

A CODELESS INTRODUCTION TO GPU PARALLELISM

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OUTLINE

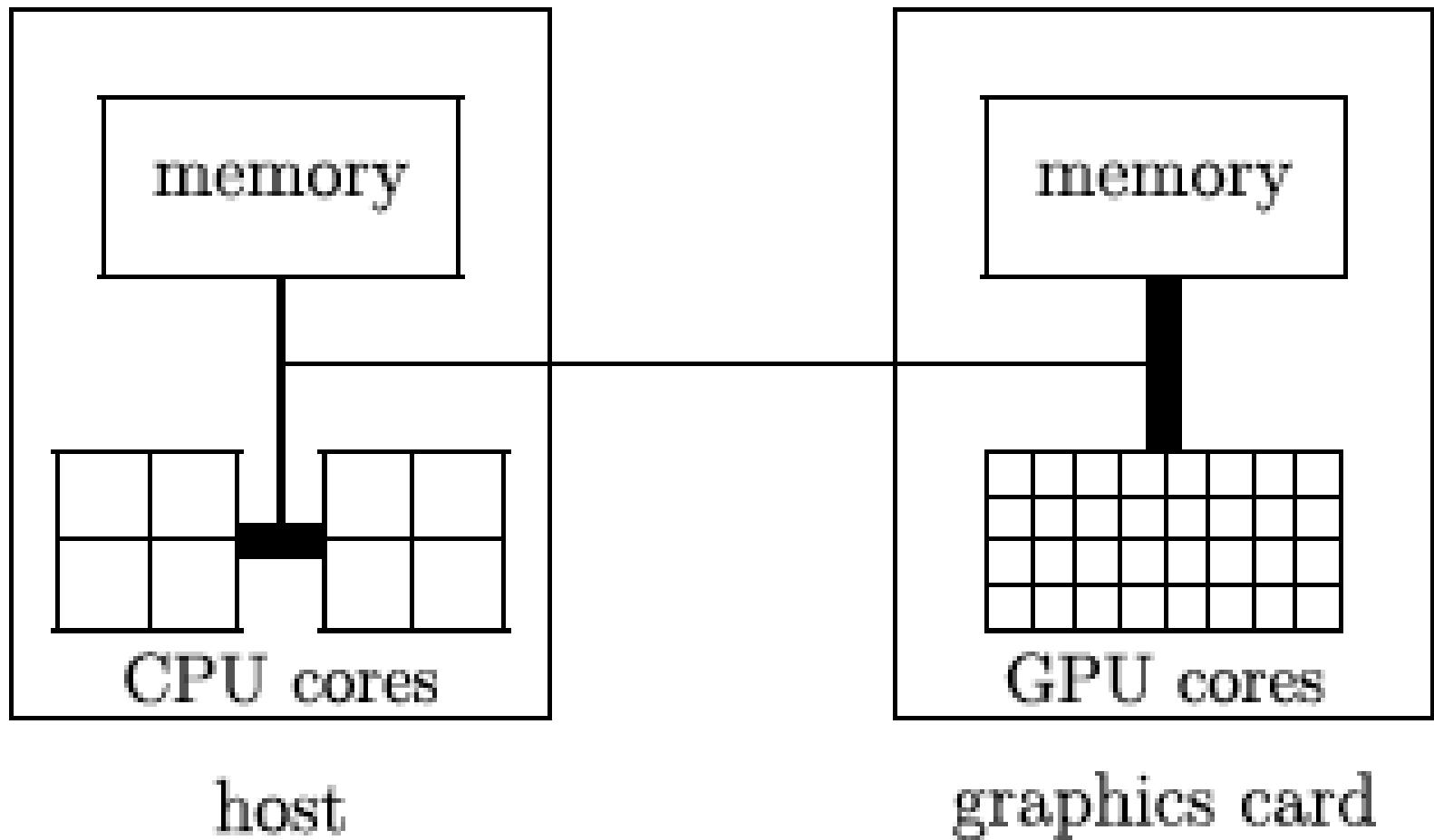
- A. A review of GPU parallelism
- B. How to GPU-parallelize the following:
 - 1. vector addition
 - 2. the pairwise (cascading) sum
 - 3. matrix multiplication
 - 4. the QR factorization

HOW THE CPU AND GPU WORK TOGETHER

A GPU can't run a whole computer on its own because it doesn't have access to all the computer's hardware.

