# Computer Graphics -Geometry Queries

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http://jjcao.github.io/ComputerGraphics/

# Last time: Geometry Processing

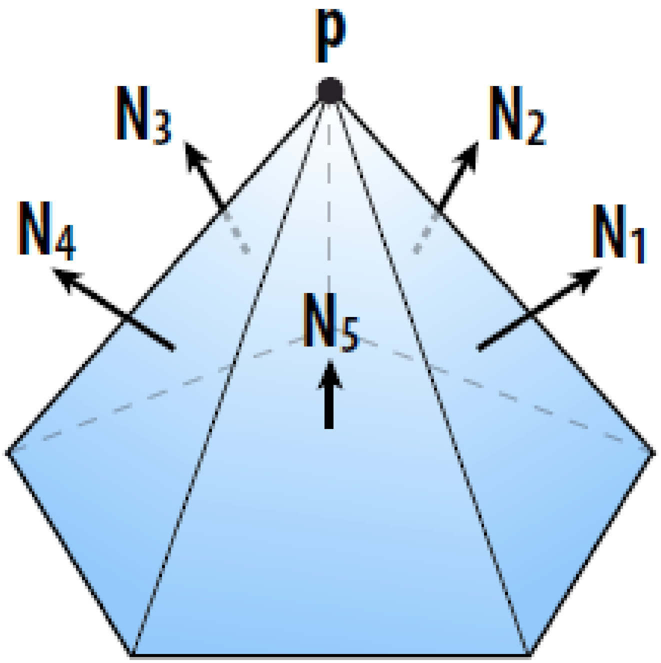
- **Extend signal processing to curved shapes**
  - **encounter familiar issues (sampling, aliasing, etc.)**
  - **some new challenges (irregular sampling, no FFT, etc.)**
- **Focused on resampling triangle meshes**
  - **local: edge flip, split, collapse**
  - **global: subdivision, quadric error, isotropic remeshing**
- **Today: what kind of geometric queries *can't* we answer yet?**

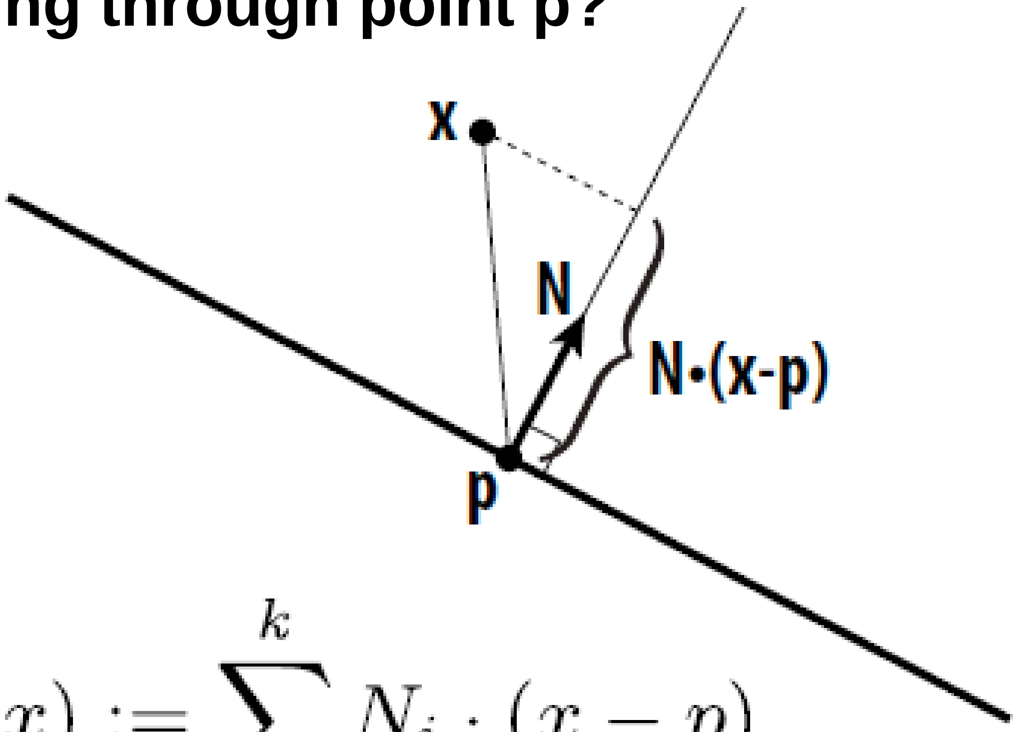
# Simplification via Quadric Error Metric

- One popular scheme: iteratively collapse edges
- Which edges? Assign score with *quadric error metric**
  - approximate distance to surface as sum of distance to
  - aggregated triangles
  - iteratively collapse edge with smallest score
  - greedy algorithm... great results!

