

What is Your Estimand?

Defining the Target Quantity
Connects Statistical Evidence
to Theory



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on [Dataverse](https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7927/H73K-N391). Research reported in this publication was supported by The Eunice
Kennedy Shriver National Institute of Child Health & Human Development of the Na-
tional Institutes of Health under Award Number P2CHD047879.

There is a question every quantitative study must answer:

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The **purpose** of the
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A common answer:




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A common answer:

— We took [data source]



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There is a question every quantitative study must answer:

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A common answer:

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$$Y = \beta_0 + X_1\beta_1 + X_2\beta_2 + \epsilon$$



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**Epistemological
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A **unit-specific**
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$$Y_i(t)$$

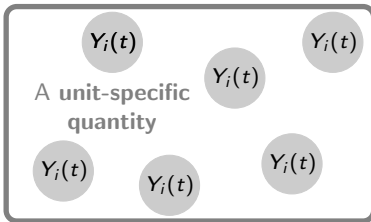
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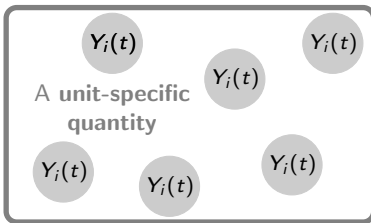
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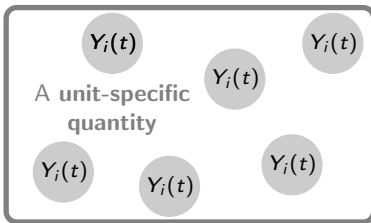


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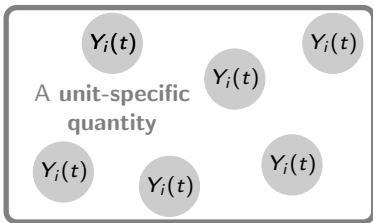


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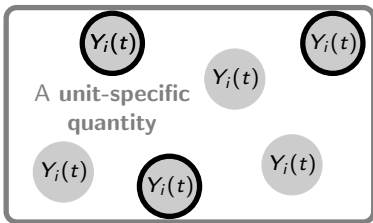


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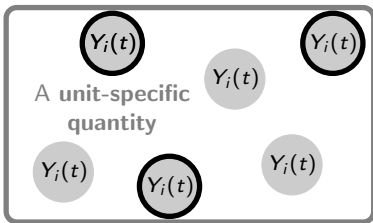


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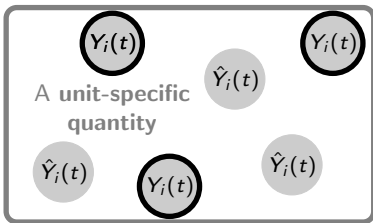


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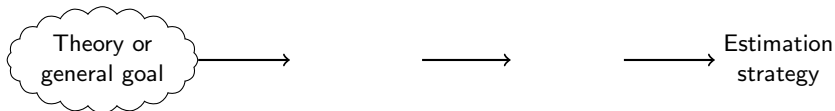
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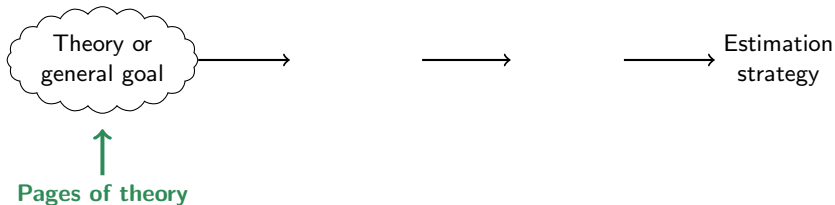
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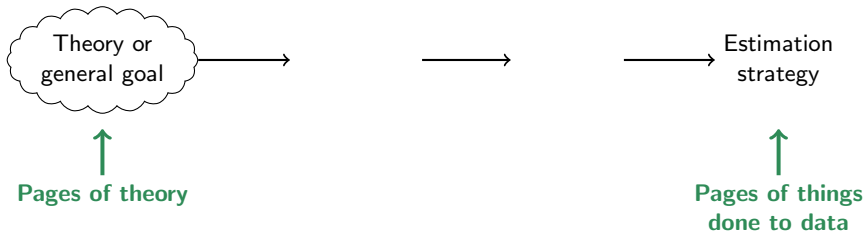
Research framework: Estimands connect theory to evidence