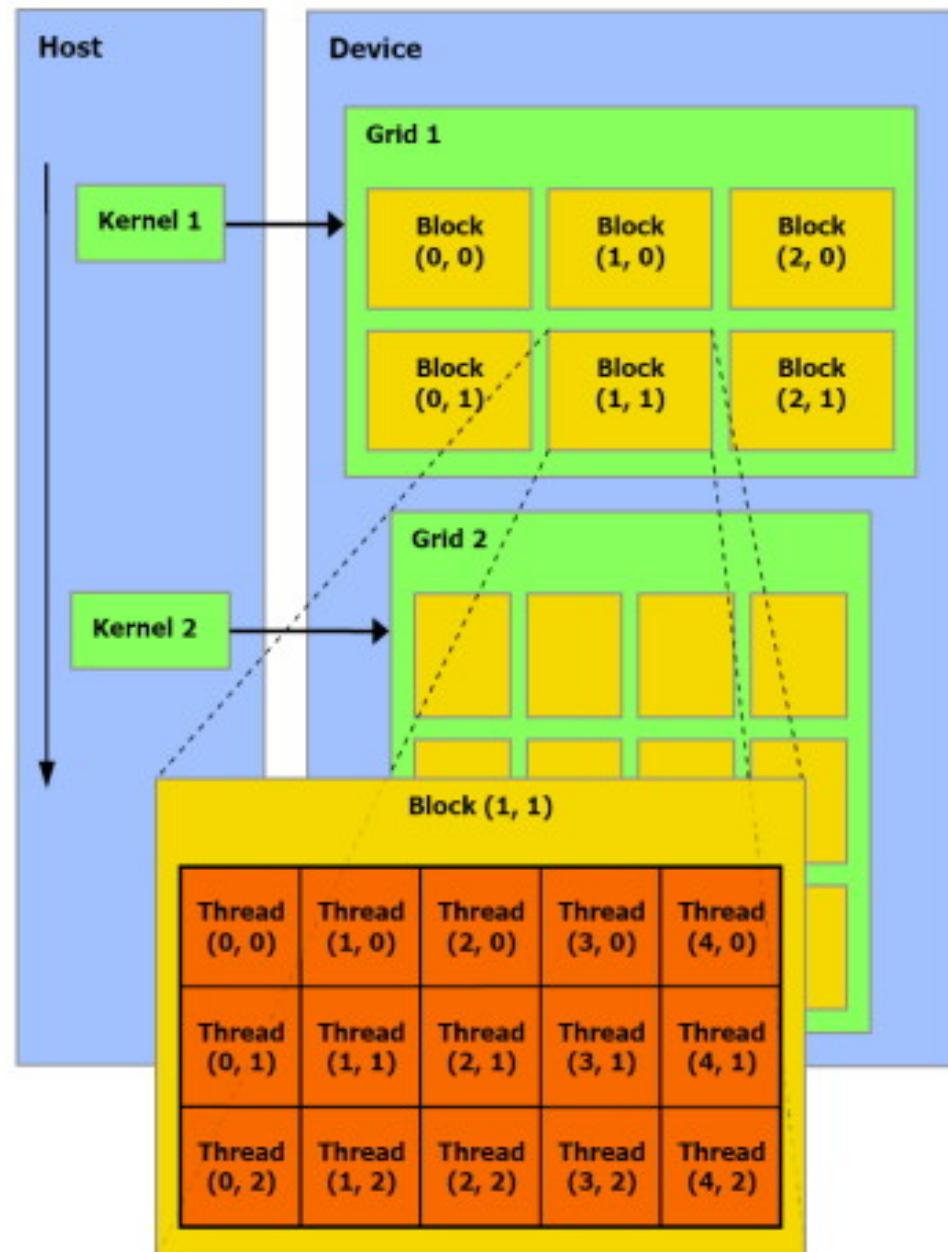
In a GPU-capable computer, the CPU is the main processor, and the GPU is an optional hardware add-on.

The CPU uses the GPU like a human would use a hand-held calculator: the CPU does all the main thinking and the GPU does the most cumbersome bits and pieces of number-crunching.



GPU PARALLELISM

1. The CPU sends a CPU-to-GPU command called a **kernel** to a single GPU core.
2. The GPU core multitasks to execute the command:
 - a. The GPU makes $B \cdot T$ **copies** of the kernel's code, and then runs all those copies simultaneously. Those parallel copies are called **threads**.
 - b. The $B \cdot T$ threads are partitioned into B groups, called **blocks**, of T threads each.
 - c. The sum total of all the threads from a kernel call is a **grid**.



WHEN TO PARALLELIZE

Calculations you want to parallelize:

- Highly repetitive floating point arithmetic procedures that can all be done simultaneously.

Calculations you don't want to parallelize:

- Inherently sequential calculations, such as recursions.
- Lengthy control flow: if-then statements, etc.
- CPU system routines, such as printing to the console.

EXAMPLES OF EASILY PARALLELIZABLE ALGORITHMS

Linear algebraic algorithms are particularly amenable to GPU computing because they involve a high volume of simple arithmetic.

I will showcase:

1. vector addition
2. the pairwise (cascading) sum
3. matrix multiplication
4. the QR factorization

VECTOR ADDITION

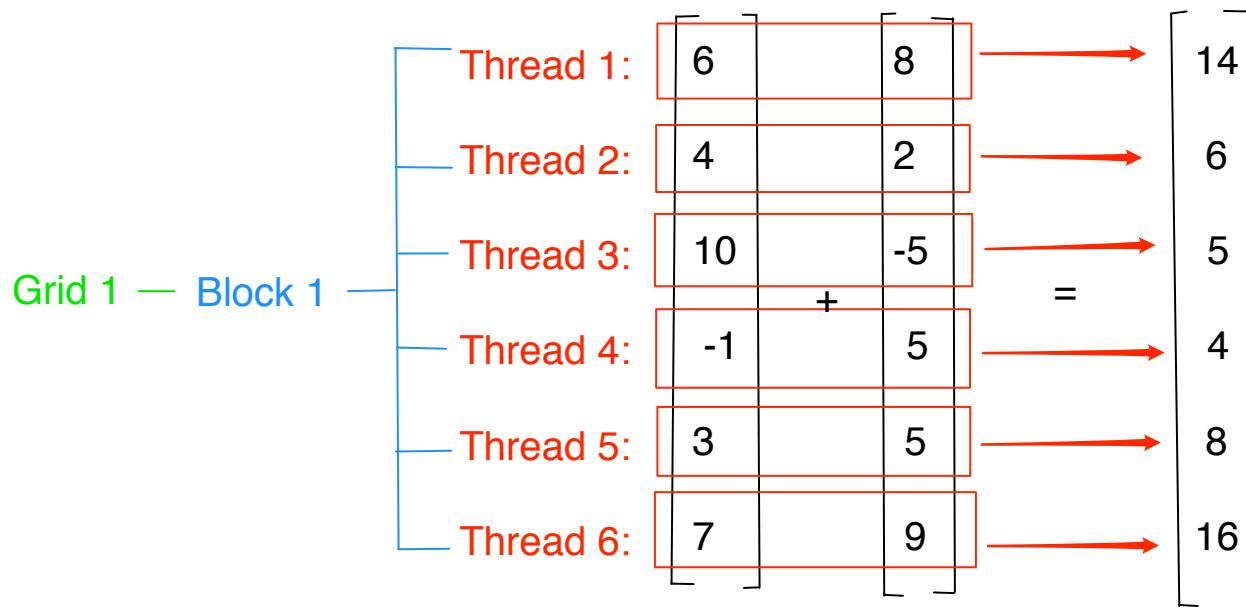
Say I have two vectors:

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

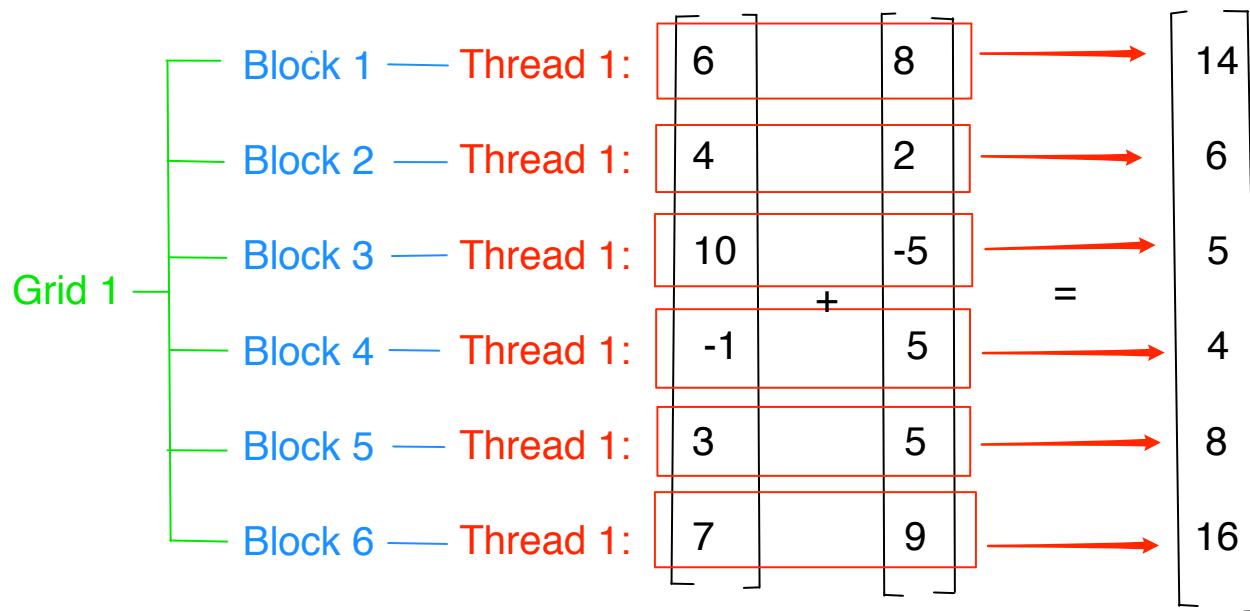
I compute their sum, $c = a + b$, by:

$$c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

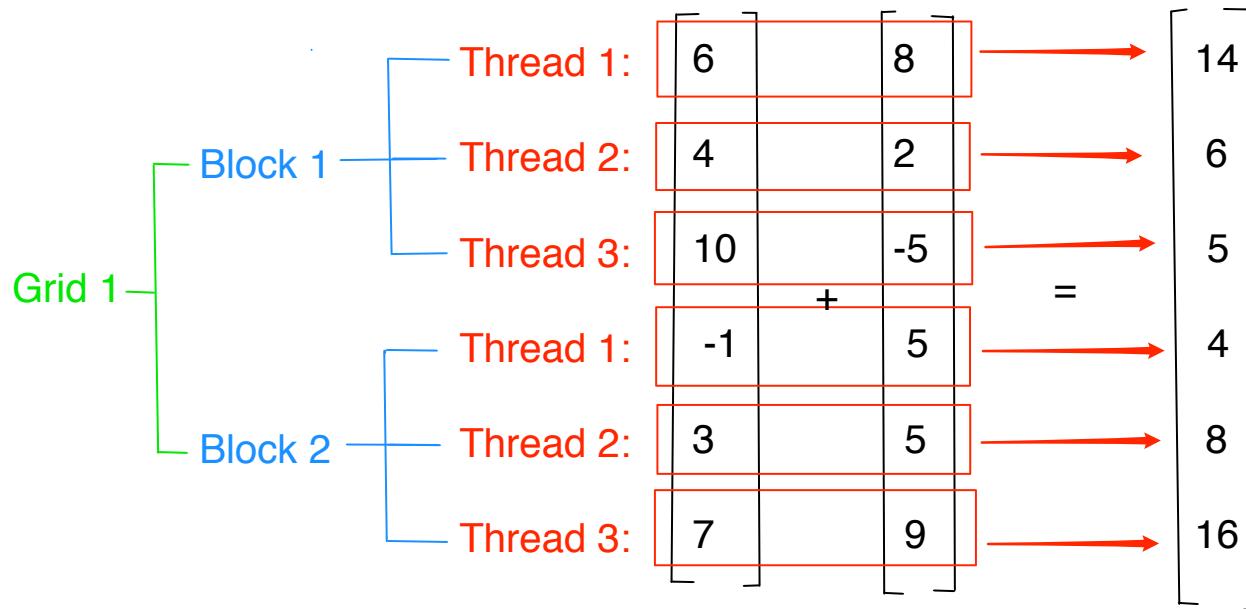
PARALLELIZING VECTOR ADDITION: METHOD 1 OF 3



