

# Primal and Dual Linear Programming Problems

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## 1 Primal to Dual Conversion

### 1.1 The Primal Problem

$$\begin{aligned} \max \quad & 5x + 3y + 2z \\ \text{Subject to} \quad & \end{aligned}$$

$$\begin{aligned} & 2x + 3y + z \leq 10 \\ & 4x + y + 2z \leq 12 \\ & x, y, z \geq 0 \end{aligned}$$

## 1.2 Standard Form of the Primal Problem

$$\max \quad 5x + 3y + 2z$$

Subject to

$$2x + 3y + z + s_1 = 10$$

$$4x + y + 2z + s_2 = 12$$

$$x, y, z, s_1, s_2 \geq 0$$

## 1.3 The Dual Problem

$$\min \quad 10y_1 + 12y_2$$

Subject to

$$2y_1 + 4y_2 \geq 5$$

$$3y_1 + 1y_2 \geq 3$$

$$1y_1 + 2y_2 \geq 2$$

$$y_1, y_2 \geq 0$$

# 2 Problem 1

## 2.1 The Definition of the Primal Problem

$$\max \quad 3x + 4y$$

Subject to  $3x + y \leq 24$

$$x + 4y \leq 36$$

$$x \geq 0$$

$$y \geq 0$$

## 2.2 Standard Form of the Primal Problem

$$\max \quad 3x + 4y$$

Subject to  $3x + y + s_1 = 24$

$$x + 4y + s_2 = 36$$

$$x \geq 0$$

$$y \geq 0$$

$$s_1 \geq 0$$

$$s_2 \geq 0$$

### 2.3 The Dual Problem

$$\begin{aligned}
 \min \quad & 24y_1 + 36y_2 \\
 \text{Subject to} \quad & 3y_1 + y_2 \geq 3 \\
 & 1y_1 + 4y_2 \geq 4 \\
 & y_1 \geq 0 \\
 & y_2 \geq 0
 \end{aligned}$$

### 2.4 Last Simplex Tableau for the Primal Problem

	x1	x2	x3	x4	Solution
z	-0.0	-0.0	0.727	0.818	46.909
x1	1.0	0.0	0.364	-0.091	5.455
x2	0.0	1.0	-0.091	0.273	7.636

Table 1: Last Simplex Tableau for the Primal Problem

### 2.5 Last Simplex Tableau for the Dual Problem

	x1	x2	x3	x4	x5	x6	Solution
z	-0.0	-0.0	-5.455	-354.545	-7.636	-352.364	46.909
x1	1.0	0.0	-0.364	0.364	0.091	-0.091	0.727
x2	0.0	1.0	0.091	-0.091	-0.273	0.273	0.818

Table 2: Last Simplex Tableau for the Dual Problem, Surplus: [3, 5] Artificial: [4, 6] Basic Variables: [1, 2]

## 3 Problem 2

### 3.1 The Definition of the Primal Problem

$$\begin{aligned}
 \min \quad & 2x + 3y + 4z \\
 \text{Subject to} \quad & x + y + z \geq 12 \\
 & 2x + 3y + z \geq 24 \\
 & x \geq 0 \\
 & y \geq 0 \\
 & z \geq 0
 \end{aligned}$$

### 3.2 Standard Form of the Primal Problem

$$\begin{aligned}
 \min \quad & 2x + 3y + 4z \\
 \text{Subject to} \quad & x + y + z - s_1 = 12 \\
 & 2x + 3y + z - s_2 = 24 \\
 & x \geq 0 \\
 & y \geq 0 \\
 & z \geq 0 \\
 & s_1 \geq 0 \\
 & s_2 \geq 0
 \end{aligned}$$

### 3.3 The Dual Problem

$$\begin{aligned}
 \max \quad & 12y_1 + 24y_2 \\
 \text{Subject to} \quad & 1y_1 + 2y_2 \leq 2 \\
 & 1y_1 + 3y_2 \leq 3 \\
 & 1y_1 + 1y_2 \leq 4 \\
 & y_1 \geq 0 \\
 & y_2 \geq 0
 \end{aligned}$$

### 3.4 Last Simplex Tableau for the Primal Problem

	x1	x2	x3	x4	x5	x6	x7	Solution
z	-0.0	-0.0	-3.0	-0.0	-240.0	-1.0	-239.0	24.0
x1	1.0	0.0	2.0	-3.0	3.0	1.0	-1.0	12.0
x2	0.0	1.0	-1.0	2.0	-2.0	-1.0	1.0	0.0

Table 3: Last Simplex Tableau for the Primal Problem: Surplus: [4, 6] Artificial: [5, 7] Basic Variables: [1, 2]

### 3.5 Last Simplex Tableau for the Dual Problem

	x1	x2	x3	x4	x5	Solution
z	-0.0	-0.0	12.0	-0.0	-0.0	24.0
x2	0.5	1.0	0.5	0.0	0.0	1.0
x4	-0.5	0.0	-1.5	1.0	0.0	0.0
x5	0.5	0.0	-0.5	0.0	1.0	3.0

Table 4: Last Simplex Tableau for the Dual Problem: Slack: [3, 4, 5] Basic Variables: [2, 4, 5]