

CUDA C: THE SIMD PARADIGM, SHARED MEMORY, AND THE DOT PRODUCT

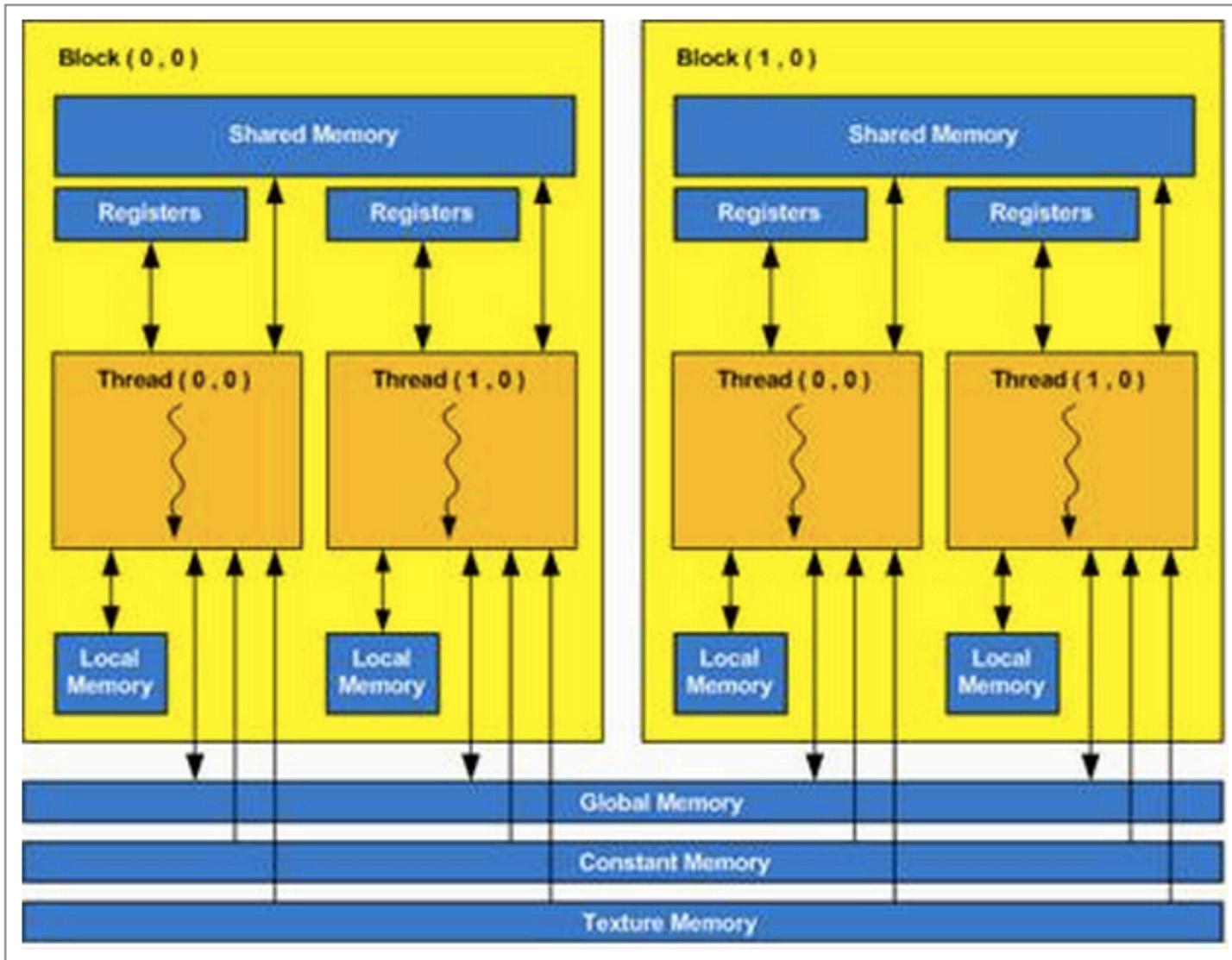
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OUTLINE

- Global vs. local vs. shared memory
- Implementing the dot product

Featured examples:

- `dot_product.cu`



EXAMPLE: USING GLOBAL, SHARED, AND LOCAL MEMORY

Let's say we have a kernel:

```
--global__ void kernel(int *a){  
    *a = blockIdx.x * blockDim.x + threadIdx.x;  
    int b = blockIdx.x * blockDim.x + threadIdx.x;  
    __shared__ int c = blockIdx.x * blockDim.x + threadIdx.x;  
}
```

What are *a, b, and c after a call to `kernel<<<3, 2>>>(a)`?

