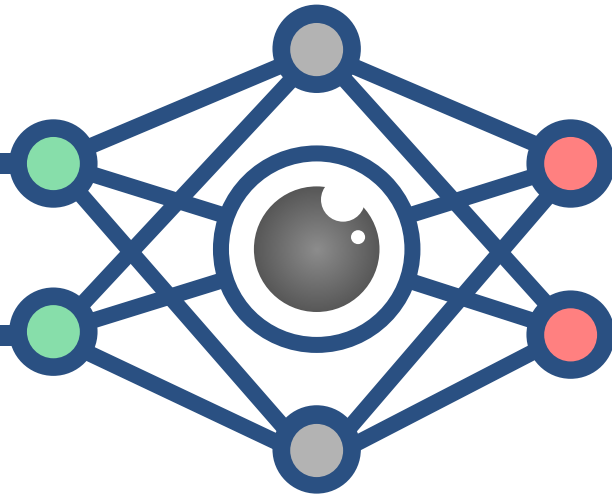


CS3485

# Deep Learning for Computer Vision



*Lec 2: Linear Classification and Perceptron*

# Announcements

- Lab 1 is out:
  - Make sure to find a pair to work on it with. If you can't find one, let me know **by Tuesday**.
  - It is an easy lab: you'll just need the basis of Python/Numpy + this slide deck. Feel free to ask me or come to my office hours if you have questions about either Python or Numpy.
  - The instructions carry a little info on what I expect in the report. I'll go easy on the grade this time, so you know what to improve for the next lab.
  - Keep in mind your late day budget (4 for **all labs**).
- Lecture attendance:
  - I won't take attendance for most lectures and you are not required to make to all lectures.
  - However, if I notice a student missing many consecutive lectures, that will **heavily** impact their participation grade.
- Our LA, Vaishali, created this form for you guys to fill out on what the best LA hours times:  
<https://docs.google.com/forms/d/e/1FAIpQLSejUZ62Dno9kBc7sgtGsQGFzBlauAcRdm9aMDXHlqUeyJoxSg/viewform>

# Announcements

- Amazon Go is a flop! It seems that it won't grow bigger than small shops

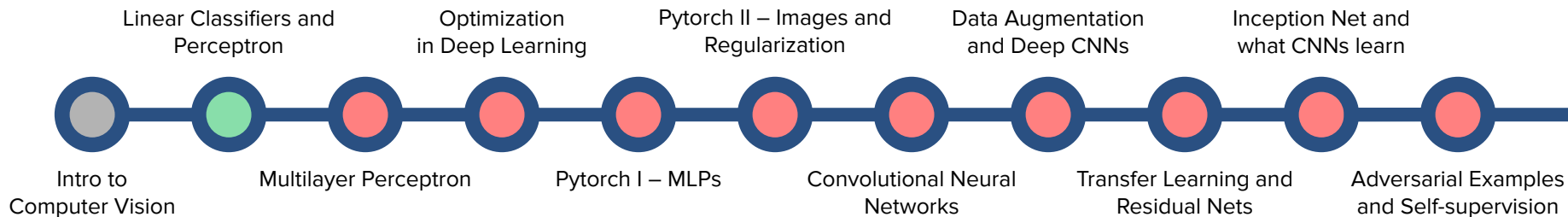
*In a statement, Amazon said it will continue using the Just Walk Out technology in Amazon Go stores, at smaller format Fresh stores in the UK, and third-party locations such as certain sports stadiums and college campuses*

- On the other hand, another CV technology is taking place: Dash Carts!

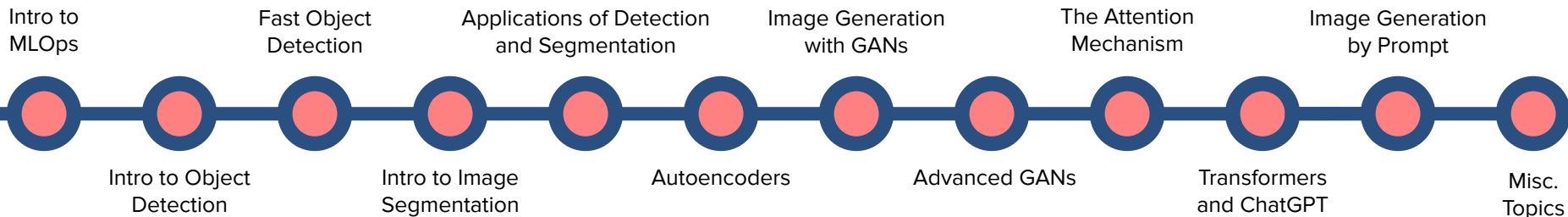


# (Tentative) Lecture Roadmap

## Basics of Deep Learning



## Deep Learning and Computer Vision in Practice



# The image classification problem

- The first task in Computer Vision we are tackling is that of Image Classification:

Image Classification the process of recognition, understanding, and grouping of images into preset categories or classes.
- We want a model (or rule) that effectively categorizes (unseen) images into a set of target classes.
- In order to find this rule, we have a set of **labeled example images** at our disposal that we can **train** our model on and **learn** that rule.
- This process of finding such a model from labeled data is called **Supervised Learning**.

Labeled  
images of cats



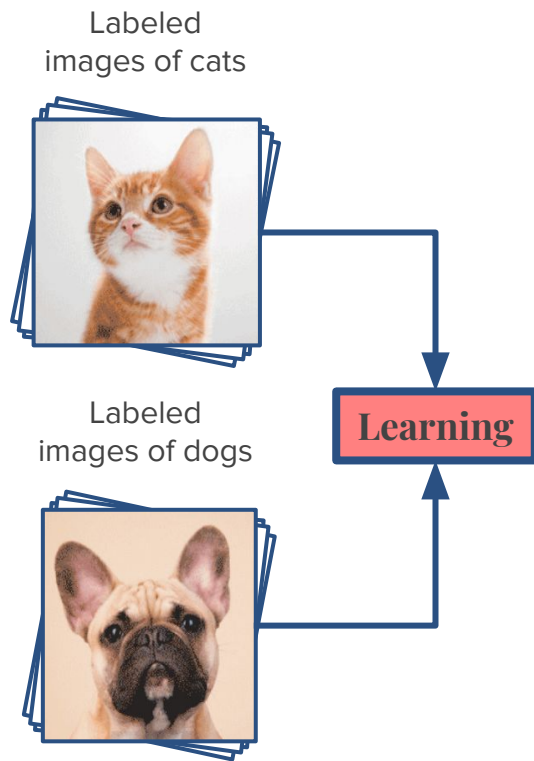
Labeled  
images of dogs



# The image classification problem

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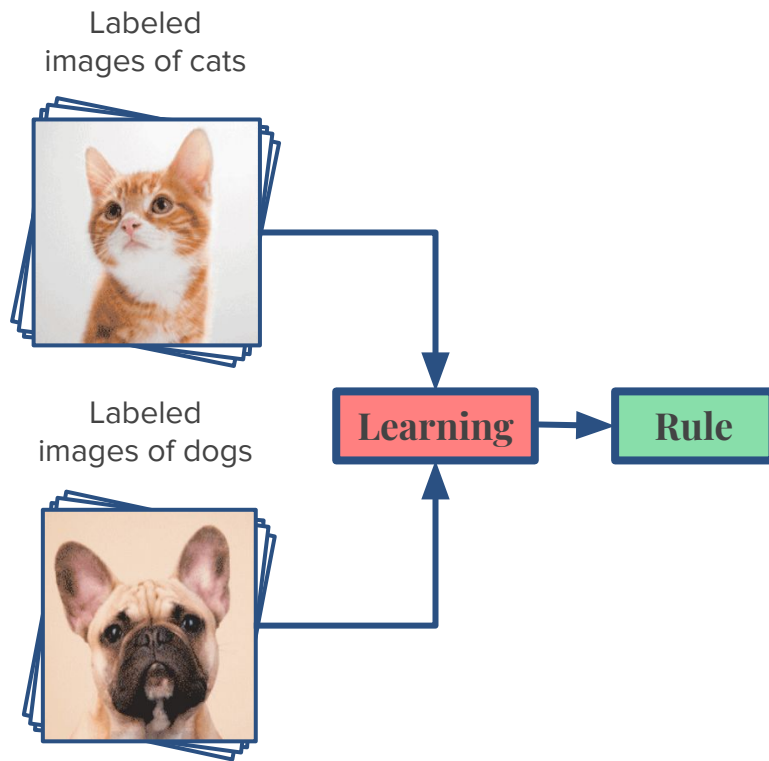
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Unseen (unlabeled) images



