Pressureless Navier-Stokes Formulation

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Compressible

We can derive the compressible Navier-Stokes equations in terms of the Cauchy stress tensor. First note that

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}\,,$$

and

$$\begin{aligned}\sigma_{ii} &= 2\mu\varepsilon_{ii} + N\lambda\varepsilon_{ii}\\ &= (2\mu + N\lambda)\varepsilon_{ii}\,,\end{aligned}$$

where N is the dimension. Then

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{2\mu}\varepsilon_{kk}\delta_{ij}\\ &= \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{2\mu(2\mu+N\lambda)}\sigma_{kk}\delta_{ij}\\ &= \frac{1}{2\mu}\sigma_{ij} - \frac{1}{2\mu(\frac{2\mu}{\lambda}+N)}\sigma_{kk}\delta_{ij}\,.\end{aligned}$$

If we let $\lambda \to \infty$, then

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2\mu}\sigma_{ij} - \frac{1}{2N\mu}\sigma_{kk}\delta_{ij}\\ &= \frac{1}{2\mu}\left[\sigma_{ij} - \frac{1}{N}\sigma_{kk}\delta_{ij}\right]\,.\end{aligned}$$

Which corresponds to the incompressible case. Alternatively, if we assume the Stokes hypothesis that $\lambda = -\frac{2}{3}\mu$, we instead get

$$\varepsilon_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{1}{2\mu(N-3)}\sigma_{kk}\delta_{ij}\,.$$