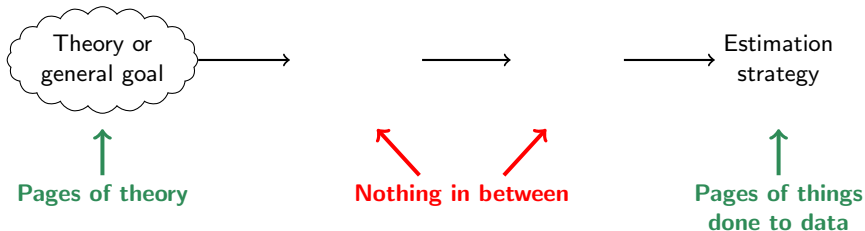
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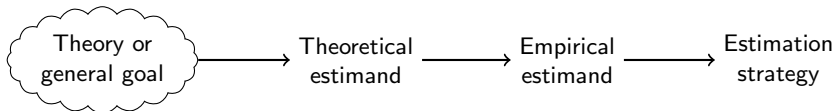
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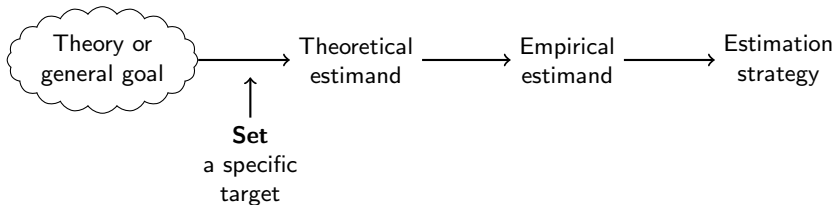
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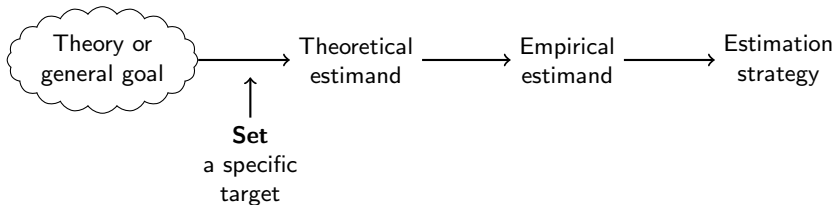
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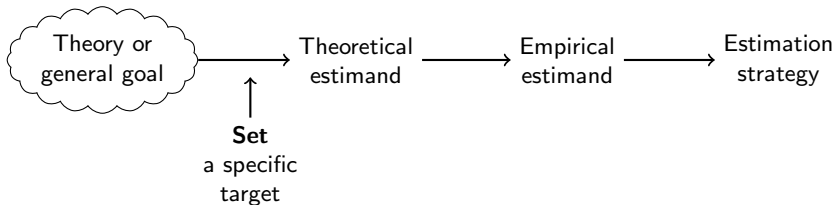
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Definition

A **unit-specific quantity**
aggregated over a
target population

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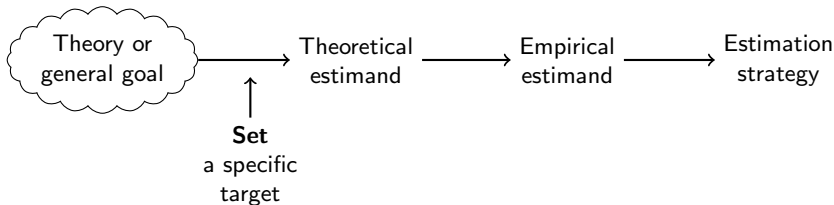
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Example

$$\frac{1}{\text{Size of U.S. adult population}} \sum_{i \text{ in U.S. adult population}} \left(\text{Employed}_i \right)$$

Research framework: Estimands connect theory to evidence



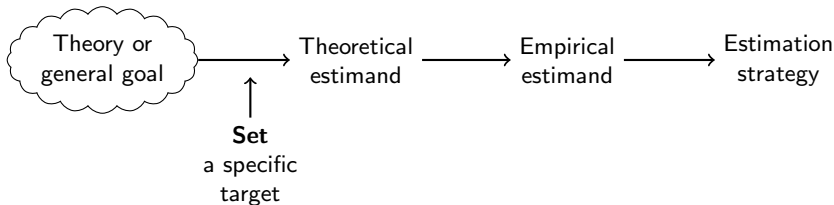
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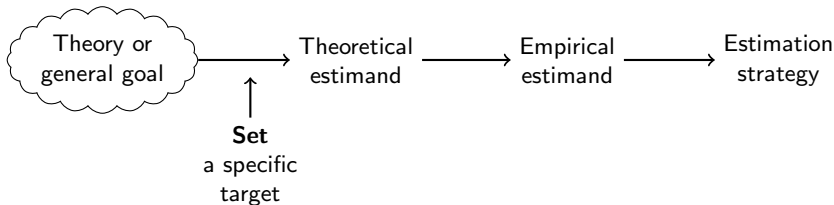
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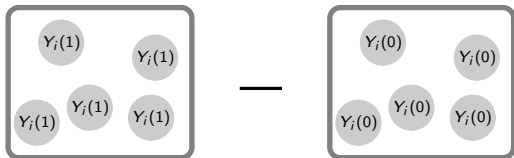
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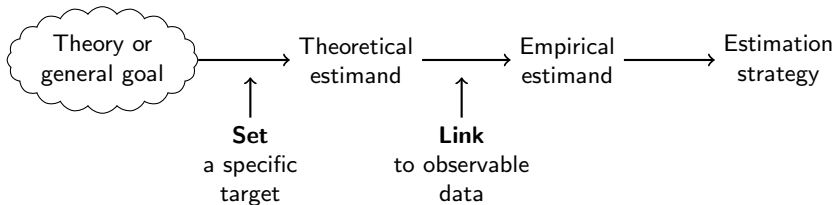
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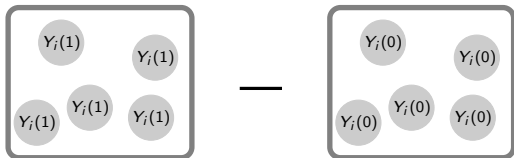
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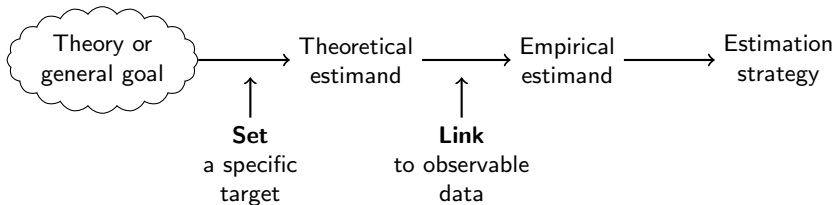
Definition

