Rutherford Schools Physics Project: Algebra in Physics

1 Algebraic manipulation

It is assumed that students will be familiar with the following concepts:

- Basic algebraic manipulation, including rearranging an equation to change the subject of that equation.

  For example: rearrange the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to find an expression for u in terms of f and v.
  So, starting with the equation

  $$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

  obtain an expression for $1/u$ by subtracting $1/v$ from both sides i.e.

  $$\frac{1}{f} - \frac{1}{v} = \frac{1}{u} + \frac{1}{v} - \frac{1}{v} = \frac{1}{u}.$$

  Before taking the reciprocal to find u it is easiest to put the left-hand side over a common factor fv i.e.

  $$\frac{v-f}{fv} = \frac{1}{u}.$$

  Now take the reciprocal of both sides to give

  $$u = \frac{fv}{v-f}.$$

  For example: rearrange the equation $F = mv^2/r$ to find an expression for v in terms of F, m and r.
  So, starting with the equation

  $$F = \frac{mv^2}{r},$$

  to obtain an expression for v^2 multiply both sides by r and divide by m

  $$\frac{r}{m}F = \frac{r}{m}\frac{mv^2}{r}.$$

  Thus, cancelling m and r on the top and bottom of the expression on the right-hand side gives

  $$\frac{r}{m}F = v^2,$$

  and taking the square root of both sides we obtain

  $$v = \sqrt{\frac{Fr}{m}}.$$

- Substituting numerical values into algebraic equations and using appropriate units.

  For example: find the kinetic energy of a particle with a mass of 0.61 kg and a speed of 13 m s^{-1}.
  The kinetic energy of a particle is given by $E_k = \frac{1}{2}mv^2$, thus

  $$E_k = \frac{1}{2}(0.61\ \text{kg})(13\ \text{m s}^{-1})^2 = 52\ \text{J}$$

since energy has the units of joules.
For example: find the force (in newtons) on a particle of mass 11 g if it has an acceleration of 3.2 cm s^{-2}.
The force on an accelerating particle is given by $F = ma$. To calculate the force in newtons the units of m are kg and of a are m s^{-2}; thus in this case, to convert from g to kg multiply by 10^{-3} and from cm s^{-2} to m s^{-2} multiply by 10^{-2}, so

$$F = (11 \times 10^{-3}\ \text{kg})(3.2 \times 10^{-2}\ \text{m s}^{-2}) = 3.5 \times 10^{-4}\ \text{N}$$

Exercise 1: In this problem, rearrange the physical formulae and relationships given to obtain an expression for the variable/variables indicated (for example in (i) $a = F/m$).
(i) $F = ma$, subject: a.
(ii) $W = mg$, subject: m.
(iii) $\rho = \frac{m}{V}$, subject: m.
(iv) $V = IR$, subjects: I, R.
(v) $F = \frac{\Delta p}{\Delta t}$, subject: Δp.
(vi) $pV = nRT$, subject: n.
(vii) $F = \frac{Qq}{4\pi\epsilon_0 r^2}$, subject: q.
(viii) $F = \frac{mv^2}{r}$, subject: r.
(ix) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, subjects: R (not $1/R$), R_1 (not $1/R_1$).
(x) $E_k = \frac{1}{2}mv^2$, subject: v.
(xi) $P = \frac{V^2}{R}$, subject V.
(xii) $F = -\frac{GMm}{r^2}$, subject: r.
(xiii) $F = BIL\sin\theta$, subject: θ.

Solution 1:

Exercise 2: In this problem, rearrange the equations given to obtain an expression for one variable in terms of the others.
(i) $v = u + at$, t (in terms of u, v and a) and a (in terms of u, v and t).
(ii) $s = \frac{1}{2}(u + v)t$, u (in terms of s, v and a) and t (in terms of s, u and v).
(iii) $s = ut + \frac{1}{2}at^2$, u (in terms of s, a and t), a (in terms of s, u and t), t (in terms of s and a assuming $u = 0$) and t (in terms of s, u and a).
(iv) $v^2 = u^2 + 2as$, u (in terms of s, v and a), a (in terms of u, v and s) and s (in terms of u, v and a).
Starting with the equations $v = u + at$ and $s = ut + \frac{1}{2}at^2$ eliminate t to show that $v^2 = u^2 + 2as$.
Starting with the equations $v = u + at$ and $s = \frac{1}{2}(u + v)t$ show that $s = ut + \frac{1}{2}at^2$.

Solution 2:

Exercise 3: In this problem, calculate the values of the specified quantity using appropriate units.
(i) Using $v = u + at$, find v if $u = 3.0\,\text{m s}^{-1}$, $a = 9.8\,\text{m s}^{-2}$ and $t = 2.0\,\text{s}$.
(ii) Using $v = u + at$, find v if $u = 3.0\,\text{cm s}^{-1}$, $a = 9.8\,\text{m s}^{-2}$ and $t = 2.0\,\text{ms}$.

Solution 3: