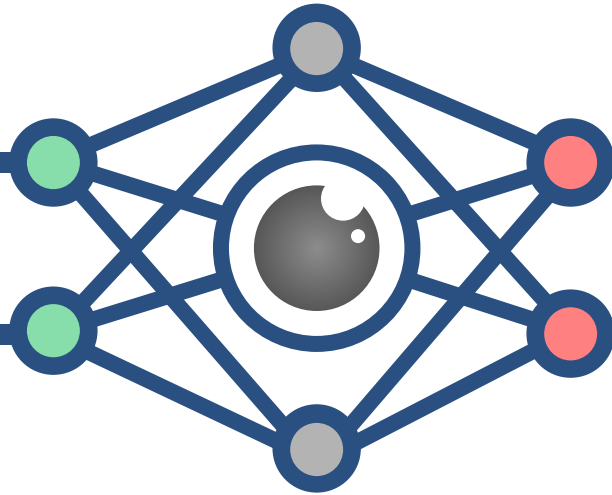


CS3485

Deep Learning for Computer Vision



Lec 2: Linear Classification and Perceptron

Announcements

- Lab 1 is out:
 - Make sure to find a pair to work on it with. If you can't find one, let me know **by Tuesday**.
 - It is an easy lab: you'll just need the basis of Python/Numpy + this slide deck. Feel free to ask me or come to my office hours if you have questions about either Python or Numpy.
 - The instructions carry a little info on what I expect in the report. I'll go easy on the grade this time, so you know what to improve for the next lab.
 - Keep in mind your late day budget (4 for **all labs**).
 - **Suggestion:** Use [Overleaf](#) for starting off with Latex.
- Starting today, we'll start seeing a bit of math. Make sure to check out our preliminary [math materials](#), if you need help over there

Announcements

- Amazon Go is a flop! It seems that it won't grow bigger than small shops

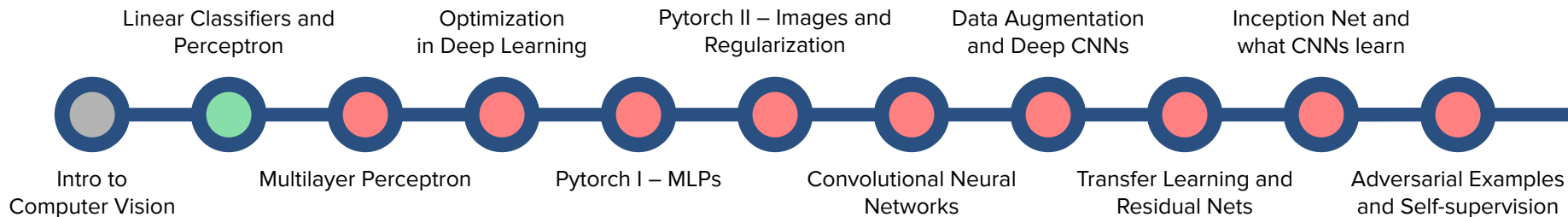
In a statement, Amazon said it will continue using the Just Walk Out technology in Amazon Go stores, at smaller format Fresh stores in the UK, and third-party locations such as certain sports stadiums and college campuses

- On the other hand, another CV technology is taking place: Dash Carts!

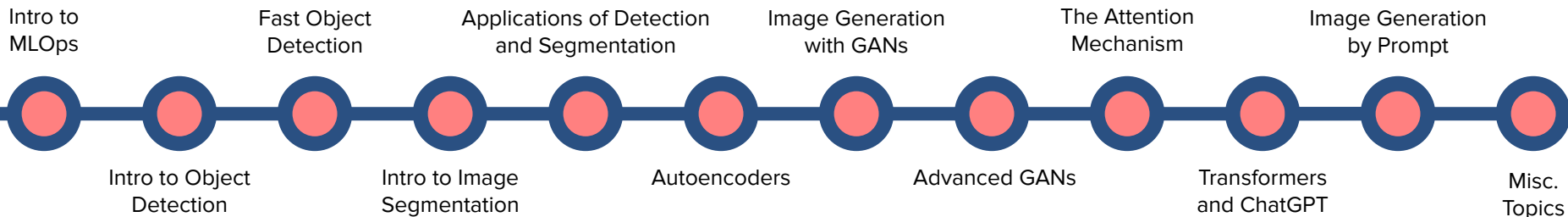


(Tentative) Lecture Roadmap

Basics of Deep Learning



Deep Learning and Computer Vision in Practice



The image classification problem

- The first task in Computer Vision we are tackling is that of Image Classification:

Image Classification the process of recognition, understanding, and grouping of images into preset categories or classes.
- We want a model (or rule) that effectively categorizes (unseen) images into a set of target classes.
- In order to find this rule, we have a set of **labeled example images** at our disposal that we can **train** our model on and **learn** that rule.
- This process of finding such a model from labeled data is called **Supervised Learning**.

Labeled
images of cats



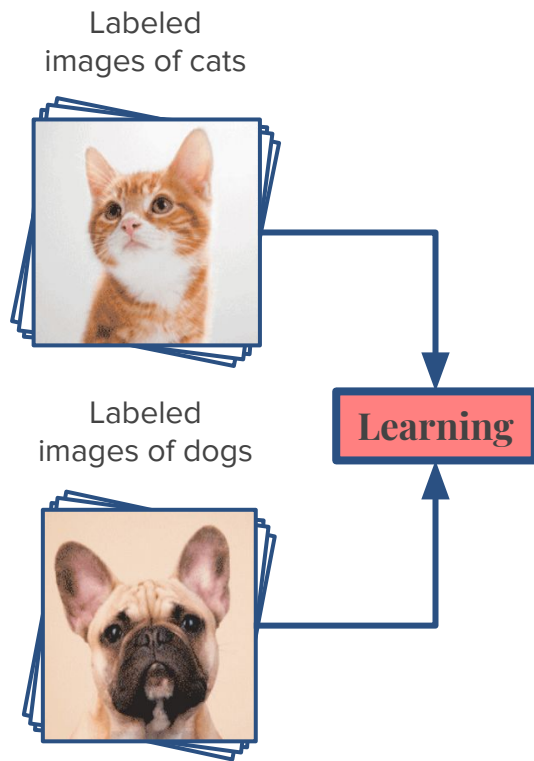
Labeled
images of dogs



The image classification problem

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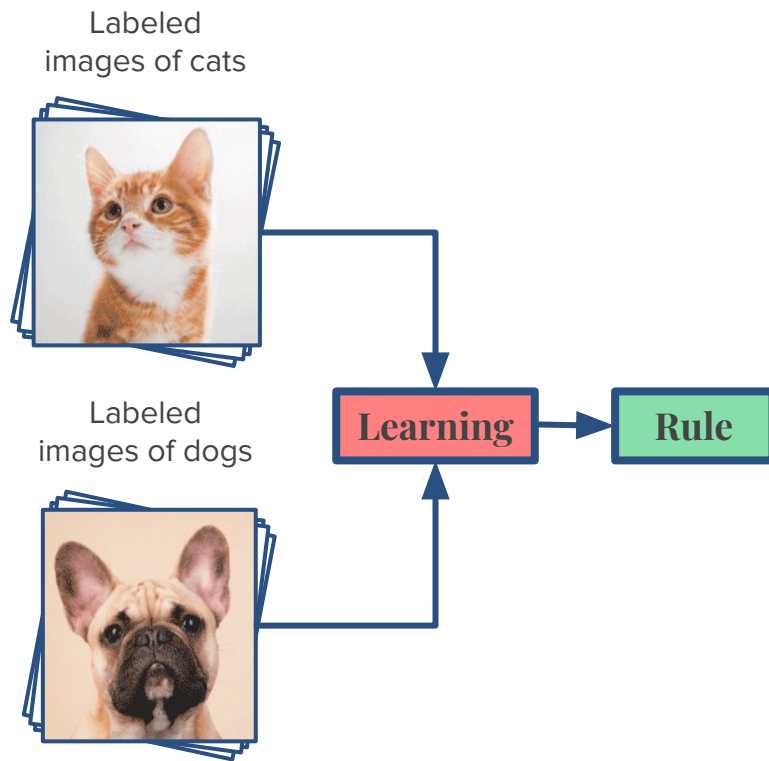
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Unseen (unlabeled) images



(Predicted)
classes

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(Predicted)
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Unseen (unlabeled) images



Rule

(Predicted) classes

Cat

The image classification problem

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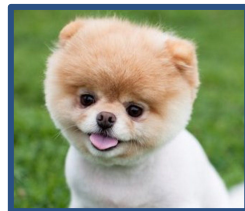
Unseen (unlabeled) images

(Predicted) classes



Rule

Cat



Rule

Dog

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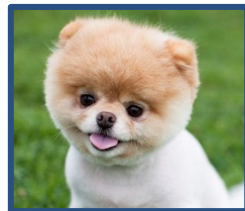
Unseen (unlabeled) images

