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An improved relay auto tuning of PID controllers for unstable FOPTD systems

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Abstract

Using a single symmetric relay feedback test, a method is proposed to identify all the three parameters of a first order plus time delay (FOPTD) unstable model. It is found by simulation that, the relay autotune method gives −23% error in the calculation of k_u, when $D/\tau = 0.6$. In the present work, a method is proposed, by incorporating the higher order harmonics to explain the error in the k_u calculation. A method is proposed to estimate all the parameters of an unstable FOPTD system. Two simulation results are given on unstable first and second order plus time delay transfer function models. The estimated model parameters of the unstable FOPTD model are compared with the methods of Majhi, S. & Atherton, D.P. [(2000). Online tuning of controllers for an unstable FOPTD process. *IEE Proceedings Control Theory Applications*, *147*(4), 421–427] and the Thyagarajan, T., & Yu, C.C. [(2003). Improved autotuning using the shape factor from relay feedback, *Industrial engineering Chemistry Research*, *42*, 4425–4440] PID controllers are designed for the identified model and for the actual system. The proposed method gives performance close to that of the actual system. For a second order plus time delay system, the latter two methods fail to identify the FOPTD model. Simulation results are also given for a nonlinear bioreactor system. The PID controller designed on the model identified by the proposed method gives a performance close to that of the controller designed on the locally linearized model.

Keywords: Unstable system; Relay tuning; Identification

1. Introduction

Astrom and Hagglund (1984) have suggested the use of an ideal (on–off) relay to generate a sustained oscillation of the controlled variable and to get the ultimate gain (k_u) and the ultimate frequency (ω_u) directly from the relay experiment. The relay feedback method has become very popular because, it is time efficient as compared to the conventional method. The amplitude (a) and the period of oscillation (p_u) are noted from the sustained oscillation of the system output. The ultimate gain (k_u) and ultimate frequency (ω_u), are calculated from the principal harmonics approximation as

$$K_u = \frac{4h}{\pi a} \tag{1}$$

$$\omega = \frac{2\pi}{p_u} \tag{2}$$

where a is the amplitude, h the relay height and p_u is the period of oscillations in the system output.

Luyben (1987) has suggested the use of relay testing for identifying a transfer function model. Using k_u and ω_u in the phase angle and amplitude criteria for an unstable FOPTD model, the following two equations relating three model parameters are obtained

$$\frac{k_u k_p}{(\tau^2 \omega_u^2 + 1)^{0.5}} = 1 \tag{3}$$

$$-D\omega_u + \tan^{-1}(\tau\omega_u) = 0 \tag{4}$$

Since only k_u and ω_u are available, additional information such as the steady state gain, or the time delay should be a known priori in order to fit a typical transfer function model,

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such as unstable FOPTD. The above Eqs. (1) and (2), assume that, the higher order harmonics are neglected.

Majhi and Atherton (1999) have proposed an improved method for calculating the FOPTD model parameters by a symmetric relay tuning. In this method, the output response is aligned with the input response by shifting to the left. The area under the curve for the process output response over a half period of the cycle is calculated (a_y). The corresponding net area for the process input response over the half period of the output response is also calculated (a_u). The steady state gain k_p is estimated as

$$k_p = \frac{a_y}{a_u} \tag{5}$$

After obtaining k_p, the time delay (D) and time constant (τ) are calculated as

$$\tau = \frac{\frac{p_u}{2}}{\ln\left(\frac{k_p h + a}{k_p h - a}\right)} \tag{6}$$

$$D = \frac{\frac{p_u}{2}\ln\left(\frac{k_p h + a}{k_p h}\right)}{\ln\left(\frac{k_p h + a}{k_p h - a}\right)} \tag{7}$$

Recently Thyagarajan and Yu (2003) have proposed a method of identifying a FOPTD unstable model based on the shape of the response of the process using a symmetric relay. In this method, the output response is aligned with the input response by shifting to the left. Then, the time to peak amplitude, the peak amplitude and the period of oscillation are noted. The time delay is considered as the time to the peak value. From the derived analytical expression of the process output response of an unstable FOPTD system for a symmetric relay input, the time constant and gain are calculated as

$$\tau = \frac{\frac{p_u}{2}}{\ln\left[\frac{1}{(2e^{-D/\tau} - 1)}\right]} \tag{8}$$

$$k_p = \frac{a}{h(e^{D/\tau} - 1)} \tag{9}$$

It is to be noted that, for higher order systems, the recorded time to peak value from the response (D) will not match with that of the actual time delay of the process.

Li, Eskinat, and Luyben (1991) have reported that the model identified by the symmetry relay auto tune method gives error as high as 27 to −18% in the value of k_u for stable FOPTD systems. Recently, Srinivasan and Chidambaram (2004) proposed a method of considering higher order harmonics, to explain the reported error of 27 to −18% in k_u calculations for stable systems. Srinivasan and Chidambaram (2004) have also suggested a method to estimate the model parameters of stable FOPTD system. In the present work, the Srinivasan and Chidambaram method is extended to an unstable FOPTD system using a single symmetric relay test.

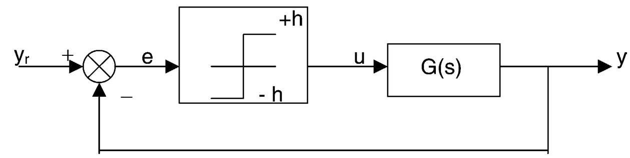

Fig. 1. Block diagram for the symmetric relay feedback system.

2. Problem description

Let us consider an unstable first order plus time delay (FOPTD) system

$$\frac{y(s)}{u(s)} = \frac{k_p \exp(-Ds)}{\tau s - 1} \tag{10}$$

where, k_p is the process gain, D the time delay and τ is the time constant are the parameters to be estimated. Consider a relay feedback system (refer Fig. 1) where $G(s)$ is the process transfer function, y the output, y_r the set point, e the error and u is the manipulated variable. Here, the feedback loop uses a symmetric relay.

3. Consideration of higher order harmonics

From the Fourier series analysis, it can be easily shown (Astrom & Hagglund, 1984; Yu, 1999) that a relay consists of many sinusoidal waves of odd multiples of fundamental frequency ω and with the amplitude $4h/(n\pi)$ ($n = 1, 3, 5, \ldots$). The input to the process thus consists of many sine waves. For a FOPTD system, the output wave is also a sinusoidal wave with different amplitude and frequency. Here $y(t)$ is the combined response due to many of the sine waves

$$y(t) = [a_1 \sin(\omega_u t + \phi_1) + (1/3)a_3 \sin(3\omega_u t + \phi_3) + \cdots] \tag{11}$$

where

$$a_1 = \frac{1}{[1 + (\tau\omega_u)^2]^{0.5}}; \qquad a_3 = \frac{1}{[1 + (3\tau\omega_u)^2]^{0.5}};$$
$$a_5 = \frac{1}{[1 + (5\tau\omega_u)^2]^{0.5}}; \ldots; \text{etc.} \tag{12}$$

$$\phi_1 = -D\omega_u - \pi + \tan^{-1}(\tau\omega_u) = -\pi \tag{13a}$$

$$\phi_3 = -3D\omega_u - \pi + \tan^{-1}(3\tau\omega_u);$$
$$\phi_j = -jD\omega_u - \pi + \tan^{-1}(j\tau\omega_u) \tag{13b}$$

Eq. (11) can be written as

$$y(t) = a_1[\sin(\omega_u t + \phi_1) + (1/3)b_3 \sin(3\omega_u t + \phi_3) + \cdots] \tag{14}$$

where

$$b_3 = \left\{ \frac{[1 + (\tau\omega_u)^2]}{[1 + (3\tau\omega_u)^2]} \right\}^{0.5};$$

$$b_5 = \left\{ \frac{[1 + (\tau\omega_u)^2]}{[1 + (5\tau\omega_u)^2]} \right\}^{0.5}; \text{ etc.,} \tag{15}$$

If $\tau\omega_u$ is assumed very large (∞), then $y(t)$ will consists of only the fundamental harmonics. In the Eq. (11), the terms containing $a_3 \sin(3\tau\omega_u + \phi_3)$, $a_5 \sin(5\tau\omega_u + \phi_5)$, etc., will be neglected. As stated earlier, this assumption gives a large error in the calculation of k_u and hence in the estimated model parameters of FOPTD model. In what follows, we will consider the higher order dynamics for the calculation of k_u and in the identification of the model parameters. Since for the unstable FOPTD systems, the value of D/τ should be less than one (Venkatashankar & Chidambaram, 1994) for stabilization using a proportional controller and hence using a relay, let us derive approximate evaluation of $y(t)$ for the limiting cases of larger $\tau\omega_u$.

3.1. Case: Limiting case of larger $\tau\omega_u$

Eq. (13a) becomes

$$\phi_1 = -D\omega_u + (\pi/2) = 0 \tag{16}$$

Hence,

$$D\omega_u = 0.5\pi \tag{17}$$

$$\phi_3 = -3D\omega_u - \pi + \tan^{-1}(3\tau\omega_u) \tag{18a}$$

$$\phi_3 = -3D\omega_u - (\pi/2) = -2\pi \tag{18b}$$

Similarly,

$$\phi_5 = -\frac{6\pi}{2}; \ldots; \phi_N = -\frac{(N+1)\pi}{2} \tag{18c}$$

Hence, Eq. (14) can be written as

$$y(t) = a_1[-\sin(\omega_u t) - (1/3)b_3 \sin(3\omega_u t) - \cdots] \tag{19}$$

$$b_3 = \left\{ \frac{[1 + (\tau\omega_u)^2]}{1 + (3\tau\omega_u)^2} \right\}^{0.5}; \qquad b_5 = \left\{ \frac{[1 + (\tau\omega_u)^2]}{1 + (5\tau\omega_u)^2} \right\}^{0.5} \tag{20a}$$

For the larger values of $\tau\omega$, the value of one can be neglected when compared to the value of $\tau\omega_u$ and hence the values of b_1, b_3, can be approximated as

$$b_3 = \frac{1}{3}; \qquad b_5 = \frac{1}{5}; \qquad b_j = \frac{1}{7}, \text{ etc.} \tag{20b}$$

Hence, Eq. (19) becomes

$$y(t) = a_1[-\sin(\omega_u t) - (1/9)\sin(3\omega_u t) - (1/25)\sin(5\omega_u t) - \cdots] \tag{21}$$

