Models of Computation: Assessed Coursework II

This coursework is assessed. It must be submitted in hard copy by **2pm** on **Monday 14th December 2015**.

The work must be submitted individually. Although discussion of questions permitted to the 'usual extents'.

1. Consider the register machine program P, given by the following code

$$
\begin{aligned}
&L_0 : R_1^- \to L_2, L_1 \\
&L_1 : HALT \\
&L_2 : R_1^- \to L_3, L_4 \\
&L_3 : R_1^- \to L_5, L_4 \\
&L_4 : HALT \\
&L_5 : R_0^+ \to L_0
\end{aligned}
$$

   (a) Give the computation of P when run from the initial configuration $(0, 0, 7)$.

   (b) Describe the function of one argument $f(x)$ that is computed by P.

   (c) The code of P, written $\ulcorner P \urcorner$, has the form $\ulcorner [\ulcorner B_0 \urcorner, \ldots, \ulcorner B_5 \urcorner] \urcorner$ where B_i is the body of L_i for each i. Give the value of $\ulcorner B_i \urcorner$ for each i.

2. Consider the natural number $2^{46} \times 20483$. What register machine program is represented by this number? Give both the program and its graphical representation.

3. An n-instruction *busy beaver* register machine is a register machine with n instructions that is designed to halt with register R_0 as large as possible, starting with all registers set to zero. You might like to try constructing busy beavers for various values of n.

   The function $S(n)$ is defined to be the maximum value of R_0 achievable by an n-instruction busy beaver.

   (a) Show that, for every $n \in \mathbb{N}$, there is a machine with at most $n + 3$ instructions that computes $2n$, starting with all registers at 0. (You need not consider instructions containing bodies of the form $HALT$ as part of the machine, since the machine would also halt if it jumped to an undefined instruction. This allows for the number of instructions to be smaller.) It is enough to show how such a machine is constructed for arbitrary n; you do not need to prove that the machine computes the required answer.

   (b) Suppose that $\phi : \mathbb{N} \to \mathbb{N}$ is a computable function, which is computed by register machine M_ϕ. Let n_ϕ be the number of instructions in M_ϕ. Show that there is a register machine with $2n_\phi + 8$ instructions that computes $\phi(2n_\phi + 8) + 1$ on R_0 with all registers initialised to 0.

   (c) Explain why this proves that S is not a computable function.