(Predicted) classes



Rule

Cat



Rule

Dog

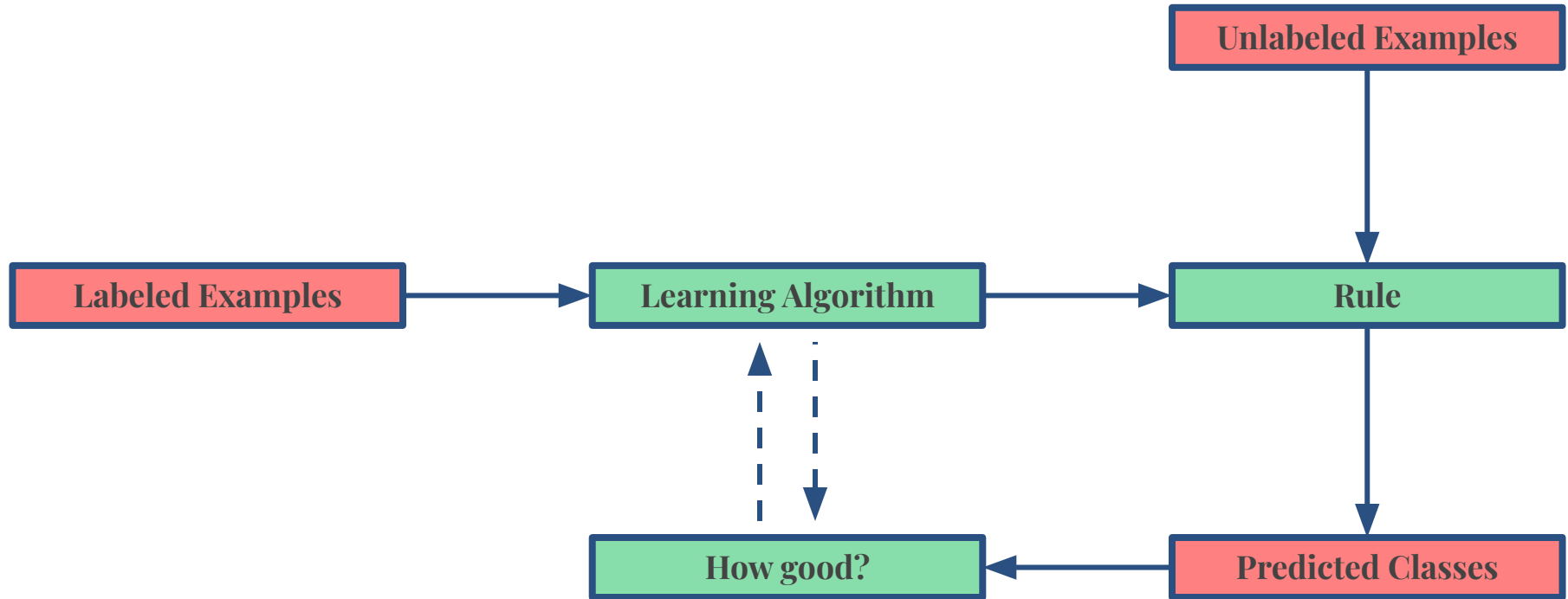


Rule

???

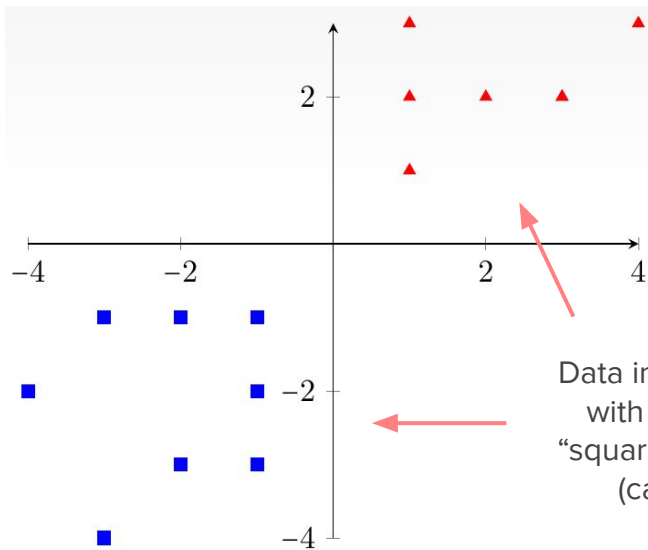
In some settings, the rule can also output “don’t know”!

Supervised Classification Pipeline



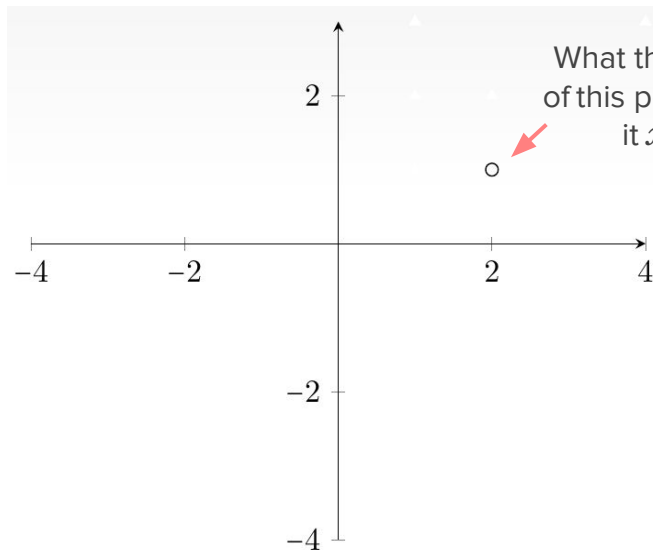
Example of Classification Problem

Original Data



Data in 2 dimensions
with labels either
"square" or "triangle"
(call them y).

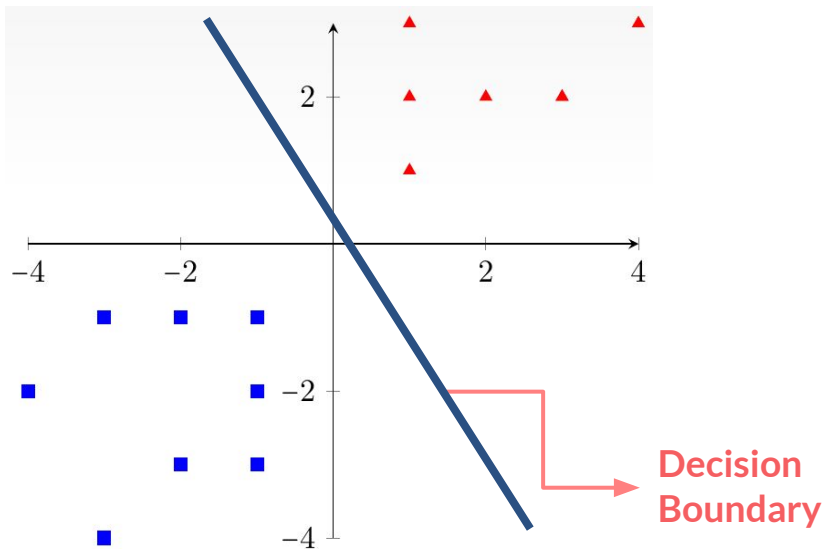
New Unlabeled Datapoint



What the class
of this point (call
it x)?

Linear Classifiers

Original Data

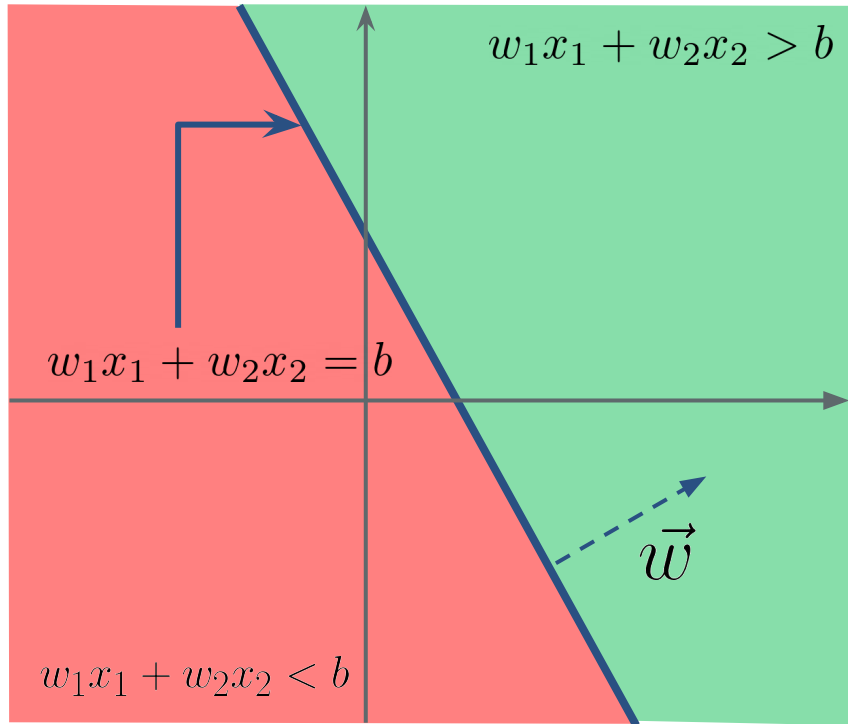


- We need to find a classification rule (**decision boundary**) based on the labeled data.
- Today's choice:

Linear Classifiers

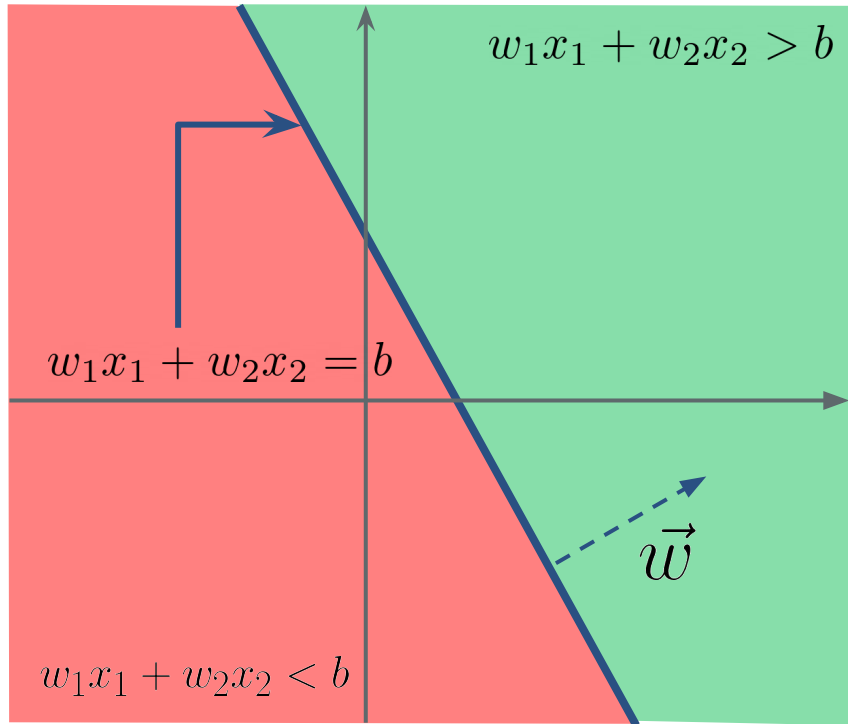
- Which means: “If x is on one side of the line, it is a triangle, otherwise it is a square”.
- How to define the line and its sides **mathematically**, so we can come up with algorithms?

A linear classifier in 2D

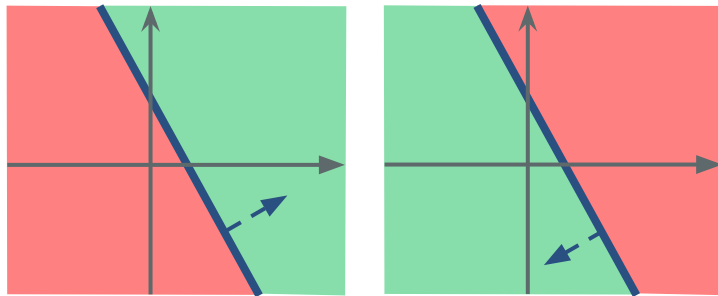


- In 2D, we represent a line using **three numbers**:
 - Two to form a vector $w = [w_1, w_2]$ called **weight vector**;
 - One number called **bias**, b .
- If a new point $x = [x_1, x_2]$ comes in, we just check whether:
$$w_1x_1 + w_2x_2 > b$$
- If **True**, x lies on one side of the plane, if **False** it belongs to the other side
- If equal, it x is exactly **on the line**, and it can be classified as either True or False.

A linear classifier in 2D

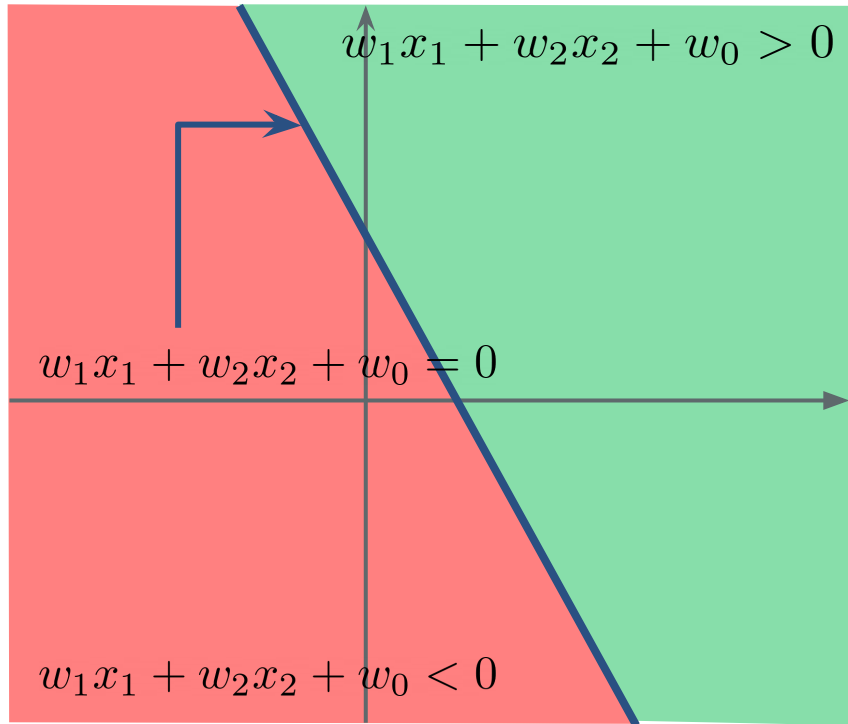


- The **direction of the vector of weights** plays a role here too.
- It always points to the side where the value of $w_1x_1 + w_2x_2 > b$ is **True**:



- The boundary is, however, the same in both cases, and one can change the direction of w by setting $w = -w$.

A linear classifier in 2D



- Now, we can also define the weight vector to include b , making:

$$w = [w_0, w_1, w_2]$$

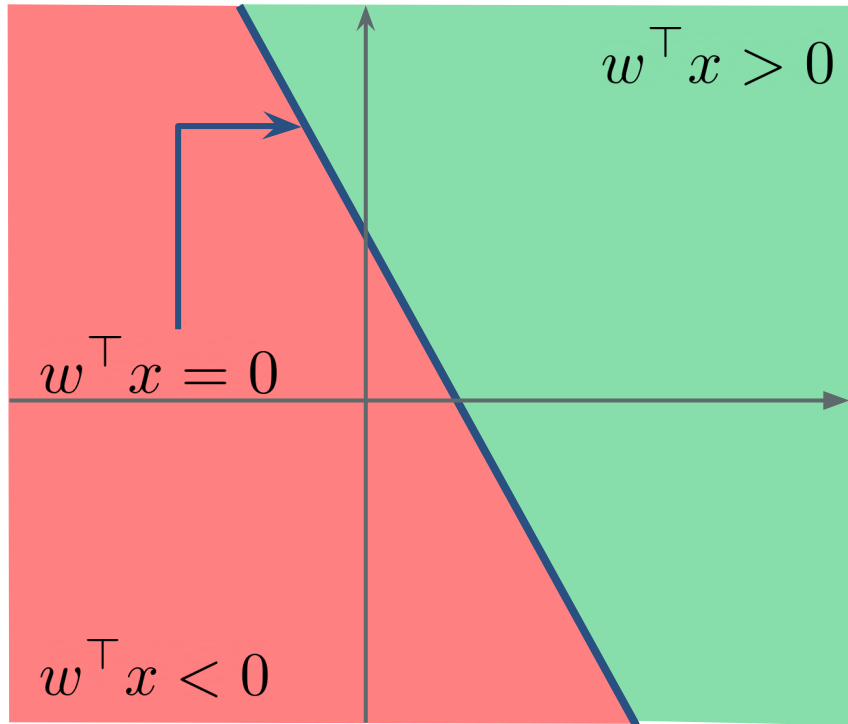
where $b = -w_0$.

- Now, **very importantly**, because of that change in w , we need to add a new dimension with a “1” to all data points x :

$$x = [1, x_1, x_2]$$

- For example, if x was $[5, 7]$ initially, now it will be $[1, 5, 7]$.
- We'll use this change in today's examples.

A linear classifier in 2D



- Finally, we can use the following notation:

$$\begin{aligned}w^T x &= [w_0 \ w_1 \ w_2]^T [1 \ x_1 \ x_2] \\ &= w_0 + w_1 x_1 + w_2 x_2\end{aligned}$$

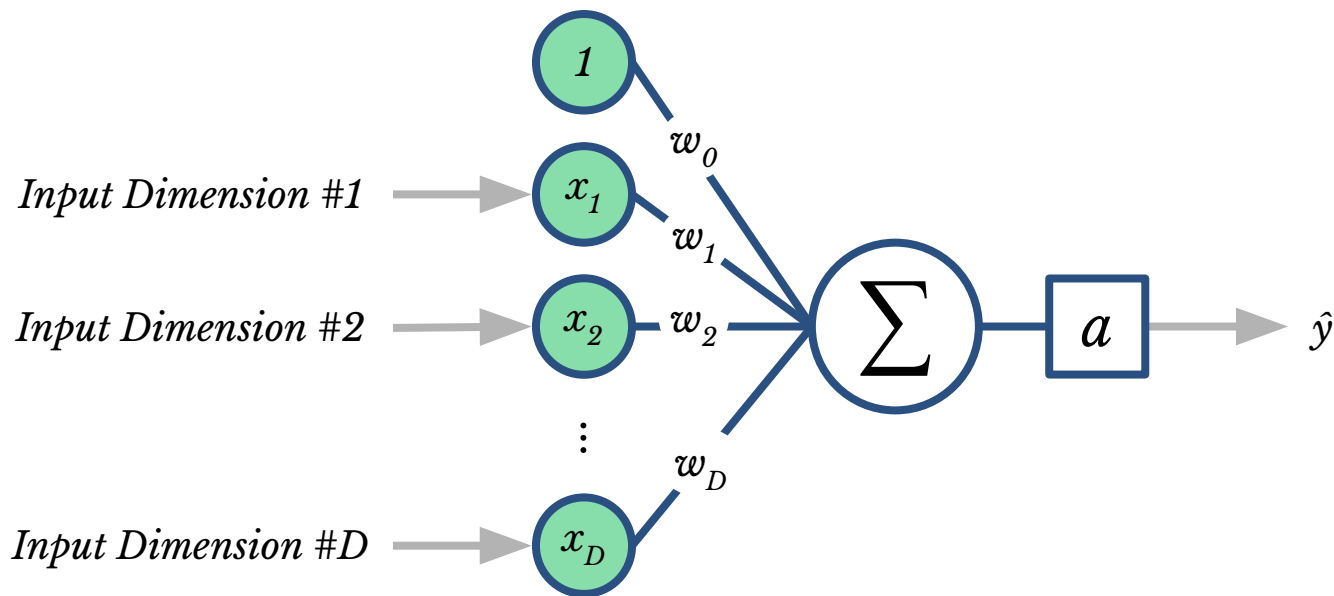
where T is the transpose operation.

