Project: Kinematics Pick & Place

Kinematic Analysis

1. Run the forward_kinematics demo and evaluate the kr210.urdf.xacro file to perform kinematic analysis of Kuka KR210 robot and derive its DH parameters.

This is the table containing the Modified DH parameters.

$\mathbf{i}$	α_{i-1}	$\mathbf{a}_{i-1}$	$\mathbf{d}_i$	θ_i
1	0	0	0.75	
2	$-\frac{\pi}{2}$	0.35	0	$\theta_2 = \theta_2 - \frac{\pi}{2}$
3	0	1.25	0	
4	$-\frac{\pi}{2}$	-0.054	1.50	
5	$\frac{\pi}{2}$	0	0	
6	$-\frac{\pi}{2}$	0	0	
7	0	0	0.303	$\theta_7 = 0$

2. Using the DH parameter table you derived earlier, create individual transformation matrices about each joint. In addition, also generate a generalized homogeneous transform between base_link and gripper_link using only end-effector(gripper) pose.

$$T_1^0 = \begin{bmatrix} cos(\theta_1) & -sin(\theta_1) & 0 & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_2^1 = \begin{bmatrix} sin(\theta_2) & cos(\theta_2) & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ cos(\theta_2) & -sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} cos(\theta_3) & -sin(\theta_3) & 0 & 1.25 \\ sin(\theta_3) & cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_4^3 = \begin{bmatrix} cos(\theta_4) & -sin(\theta_4) & 0 & -0.054 \\ 0 & 0 & 1 & 1.50 \\ -sin(\theta_4) & -cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} cos(\theta_5) & -sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sin(\theta_5) & cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_6^5 = \begin{bmatrix} cos(\theta_6) & -sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin(\theta_6) & -cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_G^6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Given the target orientation $roll$, $pitch$ and yaw we have the following rotation matrices:

$$R_{roll} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(roll) & -sin(roll) \\ 0 & sin(roll) & cos(roll) \end{bmatrix}$$

$$R_{pitch} = \begin{bmatrix} cos(pitch) & 0 & sin(pitch) \\ 0 & 1 & 0 \\ -sin(pitch) & 0 & cos(pitch) \end{bmatrix}$$

$$R_{yaw} = \begin{bmatrix} cos(yaw) & -sin(yaw) & 0 \\ sin(yaw) & cos(yaw) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And thus:

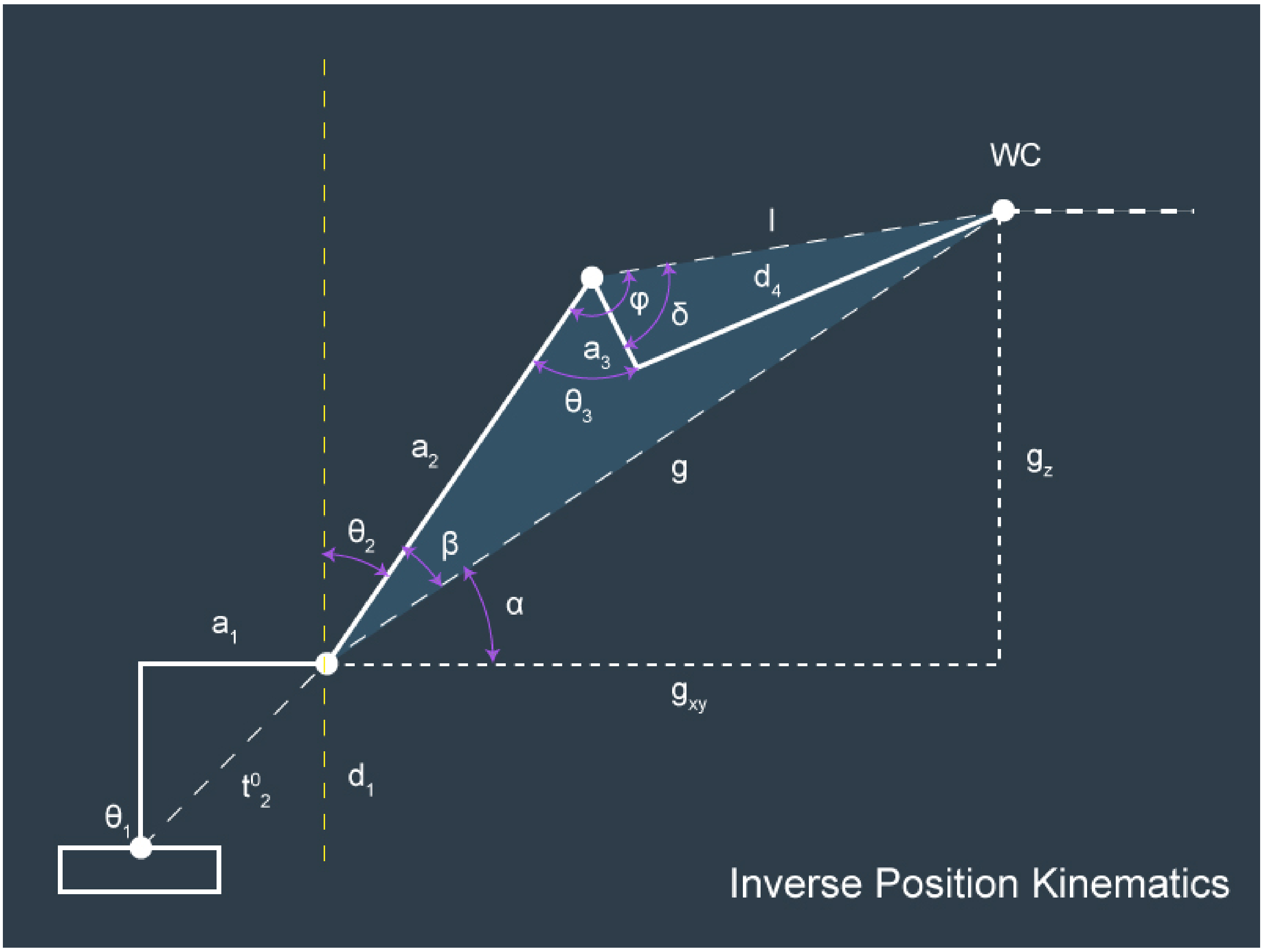
$$R_6^0 = R_{yaw} \cdot R_{pitch} \cdot R_{roll} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \tag{2}$$