- *a:** There is one copy of ***a** in GLOBAL MEMORY common to all threads and blocks. Hence, the value of ***a** depends on which thread finishes last.

 - b:** There are $3 \cdot 2 = 6$ copies of **b** in LOCAL MEMORY, one for each of the six threads. The values of **b** will be:
- | | | | | | | |
|-----------------------|--------|--------|--------|--------|--------|--------|
| (Block ID, Thread ID) | (0, 0) | (0, 1) | (1, 0) | (1, 1) | (2, 0) | (2, 1) |
| Value of b | 0 | 1 | 2 | 3 | 4 | 5 |
- c:** There are three copies of **c** in SHARED MEMORY, one for each block. The values of **c** might be:

(Block ID, Thread ID)	(0, 0)	(0, 1)	(1, 0)	(1, 1)	(2, 0)	(2, 1)
Value of b	0	0	3	3	4	4

depending on which thread finishes last within each block.

NOW, WE'RE READY FOR THE DOT PRODUCT

$$(a_0, a_1, \dots, a_{15}) \bullet (b_0, b_1, \dots, b_{15}) = a_0b_0 + a_1b_1 + \dots + a_{15}b_{15}$$

The basic workflow is:

- Pass vectors **a** and **b** to the GPU.
- Give each block a sub vector of **a** and the analogous subvector of **b**, For example:

Block 0 works on:

$$\begin{aligned}(a_0, a_1, \dots, a_7) \\ (b_0, b_1, \dots, b_7)\end{aligned}$$

Block 1 works on:

$$\begin{aligned}(a_8, a_9, \dots, a_{15}) \\ (b_8, b_9, \dots, b_{15})\end{aligned}$$

- Within each block, compute a vector of pairwise products:

Block 0:

$$\text{cache} = (a_0 \cdot b_0, a_1 \cdot b_1, \dots, a_7 \cdot b_7)$$

Block 1:

$$\text{cache} = (a_8 \cdot b_8, a_9 \cdot b_9, \dots, a_{15} \cdot b_{15})$$

where **cache** is an array in shared memory.

- Within each block, compute the pairwise sum of `cache` and write it to `cache[0]`. In our example:

Block 0:

$$\text{cache}[0] = a_0 \cdot b_0 + a_1 \cdot b_1 + \cdots + a_7 \cdot b_7$$

Block 1:

$$\text{cache}[0] = a_8 \cdot b_8 + a_9 \cdot b_9 + \cdots + a_{15} \cdot b_{15}$$

- Fill a new vector in global memory, `partial_c`, with these partial dot products:

$$\text{partial_c} = (\text{block 0 cache}[0], \text{block 1 cache}[0])$$

- Return to the CPU and compute the linear sum of `partial_c` and write it to `partial_sum[0]`. Then:

$$\text{partial_sum}[0] = a_0 \cdot b_0 + a_1 \cdot b_1 + \dots + a_{15} \cdot b_{15}$$

First part of the code:

```
#include "../common/book.h"
#include <stdio.h>
#include <stdlib.h>
#define imin(a,b) (a<b?a:b)

const int N = 33 * 1024;
const int threadsPerBlock = 256;
const int blocksPerGrid = imin( 32, (N+threadsPerBlock-1)
    / threadsPerBlock );

__global__ void dot( float *a, float *b, float *partial_c
) {

    __shared__ float cache[threadsPerBlock];
    int tid = threadIdx.x + blockIdx.x * blockDim.x;
    int cacheIndex = threadIdx.x;
    float temp = 0;
```

```
while ( tid < N) {  
    temp += a[ tid ] * b[ tid ];  
    tid += blockDim.x * gridDim.x;  
}  
  
// set the cache values  
cache[ cacheIndex ] = temp;
```