- This notation is called the **inner product**, and it is handy since it is the same even if our data points are of $D > 2$ dimensions.
- Mathematically**, the predicted class \hat{y} of a point x by a linear classifier given by w is:

$$\hat{y} = \text{sign}(w^T x) = \begin{cases} 1, & \text{if } w^T x \geq 0 \\ -1, & \text{if } w^T x < 0 \end{cases}$$

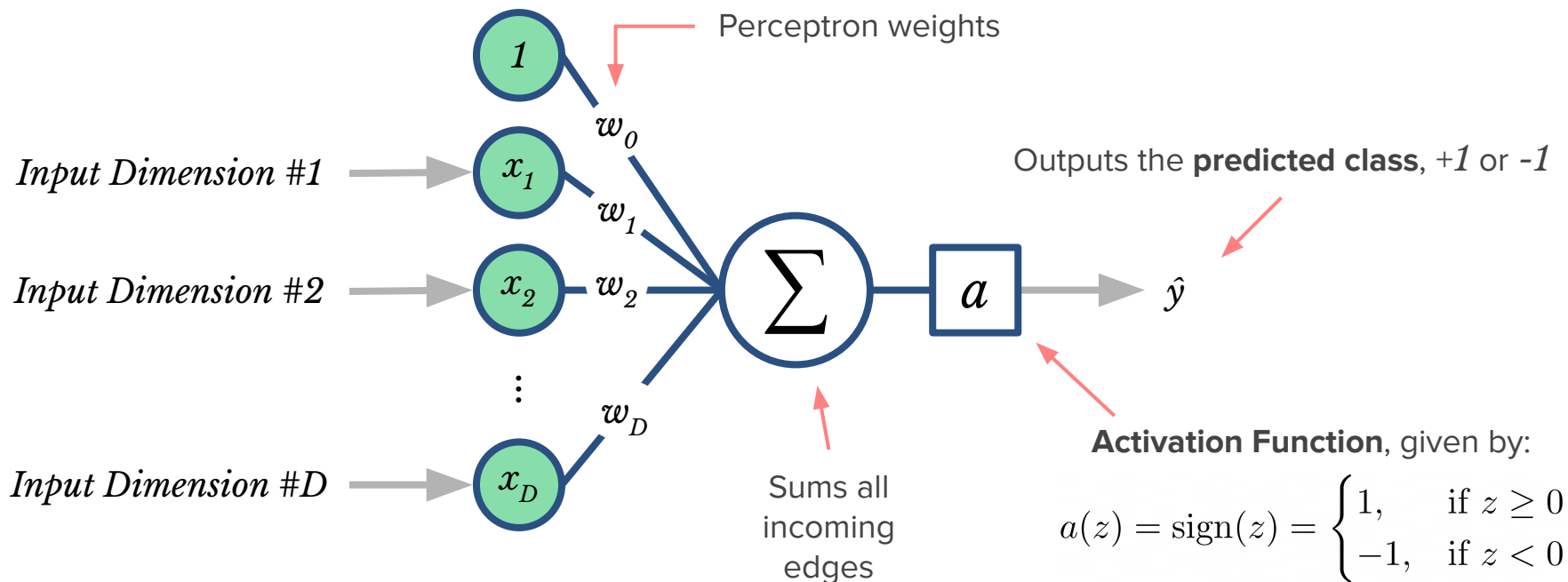
Linear Classifier Model: Perceptron

- Using these concepts, we can build a **model for classification** called **perceptron**!



Linear Classifier Model: Perceptron

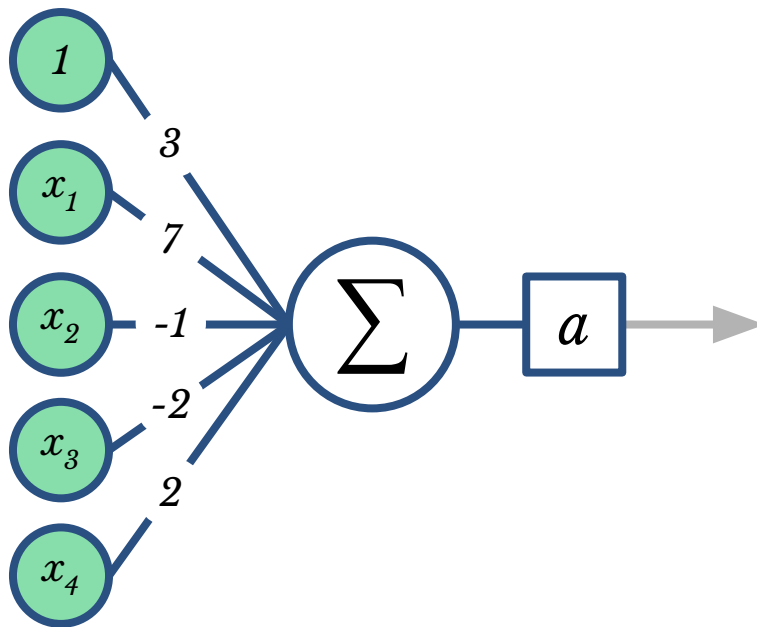
- Below, you have some important nomenclature of the inner workings of the perceptron:



Linear Classifier Model: Perceptron

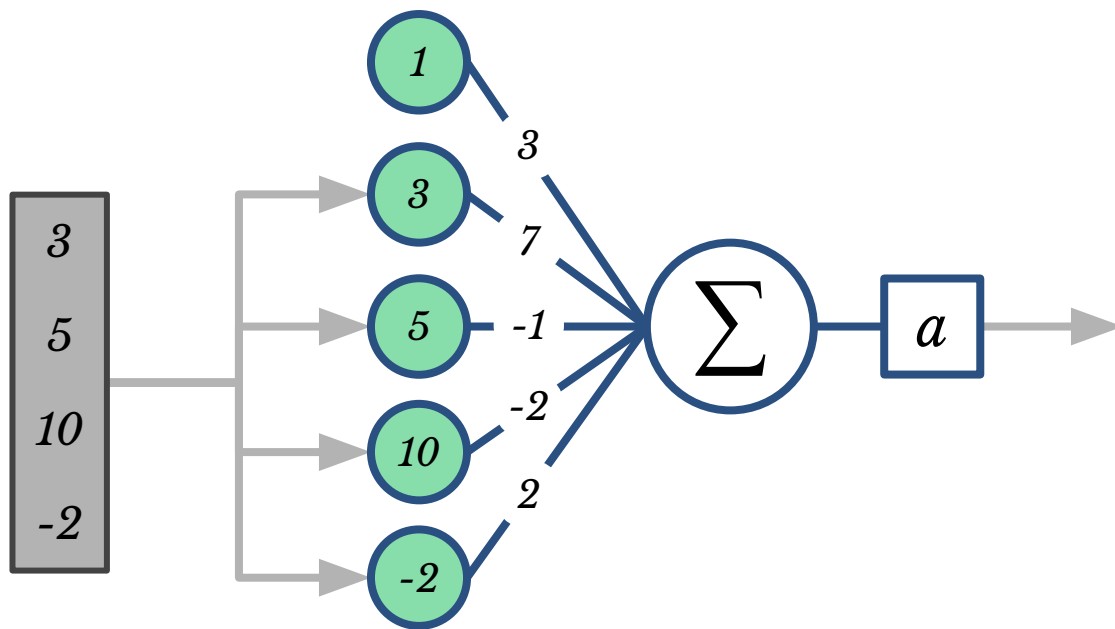
- Say **we know the perceptron weights**, then classifying a new point is easy. For example:

A data point to
be classified



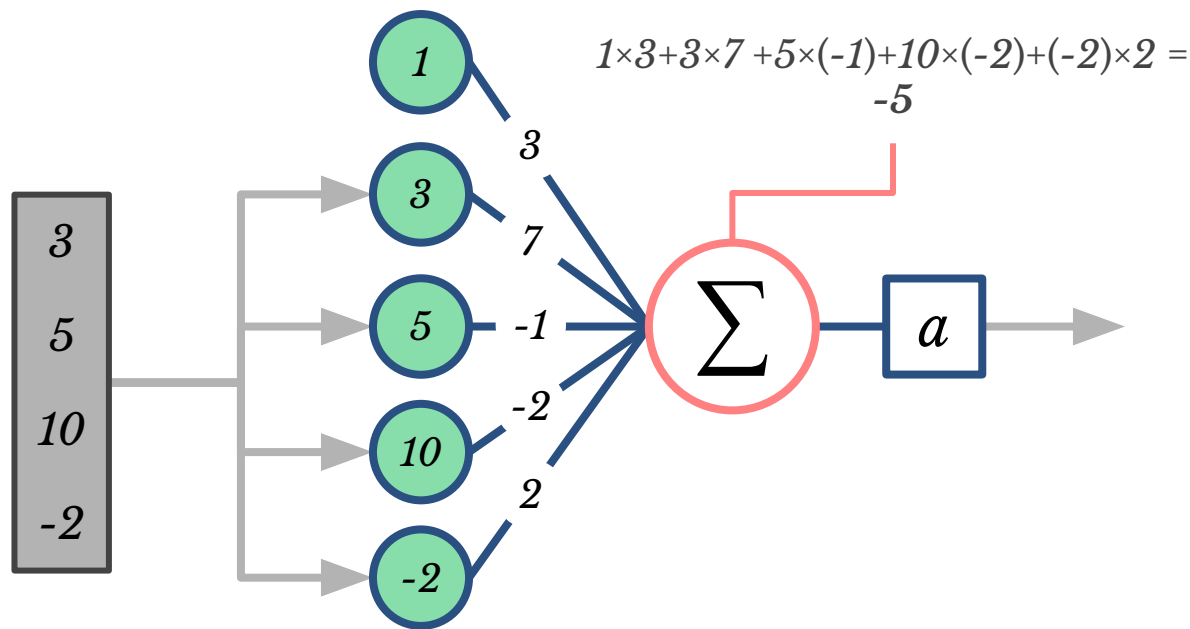
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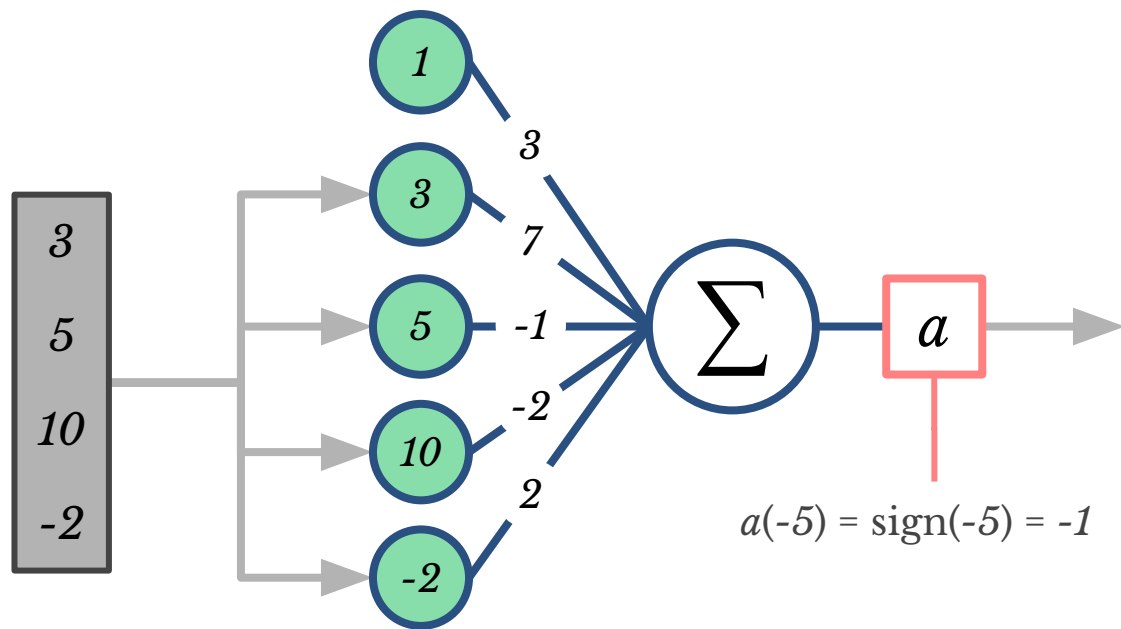
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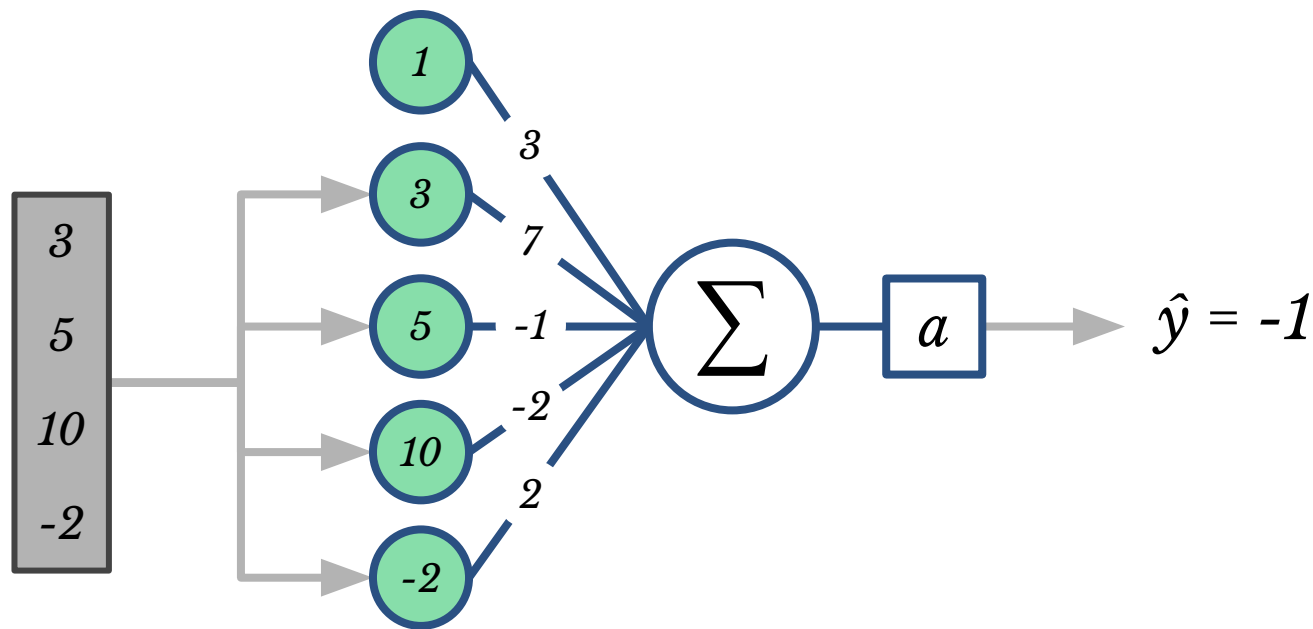
Linear Classifier Model: Perceptron

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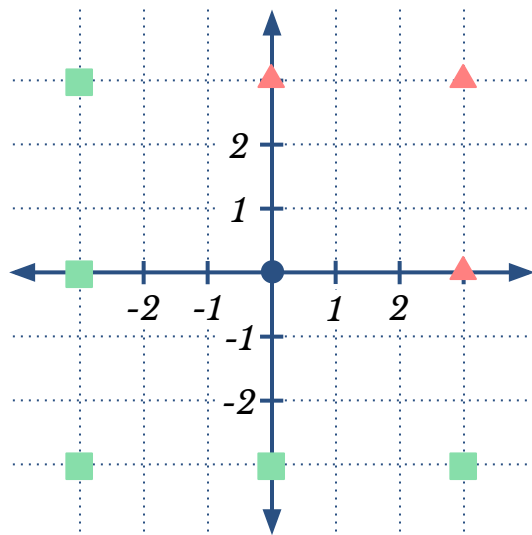
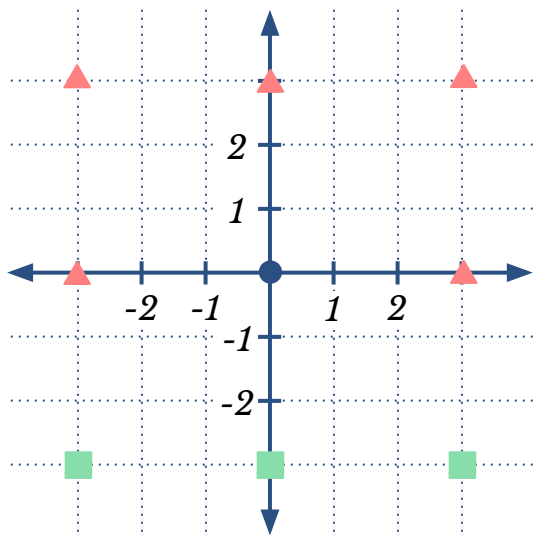
Linear Classifier Model: Perceptron

- This process is called **Forward Pass**.