(Predicted)  
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Rule

(Predicted)  
classes

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Unseen (unlabeled) images



Rule

(Predicted) classes

Cat

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Unseen (unlabeled) images

(Predicted) classes



Rule

Cat



Rule

Dog

# The image classification problem

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Unseen (unlabeled) images

(Predicted) classes



Rule

Cat



Rule

Dog

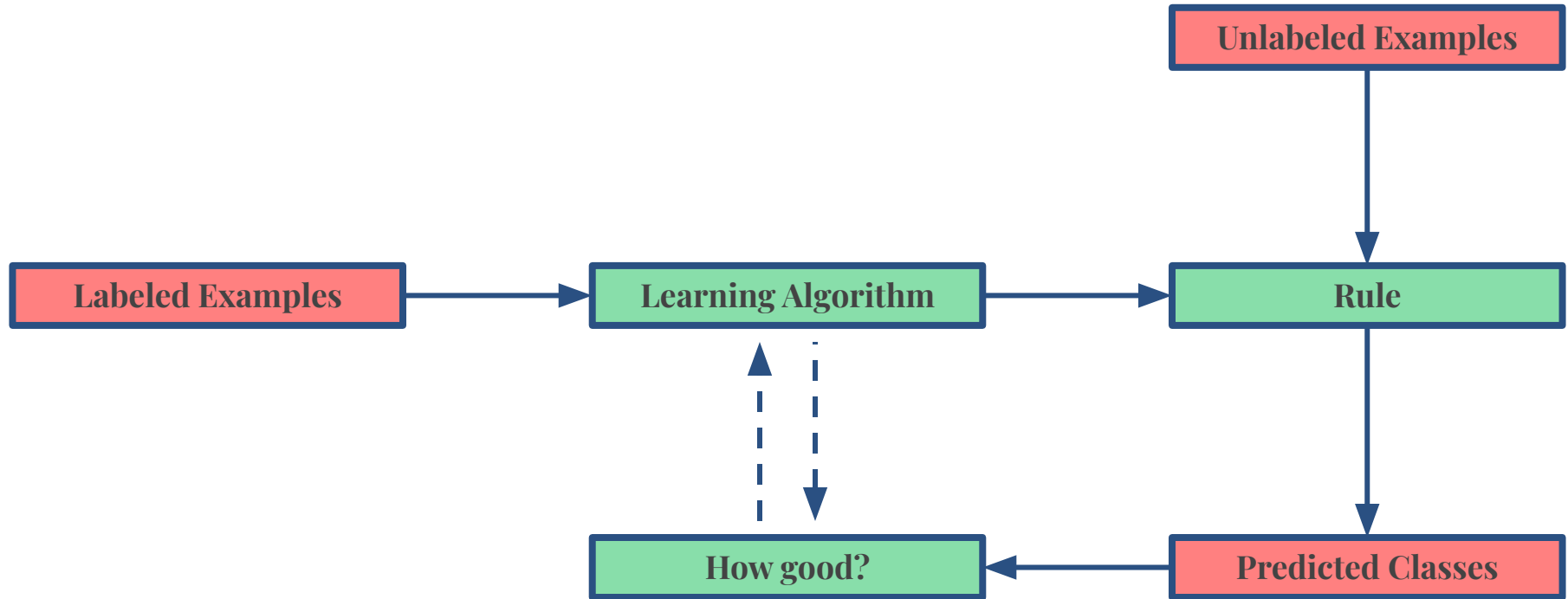


Rule

???

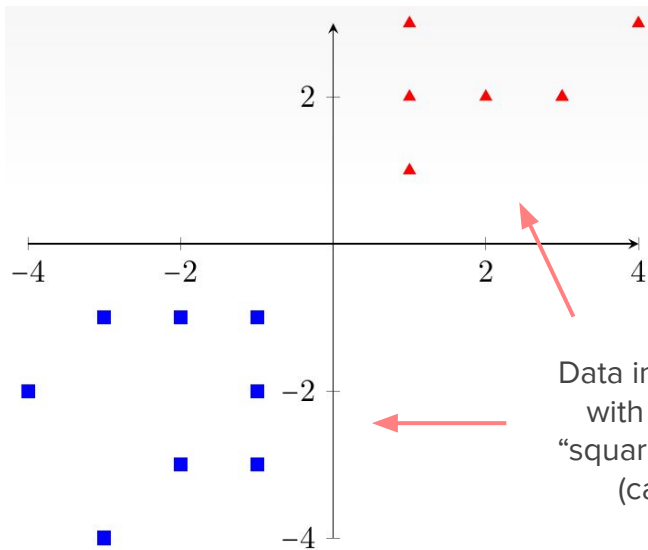
In some settings, the rule can also output “don’t know”!

# Supervised Classification Pipeline



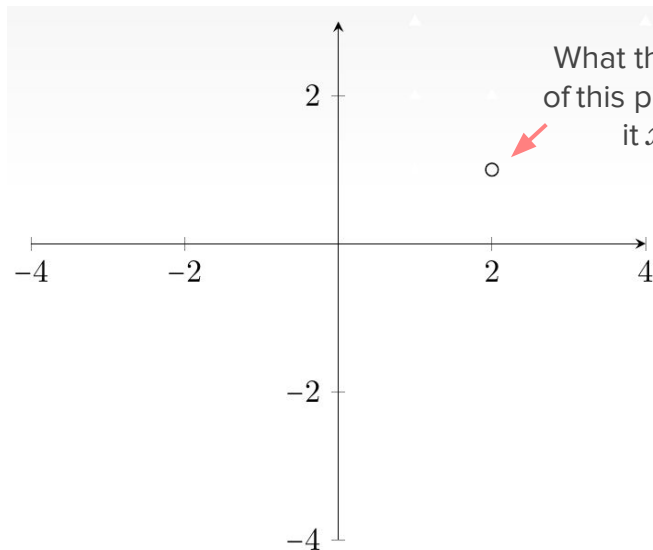
# Example of Classification Problem

Original Data



Data in 2 dimensions  
with labels either  
"square" or "triangle"  
(call them  $y$ ).

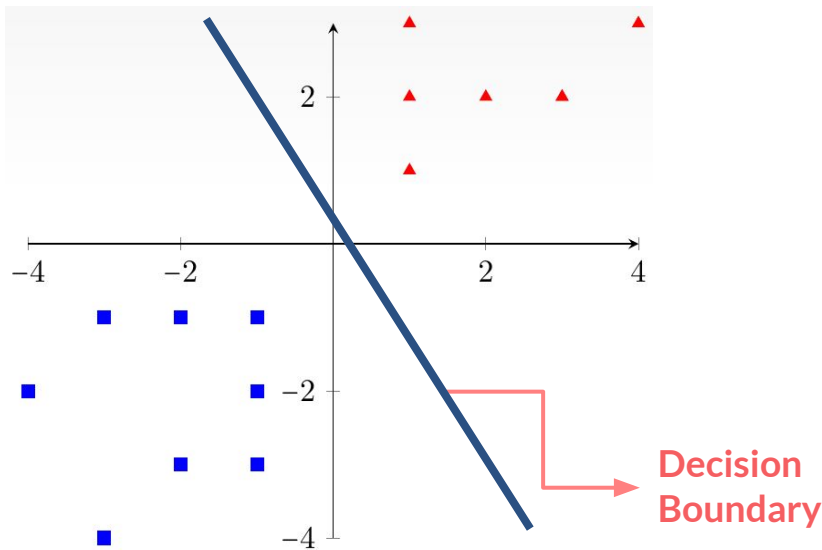
New Unlabeled Datapoint



What the class  
of this point (call  
it  $x$ )?

# Linear Classifiers

## Original Data

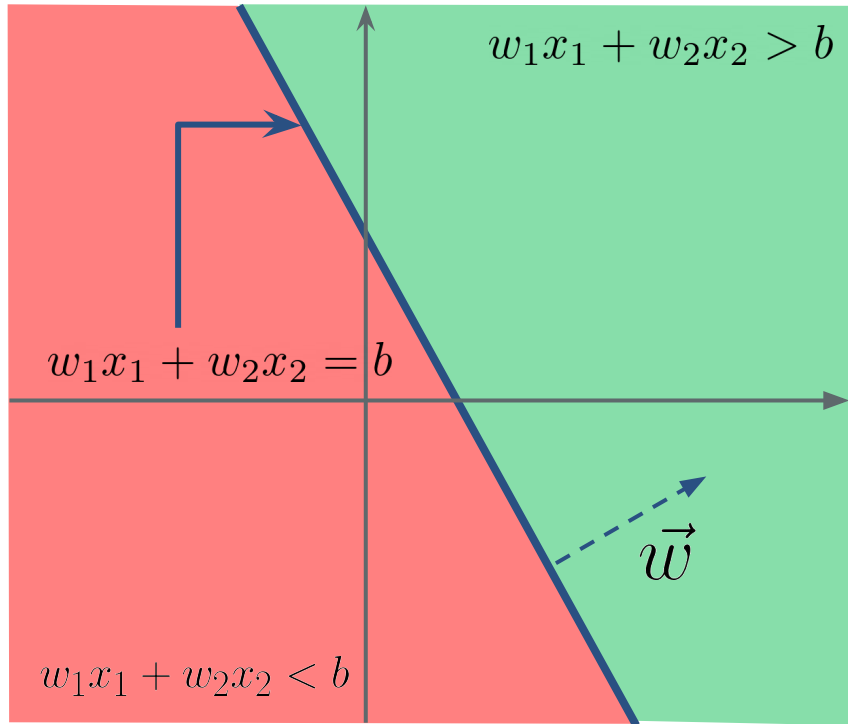


- We need to find a classification rule (**decision boundary**) based on the labeled data.
- Today's choice:

## Linear Classifiers

- Which means: *"If  $x$  is on one side of the line, it is a triangle, otherwise it is a square"*.
- How to define the line and its sides **mathematically**, so we can come up with algorithms?