Here the value of a_1 is to be calculated. The value of a_1 is not the amplitude what we observe from the output oscillation. Let us estimate the error involved in k_u by using only the principle harmonics in the analysis of relay testing. From the output oscillations, it is possible to calculate $y(t)$ at any time t. ω_u is the frequency of observed oscillations. From Eq. (21), we get

$$a_1 = \frac{y(t)}{\sum[\sin(i\omega_u t)/i^2]} \tag{22}$$

Let us consider the time (t^*) at which

$$\omega_u t^* = 0.5\pi \tag{23}$$

Then Eq. (21) becomes

$$y(t^*) = a_1[1 + (1/9) + (1/25) + (1/49) + (1/81) + \cdots] \tag{24}$$

In above equation, let N is the number of terms considered. Using the limiting value for the summation term ($0.125\pi^2$), we get from Eq. (22)

$$a_1 = 0.810\,y(t^*) \tag{25}$$

The time at which the value of $y(t^*)$ to be noted from the response is given by $t^* = 0.5\pi/\omega_u$. In the conventional method, we have to use only the first term in Eq. (22) [i.e. $a = y(t^*)$]. The consideration of all higher order harmonics gives $a_1 = 0.81y(t^*)$. This limiting value shows that a maximum error −19% in k_u is obtained by using the conventional analysis (principle harmonics method). The observed amplitude (a) is always greater than the corrected amplitude (a_1). Later it will be shown that the actual maximum error in k_u will be of −23% (percentage error in ultimate gain is given by ($k_{u(\text{principle harmonics})} - k_{u(\text{exact})})/k_{u(\text{exact})}$). The calculated value of k_u by the conventional method is always lesser than the actual values.

In deriving the equation for the ultimate gain of the controller, it is assumed that, all the higher order harmonics of the relay output are filtered by the system and allows only the fundamental frequency of oscillation (Astrom & Hagglund, 1984). The simulation result for several D/τ ratios indicates the system is not filtering out all the higher order harmonics. For $D/\tau = 0.6$, the relay autotune method gives −23% error in the calculation of k_u by the principle harmonics assumption (refer to Table 1). In the proposed work, we suggest a method to incorporate adequate higher order harmonics to get improved estimates of the ultimate gain. It is better to incorporate $N = 3$ than 1 (which is principle harmonics assumption). It is also observed that, depending upon the D/τ ratio the system will filter out some of the higher order harmonics (not all). Hence, by incorporation of the all N may also lead to an error in the estimation of the ultimate gain.

The values of a_1 are calculated for different number of terms (N) considered in Eq. (24). After calculating a_1 values, the values of k_u are calculated (corresponding to N). Table 1

Table 1
Effect of including the higher order harmonics on k_u

D/τ	$N=1$ (k_u)	$N=3$ (k_u)	$N=5$ (k_u)	$N=7$ (k_u)	$N=9$ (k_u)	$N=\propto$ (k_u)	Exact (k_u)	Percentage error (k_u)*
0.60	1.550	2.133	2.194	2.220	2.228	2.287	2.019	−23.23
0.50	1.969	2.726	2.804	2.837	2.848	2.924	2.536	−22.36
0.40	2.591	3.515	3.615	3.658	3.672	3.770	3.315	−21.84
0.20	5.862	7.266	7.473	7.562	7.590	7.792	7.229	−18.91
0.1	12.519	14.715	15.134	15.314	15.372	15.777	15.07	−16.93
0.06	21.877	24.627	25.312	25.618	25.826	26.415	25.54	−14.34
0.04	33.683	35.967	36.905	37.338	37.669	38.583	38.63	−12.81
0.02	70.735	72.756	74.458	75.339	76.241	77.636	77.90	−9.19

Percentage error in k_u is calculated as: $(k_{u(N=1)} - k_{u(\mathrm{exact})})/k_{u(\mathrm{exact})}$. Relay gives oscillations only up to $D/\tau = 0.6$.

Table 2
Details of calculations for Table 1

D/τ	Observed amplitude (a_0)	ω_u (frequency)	t^* ($0.5\pi/\omega_u$)	$Y(t^*)$	Calculated amplitude (a_1)
0.6	0.8214	1.3483	1.1650	0.6870	0.5968 ($N=3$)
0.5	0.6464	2.0268	0.7750	0.5375	0.4669 ($N=3$)
0.4	0.4913	2.9224	0.5375	0.4169	0.3622 ($N=3$)
0.2	0.2172	6.9813	0.2250	0.2017	0.1752 ($N=3$)
0.1	0.1017	14.96	0.105	0.0996	0.0841 ($N=5$)
0.06	0.0582	25.1327	0.0625	0.0595	0.0503 ($N=5$)
0.04	0.0378	39.2699	0.04	0.0408	0.033 ($N=9$)
0.02	0.018	78.5398	0.02	0.0202	0.0164 ($N=9$)

shows the effect of including higher order harmonics, for various D/τ ratios, ranging from 0.6 to 0.02. Table 2 gives the details of the intermediate calculations. For the lower values of D/τ, using only the fundamental frequency in the analysis of relay testing, the proposed method predicts a maximum error of +23% in the amplitude (a). The actual number of the higher order harmonics to be considered, depends on the dynamics of the process. When there is no initial dynamics, the system may be of FOPTD system and a value of $N=3$ is suggested. If the system response shows any initial dynamics, then the system may be second order or higher order, then $N=7$ or 9 is recommended. Table 2 shows that for smaller values of D/τ for FOPTD systems, a larger value of N (number of higher order harmonics) is to be used to get an accurate value of k_u. It is found that when $0.1 < (D/\tau) \leq 0.69$, the value of $N=3$ is recommended and when $D/\tau = 0.1$ or below, $N=5$ or 7 is recommended. Once we decide on the value of N to be used, we can get a more accurate value of k_u. The corrected k_u is calculated as $k_u = 4h/\pi a_1$ where a_1 is calculated from Eq. (24). Knowing the corrected k_u and using the measured value of ω_c, the appropriate equations are used to calculate model parameters (k_p, D and τ) of an unstable FOPTD model. Let us now discuss how to formulate the equations to identify the model parameters of an unstable FOPTD system.