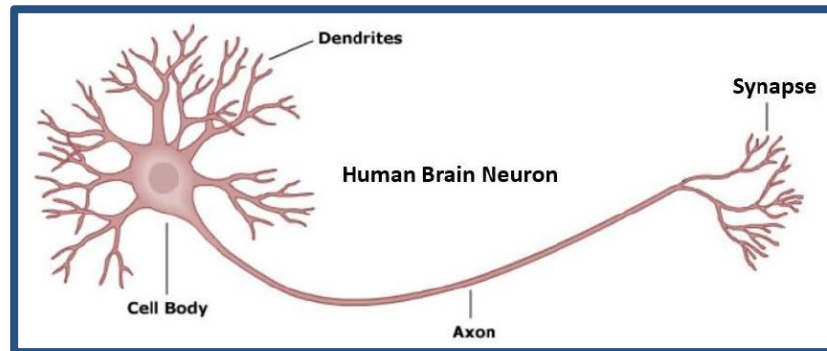
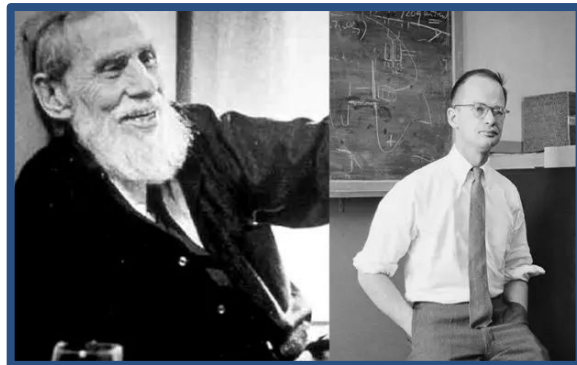
Exercise (*In pairs*)

- Find weights $w = [w_0, w_1, w_2]$ for the lines that separate the **triangles** from the **rectangles**. After that, apply a perceptron model with weights given by w to $x = [2, 2]$ and $x = [-2, -2]$.

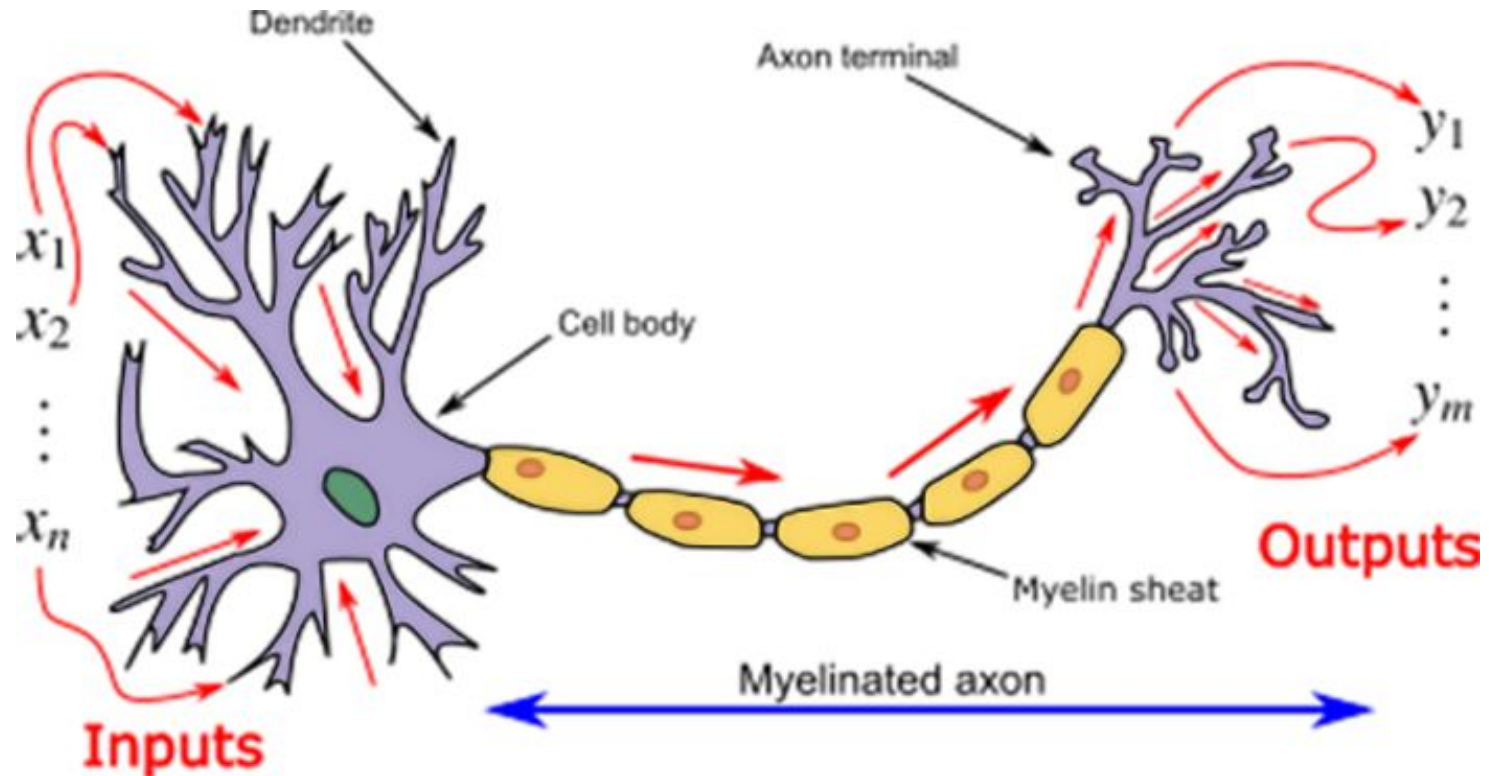


Neurons and the perceptron

- The perceptron model was developed to mathematically model **human neurons**!
- It was proposed **Warren MuCulloch** (neuroscientist) and **Walter Pitts** (logician) in 1943.
- It is considered the first **Artificial Neural Network** model and is the basis of deep learning.

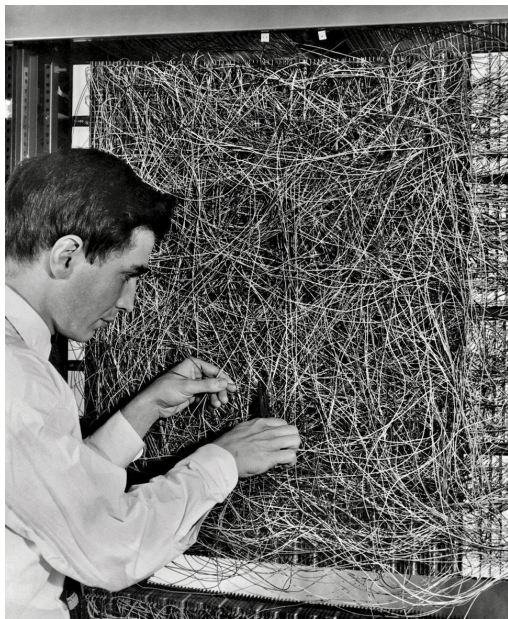


The Neuron



Supervised Learning with the Perceptron

- The **perceptron** needs a linear classifier when classifying.
- We need then a way to **compute the perceptron weights** $w_0, w_1, w_2, \dots, w_D$.
- We can **learn** them from a training dataset S using the **Perceptron Algorithm**, first implemented by **Frank Rosenblatt** in 1958.
- We can show that, if S is **linearly separable**, it always finds an optimal decision boundary.



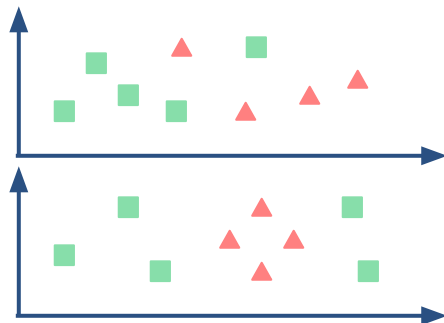
Frank Rosenblatt working on the perceptron algorithm implementation at Cornell in 1958.