# A linear classifier in 2D



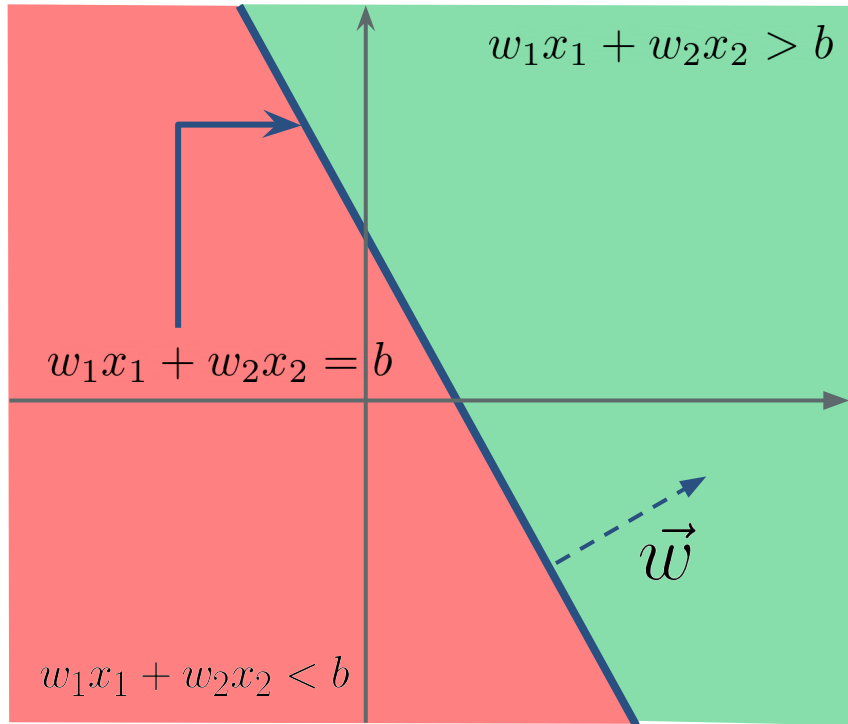
- In 2D, we represent a line using **three numbers**:
  - Two to form a vector  $w = [w_1, w_2]$  called **weight vector**;
  - One number called **bias**,  $b$ .
- If a new point  $x = [x_1, x_2]$  comes in, we just check whether:

$$w_1x_1 + w_2x_2 > b$$

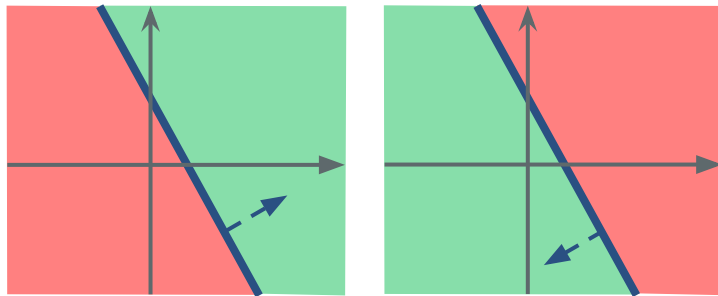
- If **True**,  $x$  lies on one side of the plane, if **False** it belongs to the other side
- If equal, it  $x$  is exactly **on the line**, and it can be classified as either True or False.



# A linear classifier in 2D

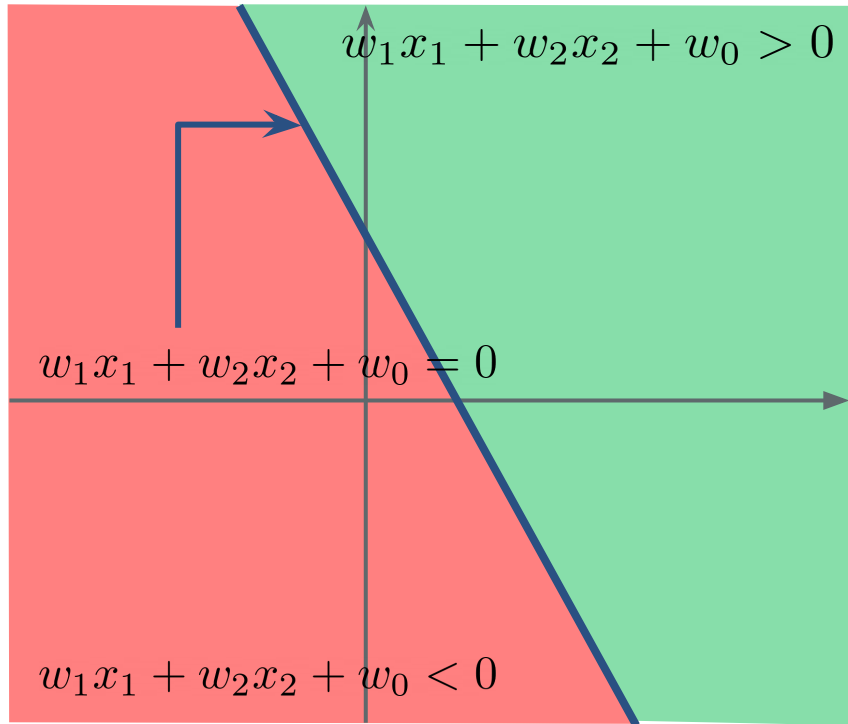


- The **direction of the vector of weights** plays a role here too.
- It always points to the side where the value of  $w_1x_1 + w_2x_2 > b$  is **True**:



- The boundary is, however, the same in both cases, and one can change the direction of  $w$  by setting  $w = -w$ .

# A linear classifier in 2D



- Now, we can also define the weight vector to include  $b$ , making:

$$\mathbf{w} = [w_0, w_1, w_2]$$

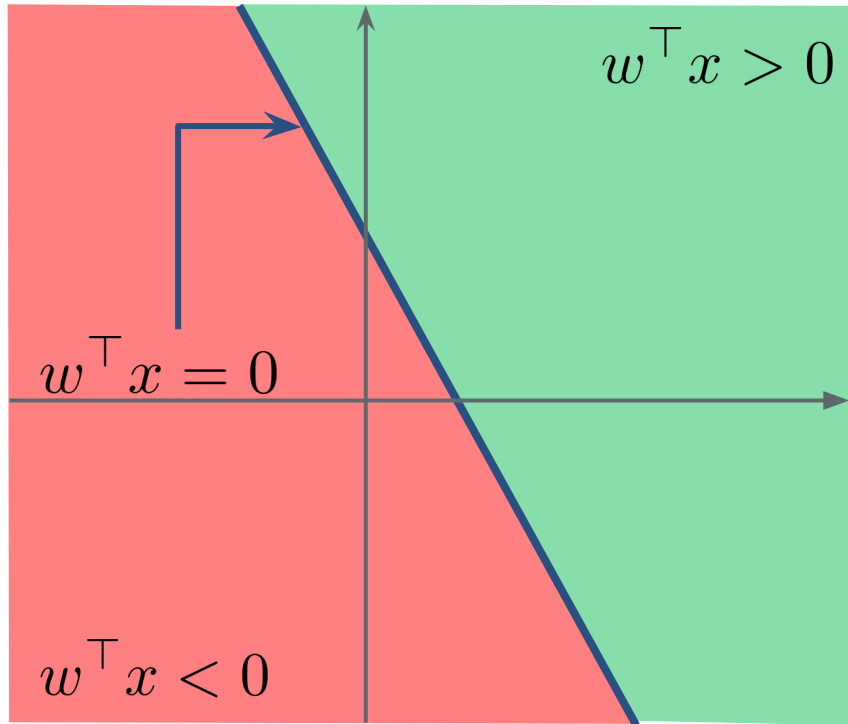
where  $b = -w_0$ .

- Now, **very importantly**, because of that change in  $\mathbf{w}$ , we need to add a new dimension with a “1” to all data points  $x$ :

$$x = [1, x_1, x_2]$$

- For example, if  $x$  was  $[5, 7]$  initially, now it will be  $[1, 5, 7]$ .
- We'll use this change in today's examples.

# A linear classifier in 2D



- Finally, we can use the following notation:

$$\begin{aligned}w^T x &= [w_0 \ w_1 \ w_2]^T [1 \ x_1 \ x_2] \\ &= w_0 + w_1 x_1 + w_2 x_2\end{aligned}$$

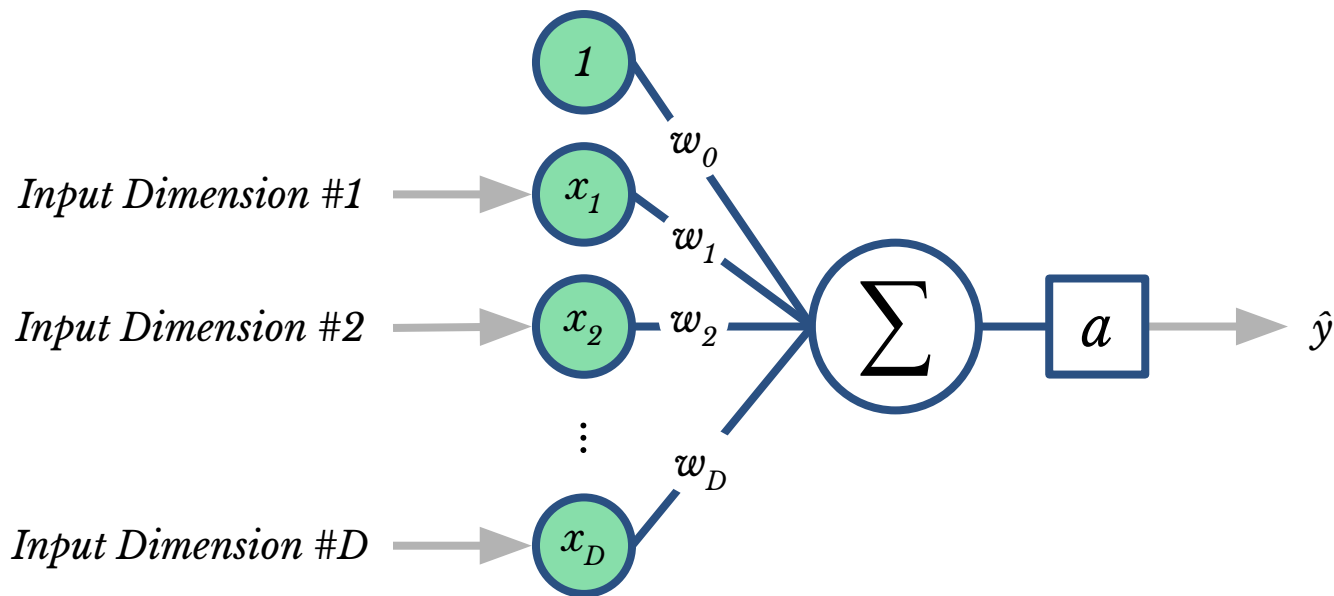
where  $^T$  is the transpose operation.