# Quadric Error Metric

- Approximate distance to a collection of triangles
- Distance is sum of point-to-plane distances
  - Q: Distance to plane w/ normal N passing through point p?
  - A: d(x) = N•x - N•p = N•(x-p)
- Sum of distances:




$$d(x) := \sum_{i=1}^{k} N_i \cdot (x - p)$$

# Quadric Error - Homogeneous Coordinates

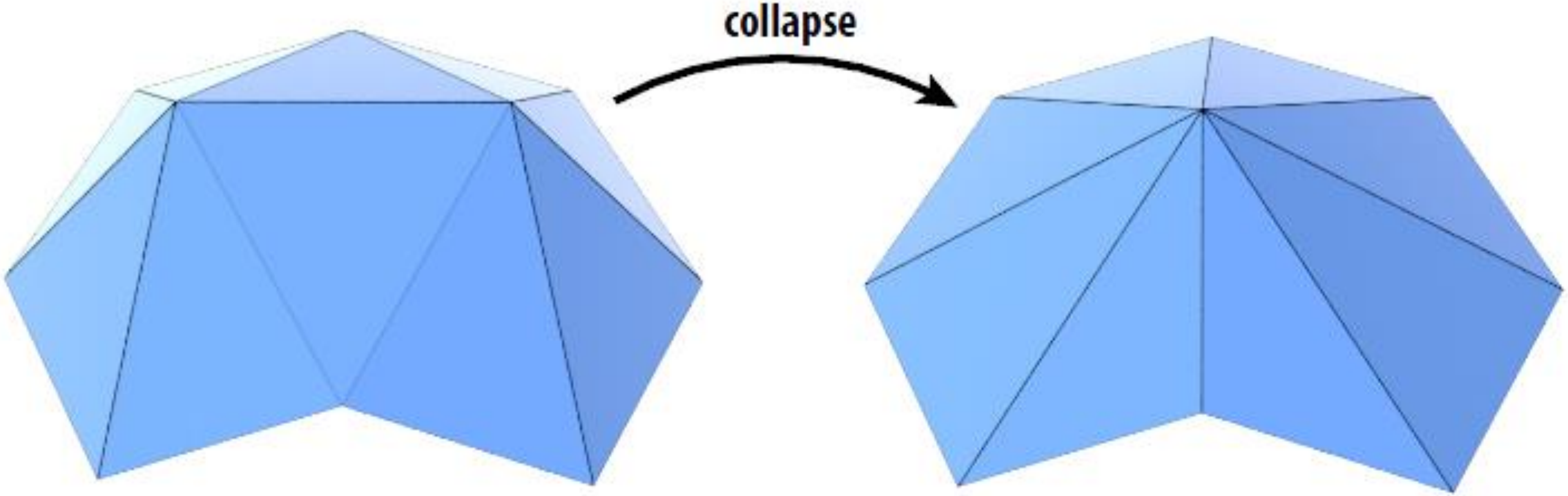
- **Suppose in coordinates we have**
  - **a query point (x,y,z)**
  - **a normal (a,b,c)**
  - **an offset d := -(x,y,z) • (a,b,c)**
- **Then in homogeneous coordinates**
  - **u := (x,y,z,1)**
  - **v := (a,b,c,d)**

$$Q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

- ***Signed* distance to plane is then just u•v = ax+by+cz+d**
- ***Squared* distance is (uTv)2 = uT(vvT)u =: uTQu**
- **Key idea: *matrix Q encodes distance to plane***
- **Q is symmetric, contains 10 unique coefficients (small storage)**

# Quadric Error of Edge Collapse

- How much does it cost to collapse an edge?
- Idea: compute edge midpoint, measure quadric error




- Better idea: use point that minimizes quadric error as new point!
- Q: How do we minimize quadric error?

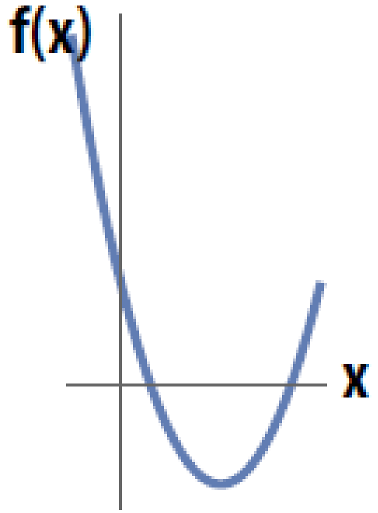
# Review: Minimizing a Quadratic Function

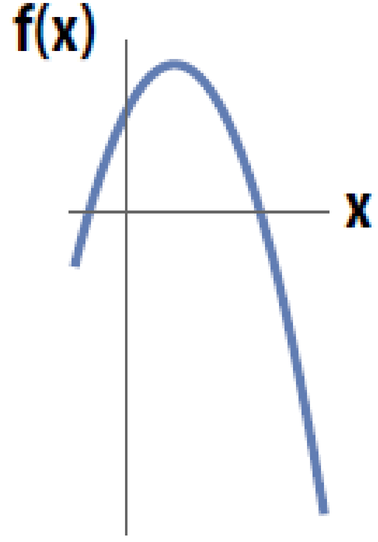
- Suppose I give you a function f(x) = ax2+bx+c
- Q: What does the graph of this function look like?
- Could also look like this!
- Q: How do we find the *minimum?*
- A: Look for the point where the function isn't
- changing (if we look "up close")
- I.e., find the point where the *derivative* vanishes

$$f'(x) = 0$$

$$2ax + b = 0$$

$$x = -b/2a$$



# Minimizing a Quadratic Form

- A *quadratic form* is just a generalization of our quadratic
- polynomial from 1D to nD
- E.g., in 2D: f(x,y) = ax2 + bxy + cy2 + dx + ey + g
- Can always (always!) write quadratic polynomial using a
- *symmetric* matrix (and a vector, and a constant):

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + g$$

$$= \mathbf{x}^{\mathsf{T}} A \mathbf{x} + \mathbf{u}^{\mathsf{T}} x + g$$ (this expression works for *any* n!)