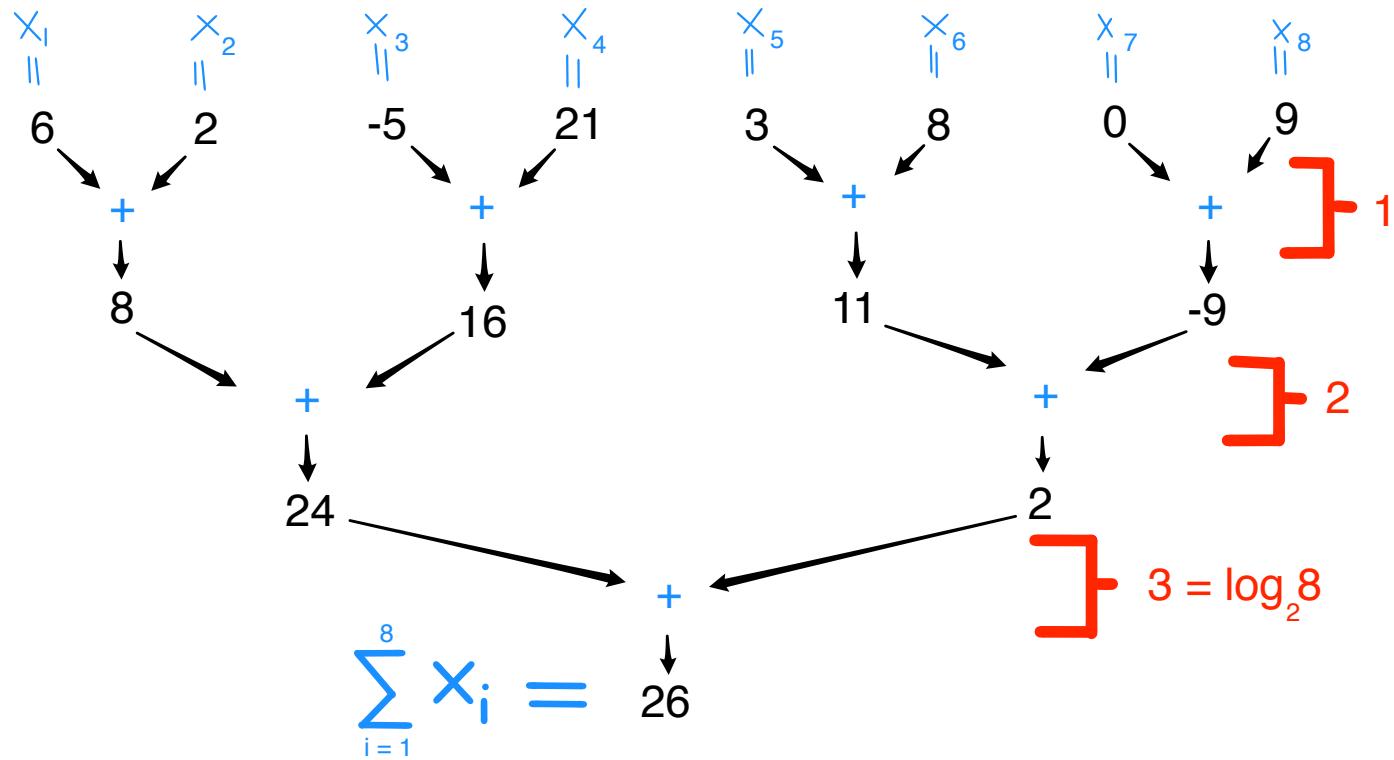
PARALLELIZING VECTOR ADDITION: METHOD 2 OF 3



PARALLELIZING VECTOR ADDITION: METHOD 3 OF 3



2. THE PAIRWISE (CASCADING) SUM



A RIGOROUS DESCRIPTION

Suppose you have a vector $X_0 = (x_{(0,1)}, x_{(0,2)}, \dots, x_{(0,n)})$, where $n = 2^m$ for some $m > 0$.

Compute $\sum_{i=1}^n x_{(0,i)}$ in the following way:

1. Create a new vector:

$$X_1 = (\underbrace{x_{(0,1)} + x_{(0,2)}}_{x_{(1,1)}}, \underbrace{x_{(0,3)} + x_{(0,4)}}_{x_{(1,2)}}, \dots, \underbrace{x_{(0,n-1)} + x_{(0,n)}}_{x_{(1,n/2)}})$$

2. Create another new vector:

$$X_2 = (\underbrace{x_{(1,1)} + x_{(1,2)}}_{x_{(2,1)}}, \underbrace{x_{(1,3)} + x_{(1,4)}}_{x_{(2,2)}}, \dots, \underbrace{x_{(1,n/2-1)} + x_{(1,n/2)}}_{x_{(2,n/4)}})$$

3. Continue this process until you get a singleton vector:

$$X_m = (\underbrace{x_{(m-1,1)}, x_{(m-1,2)}}_{x_{(m,1)}})$$

Notice: $\sum_{i=1}^n x_{(0,i)} = x_{(m,1)}$

PARALLELIZING THE PAIRWISE SUM

Spawn one grid with a single block and $n = 2^m$ threads.
For each iteration $i = 1, 2, \dots, m$, do the following:

1. Assign thread j to compute:

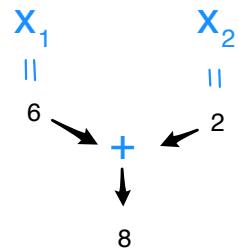
$$x_{(i,j)} = x_{(i-1, 2j-1)} + x_{(i-1, 2j)}$$

for $j = 1, 2, \dots, \frac{n}{2^i}$.

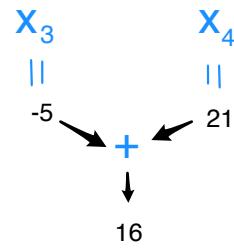
2. Wait until all the above $\frac{n}{2^i}$ threads have completed step 1.
3. Set $i = i + 1$ and repeat.

EXAMPLE

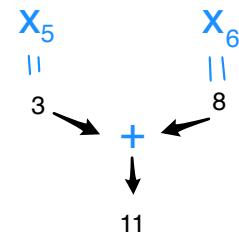
Thread 1:



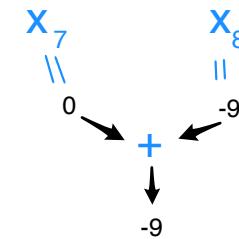
Thread 2:



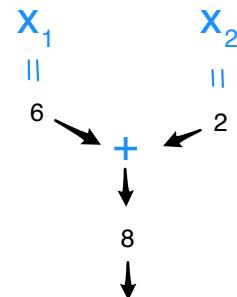
Thread 3



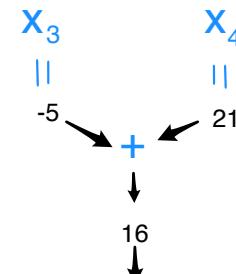
Thread 4:



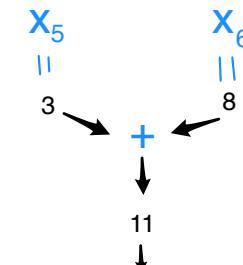
Thread 1:



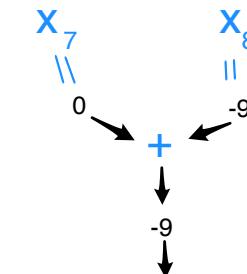
Thread 2:



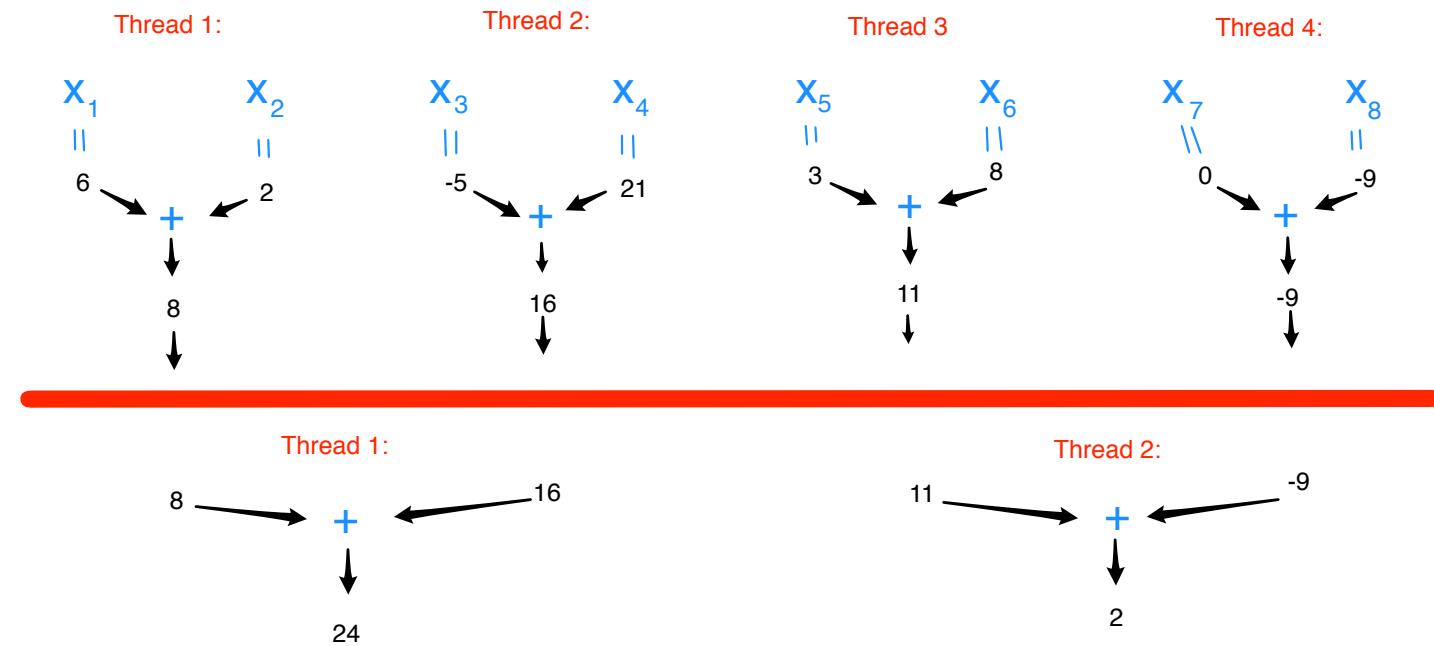
Thread 3

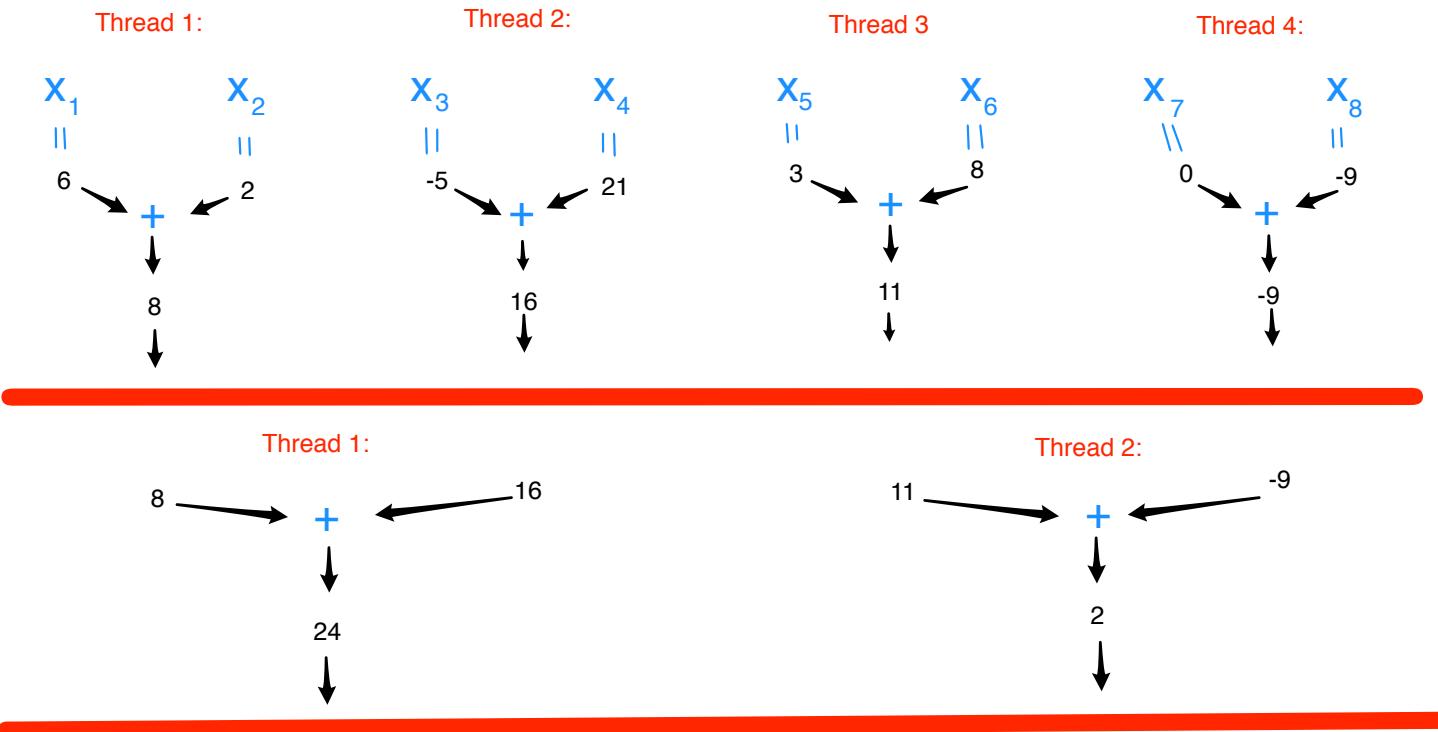


Thread 4:



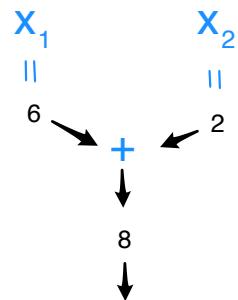
STOP HERE AND WAIT FOR ALL THREADS TO REACH THIS POINT!



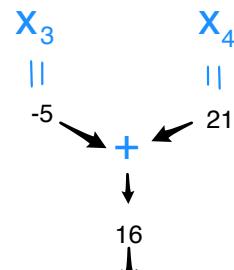


STOP AND WAIT FOR THREADS 1 AND 2 TO REACH THIS POINT!

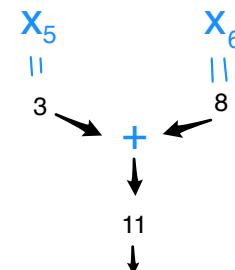
Thread 1:



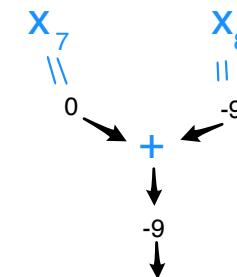
Thread 2:



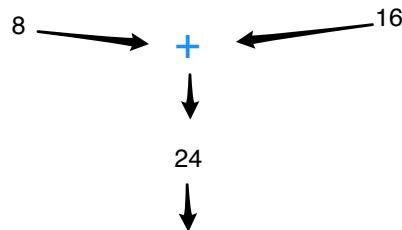
Thread 3



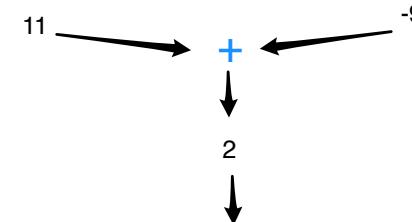
Thread 4:



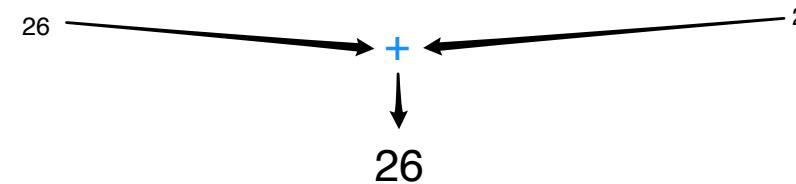
Thread 1:



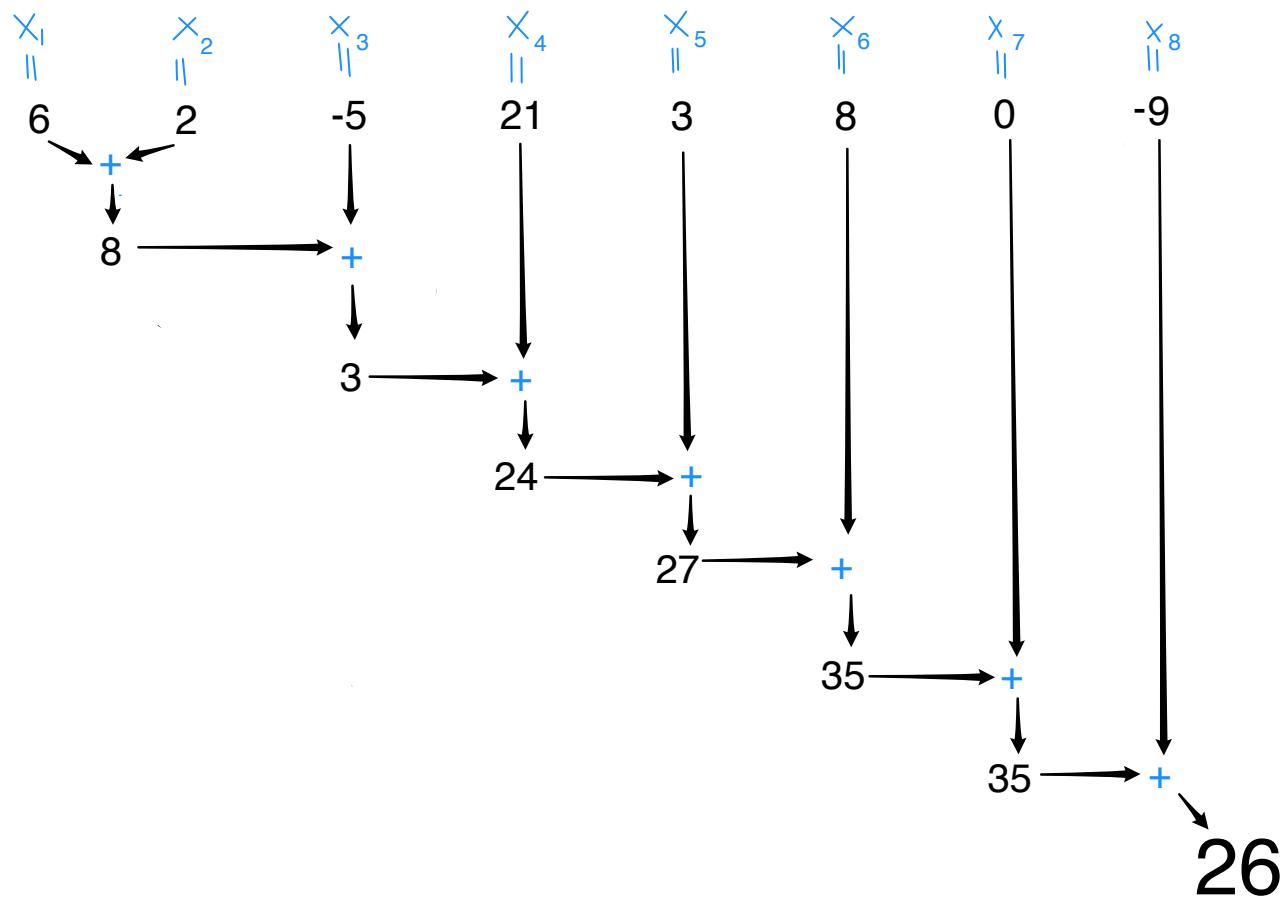
Thread 2:



