

Primal and Dual Linear Programming Problems

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1 Primal to Dual Conversion

1.1 The Primal Problem

$$\begin{aligned} \max \quad & 5x + 3y + 2z \\ \text{Subject to} \quad & \\ & 2x + 3y + z \leq 10 \\ & 4x + y + 2z \leq 12 \\ & x, y, z \geq 0 \end{aligned}$$

1.2 Standard Form of the Primal Problem

$$\begin{array}{ll}\max & 5x + 3y + 2z \\ \text{Subject to} & \\ & 2x + 3y + z + s_1 = 10 \\ & 4x + y + 2z + s_2 = 12 \\ & x, y, z, s_1, s_2 \geq 0\end{array}$$

1.3 The Dual Problem

$$\begin{array}{ll}\min & 10y_1 + 12y_2 \\ \text{Subject to} & \\ & 2y_1 + 4y_2 \geq 5 \\ & 3y_1 + 1y_2 \geq 3 \\ & 1y_1 + 2y_2 \geq 2 \\ & y_1, y_2 \geq 0\end{array}$$

2 Problem 1

2.1 The Definition of the Primal Problem

$$\begin{array}{ll}\max & 3x + 4y \\ \text{Subject to} & 3x + y \leq 24 \\ & x + 4y \leq 36 \\ & x \geq 0 \\ & y \geq 0\end{array}$$

2.2 Standard Form of the Primal Problem

$$\begin{array}{ll}\max & 3x + 4y \\ \text{Subject to} & 3x + y + s_1 = 24 \\ & x + 4y + s_2 = 36 \\ & x \geq 0 \\ & y \geq 0 \\ & s_1 \geq 0 \\ & s_2 \geq 0\end{array}$$

2.3 The Dual Problem

$$\begin{aligned}
 \min \quad & 24y_1 + 36y_2 \\
 \text{Subject to} \quad & 3y_1 + y_2 \geq 3 \\
 & 1y_1 + 4y_2 \geq 4 \\
 & y_1 \geq 0 \\
 & y_2 \geq 0
 \end{aligned}$$

2.4 Last Simplex Tableau for the Primal Problem

	x1	x2	x3	x4	Solution
z	-0.0	-0.0	0.727	0.818	46.909
x1	1.0	0.0	0.364	-0.091	5.455
x2	0.0	1.0	-0.091	0.273	7.636

Table 1: Last Simplex Tableau for the Primal Problem

2.5 Last Simplex Tableau for the Dual Problem

	x1	x2	x3	x4	x5	x6	Solution
z	-0.0	-0.0	-5.455	-354.545	-7.636	-352.364	46.909
x1	1.0	0.0	-0.364	0.364	0.091	-0.091	0.727
x2	0.0	1.0	0.091	-0.091	-0.273	0.273	0.818

Table 2: Last Simplex Tableau for the Dual Problem, Surplus: [3, 5] Artificial: [4, 6] Basic Variables: [1, 2]

3 Problem 2

3.1 The Definition of the Primal Problem

$$\begin{aligned}
 \min \quad & 2x + 3y + 4z \\
 \text{Subject to} \quad & x + y + z \geq 12 \\
 & 2x + 3y + z \geq 24 \\
 & x \geq 0 \\
 & y \geq 0 \\
 & z \geq 0
 \end{aligned}$$

3.2 Standard Form of the Primal Problem

$$\begin{aligned}
 \min \quad & 2x + 3y + 4z \\
 \text{Subject to} \quad & x + y + z - s_1 = 12 \\
 & 2x + 3y + z - s_2 = 24 \\
 & x \geq 0 \\
 & y \geq 0 \\
 & z \geq 0 \\
 & s_1 \geq 0 \\
 & s_2 \geq 0
 \end{aligned}$$

3.3 The Dual Problem

$$\begin{aligned}
 \max \quad & 12y_1 + 24y_2 \\
 \text{Subject to} \quad & 1y_1 + 2y_2 \leq 2 \\
 & 1y_1 + 3y_2 \leq 3 \\
 & 1y_1 + 1y_2 \leq 4 \\
 & y_1 \geq 0 \\
 & y_2 \geq 0
 \end{aligned}$$

3.4 Last Simplex Tableau for the Primal Problem

	x1	x2	x3	x4	x5	x6	x7	Solution
z	-0.0	-0.0	-3.0	-0.0	-240.0	-1.0	-239.0	24.0
x1	1.0	0.0	2.0	-3.0	3.0	1.0	-1.0	12.0
x2	0.0	1.0	-1.0	2.0	-2.0	-1.0	1.0	0.0

Table 3: Last Simplex Tableau for the Primal Problem: Surplus: [4, 6] Artificial: [5, 7] Basic Variables: [1, 2]

3.5 Last Simplex Tableau for the Dual Problem

	x1	x2	x3	x4	x5	Solution
z	-0.0	-0.0	12.0	-0.0	-0.0	24.0
x2	0.5	1.0	0.5	0.0	0.0	1.0
x4	-0.5	0.0	-1.5	1.0	0.0	0.0
x5	0.5	0.0	-0.5	0.0	1.0	3.0

Table 4: Last Simplex Tableau for the Dual Problem: Slack: [3, 4, 5] Basic Variables: [2, 4, 5]