- Q: How do we find a critical point (min/max/saddle)?
- A: Set derivative to zero!

$$2A\mathbf{x} + \mathbf{u} = 0$$

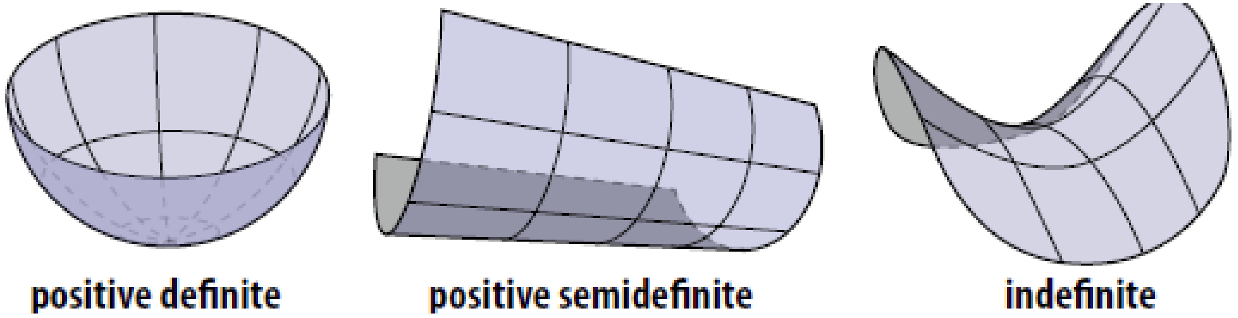
$$\mathbf{x} = -\tfrac{1}{2} A^{-1} \mathbf{u}$$

# Positive Definite Quadratic Form

- Just like our 1D parabola, critcal point is *not* always a min!
- Q: In 2D, 3D, nD, when do we get a *minimum?*
- A: When matrix A is *positive-definite:*

$$\mathbf{x}^{\mathsf{T}} A\,\mathbf{x} > 0 \qquad \forall\ \mathbf{x}$$

- 1D: Must have xax = ax2 > 0. In other words: a is positive!
- 2D: Graph of function looks like a “bowl”:




- Positive-definiteness is *extremely important* in CG: it means we can find a minimum by solving linear equations. Basis of many, many modern algorithms (geometry processing, simulation, ...).

# Minimizing Quadratic Error

- **Find "best" point for edge collapse by minimizing quad. form**

$$\min_{u} \mathbf{u}^{\mathsf{T}} K \mathbf{u}$$

- **Already know fourth (homogeneous) coordinate is 1!**
- **So, break up our quadratic function into two pieces:**

$$\begin{bmatrix} \mathbf{x}^{\mathsf{T}} & 1 \end{bmatrix} \begin{bmatrix} B & \mathbf{w} \\ \mathbf{w} & d^2 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

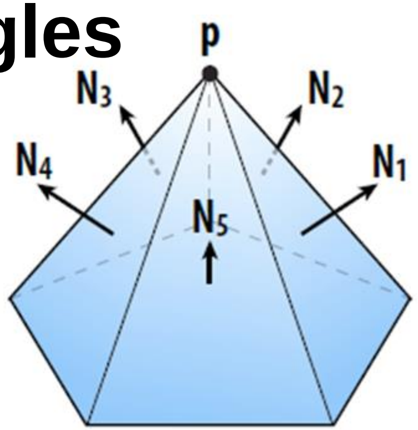
$$= \mathbf{x}^{\mathsf{T}} B \mathbf{x} + 2\mathbf{w}^{\mathsf{T}} \mathbf{x} + d^2$$

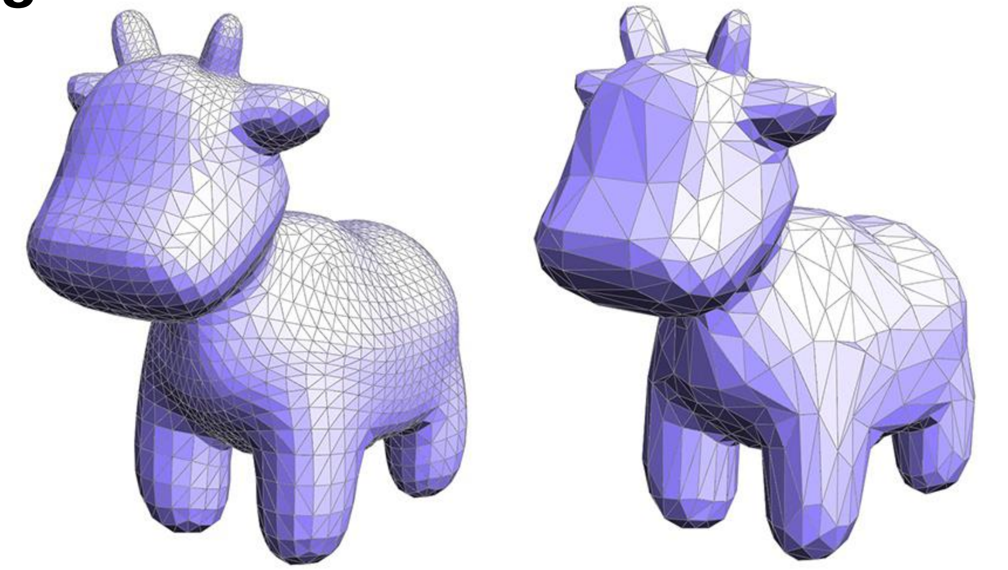
- **Now we have a quadratic form in the 3D position x.**
- **Can minimize as before:**

$$2B\mathbf{x} + 2\mathbf{w} = 0 \qquad \Longleftrightarrow \qquad \mathbf{x} = -B^{-1}\mathbf{w}$$