Thread 1:



COMPARE TO THE SEQUENTIAL VERSION



The pairwise sum requires only $\log_2(n)$ sequential steps, whereas the sequential sum requires $n - 1$ steps.

AN ASIDE: SYNCHRONIZING THREADS

Synchronization: Waiting for all parallel tasks to reach a checkpoint before allowing any of those tasks to proceed beyond that checkpoint.

NOTE:

- Threads from the same block can be synchronized easily.
- In general, do not try to synchronize threads from different blocks. It's possible, but extremely inefficient.

3. MATRIX MULTIPLICATION

Consider an $m \times n$ matrix, $A = (a_{ij})$, and an $n \times p$ matrix, $B = (b_{ij})$. Compute $A \cdot B$:

1. Break apart A into its rows: $A = \begin{bmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{m\cdot} \end{bmatrix}$, where each $a_{i\cdot} = [a_{i1} \ a_{i2} \ \cdots \ a_{in}]$
2. Break apart B into its columns: $B = [b_{\cdot 1} \ b_{\cdot 2} \ \cdots \ b_{\cdot p}]$, where each $b_{\cdot j} = \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix}$
3. Compute $C = A \cdot B$ elementwise, using the usual matrix multiplication rules to find each $a_i \cdot b_j$:

$$C = A \cdot B = \begin{bmatrix} (a_{1\cdot} \cdot b_{\cdot 1}) & (a_{1\cdot} \cdot b_{\cdot 2}) & \cdots & (a_{1\cdot} \cdot b_{\cdot p}) \\ (a_{2\cdot} \cdot b_{\cdot 1}) & (a_{2\cdot} \cdot b_{\cdot 2}) & & (a_{2\cdot} \cdot b_{\cdot p}) \\ \vdots & & \ddots & \vdots \\ (a_{m\cdot} \cdot b_{\cdot 1}) & (a_{m\cdot} \cdot b_{\cdot 2}) & \cdots & (a_{m\cdot} \cdot b_{\cdot p}) \end{bmatrix}$$

i.e.:

$$C_{(i,j)} = a_{i\cdot} \cdot b_{\cdot j}$$

PARALLELIZING MATRIX MULTIPLICATION

Spawn one grid with $m \cdot p$ blocks. Assign block (i, j) to compute $C_{(i,j)} = a_{i\cdot} \cdot b_{\cdot j}$ using the following steps:

1. Spawn n threads.
2. Tell the k 'th thread to compute $c_{ijk} = a_{ik}b_{kj}$.
3. Synchronize the n threads to make sure we have finished calculating all of $c_{ij1}, c_{ij2}, \dots, c_{ijn}$ before proceeding.
4. Compute $C_{(i,j)} = \sum_{k=1}^n c_{ijk}$ as a pairwise sum.

EXAMPLE

Say I want to compute $A \cdot B$, where:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 5 \\ 7 & -9 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 8 & 7 \\ 3 & 5 & 2 \end{bmatrix}$$

which I'm setting up as:

$$C = A \cdot B = \begin{bmatrix} \left([1 \ 2] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([1 \ 2] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([1 \ 2] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right) \\ \left([-1 \ 5] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([-1 \ 5] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([-1 \ 5] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right) \\ \left([7 \ -9] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([7 \ -9] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([7 \ -9] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right) \end{bmatrix}$$

$$\begin{bmatrix}
\text{Block (1, 1)} & \text{Block (1, 2)} & \text{Block (1, 3)} \\
\left([1 \ 2] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([1 \ 2] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([1 \ 2] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right) \\
\text{Block (2, 1)} & \text{Block (2, 2)} & \text{Block (2, 3)} \\
\left([-1 \ 5] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([-1 \ 5] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([-1 \ 5] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right) \\
\text{Block (3, 1)} & \text{Block (3, 2)} & \text{Block (3, 3)} \\
\left([7 \ -9] \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix} \right) & \left([7 \ -9] \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} \right) & \left([7 \ -9] \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right)
\end{bmatrix}$$

We don't need to synchronize the blocks because we can compute them independently.

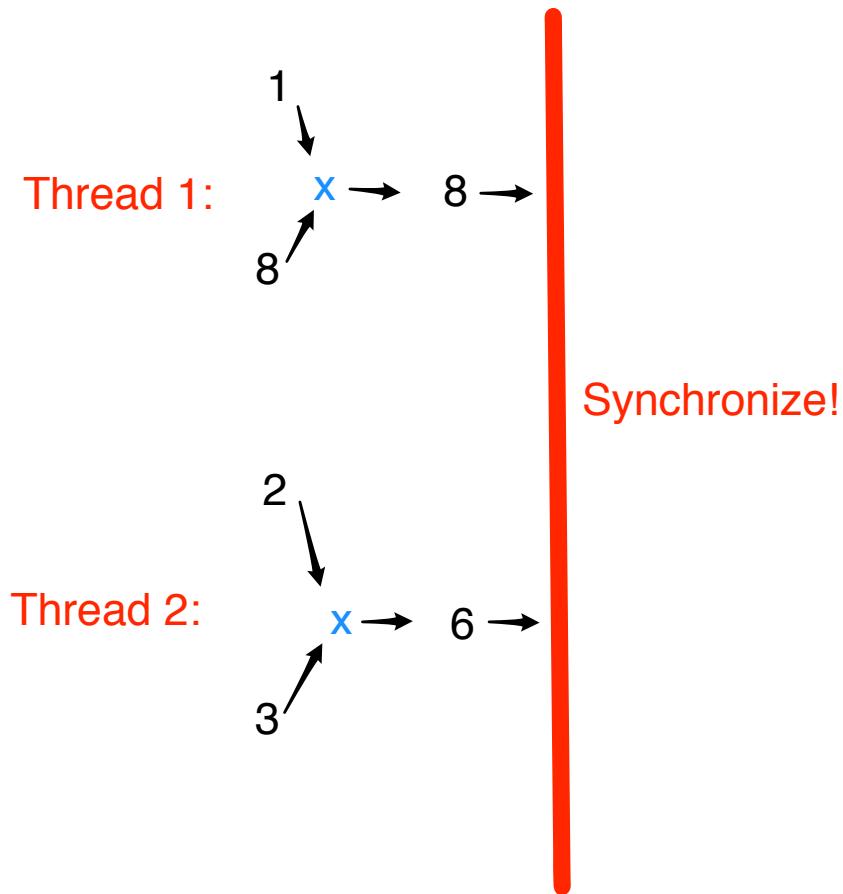
Consider Block (1,1), which computes $\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 3 \end{bmatrix}$:

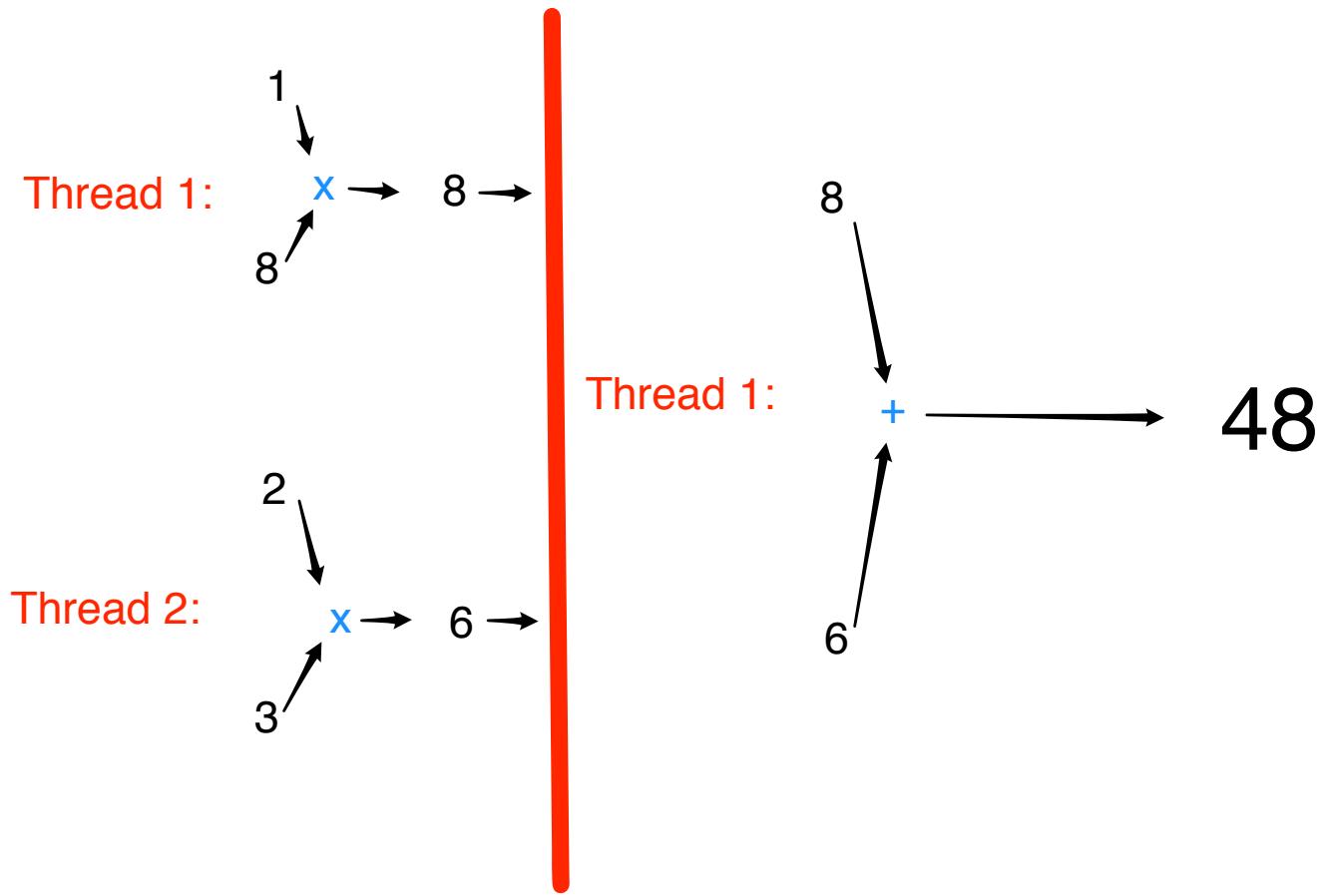
Thread 1:

$$\begin{array}{ccc} 1 & \downarrow & \\ & x \rightarrow & 8 \\ 8 & \nearrow & \end{array}$$

Thread 2:

$$\begin{array}{ccc} 2 & \downarrow & \\ & x \rightarrow & 6 \\ 3 & \nearrow & \end{array}$$





OUTLINE

- A. A review of GPU parallelism
- B. How to GPU-parallelize the following:
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 - 2. the pairwise (cascading) sum
 - 3. matrix multiplication
 - 4. the QR factorization

MATERIALS

These slides, a tentative syllabus for the whole lecture series, and code are available at:

<https://github.com/wlandau/gpu>.

After logging into you home directory on impact1, type:

```
git clone https://github.com/wlandau/gpu
```

into the command line to download all the materials.

REFERENCES

Lay, David C. *Linear Algebra and Its Applications*. 3rd Ed. Addison Wesley, 2006.

J. Sanders and E. Kandrot. *CUDA by Example*. Addison-Wesley, 2010.