What the code would do in a call to `dot<<<2, 4>>>(a, b, c)` with $N = 16$:

```
dot<<2,4>>(a, b, c)  
  
blockDim.x = 4  
gridDim.x = 2
```

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)

Block 0	Block 1
cache[0] = cache[1] = cache[2] = cache[3] =	cache[0] = cache[1] = cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 0
blockIdx.x = 0

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)

*

+

*

cache[0] = 47

Block 0	Block 1
cache[0] = 47 cache[1] = cache[2] = cache[3] =	cache[0] = cache[1] = cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 1
blockIdx.x = 0

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)

*

+

*

cache[1] = 14

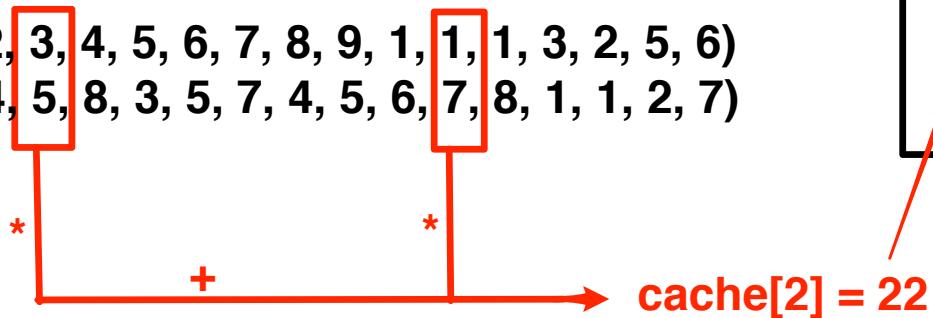
Block 0	Block 1
cache[0] = 47 cache[1] = 14 cache[2] = cache[3] =	cache[0] = cache[1] = cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 2
blockIdx.x = 0

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)



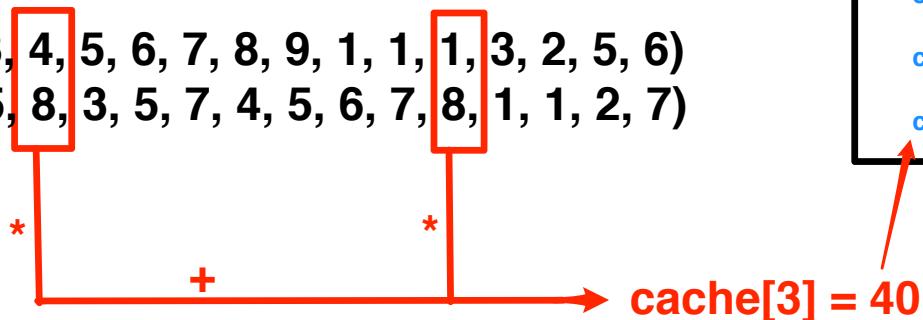
Block 0	Block 1
cache[0] = 47 cache[1] = 14 cache[2] = 22 cache[3] =	cache[0] = cache[1] = cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 3
blockIdx.x = 0

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)



Block 0	Block 1
cache[0] = 47	cache[0] =
cache[1] = 14	cache[1] =
cache[2] = 22	cache[2] =
cache[3] = 40	cache[3] =