4. Proposed method for estimation of k_p, D and τ

From the definition of Laplace transform (Kreyszig, 1996) we get

$$y(s) = \int_0^\infty y(t)\ \exp(-st)\,\mathrm{d}t \tag{26}$$

The above integral can be evaluated for a particular value of s (say s_1). It is suggested to use the value $s_1 = 8/t_s$, where t_s is the time at which three repeated cycles of oscillations appear in the output. The reason for taking $s_1 = 8/t_s$ is that, for $t > t_s$, because of very small value of the term $\exp(-s_1 t)$, the contributions by subsequent terms is negligible while evaluating the integral. Let the above resulting integral value be denoted as $y(s_1)$. Similarly,

$$u(s) = \int_0^\infty u(t)\ \exp(-st)\,\mathrm{d}t \tag{27}$$

Eq. (27) is also evaluated using $s_1 = 8/t_s$. With the numerical values of $y(s_1)$ and $u(s_1)$, using the assumed model (Eq. (1)), an equation is formulated as given below

$$\frac{y(s_1)}{u(s_1)} = \frac{k_p\ \exp(-Ds_1)}{\tau s_1 - 1} \tag{28}$$

on cross-multiplying we get,

$$\frac{y(s_1)}{u(s_1)}(\tau s_1 - 1) - k_p\ \exp(-D\,s_1) = 0 \tag{29}$$

From the amplitude criterion we get

$$\frac{k_p k_u}{[(\tau\omega_u)^2 + 1]^{0.5}} = 1 \tag{30}$$

k_u used in Eq. (30) is the corrected k_u giving

$$v = \tau\omega_u = [(k_p k_u)^2 - 1]^{0.5} \tag{31}$$

From the phase angle criterion, we get

$$D = \frac{\tan^{-1} v}{\omega_u} \tag{32}$$

Eq. (29) becomes

$$\frac{y(s_1)}{u(s_1)}((v/\omega_u)s_1 - 1) - k_p \exp[-(s_1/\omega_u)\tan^{-1} v] = 0 \quad (33)$$

Solving numerically, Eq. (33) we get the value for v. Since

$$v = [(k_p k_u)^2 - 1]^{0.5} \quad (34)$$

using $v = \tau\omega_u$ and Eq. (32) we get τ and D.

5. Simulation results

5.1. Case study 1

Let us consider an unstable FOPTD system as

$$G(s) = \frac{\exp(-0.2s)}{s-1} \quad (35)$$

With the symmetric relay test of height ±1 and at $N = 3$, the intermediate values are calculated as $k_u = 7.2664$, $s_1 = 1.6327$, $y(s_1) = 0.0173$, $u(s_1) = 0.0155$ and $\omega_u = 6.9813$. The proposed method at ($N = 3$) gives model parameters as $k_p = 0.9614$, $D = 0.2044$ and $\tau = 0.9903$. At $N = 1$, the intermediate values are calculated as $k_u = 5.8621$, $s_1 = 1.6327$, $y(s_1) = 0.0173$, $u(s_1) = 0.0155$ and the frequency as 6.9813. Model parameters obtained at $N = 1$ (conventional method) are $k_p = 1.3853$, $D = 0.2073$ and $\tau = 1.1544$. Based on the FOPTD model identified, the PID controller is tuned using the tuning rules suggested by Padmasree, Srinivas, and Chidambaram (2004) (refer to Appendix A for the tuning rule). Table 3 shows model parameters identified for the unstable FOPTD for the D/τ ranging from 0.6 to 0.2. Table 4 gives the identified model and the controller settings. Fig. 2a shows the system output response for the symmetric relay test ($D/\tau = 0.2$) and Fig. 2b shows closed loop servo response of unstable FOPTD system considered. It shows that the identified model by the proposed method gives a response closer to that of the actual system.

Based on the method proposed by Thyagarajan and Yu (2003), time to peak amplitude (the delay) is obtained as 0.2.

