Simplex Examples and Solutions

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March 7, 2025

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1 Linear Programming Problem 1 and the Simplex Solution

1.1 The Problem

$$\begin{aligned} \max z &= 5x_1 + 10x_2 \\ \text{Subject to:} & \\ & 2x_1 + 4x_2 \leq 640 \\ & 6x_1 + 2x_2 \leq 480 \\ & x_1, x_2 \geq 0 \end{aligned}$$

1.2 Initial Table

	x1	x2	x3	x4	Solution
z	-5.0	-10.0	-0.0	-0.0	0.0
x3	2.0	4.0	1.0	0.0	640.0
x4	6.0	2.0	0.0	1.0	480.0

Table 1: x_3 and x_4 are slack variables

1.3 Iteration 1

	x1	x2	x3	x4	Solution
z	-0.0	-0.0	2.5	-0.0	1600.0
x2	0.5	1.0	0.25	0.0	160.0
x4	5.0	0.0	-0.5	1.0	160.0

Table 2: No entering variables, algorithm terminates

1.4 Solution Set

$$\text{Solution} = \left\{ \begin{array}{rcl} x_1 & = & 0 \\ x_2 & = & 160 \\ x_3 & = & 0 \\ x_4 & = & 160 \\ z_{\max} & = & 1600 \end{array} \right\}$$

2 Linear Programming Problem 2 and the Simplex Solution

2.1 The Problem

$$\begin{aligned} &\max z = 5x_1 + 6x_2 + 4x_3 \\ &\text{Subject to:} \\ &\qquad 2x_1 + 4x_2 + 6x_3 \leq 640 \\ &\qquad 2x_1 + 3x_2 + x_3 \leq 960 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

2.2 Initial Table

	x1	x2	x3	x4	x5	Solution
z	-5.0	-6.0	-4.0	-0.0	-0.0	0.0
x4	2.0	4.0	6.0	1.0	0.0	640.0
x5	2.0	3.0	1.0	0.0	1.0	960.0

Table 3: x_4 and x_5 are slack variables

2.3 Iteration 1

	x1	x2	x3	x4	x5	Solution
z	-2.0	-0.0	5.0	1.5	-0.0	960.0
x2	0.5	1.0	1.5	0.25	0.0	160.0
x5	0.5	0.0	-3.5	-0.75	1.0	480.0

Table 4:

2.4 iteration 2

	x1	x2	x3	x4	x5	Solution
z	-0.0	4.0	11.0	2.5	-0.0	1600.0
x1	1.0	2.0	3.0	0.5	0.0	320.0
x5	0.0	-1.0	-5.0	-1.0	1.0	320.0

Table 5: No entering variables, algorithm terminates

2.5 Solution Set

$$\text{Solution} = \left\{ \begin{array}{lcl} x_1 & = & 320 \\ x_2 & = & 0 \\ x_3 & = & 0 \\ x_4 & = & 0 \\ x_5 & = & 320 \\ z_{\max} & = & 1600 \end{array} \right\}$$

3 Linear Programming Problem 3 and the Simplex Solution

3.1 The Problem

$$\begin{aligned} &\max z = 3x_1 + 5x_2 + 4x_3 \\ &\text{Subject to:} \\ &\qquad x_1 + 2x_2 + 5x_3 \leq 120 \\ &\qquad 3x_1 + x_2 + 4x_3 \leq 240 \\ &\qquad 2x_1 + 2x_2 + x_3 \leq 360 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

3.2 Initial Table

	x1	x2	x3	x4	x5	x6	Solution
z	-3.0	-5.0	-4.0	-0.0	-0.0	-0.0	0.0
x4	1.0	2.0	5.0	1.0	0.0	0.0	120.0
x5	3.0	1.0	4.0	0.0	1.0	0.0	240.0
x6	2.0	2.0	1.0	0.0	0.0	1.0	360.0

Table 6: x_4, x_5, x_6 are slack variables

3.3 Iteration 1

	x1	x2	x3	x4	x5	x6	Solution
z	-0.5	-0.0	8.5	2.5	-0.0	-0.0	300.0
x2	0.5	1.0	2.5	0.5	0.0	0.0	60.0
x5	2.5	0.0	1.5	-0.5	1.0	0.0	180.0
x6	1.0	0.0	-4.0	-1.0	0.0	1.0	240.0

Table 7:

3.4 Iteration 2

	x1	x2	x3	x4	x5	x6	Solution
z	-0.0	-0.0	8.8	2.4	0.2	-0.0	336.0
x2	0.0	1.0	2.2	0.6	-0.2	0.0	24.0
x1	1.0	0.0	0.6	-0.2	0.4	0.0	72.0
x6	0.0	0.0	-4.6	-0.8	-0.4	1.0	168.0

Table 8: Final Table

3.5 Solution Set

$$\text{Solution} = \left\{ \begin{array}{rcr} x_1 & = & 72 \\ x_2 & = & 24 \\ x_3 & = & 0 \\ x_4 & = & 0 \\ x_5 & = & 0 \\ x_6 & = & 168 \\ z_{\max} & = & 336 \end{array} \right\}$$