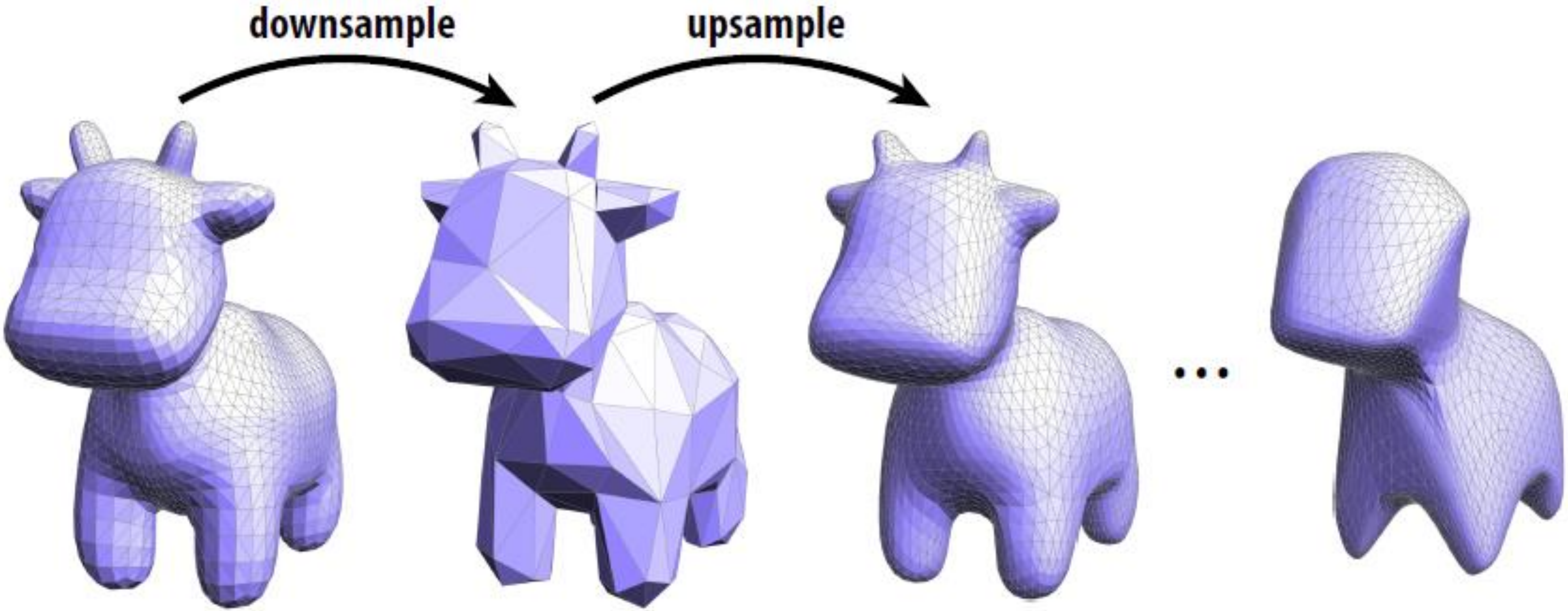
# Quadric Error Simplification: Final Algorithm

- **Compute K for each triangle (distance to plane)**
- **Set K at each vertex to sum of Ks from incident triangles**
- **Set K at each edge to sum of Ks at endpoints**
- **Find point at each edge minimizing quadric error**
- **Until we reach target # of triangles:**
  - **collapse edge (i,j) with smallest cost to get new vertex m**
  - **add Ki and Kj to get quadric Km at m**
  - **update cost of edges touching m**
- ***More details in assignment writeup!***



# Demo: Danger of Resampling




(Q: What happens with an image?)

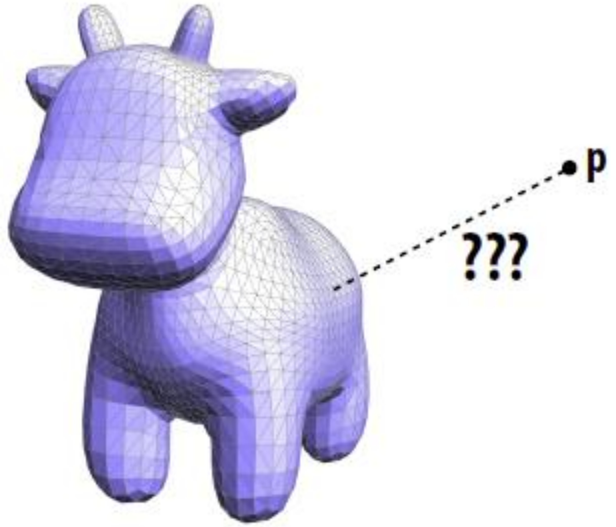
**But wait: we have the original mesh.**

**Why not just project each new sample point**

**onto the closest point of the original mesh?**

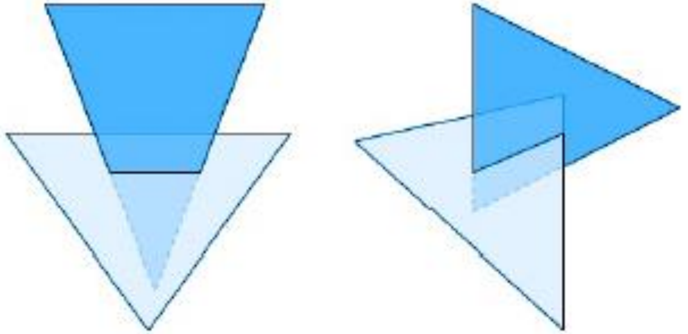
# Geometric Queries

- Q: Given a point, in space (e.g., a new sample point), how do we find the closest point on a given surface?
- Q: Does implicit/explicit representation make this easier?
- Q: Does our halfedge data structure help?
- Q: What's the cost of the naïve algorithm?
- Q: How do we find the distance to a single triangle anyway?
- So many questions!