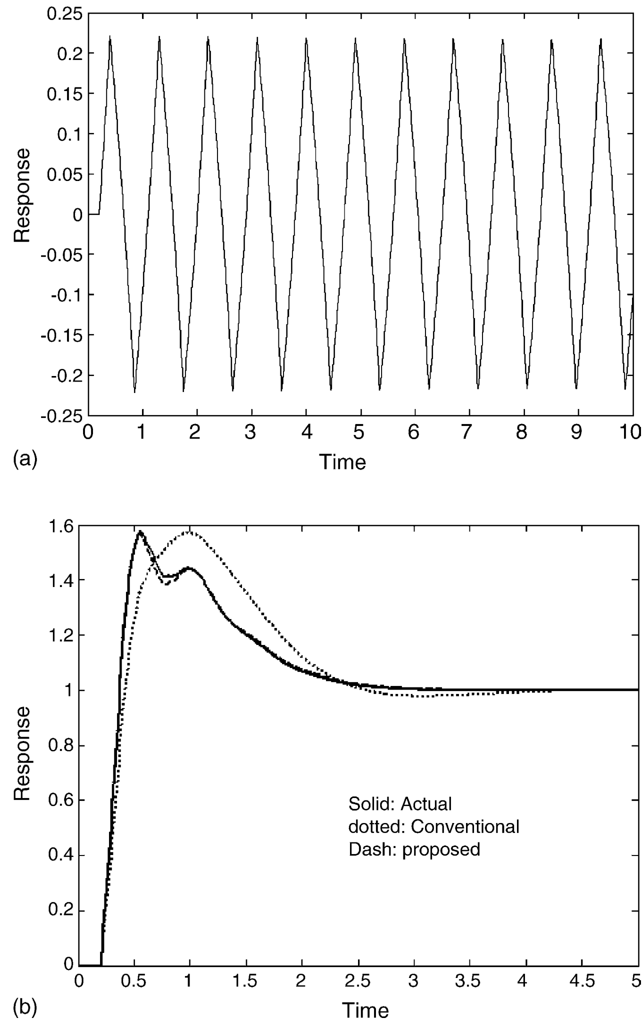

Fig. 2. (a) Relay oscillations for $(D/\tau) = 0.2$ for case study 1; (b) comparison of set point responses for case study 1.

Using Eqs. (8) and (9), the model parameters are identified as $k_p = 0.9855$ and $\tau = 1.0041$. For the example considered, based on the method proposed by Majhi and Atherton (2000), the required intermediate values are noted as: observed

Table 3
Model parameters identified for unstable FOPTD systems

	$D/\tau = 0.6$			$D/\tau = 0.5$			$D/\tau = 0.4$			$D/\tau = 0.2$		
	k_p	τ	D	k_p	τ	D	k_p	τ	D	k_p	τ	D
Proposed method	0.9782	1.35	0.79	1.0089	1.25	0.58	0.9841	1.1332	0.4372	0.9614	0.99	0.2044
Actual parameters	1.0	1.0	0.6	1.0	1.0	0.5	1.0	1.0	0.4	1.0	1.0	0.2

Table 4
Comparison of identified model parameters and controller parameters for case study 1

	k_p	τ	D	k_c	τ_I	τ_D	IAE
Actual	1	1	0.2	4.8968	1.1084	0.1042	0.815
Conventional ($N = 1$)	1.3853	1.1544	0.207	3.8665	1.1481	0.1083	0.9773
Proposed ($N = 3$)	0.9614	0.9903	0.2044	4.9616	1.1330	0.1064	0.815
Thyagarajan and Yu	0.9855	1.004	0.2	4.9854	1.1084	0.1092	0.817
Majhi and Atherton	0.9244	0.9397	0.1983	5.0659	1.0993	0.1032	0.799

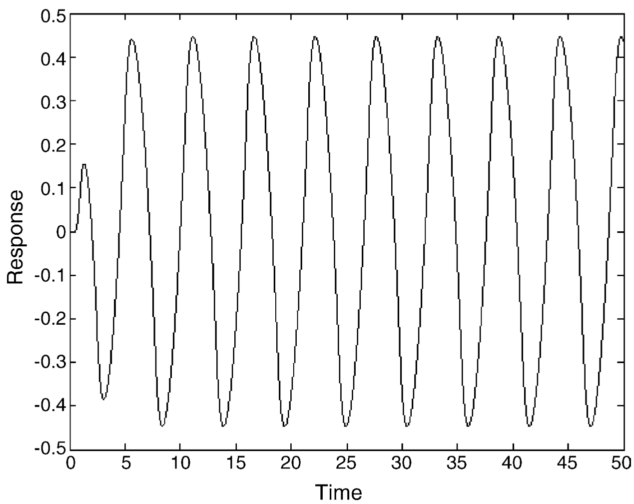

Fig. 3. Relay oscillations for case study 2.

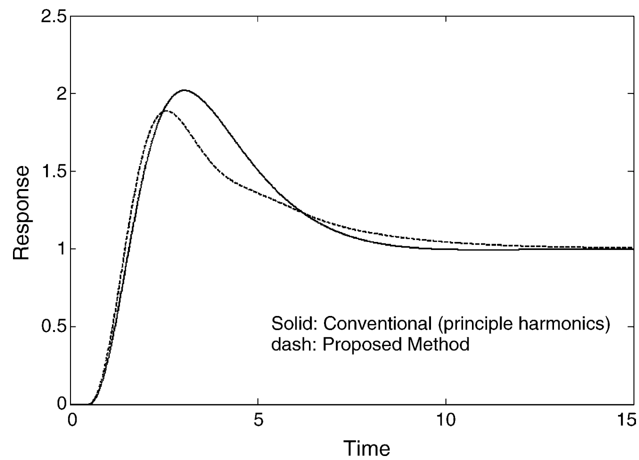

Fig. 4. Comparison of set point responses for case study 2.

amplitude $a = 0.2172$, $a_y = 0.0501$ and $a_u = 0.0542$. The identified parameters are $k_p = 0.9244$, $D = 0.1983$ and $\tau = 0.9397$. All the three methods discussed above give the model parameters close to that of the actual system (refer Table 4).