Given also the target position p_x, p_y and p_z , the complete target matrix can now be defined as follows:

$$T_G^0 = \left[\begin{array}{c|c} & p_x \\ R_6^0 & p_y \\ & p_z \\ \hline 0 & 1 \end{array}\right] = \begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

3. Decouple Inverse Kinematics problem into Inverse Position Kinematics and inverse Orientation Kinematics; doing so derive the equations to calculate all individual joint angles.

Inverse Position Kinematics



First we need to calculate the Wrist Center vector WC.

$$WC = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - d_7 \cdot R_6^0 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \implies \begin{cases} WC_x = p_x - (d_7 \cdot n_x) \\ WC_y = p_y - (d_7 \cdot n_y) \\ WC_z = p_z - (d_7 \cdot n_z) \end{cases} \tag{4}$$

Now that we have the WC, calculating θ_1 is fairly trivial.

$$\theta_1 = atan2(WC_y, WC_x) \tag{5}$$

Before calculating θ_2 and θ_3 we need to calculate the length of the sides of the triangle $[a_2, l, g]$:

$$l = \sqrt{a_3^2 + d_4^2} \tag{6}$$

$$g = WC - t_2^0 = WC - \begin{bmatrix} a_1 \cdot cos(\theta_1) \\ a_1 \cdot sin(\theta_1) \\ d_1 \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

$$|g| = \sqrt{g_x^2 + g_y^2 + g_z^2}$$

Using the cosine rule...

$$A^2 = B^2 + C^2 - 2BCcos(\alpha)$$

$$\alpha = arccos\left(\frac{B^2 + C^2 - A^2}{2BC}\right)$$

... and the conversion into $atan2$ as follows ...

$$arccos(x) = atan2(\sqrt{1-x^2}, x)$$

We can now start finding θ_3:

$$\begin{aligned} \varphi &= atan2(d_4, a_3) \\ D &= \left(\frac{l^2 + a_2^2 - |g|^2}{2la_2}\right) \\ \delta &= atan2(\sqrt{1-D^2}, D) \\ \theta_3 &= \varphi - \delta \end{aligned} \tag{7}$$

And also θ_2:

$$\begin{aligned} g_{xy} &= \sqrt{g_x^2 + g_y^2} \\ \alpha &= atan2(g_z, g_{xy}) \\ D &= \left(\frac{|g|^2 + a_2^2 - l^2}{2|g|a_2}\right) \\ \beta &= atan2(\sqrt{1-D^2}, D) \\ \theta_2 &= \frac{\pi}{2} - \alpha - \beta \end{aligned} \tag{8}$$

Inverse Orientation Kinematics

Given our target matrix R_6^0 and R_3^0 created using θ_1, θ_2 and θ_3 we can find R_6^3:

$$R_6^3 = (R_3^0)^T \cdot R_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{9}$$

From the Forward Kinematics we can also find the following:

$$R_6^3 = \begin{bmatrix} -sin(\theta_4)*sin(\theta_6)+cos(\theta_4)*cos(\theta_5)*cos(\theta_6) & -sin(\theta_4)*cos(\theta_6)-sin(\theta_6)*cos(\theta_4)*cos(\theta_5) & sin(\theta_5)*cos(\theta_4) \\ sin(\theta_5)*cos(\theta_6) & -sin(\theta_5)*sin(\theta_6) & cos(\theta_5) \\ -sin(\theta_4)*cos(\theta_5)*cos(\theta_6)-sin(\theta_6)*cos(\theta_4) & sin(\theta_4)*sin(\theta_6)*cos(\theta_5)-cos(\theta_4)*cos(\theta_6) & sin(\theta_4)*sin(\theta_5) \end{bmatrix} \tag{10}$$

So, first we calculate θ_5:

$$\theta_5 = atan2(\sqrt{r_{13}^2 + r_{33}^2}, r_{23}) \tag{11}$$

This gives us 2 two possibilities:

$$\begin{aligned} &\text{if } sin(\theta_5) < 0 \begin{cases} \theta_4 = atan2(-r_{33}, r_{13}) \\ \theta_6 = atan2(r_{22}, -r_{21}) \end{cases} \\ &\text{if } sin(\theta_5) \geq 0 \begin{cases} \theta_4 = atan2(r_{33}, -r_{13}) \\ \theta_6 = atan2(-r_{22}, r_{21}) \end{cases} \end{aligned} \tag{12}$$

Project Implementation

1. Fill in the `IK_server.py` file with properly commented python code for calculating Inverse Kinematics based on previously performed Kinematic Analysis. Your code must guide the robot to successfully complete 8/10 pick and place cycles. Briefly discuss the code you implemented and your results.

Here I'll talk about the code, what techniques I used, what worked and why, where the implementation might fail and how I might improve it if I were going to pursue this project further.

References

Rubric Points from The Udacity Robot Nano Degree program.