A quantity involving
observable data

Example



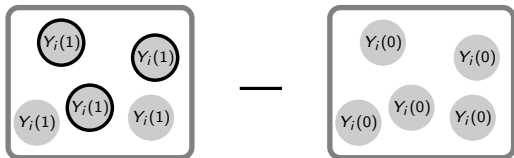
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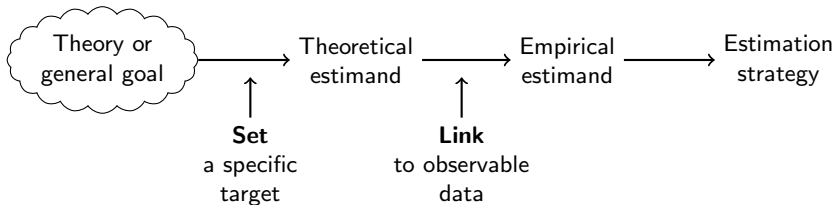
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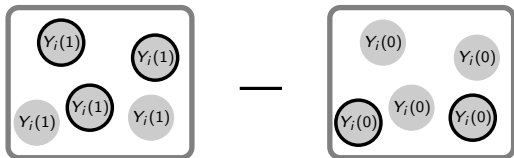
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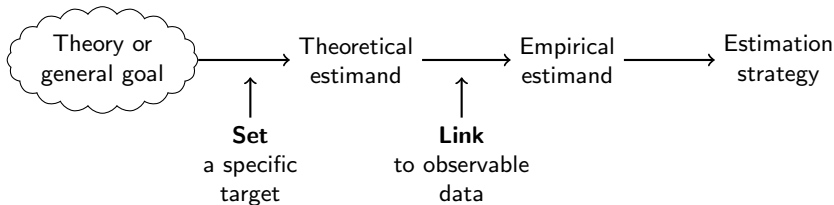
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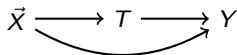
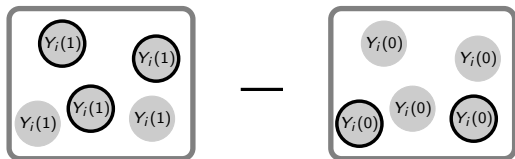
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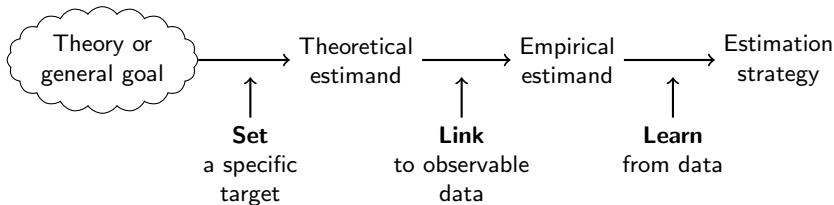
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Pearl 2009, Imbens and Rubin 2015,
Morgan and Winship 2015, Elwert and Winship 2014

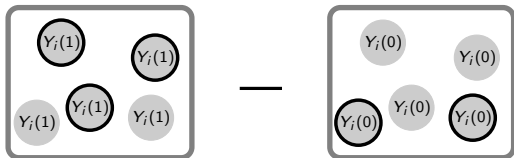
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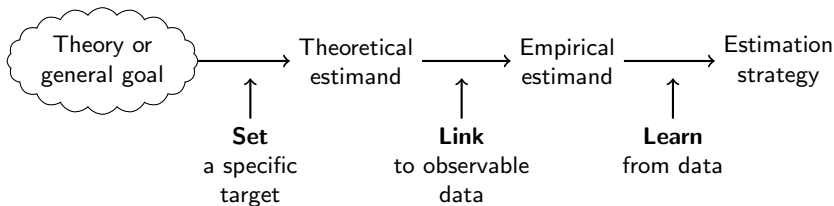
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An algorithm applied to data

