CMPS102: Homework #7

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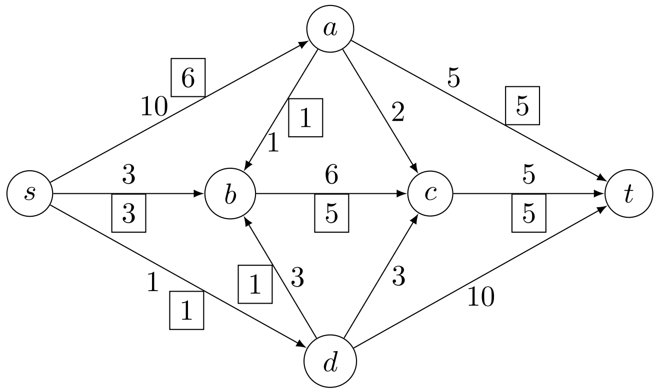
1. Consider the longest common subsequence problem between two strings $x_1, ..., x_n$ and $y_1, ..., y_m$. Define a graph over the $n \times m$ grid (plus possibly some vertices around the edges) s.t. the longest common subsequence corresponds to the longest path in this graph.

   - Clearly describe the condition for the presence of an edge between two vertices on the grid.
   - How should the edges be labeled?
   - How do you find the longest path?
   - Is this algorithm more efficient than the dynamic programming algorithm

2. Kleinberg & Tardos, Ch. 7, #3



(a) What is the value of this flow? Is this a maximum (s, t) flow in this graph?

(b) Find a minimum $s - t$ cut in the flow network pictured in Figure 7.27[above], and also say what its capacity is.

3. Kleinberg & Tardos, Ch. 7, #4

   Decide whether you think the following statement is true, give a short explanation. If it is false, give a counterexample.

   > *Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).*

4. Kleinberg & Tardos, Ch. 7, #5

   Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   > *Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A, B) be a minimum s-t cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.*

5. Kleinberg & Tardos, Ch. 7, #10
   Suppose you are given a direced graph $G = (V, E)$, with a positive integer capacity c_e on each edge e,... a maximum $s - t$ flow in G, defined by a flow value f_e on each edge

   Now, suppose we pick a specific edge $e^* \in E$ and reduce its capacity by 1 unit. Show how to find a maximum flow in the resulting capacitated graph in time $\mathcal{O}(m + n)$, where m is the number of edges in G and n is the number of nodes.

6. CRLS, Ch. 26.1, exercise 6
   Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town. Show how to formulate the problem of determining whether both his children can go to the same school as maximum-flow problem.