- This notation is called the **inner product**, and it is handy since it is the same even if our data points are of  $D > 2$  dimensions.
- **Mathematically**, the predicted class  $\hat{y}$  of a point  $x$  by a linear classifier given by  $w$  is:

$$\hat{y} = \text{sign}(w^T x) = \begin{cases} 1, & \text{if } w^T x \geq 0 \\ -1, & \text{if } w^T x < 0 \end{cases}$$

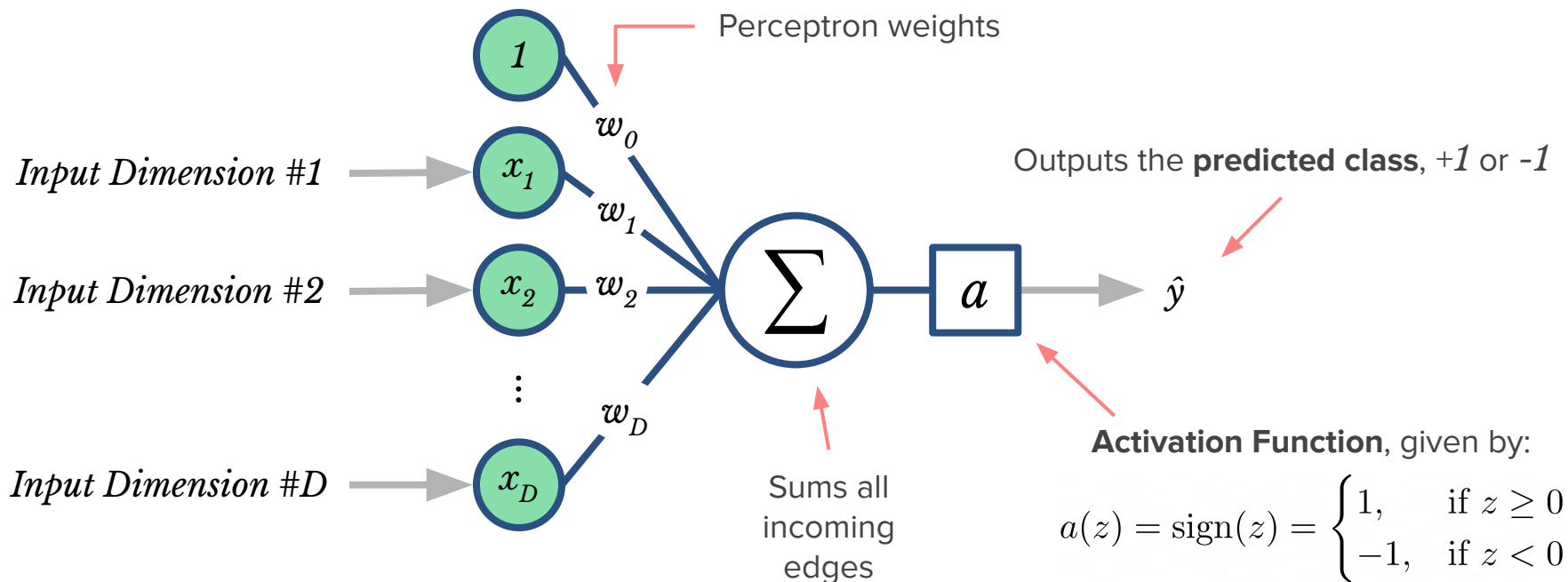
# Linear Classifier Model: Perceptron

- Using these concepts, we can build a **model for classification** called **perceptron**!



# Linear Classifier Model: Perceptron

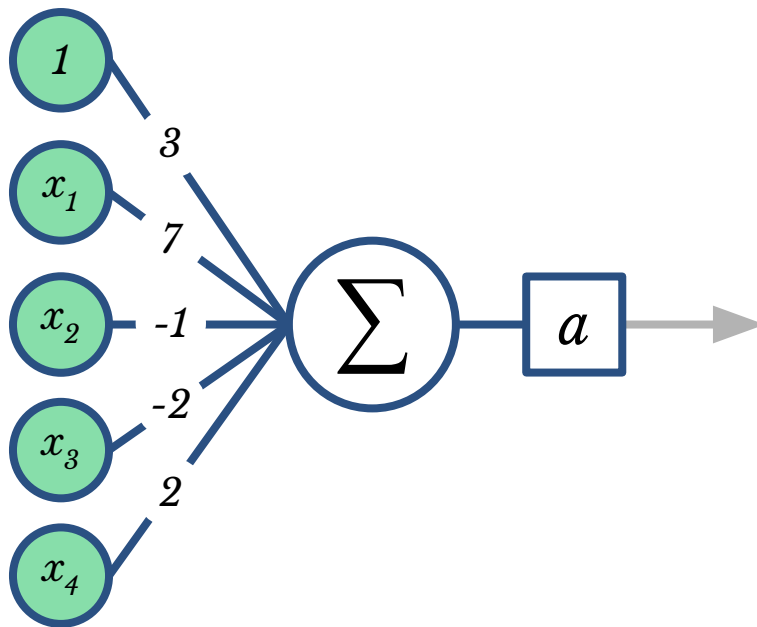
- Below, you have some important nomenclature of the inner workings of the perceptron:



# Linear Classifier Model: Perceptron

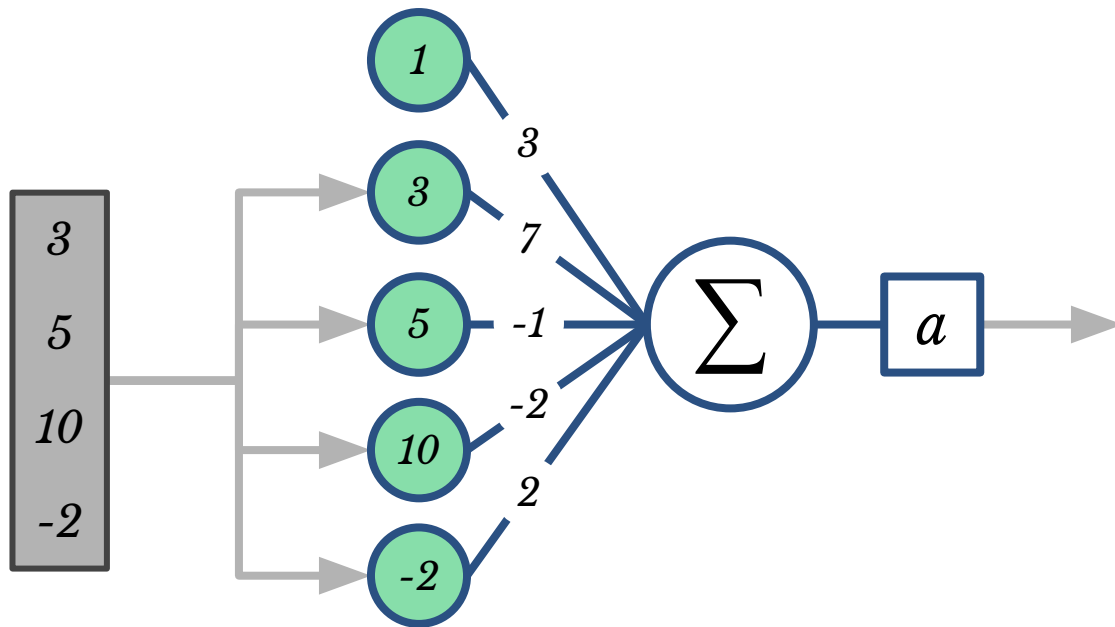
- Say **we know the perceptron weights**, then classifying a new point is easy. For example:

A data point to  
be classified



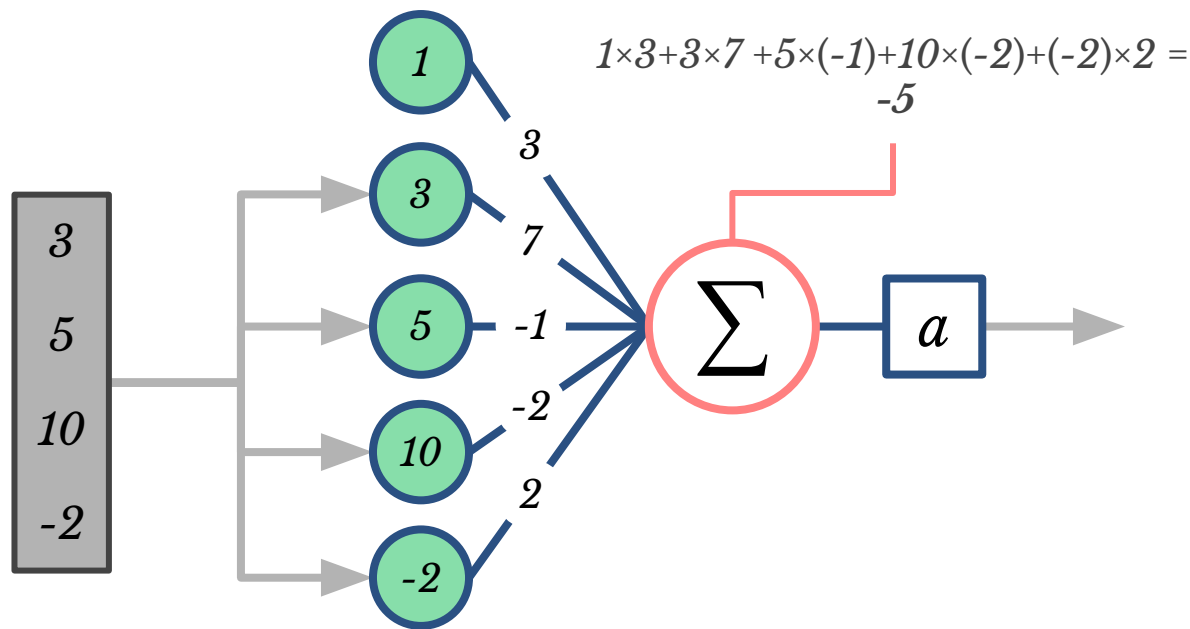
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# Linear Classifier Model: Perceptron

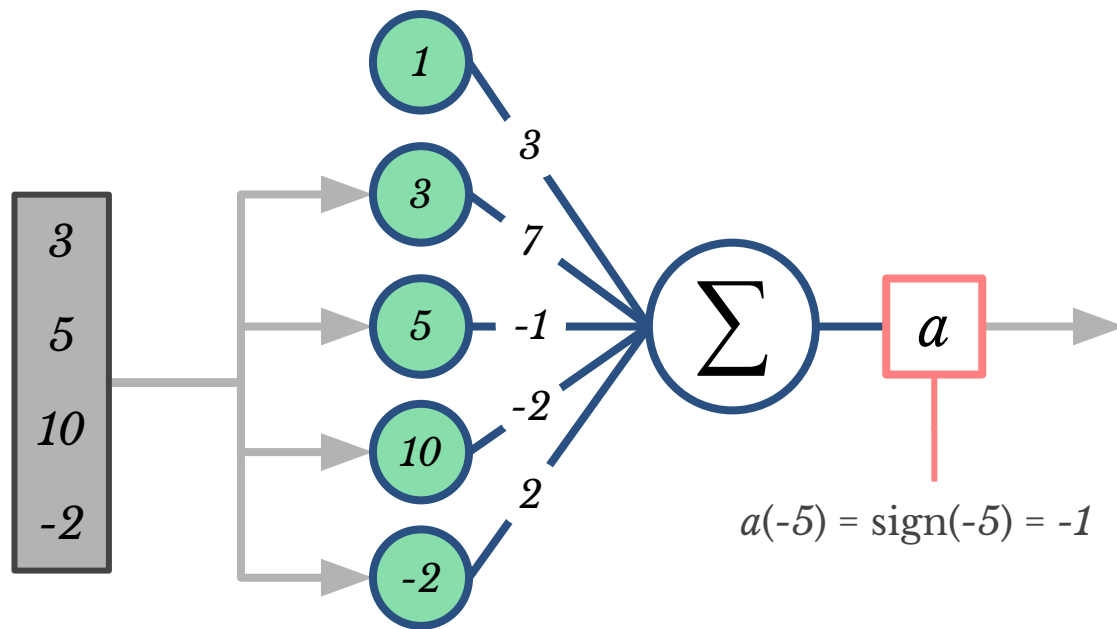
- Say **we know the perceptron weights**, then classifying a new point is easy. For example:





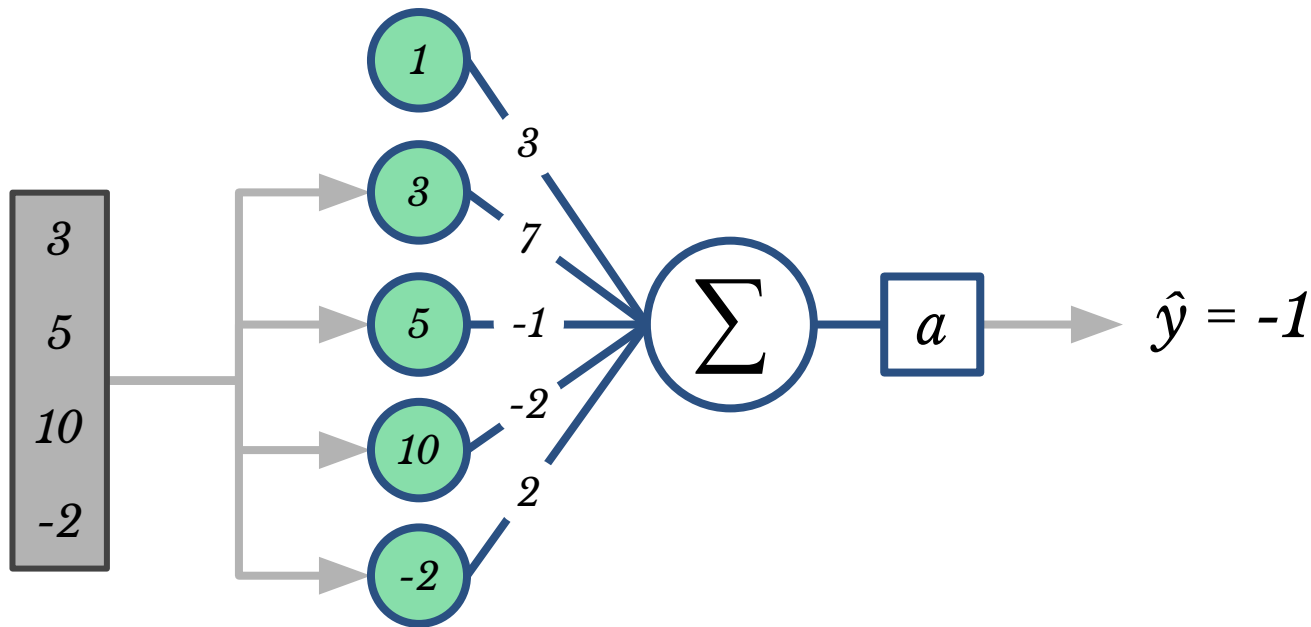
# Linear Classifier Model: Perceptron

- Say **we know the perceptron weights**, then classifying a new point is easy. For example:



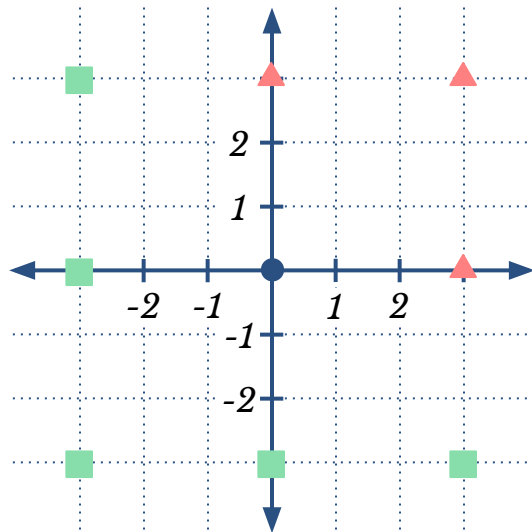
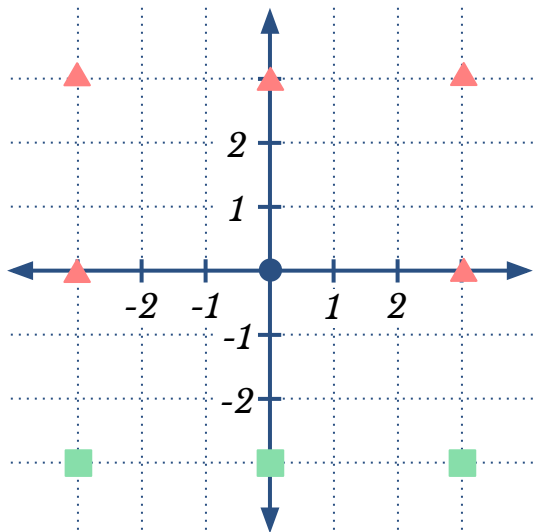
# Linear Classifier Model: Perceptron

- This process is called **Forward Pass**.