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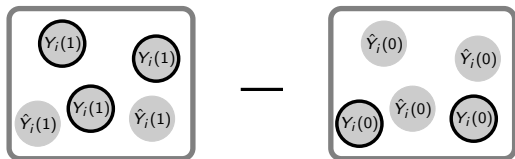
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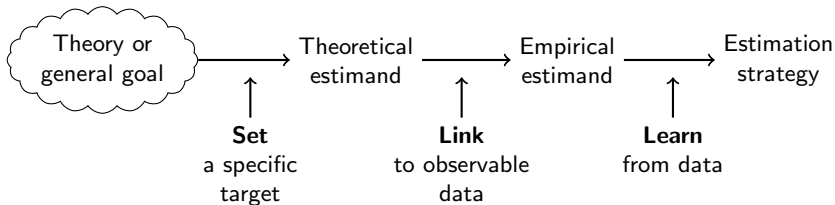
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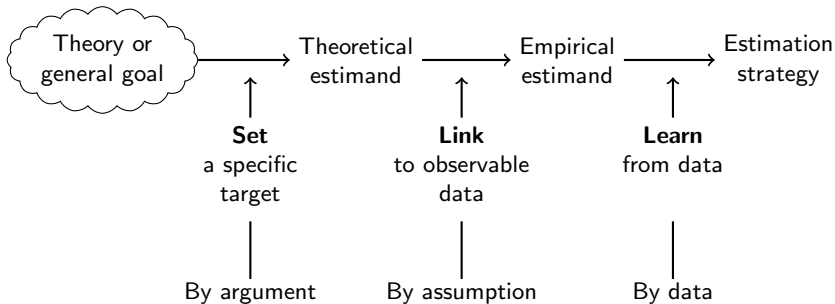


Young 2009, Watts 2014, Berk et al. 2019, Molina and Garip 2019

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An Empirical Analysis of Racial Differences in Police Use of Force

Roland G. Fryer Jr.

Harvard University and National Bureau of Economic Research

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We can never be satisfied as long as the Negro is the victim of the unspeakable horrors of police brutality. (Martin Luther King Jr., August 28, 1963)

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TheUpshot
DATA DIVE

Surprising New Evidence Shows Bias in Police Use of Force but Not in Shootings



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DATA DIVE

Surprising New Evidence Shows Bias in Police Use of Force but Not in Shootings

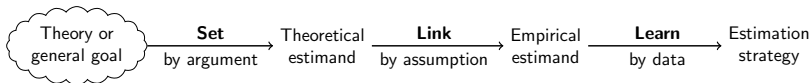
OPINION | COMMENTARY

The Myth of Systemic Police Racism **WSJ OPINION**
Hold officers accountable who use excessive force. But there's no evidence of widespread racial bias.

By Heather Mac Donald
June 2, 2020 1:44 pm ET

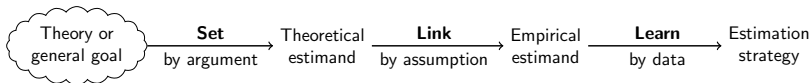
Reality check: study finds no racial bias in police shootings

The Guardian



Evidence: Police use lethal force at the same rate against black and white civilians who are stopped.

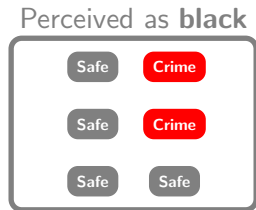
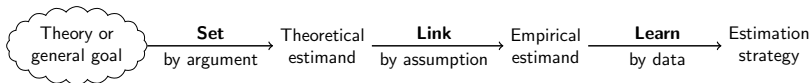
Fryer 2019. Fuller critique by Knox et al. 2020 and Durlauf and Heckman 2020.