# Many types of geometric queries

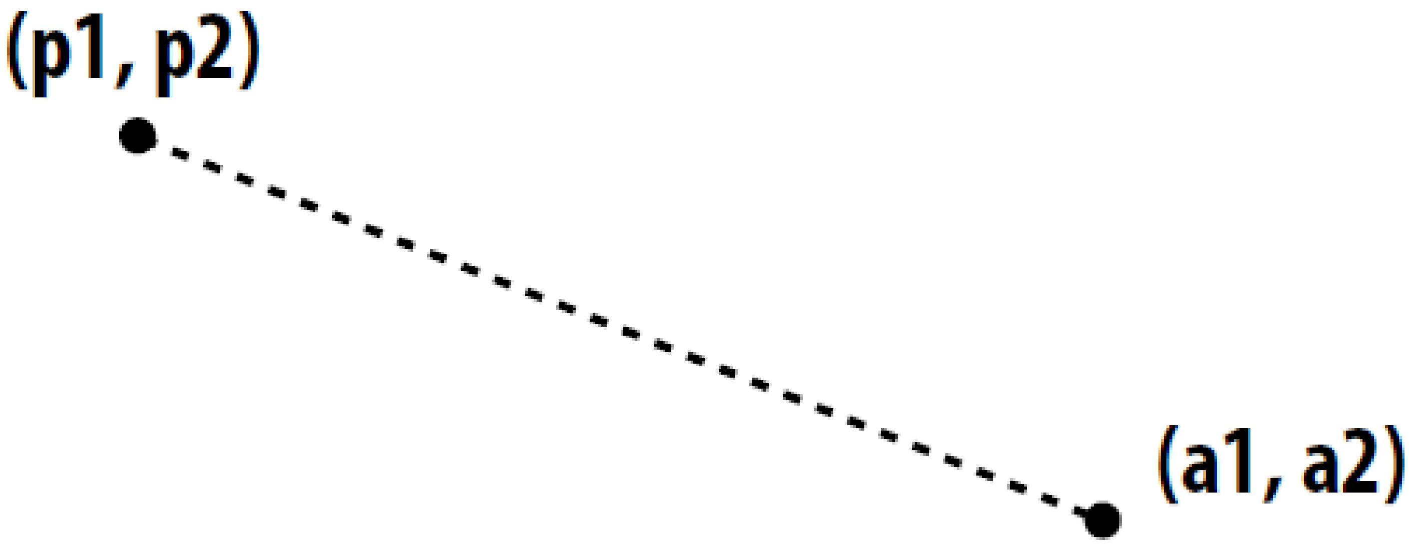
- Already identified need for “closest point” query
- Plenty of other things we might like to know:
  - Do two triangles intersect?
  - Are we inside or outside an object?
  - Does one object contain another?
  - ...



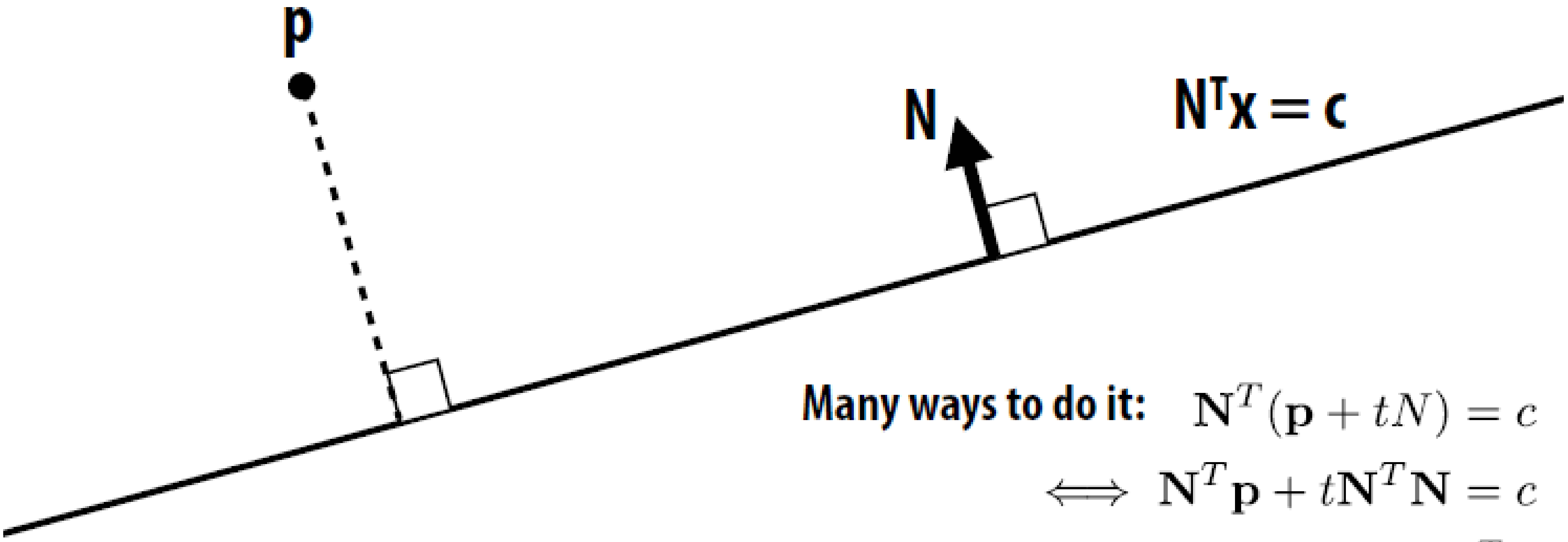
- Data structures we’ve seen so far not really designed for this...
- Need some new ideas!
- Today: come up with simple (read: slow) algorithms.
- Next lecture: intelligent ways to accelerate geometric queries.