# Exercise (*In pairs*)

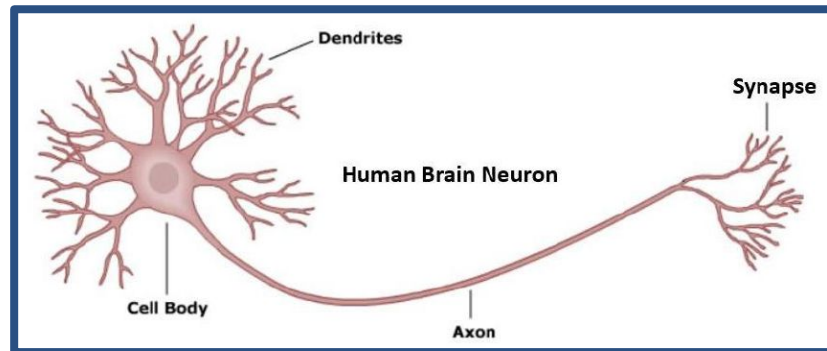
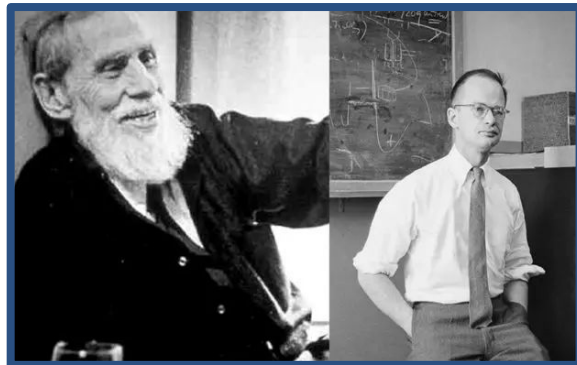
- Find weights  $w = [w_0, w_1, w_2]$  for the lines that separate the **triangles** from the **rectangles**. After that draw the vector  $[w_1, w_2]$  on the plane.



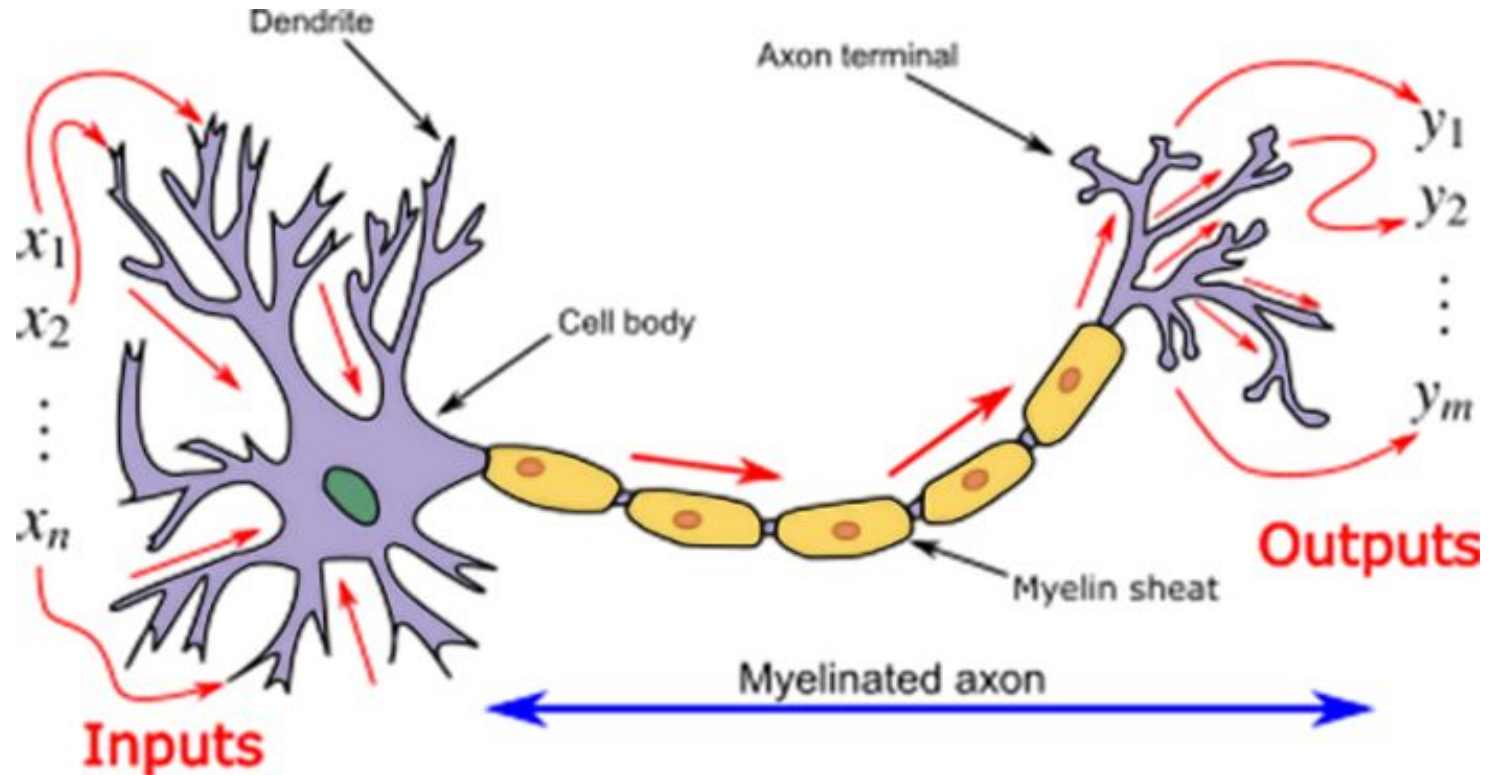
**Feed forward exercise? Just have them go through the neuron.**

# Neurons and the perceptron

- The perceptron model was developed to mathematically model **human neurons**!
- It was proposed **Warren MuCulloch** (neuroscientist) and **Walter Pitts** (logician) in 1943.
- It is considered the first **Artificial Neural Network** model and is the basis of deep learning.

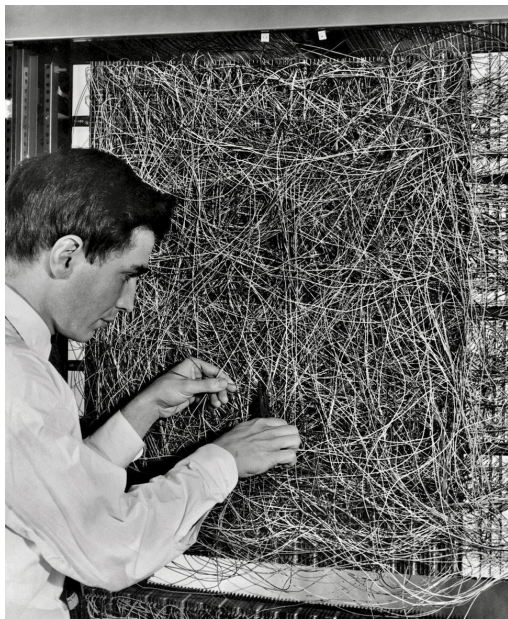


# The Neuron



# Supervised Learning with the Perceptron

- The **perceptron** needs a linear classifier when classifying.
- We need then a way to **compute the perceptron weights**  $w_0, w_1, w_2, \dots, w_D$ .
- We can **learn** them from a training dataset  $S$  using the **Perceptron Algorithm**, first implemented by **Frank Rosenblatt** in 1958.
- We can show that, if  $S$  is **linearly separable**, it always finds an optimal decision boundary.



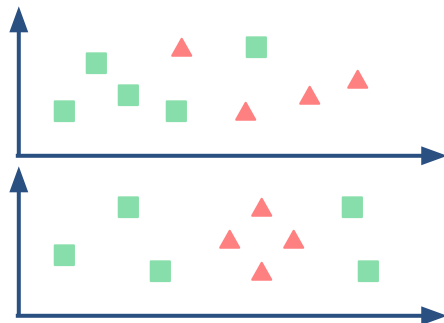
Frank Rosenblatt working on the perceptron algorithm implementation at Cornell in 1958.

## Examples of linear separability in datasets

### Linearly separable dataset

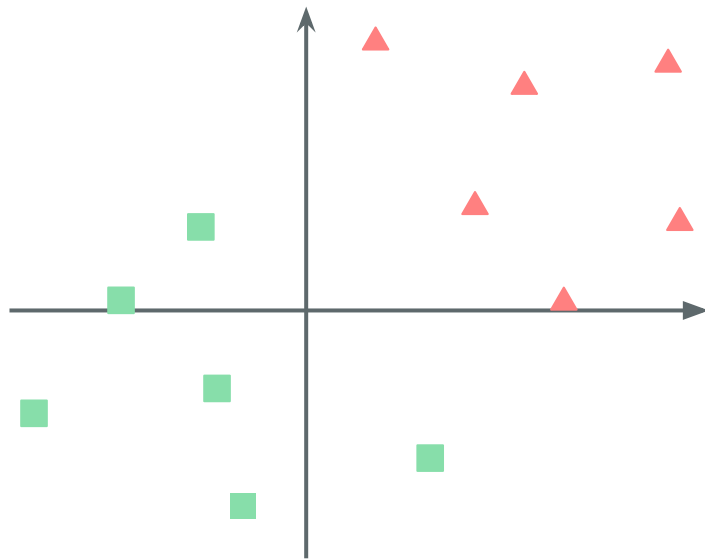


### Non-Linearly separable datasets



# The Perceptron Algorithm

- There are  $n$  points  $x^{(1)}, \dots, x^{(n)}$  in  $D$  dimensions\*, each with a class  $y^{(1)}, \dots, y^{(n)}$  of either  $-1$  or  $+1$ .
- The perceptron algorithm is:
  1. Start with a random  $w$  in  $D+1$  dimensions\*.
  2. For  $i$  in  $1$  to  $n$ , do:
    - a. Find the **predicted class**,  $\hat{y}^{(i)} = a(w^T x^{(i)})$ .
    - b. If  $y^{(i)} = \hat{y}^{(i)}$ , keep  $w$  the same ( $x^{(i)}$  is correctly classified in this case).
    - c. If  $y^{(i)} = +1$  and  $\hat{y}^{(i)} = -1$ : Do  $w = w + x^{(i)}$
    - d. If  $y^{(i)} = -1$  and  $\hat{y}^{(i)} = +1$ : Do  $w = w - x^{(i)}$
  3. Repeat step 2 (go over the dataset again) until all points are correctly classified.

