**Directions**: Examine the following relations.

1. Using Table 1, determine if  $A \to B$ ,  $B \to C$ , and  $AB \to D$ . In words, is B dependent on A? is C dependent on B? and is D dependent on AB?

Solution: Yes, no, yes.

Table 1: Relation in Problem 1.

$$R = \begin{bmatrix} A & B & C & D \\ a_1 & b_1 & c_1 & d_1 \\ a_1 & b_1 & c_3 & d_1 \\ a_2 & b_1 & c_3 & d_4 \\ a_2 & b_1 & c_3 & d_4 \\ a_3 & b_6 & c_3 & d_1 \end{bmatrix}$$

2. Determine if there are any transitive dependencies in Table 2. If so, specify.

Solution:  $A \rightarrow B \rightarrow D$ 

Table 2: Relation in Problem 2.

$$R = \begin{bmatrix} A & B & C & D \\ \hline a_1 & b_1 & c_1 & d_1 \\ \hline a_1 & b_1 & c_3 & d_1 \\ \hline a_2 & b_2 & c_3 & d_4 \\ \hline a_2 & b_2 & c_3 & d_2 \\ \hline a_3 & b_6 & c_3 & d_2 \\ \hline \end{bmatrix}$$

3. Determine all (nontrivial) dependencies in Table 3. Identify symmetric and transitive dependencies. Note: A functional dependency  $A \to B$  is **trivial** if  $B \subseteq A$ . *Hint*: there are five.

Table 3: Relation in Problem 3.

$$\mathbf{R} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \\ a_1 & b_2 & c_7 & d_9 & e_4 \\ a_5 & b_2 & c_8 & d_9 & e_5 \\ a_1 & b_4 & c_7 & d_9 & e_6 \\ a_5 & b_4 & c_8 & d_9 & e_4 \\ a_1 & b_6 & c_7 & d_2 & e_6 \\ a_5 & b_6 & c_8 & d_2 & e_6 \end{bmatrix}$$

**Solution**:  $A \to C$ ,  $C \to A$ ,  $B \to D$ ,  $AB \to E$ , and  $BC \to E$ . From here, we can use Armstrong's rules to obtain all other dependencies such as  $AB \to CD$ ,  $AB \to CDE$ , etc.

4. Use functional dependencies in the relation R (see Table 3) to determine a primary key for the relation. Note: functional dependencies are used to identify candidate keys (any of which can be used for the primary key). Candidate keys are determined by finding set(s) of attributes for which all other attributes are functionally dependent.

**Solution**: AB is a primary key since  $AB \to C, D, E$ . In fact,  $A \to C, B \to D$ , and  $AB \to E$ . In addition, trivially,  $A \to A$  and  $B \to B$ . Therefore,  $AB \to ABCDE$ .

Note: since  $A \leftrightarrow C$ , then BC is also a key.

5. List three anomalies and give an example of one of them.

**Solution**: Insertion, modification (or update), and deletion. As an example, consider the Flights database. If we need to update the destination LAX, we must update it in many places. Such an activity is prone to inconsistency errors and would vitiate the database.