# Warm up: closest point on point

- Goal is to find the point on a mesh closest to a given point.
- *Much* simpler question: given a query point (p1,p2), how do we find the closest point on the point (a1,a2)?



# Slightly harder: closest point on line




Many ways to do it: $\mathbf{N}^T(\mathbf{p} + tN) = c$

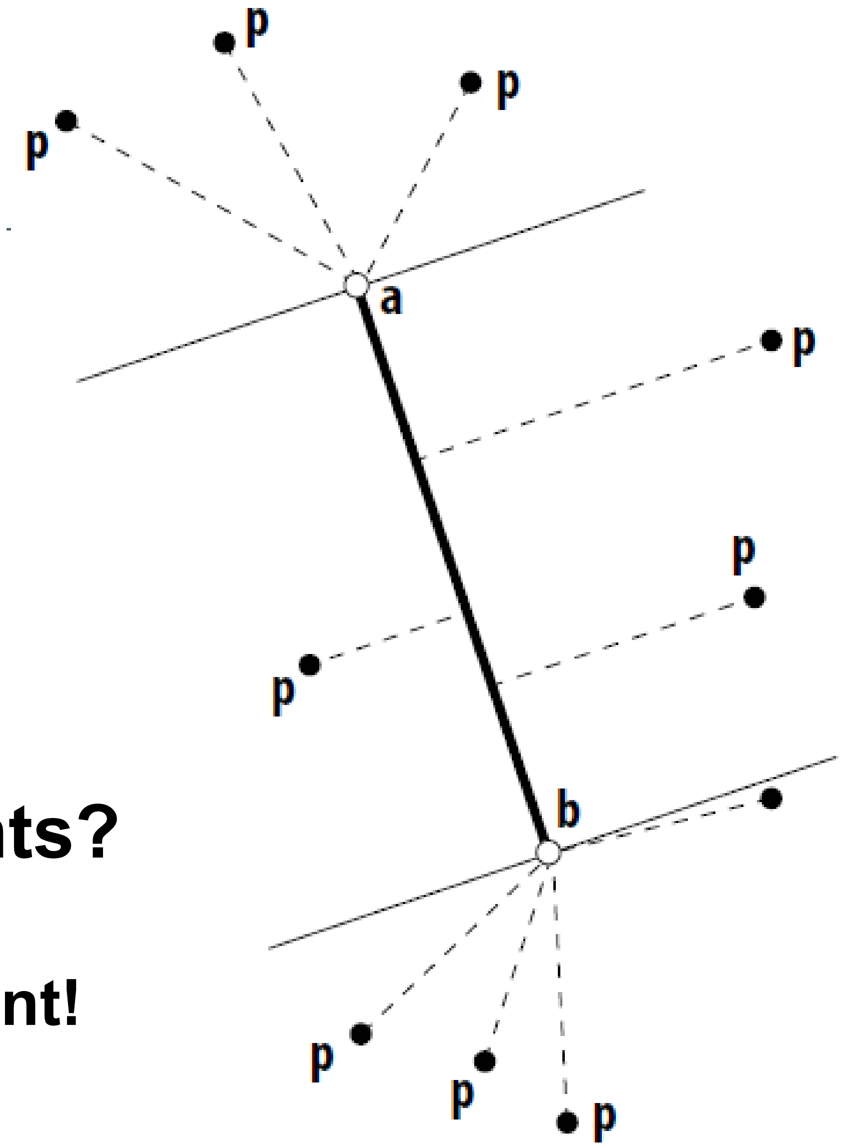
$$\iff \mathbf{N}^T\mathbf{p} + t\mathbf{N}^T\mathbf{N} = c$$

$$\iff t = c - \mathbf{N}^T\mathbf{p}$$

$$\Rightarrow \mathbf{p} + t\mathbf{N} = \boxed{\mathbf{p} + (c - \mathbf{N}^T\mathbf{p})\mathbf{N}}$$