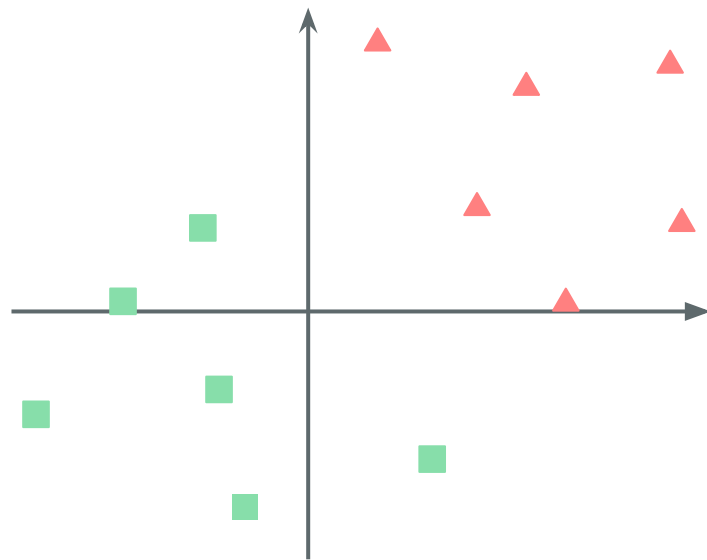


\* Remember that the points are added a new dimension with a  $1$  to account for the bias term, go [here](#) for more details.



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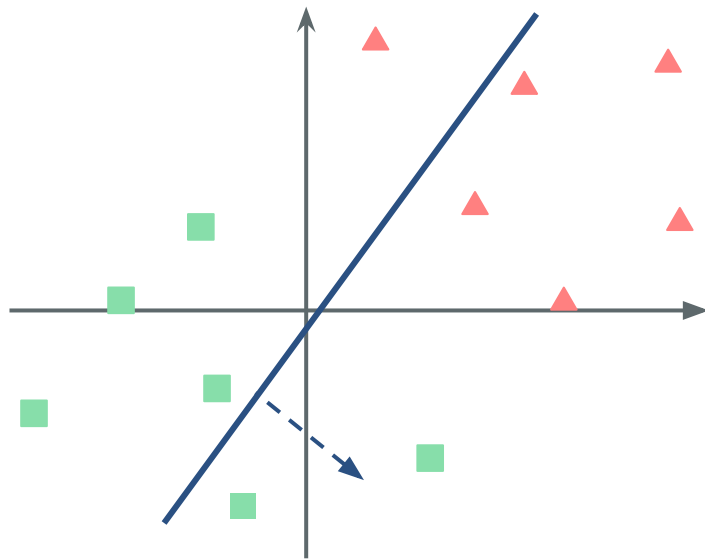


Set **triangles** to have label  $+1$  and **squares** to have label  $-1$ .

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# The Perceptron Algorithm

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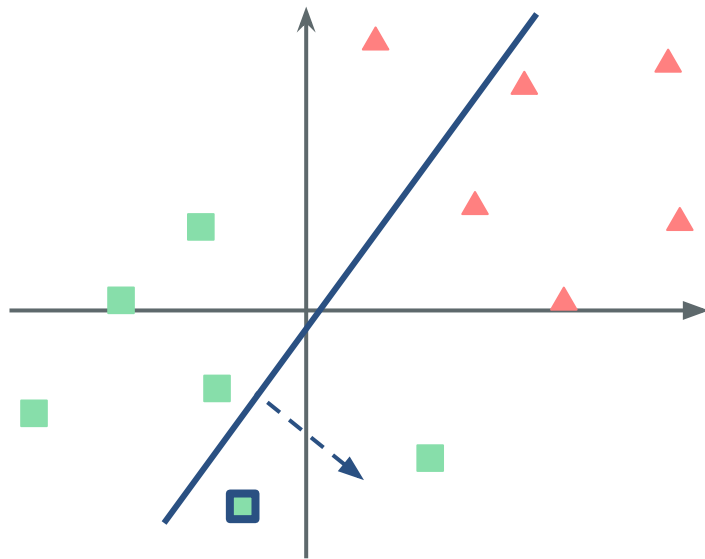


Start with a random  $w$ , which represents a random line.

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# The Perceptron Algorithm

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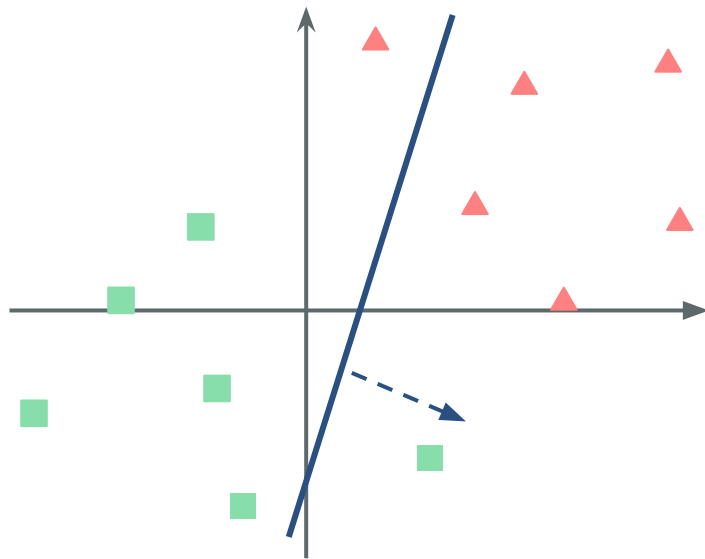


Go over the points, until you find one whose  $\hat{y}_i$  does not match with its true class,  $y_i$ .

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# The Perceptron Algorithm

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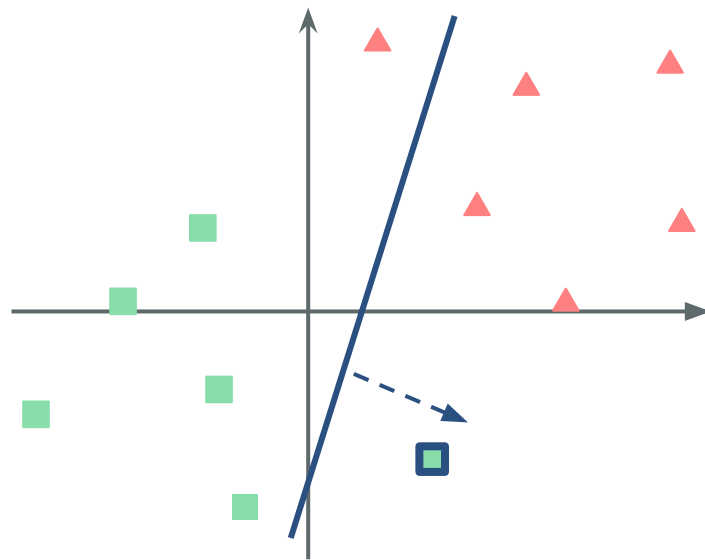


Change  $w$  according to the mismatch.

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# The Perceptron Algorithm

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    - d. If  $y^{(i)} = -1$  and  $\hat{y}^{(i)} = +1$ : Do  $w = w - x^{(i)}$
  3. Repeat step 2 (go over the dataset again) until all points are correctly classified.

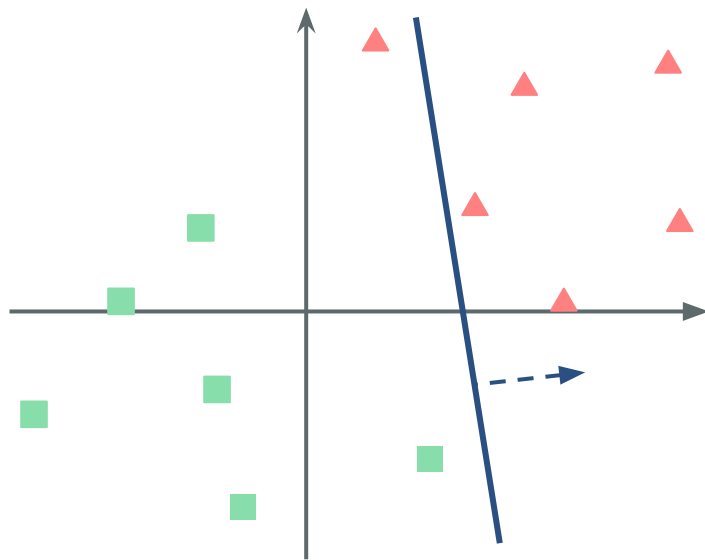


Go to the next data points where there is a mismatch.

\* Remember that the points are added a new dimension with a  $1$  to account for the bias term, go [here](#) for more details.

# The Perceptron Algorithm

- There are  $n$  points  $x^{(1)}, \dots, x^{(n)}$  in  $D$  dimensions\*, each with a class  $y^{(1)}, \dots, y^{(n)}$  of either  $-1$  or  $+1$ .
- The perceptron algorithm is:
  1. Start with a random  $w$  in  $D+1$  dimensions\*.
  2. For  $i$  in  $1$  to  $n$ , do:
    - a. Find the **predicted class**,  $\hat{y}^{(i)} = a(w^T x^{(i)})$ .
    - b. If  $y^{(i)} = \hat{y}^{(i)}$ , keep  $w$  the same ( $x^{(i)}$  is correctly classified in this case).
    - c. If  $y^{(i)} = +1$  and  $\hat{y}^{(i)} = -1$ : Do  $w = w + x^{(i)}$
    - d. If  $y^{(i)} = -1$  and  $\hat{y}^{(i)} = +1$ : Do  $w = w - x^{(i)}$
  3. Repeat step 2 (go over the dataset again) until all points are correctly classified.

