg@cornell.edu)