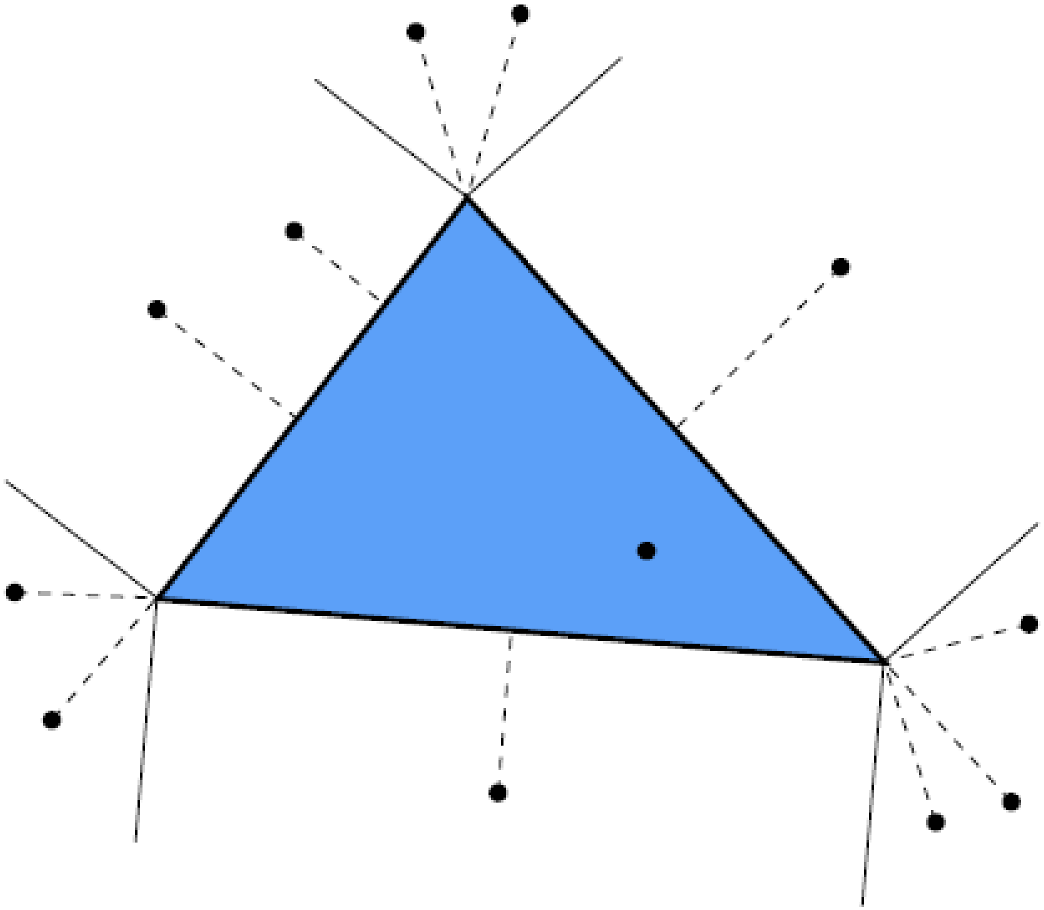
# Harder: closest point on line segment

- Two cases: endpoint or interior
- Already have basic components:
  - point-to-point
  - point-to-line
- Algorithm?
  - find closest point on line
  - check if it's between endpoints
  - if not, take closest endpoint
- How do we know if it's between endpoints?
  - write closest point on line as a+t(b-a)
  - if t is between 0 and 1, it's inside the segment!



# Even harder: closest point on triangle

- What are all the possibilities for the closest point?
- Almost just minimum distance to three segments:

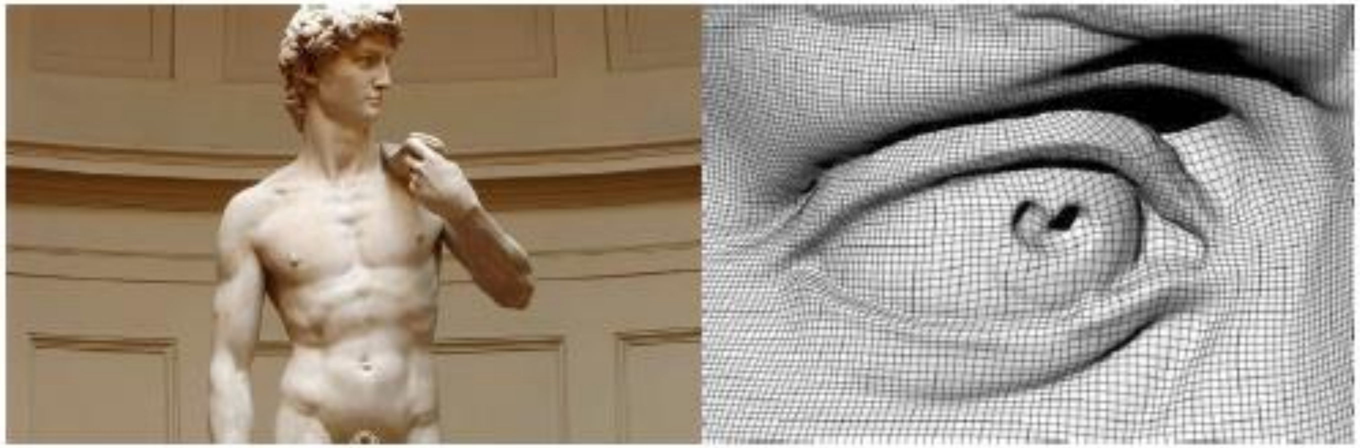


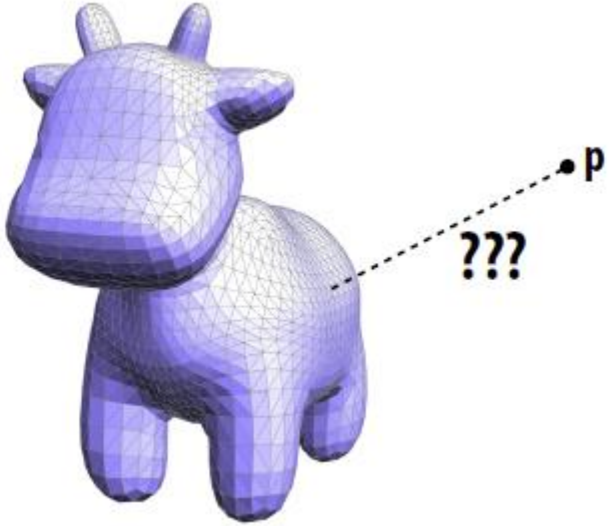
# Closest point on triangle in 3D

- Not so different from 2D case
- Algorithm?
  - project onto plane of triangle
  - use half-plane tests to classify point
  - if inside the triangle, we’re done!
  - otherwise, find closest point on associated vertex or edge
- By the way, how do we find closest point on plane?
- Same expression as closest point on a line!
- E.g., $p + ( c - N^T p ) N$