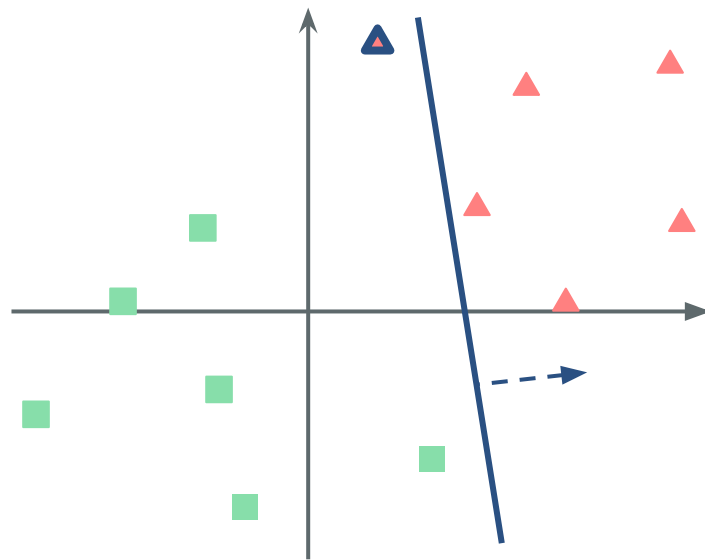


Change  $w$  according to the mismatch.

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# The Perceptron Algorithm

- There are  $n$  points  $x^{(1)}, \dots, x^{(n)}$  in  $D$  dimensions\*, each with a class  $y^{(1)}, \dots, y^{(n)}$  of either  $-1$  or  $+1$ .
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    - d. If  $y^{(i)} = -1$  and  $\hat{y}^{(i)} = +1$ : Do  $w = w - x^{(i)}$
  3. Repeat step 2 (go over the dataset again) until all points are correctly classified.

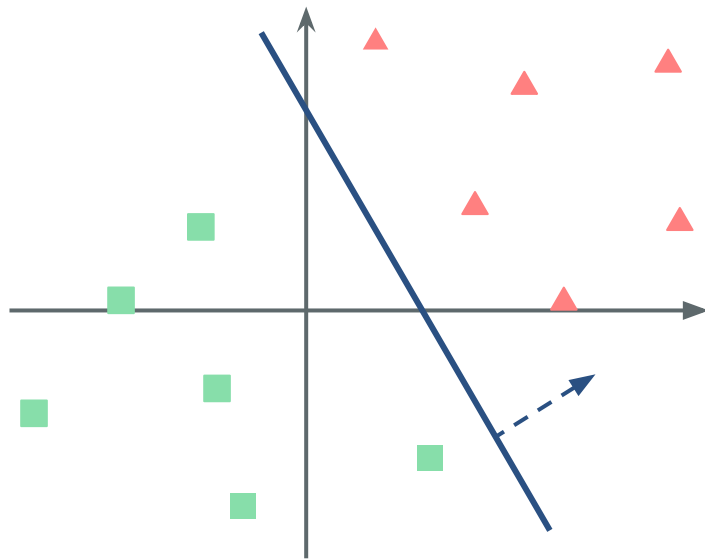


Do that until there are no mismatches.

\* Remember that the points are added a new dimension with a  $1$  to account for the bias term, go [here](#) for more details.

# The Perceptron Algorithm

- There are  $n$  points  $x^{(1)}, \dots, x^{(n)}$  in  $D$  dimensions\*, each with a class  $y^{(1)}, \dots, y^{(n)}$  of either  $-1$  or  $+1$ .
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    - d. If  $y^{(i)} = -1$  and  $\hat{y}^{(i)} = +1$ : Do  $w = w - x^{(i)}$
  3. Repeat step 2 (go over the dataset again) until all points are correctly classified.



Do that until there are no mismatches.

\* Remember that the points are added a new dimension with a  $1$  to account for the bias term, go [here](#) for more details.

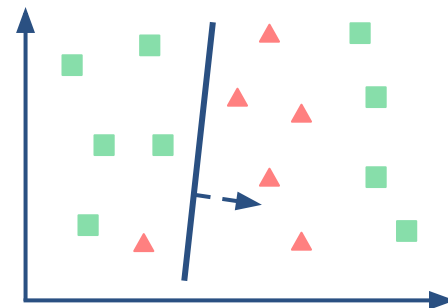


# Measuring classification efficiency

- For non-linearly separable datasets, the perceptron algorithm **won't find** a linear classifier that correctly classifies all points.
- If the classification isn't perfect, we need to find **a measure of how good it is**.
- One possible measure is our **Classification Accuracy (Acc)**:

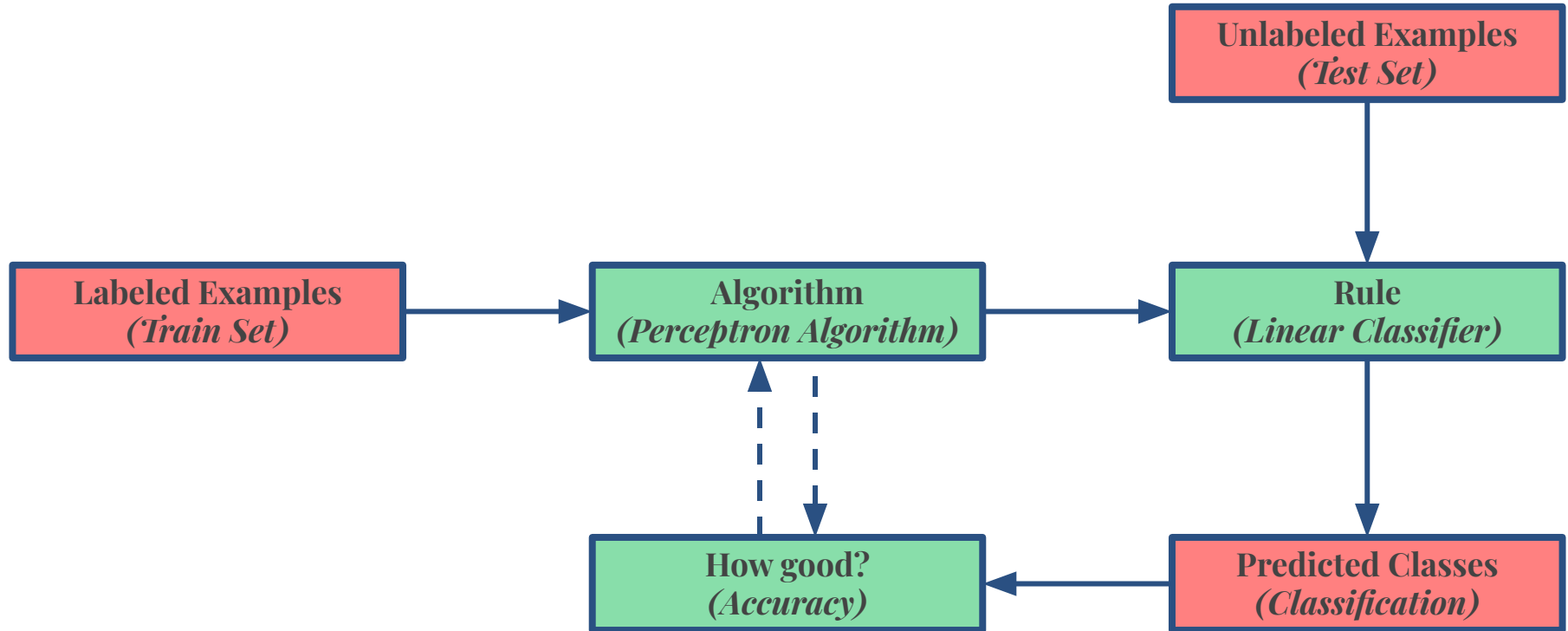
$$\text{Acc} = \frac{\text{Number of correctly classified points}}{\text{Total number of points}}$$

- It is easy to evaluate a model's performance with it, since  $0 \leq \text{Acc} \leq 1$  and the accuracy higher the better.
- However, **Acc** only assumes “discrete” values, since we have a discrete number of points, which **can be a hindrance to many learning algorithms**.
- For that reason we may use a closely related measure called **loss** (*more on it next time*).



If **triangles** are **+1** and **squares** **-1**, the above classifier has an accuracy of  $10/15 = 0.66\%$

# Classification Pipeline for the Perceptron



# Exercise (*In pairs*)

- You have the points  $x_1 = [-1, 0]$ ,  $x_2 = [0, -1]$  and  $x_3 = [1, 1]$ . Assume **rectangles** are of class **-1** and the **triangle** of class **1**. Do the following:
  - Say we start with  $w = [2, -1]$  and  $b = 0$ . Draw on the image above the linear separator that  $w$  and  $b$  generates.
  - Redefine  $w$  to be  $w = [w_0, w_1, w_2]$ . Change the definitions of  $x_1, x_2$  and  $x_3$ , accordingly.
  - Perform each step of the perceptron algorithm to find the a new value  $w$ .
  - Draw on the image above the new linear separator defined by  $w$ .
  - Draw point  $x_4 = [2, -2]$  and classify it using the new value for  $w$ .