5.2. Case study 2

Let us consider a process transfer function possessing one stable pole and one unstable pole considered by Majhi and Atherton (1999, 2000)

$$G(s) = \frac{\exp(-0.5s)}{(0.5s+1)(2s-1)} \tag{36}$$

The value for ultimate frequency and the ultimate controller gain are found by the stability analysis as 1.2614 and 3.2085, respectively.

The symmetric relay with relay height ±1 is conducted. Fig. 3 shows the output response. The Fig. 3 shows initial dynamics and the sustained oscillations. The number of terms in Eq. (24) is selected as $N = 7$. Using $N = 1$, the intermediate values are obtained as $k_u = 2.8522$, $s_1 = 0.2065$, $y(s_1) = -0.0613$, $u(s_1) = 0.0558$ and frequency = 1.1383. The proposed method (for $N = 1$) gives model parameter as $k_p = 0.8266$, $D = 0.9951$ and $\tau = 1.8757$. At $N = 7$, the required intermediate values are calculated as $k_u = 3.5292$, $s_1 = 0.2065$, $y(s_1) = -0.0613$, $u(s_1) = 0.0558$ and $\omega_u = 1.1383$. The value of k_u by the proposed method using $N = 7$ is found closer to that of the actual system. The proposed method gives model parameters as $k_p = 0.7534$, $D = 1.0412$ and $\tau = 2.1642$. Based on the FOPTD model, the PID controller is tuned using the tuning rule suggested by Padmasree et al. (2004) (refer to Table 5 for details). Fig. 4 shows the closed loop servo response of the unstable FOPTD system considered. The IAE value for using $N = 1$ is 4.4493 and 4.0 for $N = 7$. The present method gives the best performance.

Based on the method proposed by Thyagarajan and Yu (2003), we noted the values of $p_u = 5.52$ and $a = 0.4464$ and time to peak value $D = 1.1381$. Using the Eqs. (8) and (9), the value of process gain and time constant are calculated as imaginary values. Using Majhi and Atherton (2000) method the required intermediate values are calculated as $a_y = 0.7855$, $a_u = 1.7544$ and $h = \pm 1$. The process gain k_p is calculated as 0.4459. Since the identified value of k_p is less than that of

Table 5
Effect of measurement noise for case study 2

Method	σ	k_p	D	τ	k_c	τ_I	τ_D	IAE
Conventional method	Without noise	0.8266	0.9951	1.8757	2.6297	5.4258	0.5098	4.22
Proposed method	Without noise	0.7534	1.0412	2.1642	3.1299	5.7894	0.5339	4.05
	$\sigma = 0.5\%$	0.7595	1.0549	2.2041	3.1183	5.8655	0.541	4.09
	$\sigma = 1\%$	0.5332	0.8807	1.3524	3.4372	5.8783	0.4503	3.86
Thyagarajan and Yu method	Without noise	[a]	1.1381	[a]	[a]	[a]	[a]	[a]
	$\sigma = 0.5\%$	[a]	1.225	[a]	[a]	[a]	[a]	[a]
	$\sigma = 1\%$	[a]	1.192	[a]	[a]	[a]	[a]	[a]
Majhi and Atherton method	Without noise	0.4459	[b]	[b]	[b]	[b]	[b]	[b]
	$\sigma = 0.5\%$	0.4346	[b]	[b]	[b]	[b]	[b]	[b]
	$\sigma = 1\%$	0.452	[b]	[b]	[b]	[b]	[b]	[b]

$G(s) = \exp(-0.5s)/((0.5s+1)(2s-1))$.
[a] For Thyagarajan and Yu method, the obtained D value is larger. Hence, the k_p and τ are obtained as imaginary values (refer to Eqs. (8) and (9)).
[b] In the Majhi and Atherton method, since the identified value of k_p is less than that of the measured amplitude, the values for D and τ are obtained as imaginary values (refer to Eqs. (6) and (7)).

Table 6
Effect of load on identification for case studies 1 and 2

Case study	Load	Proposed method			Thyagarajan and Yu method			Majhi and Atherton method		
		k_p	τ	D	k_p	τ	D	k_p	τ	D
1	0	0.9614	0.9903	0.2044	0.9855	1.004	0.2	0.9244	0.9397	0.1983
	0.02	0.8846	0.8869	0.2021	1.0122	1.004	0.2	0.9164	0.9056	0.1973
	0.04	0.8367	0.8169	0.2001	0.989	0.9536	0.1987	0.6687	0.6302	0.1857
2	0	0.7534	2.1642	1.0412	[a]	[a]	1.1381	0.4459	[b]	[b]
	0.02	0.5697	1.372	0.8819	[a]	[a]	1.1704	0.4468	[b]	[b]
	0.04	0.5611	1.2833	0.8558	[a]	[a]	1.2095	0.4517	[b]	[b]

[a] Case study 2: using Thyagarajan and Yu method for estimation, the obtained D value is larger. Hence, the k_p and τ are obtained as imaginary values (refer to Eqs. (8) and (9)).

[b] Case study 2: using Majhi and Atherton method for estimation, the obtained k_p value is less than that of the observed amplitude of the system output. Hence, the values of D and τ are obtained as imaginary values (refer to Eqs. (6) and (7)).

the measured amplitude ($a = 0.4464$), D and τ are obtained as imaginary values [refer to Eqs. (6) and (7)).

