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SPACE-TIME DPG: DESIGNING A METHOD FOR MASSIVELY PARALLEL CFD

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Abstract. We develop a space-time discontinuous Petrov-Galerkin finite element method ideal for parallel simulation of transient fluid dynamics problems.

1 A ROBUST ADAPTIVE METHOD FOR CFD

The discontinuous Petrov-Galerkin method[1] is a novel adaptive finite element framework with exceptional stability properties. Mesh design is a time-consuming and expensive part of CFD simulations, as a domain expert has to manually design the mesh to achieve near resolution in all parts of the domain lest the numerical method become unstable. DPG in contrast does not have a pre-asymptotic regime, allowing simulations to start on the coarsest mesh that can adequately represent the domain geometry. *A posteriori* error estimation and adaptivity can also be done very naturally as DPG comes with an error representation function that indicates error in the energy norm.

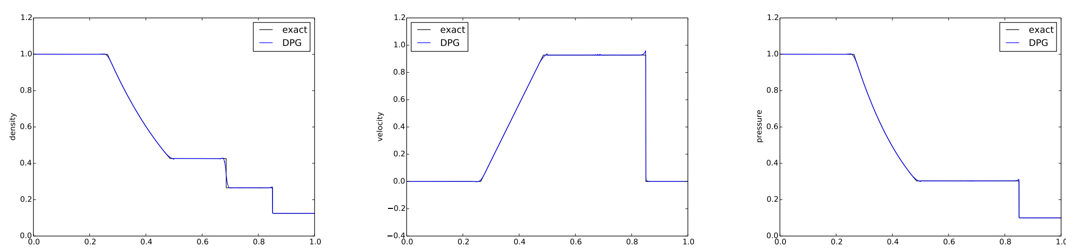
Automatic adaptivity and pre-asymptotic stability produce a powerful synergy when combined with high performance computing. Human intervention to correct or adapt a failed massively parallel simulation can be a costly endeavor, prompting the desire for a numerical technology that will automatically adapt to changing physical dynamics while avoiding "crashing" due to under-resolved meshes. An ideal parallel algorithm should be locally compute intensive while maintaining minimal memory requirements. These are all observed properties of the discontinuous Petrov-Galerkin finite element method. Locally computed *optimal test functions* ensure stability on convection-dominated flows as well as resolved viscous flow. Element contributions to the global stiffness matrix can be computed independently due to the use of discontinuous test functions. Furthermore, static condensation allows the global solve to concern only the *trace* degrees of freedom; trace variables are defined on the mesh skeleton. This allows significant reduction of the cost of the global solve. The internal degrees of freedom can then be resolved using a fully parallel post-processing step.

2 APPLICATION TO TRANSIENT FLOWS

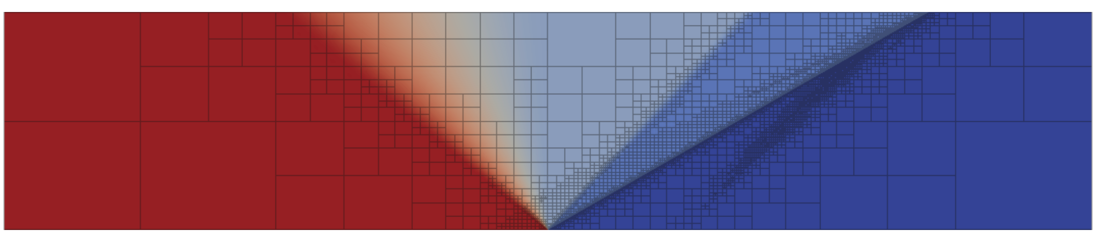
We recently began exploring the extension of these attractive DPG properties to space-time domains, allowing us to automate and localize temporal adaptivity in the same way that we already do with spatial adaptivity. Preliminary results have been generated with Camellia[2] for spatially 1D flows, and a rewrite of Camellia for higher dimensions is currently in progress. Figure 1 shows a simulation of the 1D Sod shock tube with a physical viscosity of 10^{-5} (and no artificial viscosity) until a final time of $t = 0.2$. We start on a coarse mesh of only 4 space-time elements and adaptively refine toward a resolved solution.

3 CONCLUSIONS

Initial 1D space-time results are promising. The higher dimensional version with exceptional automaticity should be an attractive method for high performance computing.



(a) Density (b) Velocity (c) Pressure



(d) Density with space-time mesh

Figure 1: Sod problem after 14 adaptive refinements

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