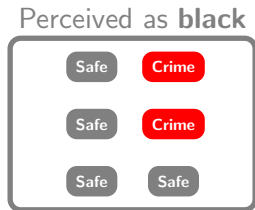
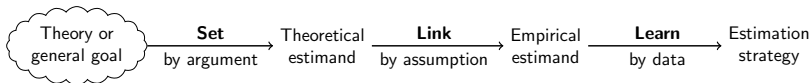
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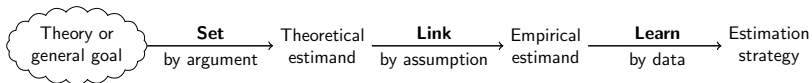
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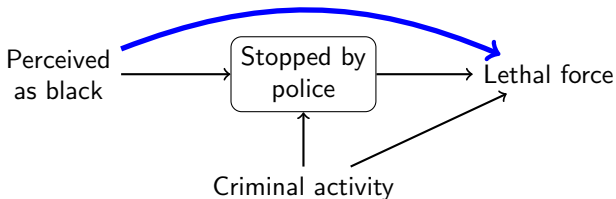
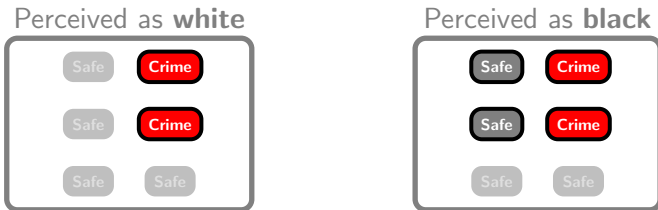
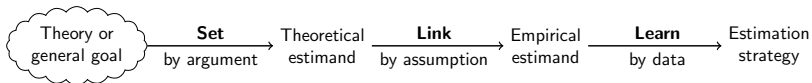
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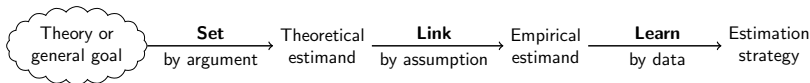
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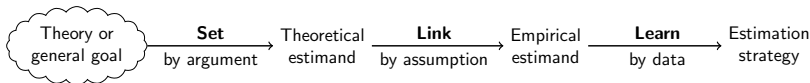
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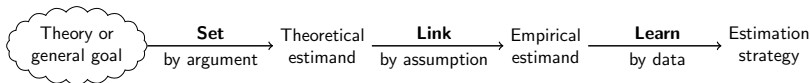


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**Fryer
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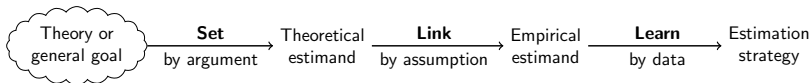


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Fryer responds:

“We use the term ‘racial differences’ 114 times in lieu of the more prescriptive wording—‘racial discrimination.’ We use the phrase ‘conditional on an interaction’ 20 times...I am not sure how many more ways we would have needed to caveat our results to satisfy [the critics].”

Fryer 2019. Fuller critique by Knox et al. 2020 and Durlauf and Heckman 2020.



Pal and Waldfogel (2016) estimate the family gap in pay.



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Is the theoretical estimand descriptive?



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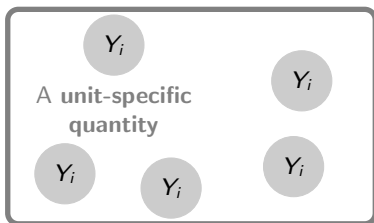
“the differential in hourly wages
between women with children
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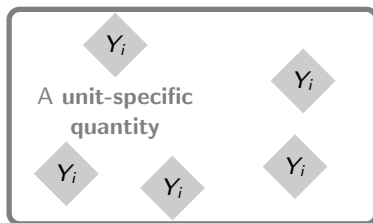
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Averaged over a
target population
of **mothers**



Averaged over a
target population
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Is the theoretical estimand descriptive? Is it causal?



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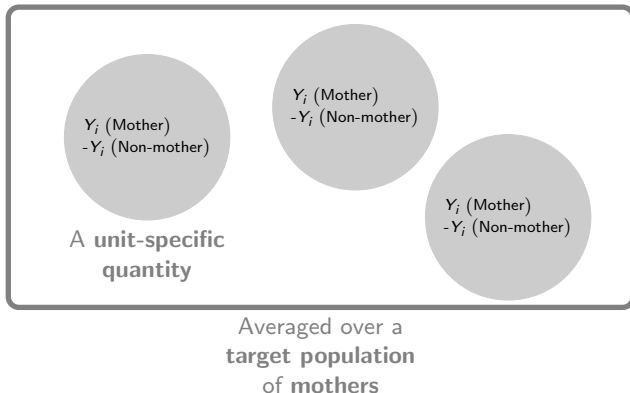
“causal estimation techniques”



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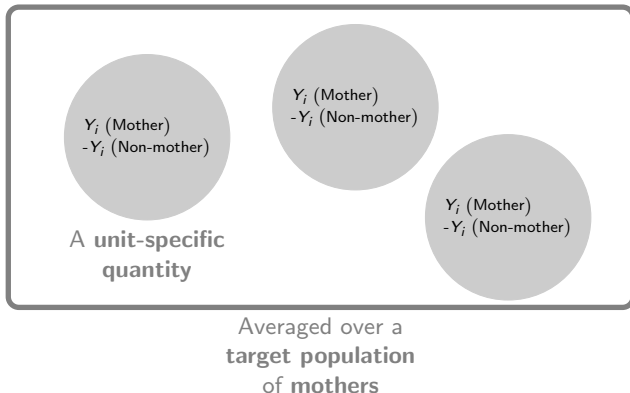




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Added complexity: Wages are undefined for the non-employed.