5.3. Effect of measurement noise on identification

The effect of measurement noise on the accuracy of the estimation of the parameters is considered for case study 2. The measurement noise with a zero mean Gaussian distribution and a standard deviation of 0.5% and separately of 1% is added to the output of the system. In the identification test, the corrupted signal is used in the feed back control and for the system output. Once the initial dynamics are died out, the amplitude and the period of oscillation are calculated by taking the average values at the various peak locations. Table 5 gives, the identified parameters using proposed method, Thyagarajan and Yu method and Majhi and Atherton method. It is to be noted that, using Majhi and Atherton method gives the identified value of k_p less than that of the observed amplitude (a). The denominator of right hand side of the Eqs. (6) and (7) becomes negative. Hence, D and τ values are obtained as imaginary values (refer to Eqs. (6) and (7)). Using Thyagarajan and Yu method, the calculated time delay is larger. Hence, the k_p and τ values are obtained as imaginary values [refer to Eqs. (8) and (9)].

Using the proposed method for parameter identification, the effect of 0.5% standard deviation noise is not significant (refer to Table 5). As the noise level increases, the parameter estimation deteriorates. It is desirable to use a filter to remove the noise. For higher order systems, Thyagarajan and Yu method and Majhi and Atherton method give imaginary values for k_p and τ and D and τ, respectively.

5.4. Effect of load on model parameter identification

The effect of the load on parameter identification is also considered for case studies 1 and 2. The load of 0.02 and separately of 0.04 is introduced. The transfer function for the load is assumed that of the process transfer function. The affected output signal is used for the feedback relay and as the system output for model identification purposes. The effect of load on model identification for the case studies 1 and 2, is reported in Table 6.

Using Majhi and Atherton method for case study 2, the identified value of k_p is found to less than that of the measured amplitude. The denominator of right hand side of the Eqs. (6) and (7) becomes negative. Hence, D and τ are an imaginary values (refer to Eqs. (6) and (7)). Similarly using Thyagarajan and Yu method for case study 2, the calculated time delay is larger. Hence, the k_p and τ values are obtained as imaginary values [refer to Eqs. (8) and (9)]. For the case study 1, for the proposed method and Thyagarajan and Yu method, there is no significant effect of the load on the parameter estimates.

6. Application to an unstable nonlinear bioreactor

The proposed method of identification of an unstable FOPTD system is applied to a nonlinear continuous bioreactor that exhibits output multiplicity. The dimensionless model equations are given by (Agarwal & Lim, 1984)

$$\frac{dX}{dt} = (\mu - D)X \tag{37}$$

$$\frac{dS}{dt} = (S_t - S)D - (\mu X/\gamma) \tag{38}$$

where

$$\mu = \frac{\mu_m S}{K_m + S + K_f S^2} \tag{39}$$

Here X, S, S_f are the dimensionless concentration of cell, substrate and feed substrate, respectively. D is the dilution rate and μ the specific generation rate. The model parameters are given by (Agrawal and Lim, 1986) as

$$\gamma = 0.4\%\,\text{g/g}; \qquad S_f = 0.4\%\,\text{g/g}; \qquad \mu_m = 0.53\,\text{h}^{-1};$$
$$k_m = 0.12\%\,\text{g/g}; \qquad k_f = 0.4545\%\,\text{g/g} \tag{40}$$

The steady state solution of Eqs. (37)–(39) gives the following three multiple steady state:

$$[X, S]_1 = [0, 4] \quad \text{(wash out condition)} \tag{41}$$

$$[X, S]_2 = [0.9951, 1.5122] \quad \text{(unstable study state)} \tag{42}$$

$$[X, S]_3 = [1.5301, 0.1746] \quad \text{(stable study state)} \tag{43}$$

Initially, the system is assumed to be at the unstable steady state condition. At time $t = 0$, $X = 0.9951$ and $S = 1.5122$. At this condition the dilution rate (D) is $0.3\ \text{h}^{-1}$. The dilution rate is considered as the manipulated variable in order to control the cell mass concentration (X) at the unstable steady state at $X = 0.9951$. A delay of 1 h is considered in the measurement of X. For the given condition of the unstable operating point, the local linearized model is obtained as an unstable FOPTD with the parameters: $k_p = -5.898$, $\tau = 5.888$ and $D = 1.0$. A symmetrical relay with relay height $(h) = 0.03$ is conducted. Relay output response (deviation value from the steady state point of $X = 0.9951$) is shown in Fig. 5. The identified unstable FOPTD model parameter values by the present method are $k_p = -5.5903$, $\tau = 5.6125$ and $D = 1.0152$. Using the Thyagarajan and Yu method, the identified unstable model parameters are $k_p = -5.9501$, $\tau = 5.895$ and $D = 1.0$. Even though the delay recorded from peak amplitude location from the relay response, matches with that of the linearized model, the shape of the system output differs significantly from that of the identified unstable FOPTD transfer function model (refer to Fig. 5) for the system response for the relay input used by Thyagarajan and Yu (2003). The response of the actual system gives a straight-line portion whereas the response used by Thyagarajan and Yu (2003) has an initial curved response. The difference in the behavior is due to the non-linearity of the bioreactor. A closed loop response of the nonlinear model with designed PID controller is evaluated for a step change in X from 0.9951 to 1.294. The identified model parameters and controller parameters are given in Table 7. A good response is obtained as shown in Fig. 6. For the present problem, using Majhi and Atherton method, the process gain is calculated as $k_p = -2.5$. After calculating k_p, the values of D and τ are obtained using Eqs. (6) and (7) as $D = 0.8505$ and $\tau = 2.3265$.