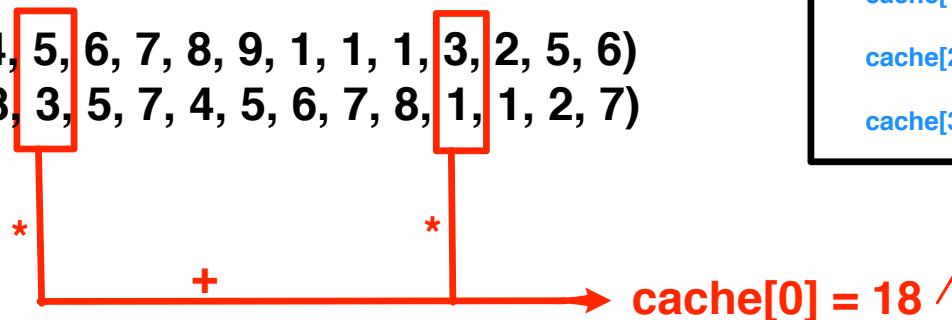
```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 0

blockIdx.x = 1

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)



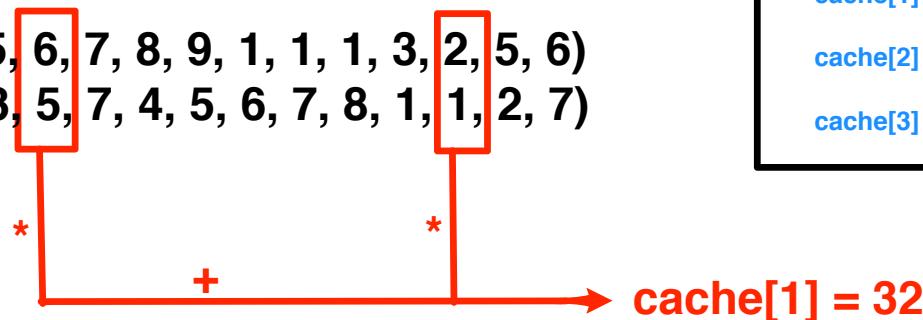
Block 0	Block 1
cache[0] = 47 cache[1] = 14 cache[2] = 22 cache[3] = 40	cache[0] = 18 cache[1] = cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

**a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)**

**threadIdx.x = 1
blockIdx.x = 1**



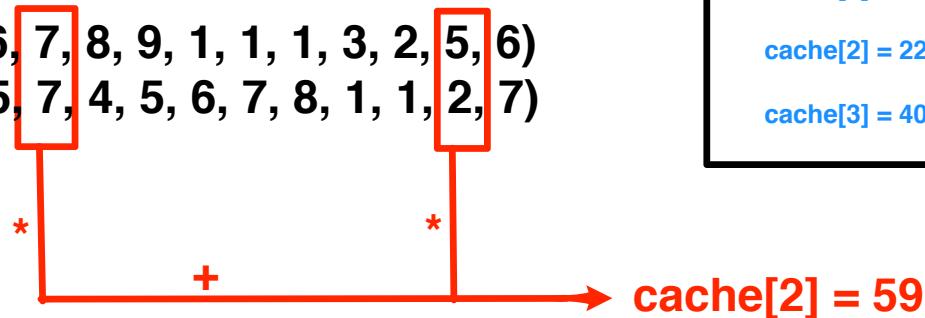
Block 0	Block 1
cache[0] = 47 cache[1] = 14 cache[2] = 22 cache[3] = 40	cache[0] = 18 cache[1] = 32 cache[2] = cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 2
blockIdx.x = 1

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)



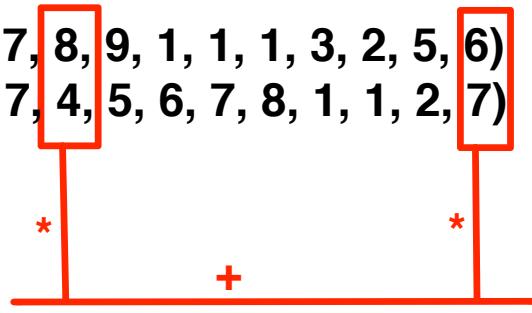
Block 0	Block 1
cache[0] = 47 cache[1] = 14 cache[2] = 22 cache[3] = 40	cache[0] = 18 cache[1] = 32 cache[2] = 59 cache[3] =

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

threadIdx.x = 3
blockIdx.x = 1

a = (1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 3, 2, 5, 6)
b = (2, 4, 5, 8, 3, 5, 7, 4, 5, 6, 7, 8, 1, 1, 2, 7)



Block 0	Block 1
cache[0] = 47	cache[0] = 18
cache[1] = 14	cache[1] = 32
cache[2] = 22	cache[2] = 59
cache[3] = 40	cache[3] = 74

We want to make sure that each block's copy of `cache` is filled up before we continue further.

Hence, the next line of code is:

```
// synchronize threads in this block  
__syncthreads();
```

NEXT, WE EXECUTE A PAIRWISE SUM ON cache FOR EACH BLOCK

```
// for reductions , threadsPerBlock must be a power of 2
// because of the following code
int i = blockDim.x/2;
while (i != 0) {
    if (cacheIndex < i)
        cache[cacheIndex] += cache[cacheIndex + i];
    __syncthreads();    i /= 2;
}
```

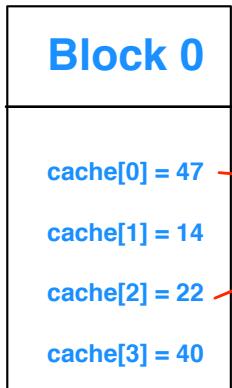
WHAT'S GOING ON IN OUR CALL TO

dot<<<2, 4>>>(a, b, c)

dot<<2,4>>(a, b, c)

blockDim.x = 4
gridDim.x = 2

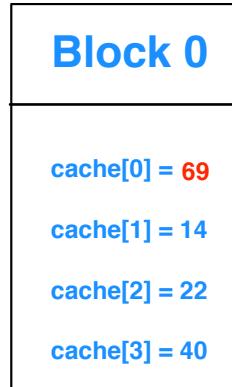
cachelIndex = threadIdx.x = 0
blockIdx.x = 0
i = 2



```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

cachelIndex = threadIdx.x = 1
blockIdx.x = 0
i = 2



```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

Block 0
cache[0] = 69 cache[1] = 54 cache[2] = 22 cache[3] = 40

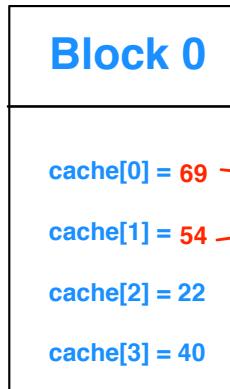
**cachelIndex = threadIdx.x = 1
blockIdx.x = 0
i = 2**

__syncthreads();

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```

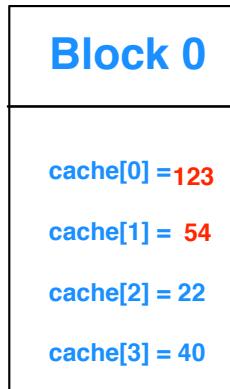
cachelIndex = threadIdx.x = 0
blockIdx.x = 0
i = 1



$$\text{cache}[0] = 69 + \text{cache}[1] = 54 = 123 \rightarrow \text{cache}[0]$$

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4  
gridDim.x = 2
```



cachelIndex = threadIdx.x = 0
blockIdx.x = 0
i = 1

__syncthreads();

```
dot<<2,4>>(a, b, c)
```

```
blockDim.x = 4
```

```
gridDim.x = 2
```

Block 0
cache[0] = 123 cache[1] = 54 cache[2] = 22 cache[3] = 40

cachelIndex = threadIdx.x = 0
blockIdx.x = 0
i = 0

i = 0, so end the pairwise sum.

The result for block 0 is cache[0] = 123.

Similarly, cache[0] for block 1 is 183.

Next:

```
if (cacheIndex == 0)
    partial_c [blockIdx.x] = cache [0];
}
```

So now, **partial_c[0]** = 123 and **partial_c[1]** is 183.

We return from the kernel, `dot()`, and compute the linear sum of the elements of `partial_c`:

```
// finish up on the CPU side
c = 0;
for (int i=0; i<blocksPerGrid; i++) {
    c += partial_c [ i ];
}
```

Now, `partial_c[0]` is the final answer.