Examples of linear separability in datasets

Linearly separable dataset

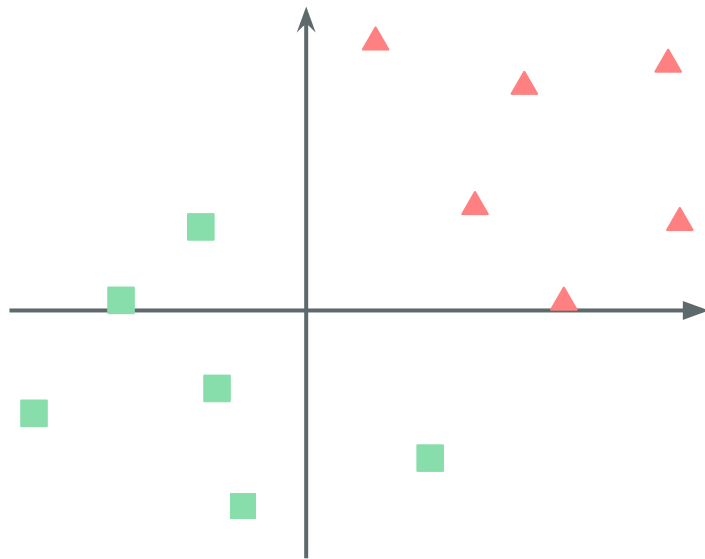


Non-Linearly separable datasets



The Perceptron Algorithm

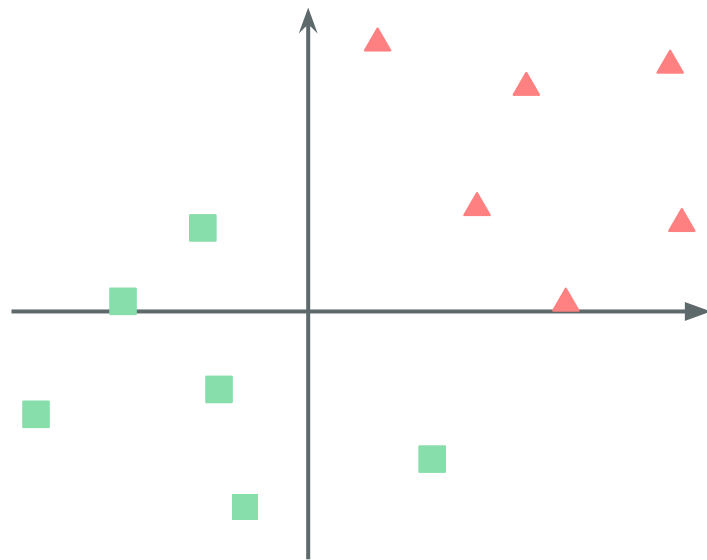
- There are n points $x^{(1)}, \dots, x^{(n)}$ in D dimensions*, each with a class $y^{(1)}, \dots, y^{(n)}$ of either -1 or $+1$.
- The perceptron algorithm is:
 1. Start with a random w in $D+1$ dimensions*.
 2. For i in 1 to n , do:
 - a. Find the **predicted class**, $\hat{y}^{(i)} = a(w^T x^{(i)})$.
 - b. If $y^{(i)} = \hat{y}^{(i)}$, keep w the same ($x^{(i)}$ is correctly classified in this case).
 - c. If $y^{(i)} = +1$ and $\hat{y}^{(i)} = -1$: Do $w = w + x^{(i)}$
 - d. If $y^{(i)} = -1$ and $\hat{y}^{(i)} = +1$: Do $w = w - x^{(i)}$
 3. Repeat step 2 (go over the dataset again) until all points are correctly classified.



* Remember that the points are added a new dimension with a 1 to account for the bias term, go [here](#) for more details.

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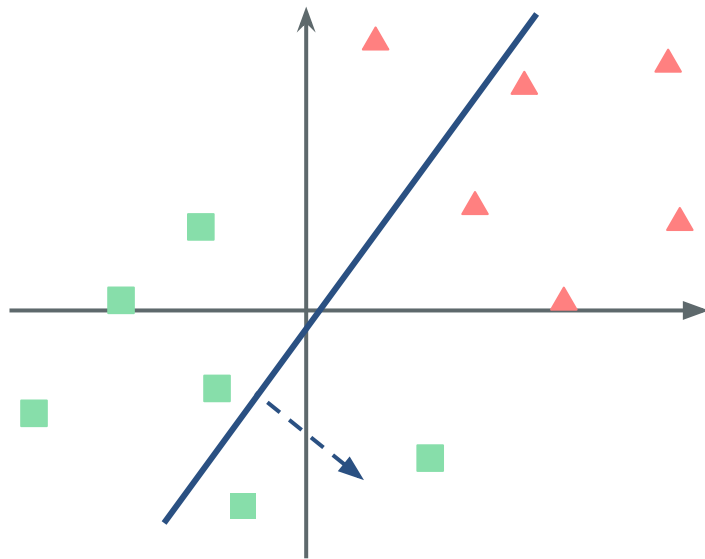


Set **triangles** to have label $+1$ and **squares** to have label -1 .

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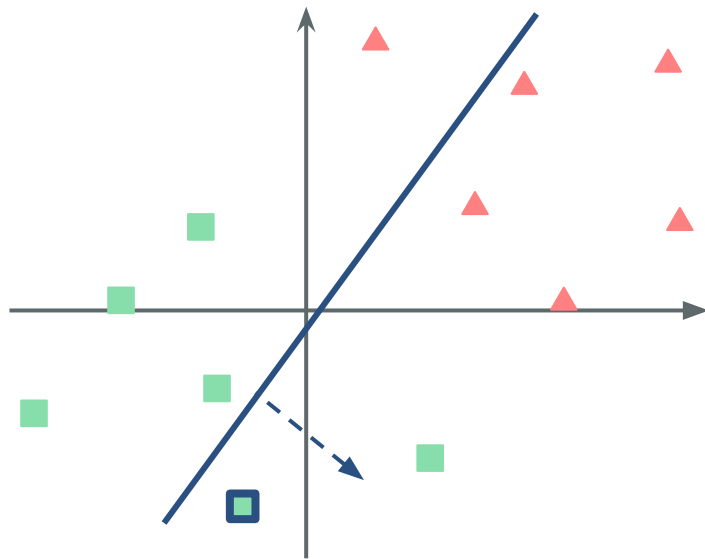


Start with a random w , which represents a random line.

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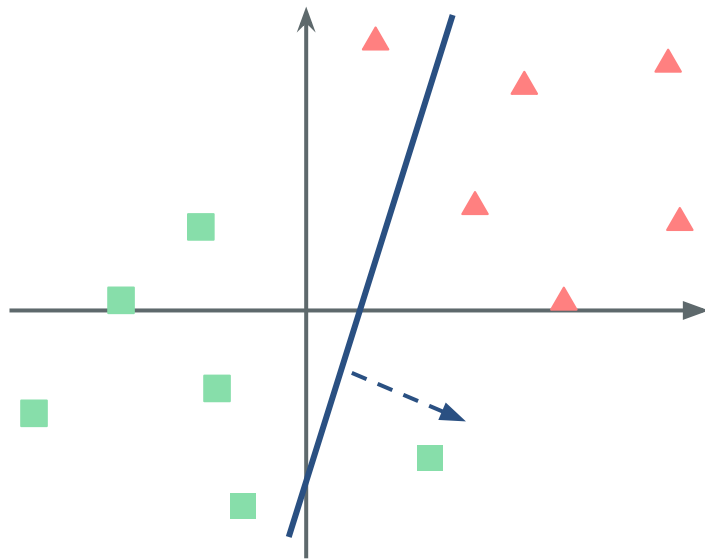


Go over the points, until you find one whose \hat{y}_i does not match with its true class, y_i .

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The Perceptron Algorithm

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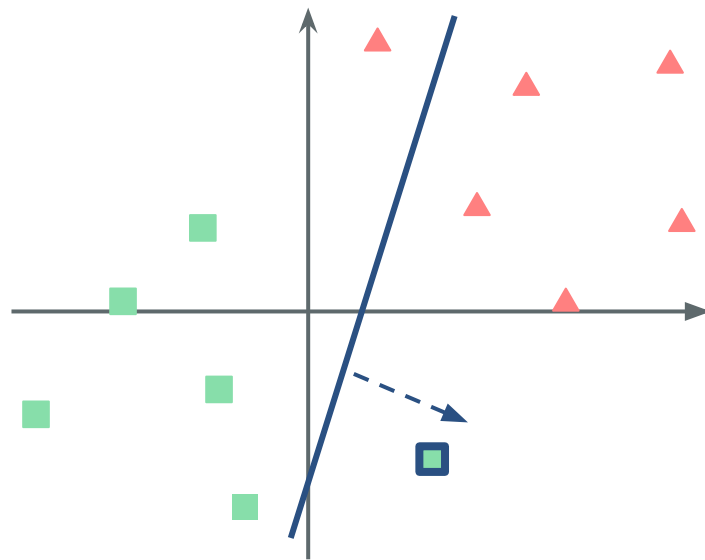


Change w according to the mismatch.

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The Perceptron Algorithm

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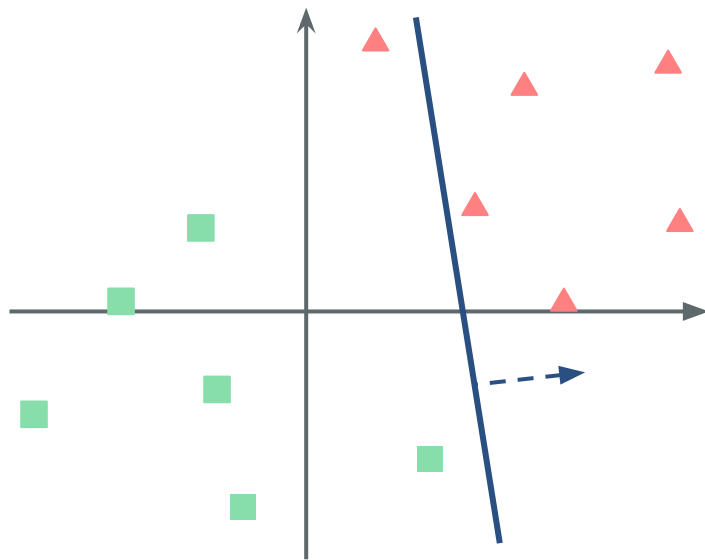


Go to the next data points where there is a mismatch.