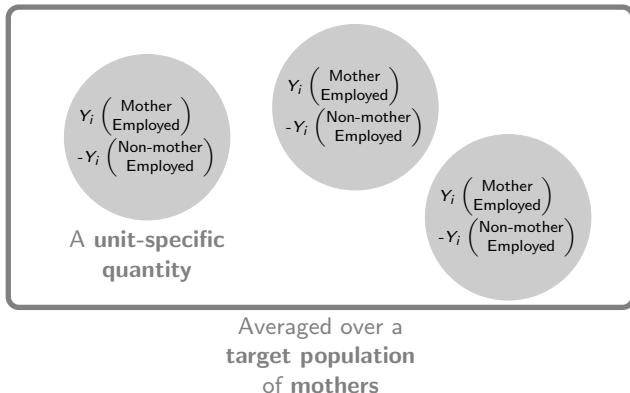


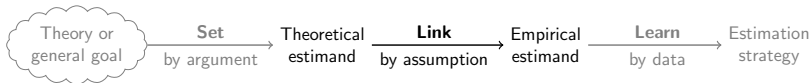


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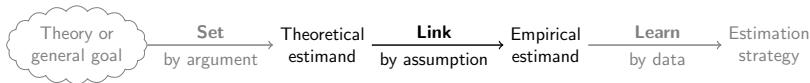
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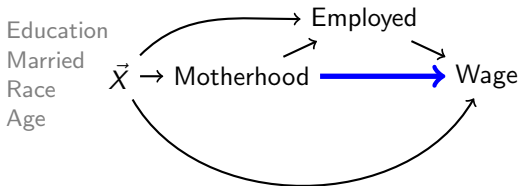


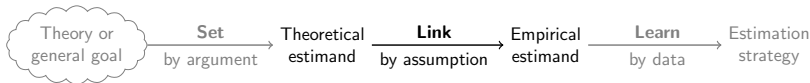


Unit-specific quantity: $Y_i \left(\begin{array}{c} \text{Mother,} \\ \text{Employed} \end{array} \right) - Y_i \left(\begin{array}{c} \text{Non-mother,} \\ \text{Employed} \end{array} \right)$

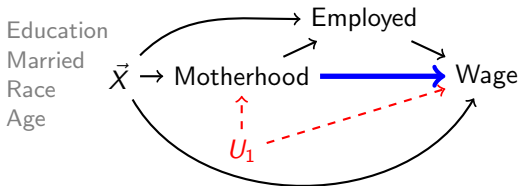


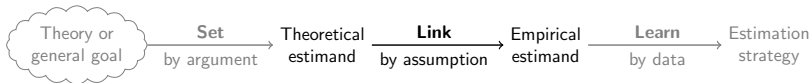
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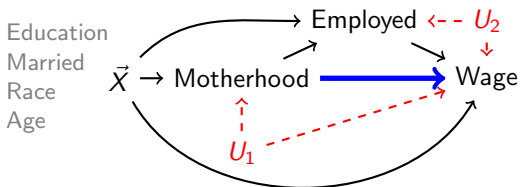


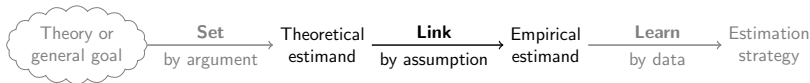
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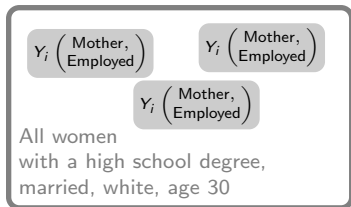
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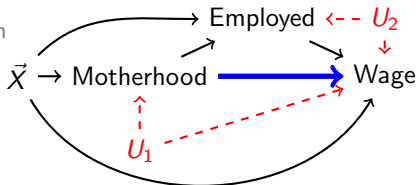
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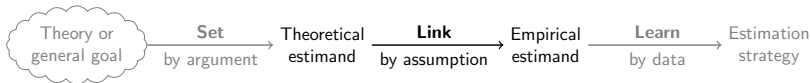
Potential outcomes



Focus on
one $\vec{X} = \vec{x}_i$

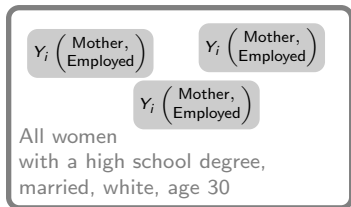
Education
Married
Race
Age





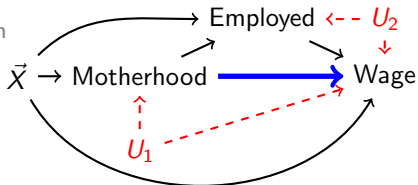
$$E \left(Y_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) \mid \vec{X} = \vec{x}_i \right)$$