COMPLETE CODE

```
#include "../common/book.h"
#include <stdio.h>
#include <stdlib.h>
#define imin(a,b) (a<b?a:b)

const int N = 33 * 1024;
const int threadsPerBlock = 256;
const int blocksPerGrid = imin( 32, (N+threadsPerBlock-1) /
    threadsPerBlock );

__global__ void dot( float *a, float *b, float *partial_c ) {

    __shared__ float cache[threadsPerBlock];
    int tid = threadIdx.x + blockIdx.x * blockDim.x;
    int cacheIndex = threadIdx.x;
    float temp = 0;

    while (tid < N) {
        temp += a[tid] * b[tid];
        tid += blockDim.x * gridDim.x;
    }
}
```

```

// set the cache values
cache[cacheIndex] = temp;

// synchronize threads in this block
__syncthreads();

// for reductions, threadsPerBlock must be a power of 2 //
// because of the following code
int i = blockDim.x/2;
while (i != 0) {
    if (cacheIndex < i)
        cache[cacheIndex] += cache[cacheIndex + i];
    __syncthreads();      i /= 2;
}

if (cacheIndex == 0)
    partial_c[blockIdx.x] = cache[0];
}

int main( void ) {
    float *a, *b, c, *partial_c;
    float *dev_a, *dev_b, *dev_partial_c;

```

```

// allocate memory on the CPU side
a = (float *)malloc( N*sizeof(float) );
b = (float *)malloc( N*sizeof(float) );
partial_c = (float *)malloc( blocksPerGrid*sizeof(float) );

// allocate the memory on the GPU
HANDLEERROR( cudaMalloc( (void**)&dev_a , N*sizeof(float) ) );
HANDLEERROR( cudaMalloc( (void**)&dev_b , N*sizeof(float) ) );
HANDLEERROR( cudaMalloc( (void**)&dev_partial_c , blocksPerGrid
    *sizeof(float) ) );

// fill in the host memory with data
for (int i=0; i<N; i++) {
    a[i] = i;
    b[i] = i*2;
}

// copy the arrays a and b to the GPU
HANDLEERROR( cudaMemcpy( dev_a , a , N*sizeof(float) ,
    cudaMemcpyHostToDevice ) );
HANDLEERROR( cudaMemcpy( dev_b , b , N*sizeof(float) ,
    cudaMemcpyHostToDevice ) );

dot<<<blocksPerGrid , threadsPerBlock>>>( dev_a , dev_b ,

```

```

    dev_partial_c ) ;

// copy the array 'c' back from the GPU to the CPU
HANDLEERROR( cudaMemcpy( partial_c , dev_partial_c ,
blocksPerGrid*sizeof(float) ,
cudaMemcpyDeviceToHost ) ) ;

// finish up on the CPU side
c = 0;
for ( int i=0; i<blocksPerGrid; i++ ) {
    c += partial_c [ i ];
}

#define sum_squares(x)( x * (x + 1) * (2 * x + 1) / 6 )

printf("Does GPU value %.6g = %.6g?\n" , c , 2 * sum_squares( (
float )(N - 1) ) ) ;

// free memory on the GPU side
cudaFree( dev_a );
cudaFree( dev_b );
cudaFree( dev_partial_c );
// free memory on the CPU side
free( a );

```

```
    free( b );
    free( partial_c );
}
```

OUTPUT

```
[landau@impact1 dot_product]$ nvcc dot_product.cu -o dot_product
[landau@impact1 dot_product]$ ./dot_product
Does GPU value 2.57236e+13 = 2.57236e+13?
[landau@impact1 dot_product]$
```

OUTLINE

- Respecting the SIMD paradigm
- Shared memory
- Implementing the dot product

Featured examples:

- `dot_product.cu`

LECTURE SERIES MATERIALS

These lecture slides, a tentative syllabus for the whole lecture series, and code are available at:

<https://github.com/wlandau/gpu>.

After logging into you home directory on impact1, type:

```
git clone https://github.com/wlandau/gpu
```

into the command line to download all the course materials.

REFERENCES

David B. Kirk and Wen-mei W. Hwu. “Programming Massively Parallel Processors: a Hands-on Approach.” Morgan Kaufman, 2010.

J. Sanders and E. Kandrot. *CUDA by Example*. Addison-Wesley, 2010.

Michael Romero and Rodrigo Urra. ”CUDA Programming.” Rochester Institute of Technology.
http://cuda.ce.rit.edu/cuda_overview/cuda_overview.html