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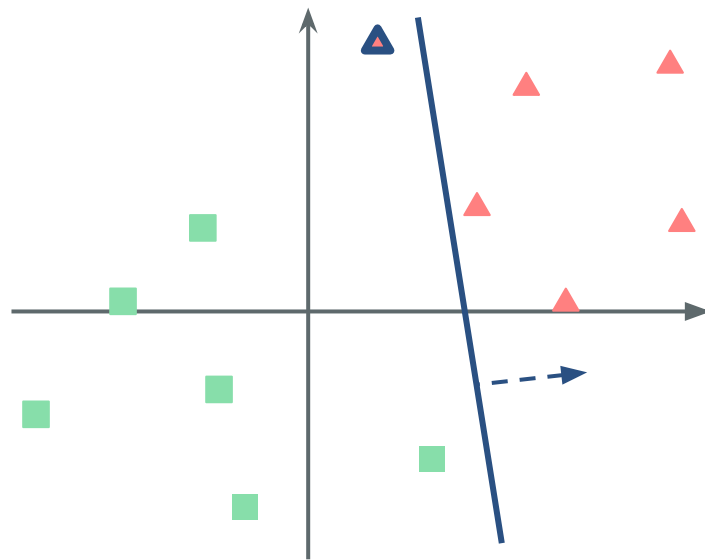


Change w according to the mismatch.

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 2. For i in 1 to n , do:
 - a. Find the **predicted class**, $\hat{y}^{(i)} = a(w^T x^{(i)})$.
 - b. If $y^{(i)} = \hat{y}^{(i)}$, keep w the same ($x^{(i)}$ is correctly classified in this case).
 - c. If $y^{(i)} = +1$ and $\hat{y}^{(i)} = -1$: Do $w = w + x^{(i)}$
 - d. If $y^{(i)} = -1$ and $\hat{y}^{(i)} = +1$: Do $w = w - x^{(i)}$
 3. Repeat step 2 (go over the dataset again) until all points are correctly classified.

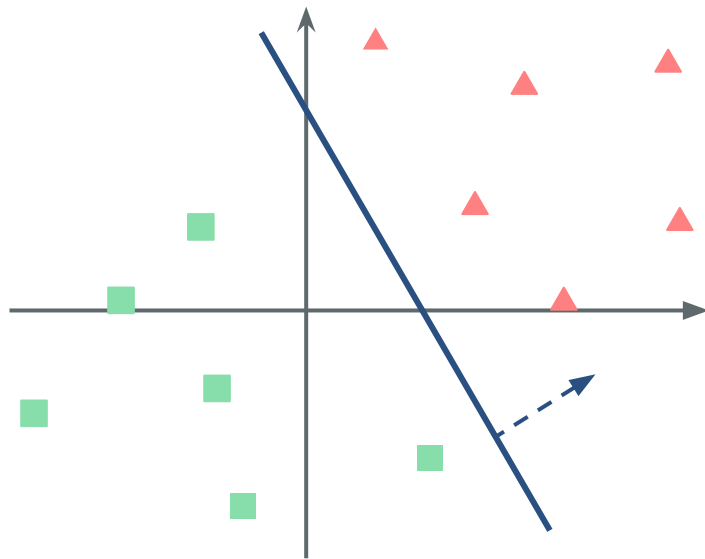


Do that until there are no mismatches.

* Remember that the points are added a new dimension with a 1 to account for the bias term, go [here](#) for more details.

The Perceptron Algorithm

- There are n points $x^{(1)}, \dots, x^{(n)}$ in D dimensions*, each with a class $y^{(1)}, \dots, y^{(n)}$ of either -1 or $+1$.
- The perceptron algorithm is:
 1. Start with a random w in $D+1$ dimensions*.
 2. For i in 1 to n , do:
 - a. Find the **predicted class**, $\hat{y}^{(i)} = a(w^T x^{(i)})$.
 - b. If $y^{(i)} = \hat{y}^{(i)}$, keep w the same ($x^{(i)}$ is correctly classified in this case).
 - c. If $y^{(i)} = +1$ and $\hat{y}^{(i)} = -1$: Do $w = w + x^{(i)}$
 - d. If $y^{(i)} = -1$ and $\hat{y}^{(i)} = +1$: Do $w = w - x^{(i)}$
 3. Repeat step 2 (go over the dataset again) until all points are correctly classified.



Do that until there are no mismatches.

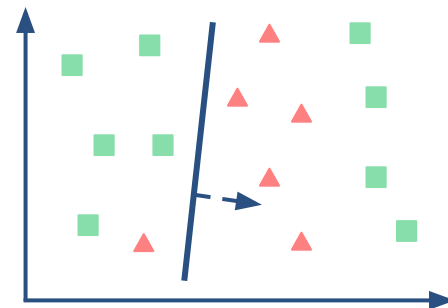
* Remember that the points are added a new dimension with a 1 to account for the bias term, go [here](#) for more details.

Measuring classification efficiency

- For non-linearly separable datasets, the perceptron algorithm **won't find** a linear classifier that correctly classifies all points.
- If the classification isn't perfect, we need to find **a measure of how good it is**.
- One possible measure is our **Classification Accuracy (Acc)**:

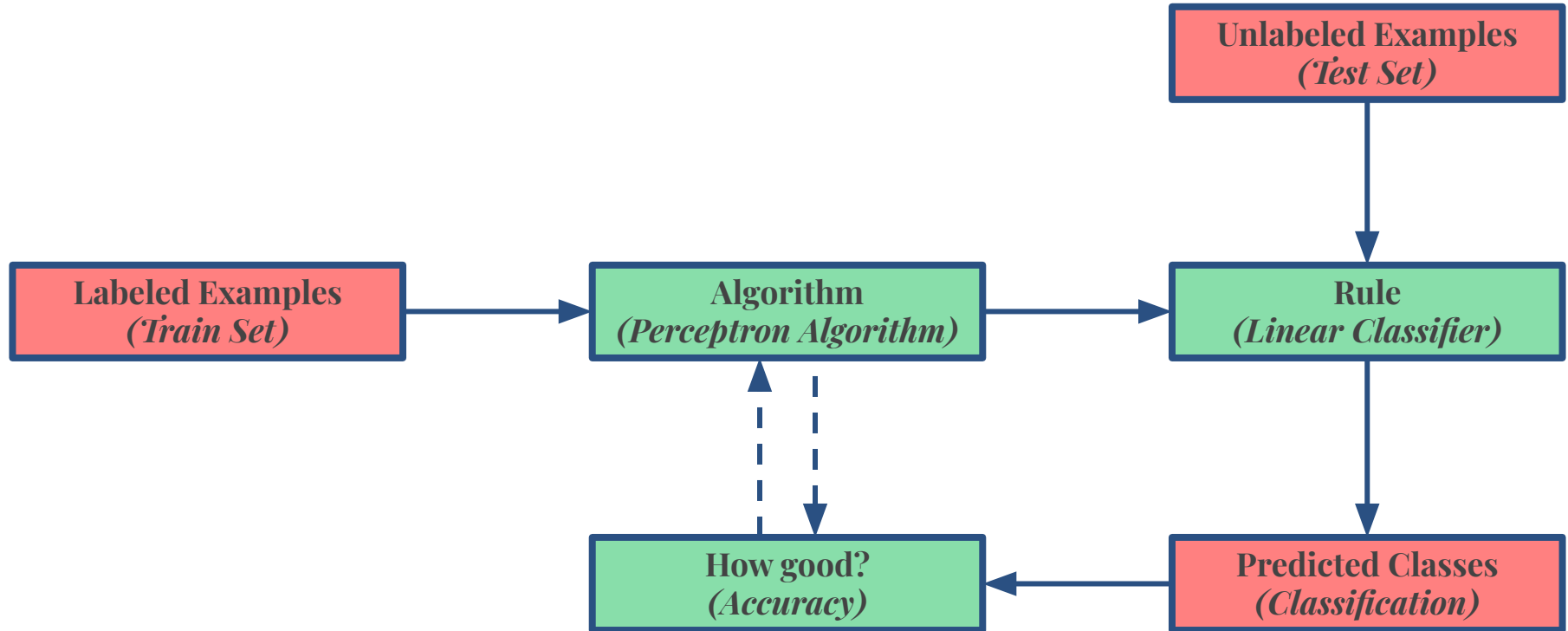
$$\text{Acc} = \frac{\text{Number of correctly classified points}}{\text{Total number of points}}$$

- It is easy to evaluate a model's performance with it, since $0 \leq \text{Acc} \leq 1$ and the accuracy higher the better.
- However, **Acc** only assumes “discrete” values, since we have a discrete number of points, which **can be a hindrance to many learning algorithms**.
- For that reason we may use a closely related measure called **loss** (*more on it next time*).



If **triangles** are **+1** and **squares** **-1**, the above classifier has an accuracy of $10/15 = 0.66\%$

Classification Pipeline for the Perceptron



Exercise (*In pairs*)

- You have the points $x_1 = [-1, 0]$, $x_2 = [0, -1]$ and $x_3 = [1, 1]$. Assume **rectangles** are of class **-1** and the **triangle** of class **1**. Do the following:
 - Say we start with $w = [2, -1]$ and $b = 0$. Draw on the image above the linear separator that w and b generates.
 - Redefine w to be $w = [w_o, w_p, w_2]$. Change the definitions of x_1, x_2 and x_3 , accordingly.
 - Perform each step of the perceptron algorithm to find the a new value w .
 - Draw on the image above the new linear separator defined by w .
 - Draw point $x_4 = [2, -2]$ and classify it using the new value for w .