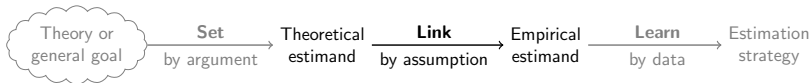
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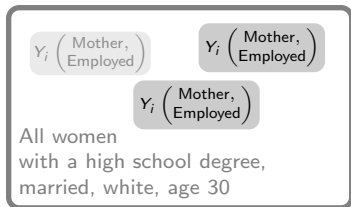
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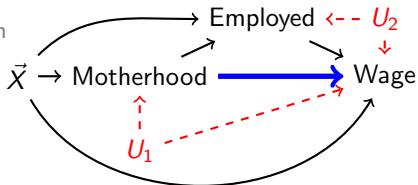
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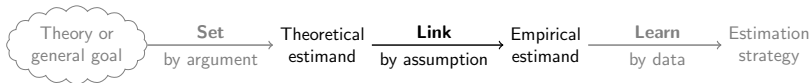
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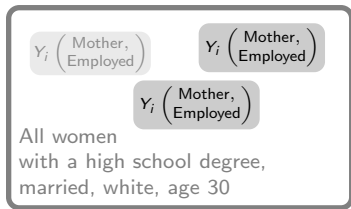
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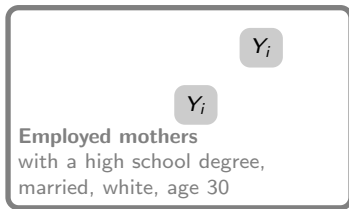


$$E \left(Y_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) \middle| \vec{X} = \vec{x}_i \right) \quad ? \quad E \left(Y_i \middle| \begin{matrix} \text{Motherhood} = \text{Mother,} \\ \text{Employment} = \text{Employed,} \\ \text{Covariates } \vec{X} = \text{Observed } \vec{x}_i \end{matrix} \right)$$

Potential outcomes

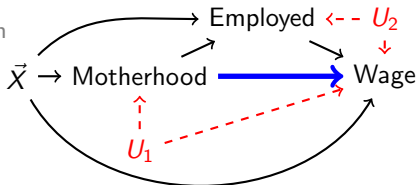


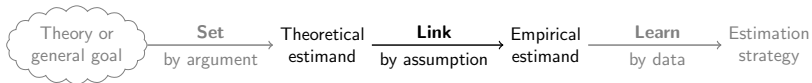
Realized outcomes



Focus on one $\vec{X} = \vec{x}_i$

Education
Married
Race
Age

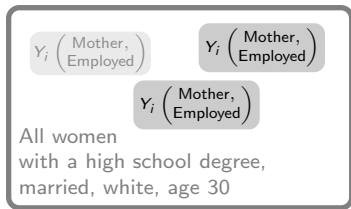




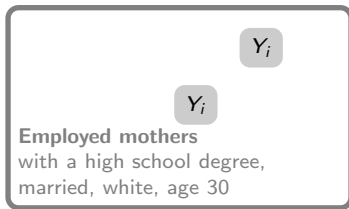
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By the DAG

Potential outcomes

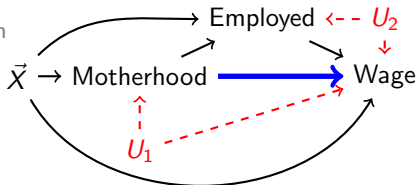


Realized outcomes



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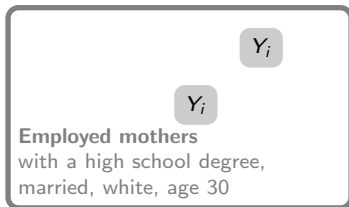
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Realized outcomes

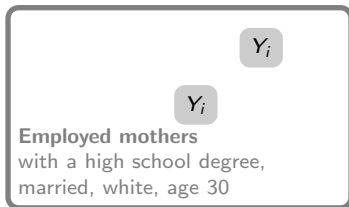




This can be estimated
by machine learning!

$$\longrightarrow E \left(Y_i \mid \begin{array}{l} \text{Motherhood} \\ \text{Employment} \\ \text{Covariates } \vec{X} \end{array} \right) = \begin{array}{l} \text{Mother,} \\ \text{Employed,} \\ \text{Observed } \vec{x}_i \end{array}$$

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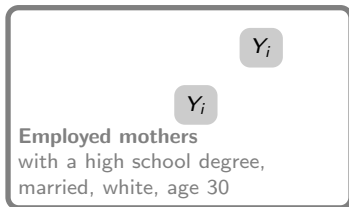


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↓
Any prediction algorithm
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Realized outcomes





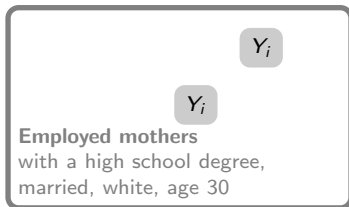
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Any prediction algorithm
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Generalized
Additive
Model

Realized outcomes





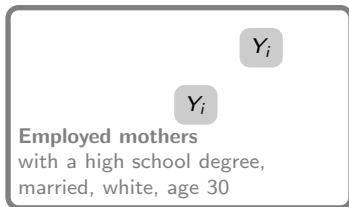
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Any prediction algorithm
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Generalized Additive Model
Random Forest