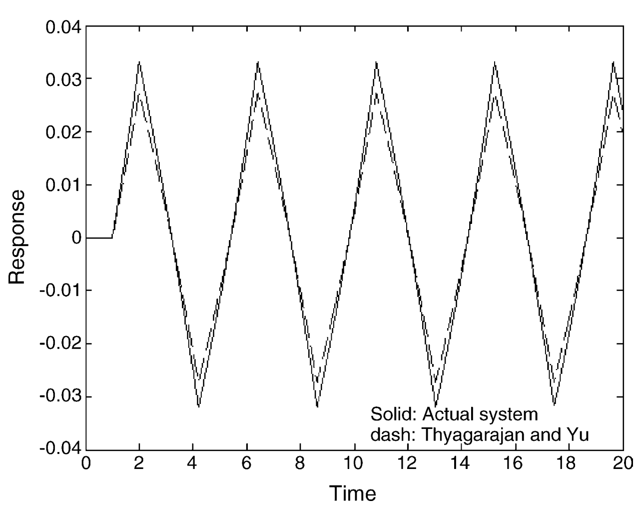

Fig. 5. Comparison of actual response (deviation variable) with that obtained by Thyagarajan and Yu method (for nonlinear bioreactor problem).

Table 7
PID settings for the identified unstable FOPTD model parameters of bioreactor

Method	k_p	τ	D	k_c	τ_1	τ_D
Linearized	−5.898	5.888	1.0	−0.9513	5.5363	0.5235
Proposed	−5.5903	5.6125	1.0152	−0.9524	5.6228	0.5304
Thyagarajan and Yu	−5.9501	5.895	1.0	−0.9439	5.5363	0.5235
Majhi and Atherton	−2.5	2.3265	0.8505	−1.1855	4.7255	0.4377

PID settings: Padmasree et al. method for unstable FOPTD system.

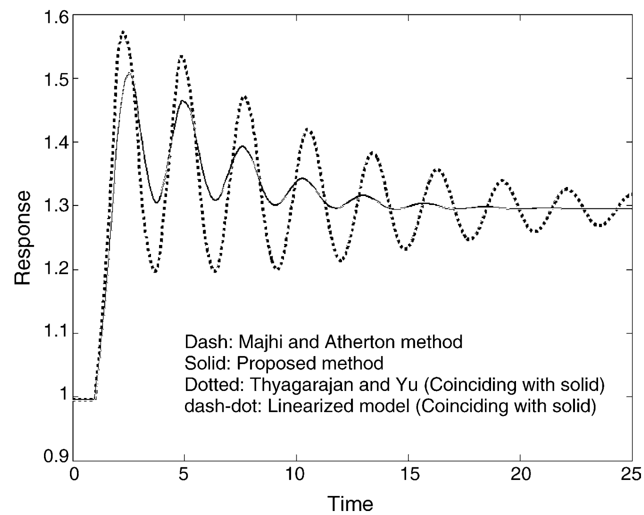

Fig. 6. Comparison of the responses of the nonlinear bioreactor using the controller settings form different identification methods.

The PID settings calculated on each of the identified models are given in Table 7. The locally linearized model (Jacob & Chidambaram, 1996) is also used for designing PID controllers (refer to Table 7). The controller settings are simulated on the nonlinear model equations of the bioreactor. The closed responses are shown in Fig. 6. The response of the controller based on the proposed identified model is very close to that of the actual system. The response based on the identified model of Majhi and Atherton (2000) gives an oscillatory response.

7. Conclusions

A method is proposed to consider higher order harmonics for estimating the parameters of an unstable FOPTD transfer function model using a single symmetric relay test. The advantages of this method are: single relay feedback test and all the three parameters can be identified. Simulation results show that, for two transfer function case studies and a nonlinear bioreactor system, the proposed method gives improved parameter estimates and hence an improved closed loop response. For FOPTD systems, the performance of the proposed method is compared with that of the Majhi and Atherton (2000) and the Thyagarajan and Yu (2003). The proposed method gives the best performance. The conventional method of analyzing the relay output for the calculation of ultimate controller gain gives a maximum error of −23% in k_u. For the unstable FOPTD systems, the value of k_u calculated is always less than that of the actual system. For higher order systems, Thyagarajan and Yu method and Majhi and Atherton method gives imaginary values for the FOPTD model parameters. The FOPTD model parameters calculated by the by the proposed method gives a good closed loop performance.

Appendix A

Equation used for PID settings (Padmasree et al., 2004): let $\varepsilon = D/\tau$,

$$k_c k_p = 1.2824\varepsilon^{-0.8325} \quad \text{for } 0.01 \le \varepsilon \le 1.2,$$

$$\tau_1/\tau = 5.573\varepsilon - 0.0063 \quad \text{for } 0.01 \le \varepsilon \le 0.5,$$

$$\tau_1/\tau = 0.483\exp(-3.3739\varepsilon) \quad \text{for } 0.5 \le \varepsilon \le 1.2,$$

$$\tau_D/\tau = 0.507\varepsilon + 0.0028 \quad \text{for } 0.01 \le \varepsilon \le 1.2.$$

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