Realized outcomes



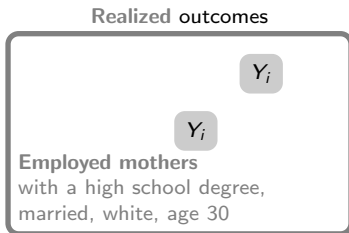


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Any prediction algorithm
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Ordinary Least Squares Generalized Additive Model Random Forest





Mechanics: How **predictive algorithms** estimate the **estimand**



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1) Learn an algorithm to predict the outcome



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

$$\hat{E} \left(Y_i \left| \begin{array}{l} \text{Motherhood} = \text{Mother,} \\ \text{Employment} = \text{Employed,} \\ \text{Covariates } \vec{X} = \text{Observed } \vec{x}_i \end{array} \right. \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

$$\hat{Y}_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) = \hat{E} \left(Y_i \left| \begin{array}{l} \text{Motherhood} = \text{Mother,} \\ \text{Employment} = \text{Employed,} \\ \text{Covariates } \vec{X} = \text{Observed } \vec{x}_i \end{array} \right. \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
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$$\hat{Y}_i \begin{pmatrix} \text{Non-mother,} \\ \text{Employed} \end{pmatrix} = \hat{E} \left(Y_i \left| \begin{array}{ll} \text{Motherhood} & = \text{Non-mother,} \\ \text{Employment} & = \text{Employed,} \\ \text{Covariates } \vec{X} & = \text{Observed } \vec{x}_i \end{array} \right. \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

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$$\hat{Y}_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) - \hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

- 1) Learn an algorithm to predict the outcome
- 2) Predict for every unit at each treatment value

$$\hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right) = \hat{E} \left(Y_i \left| \begin{matrix} \text{Motherhood} & = & \text{Non-mother,} \\ \text{Employment} & = & \text{Employed,} \\ \text{Covariates } \vec{X} & = & \text{Observed } \vec{x}_i \end{matrix} \right. \right)$$

- 3) Average over the target population

$$\frac{1}{n} \sum_{i=1}^n \left(\hat{Y}_i \left(\begin{matrix} \text{Mother,} \\ \text{Employed} \end{matrix} \right) - \hat{Y}_i \left(\begin{matrix} \text{Non-mother,} \\ \text{Employed} \end{matrix} \right) \right)$$



Mechanics: How **predictive algorithms** estimate the **estimand**

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This is called an **imputation estimator**

Hahn, 1998

Abadie & Imbens 2006

Also called the parametric *g*-formula in biostatistics, Hernán & Robins 2020



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age
+ Interaction between age and motherhood

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age
+ Interaction between age and motherhood
+ Allow a smooth curve for age rather than quadratic

Most flexible



Choose an algorithm by **predictive performance**

Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible OLS with a quadratic for age
+ Interaction between age and motherhood
+ Allow a smooth curve for age rather than quadratic
+ Include each age as a separate indicator variable

Most flexible



Choose an algorithm by **predictive performance**

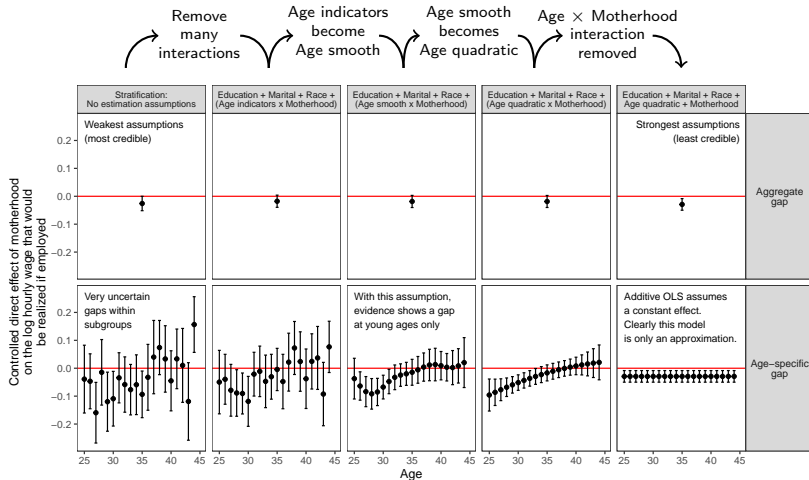
Outcome Log hourly wage

Predictors Motherhood, age, race, education, marital status

Candidate algorithms

Least flexible	OLS with a quadratic for age
	+ Interaction between age and motherhood
	+ Allow a smooth curve for age rather than quadratic
	+ Include each age as a separate indicator variable
Most flexible	+ Include all interactions among all predictors

Increasingly strong estimation assumptions: Less credibility →





What is your estimand?

← Every quantitative study should answer this question

Defining the Target Quantity
Connects Statistical Evidence
to Theory

Ian Lundberg

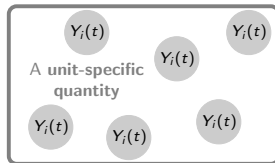
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Averaged over a
target population

Draft on [SocArxiv](#)

Code on [Dataverse](#)

American Sociological Review