# Closest point on triangle *mesh* in 3D?

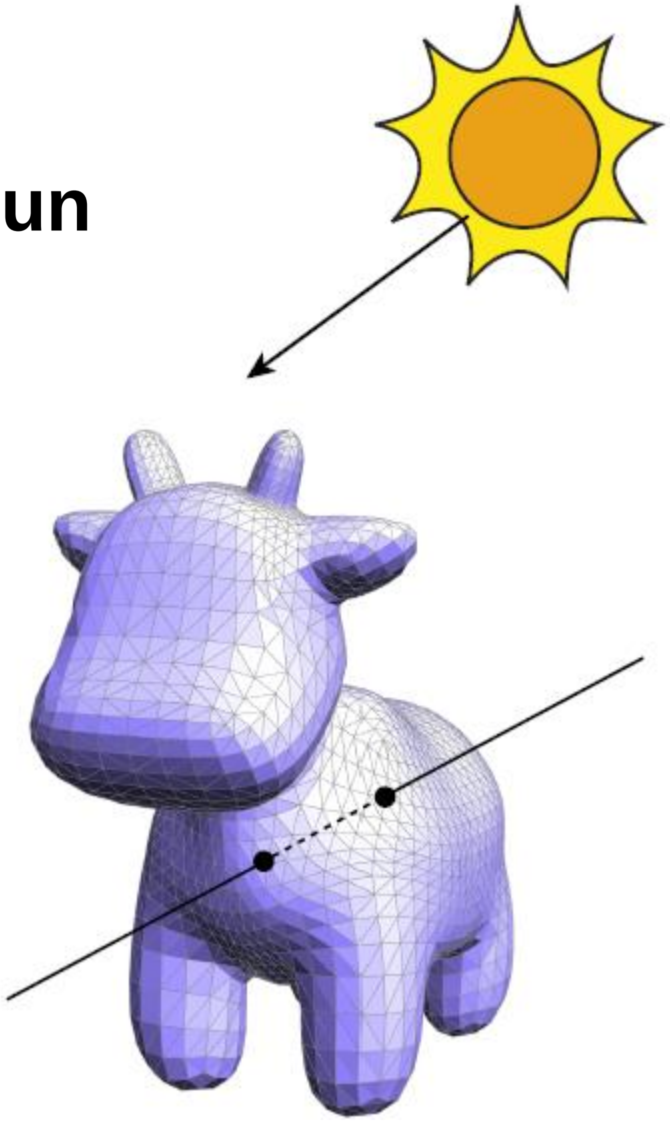
- **Conceptually easy:**
  - **loop over all triangles**
  - **compute closest point to current triangle**
  - **keep globally closest point**
- **Q: What's the cost? Does halfedge help?**
- **What if we have *billions* of faces?**
- **(Next time!)**





# Different query: ray-mesh intersection

- A “ray” is an oriented line starting at a point
- Think about a ray of light traveling from the sun
- Want to know where a ray pierces a surface
- Why?
  - GEOMETRY: inside-outside test
  - RENDERING: visibility, ray tracing
  - SIMULATION: collision detection
- Might pierce surface in many places!



# Ray equation

- **Can express ray as**

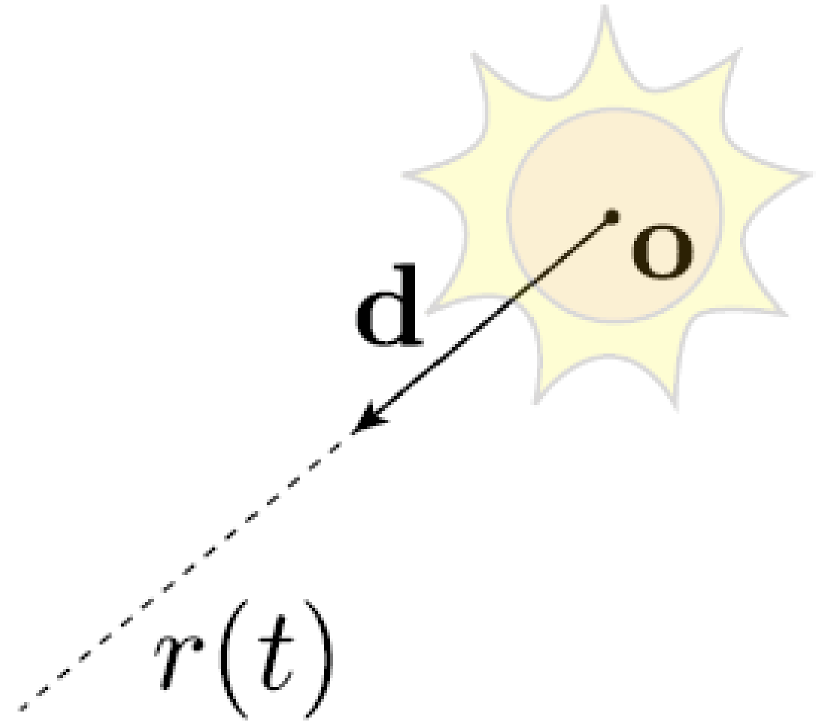
origin

unit direction

$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$$

point along ray

"time"



# Intersecting a ray with an implicit surface

- Recall implicit surfaces: all points x such that f(x) = 0
- Q: How do we find points where a ray pierces this surface?
- Well, we know all points along the ray: r(t) = o + td
- Idea: replace “x” with “r” in 1st equation, and solve for t
- Example: unit sphere

quadratic formula:

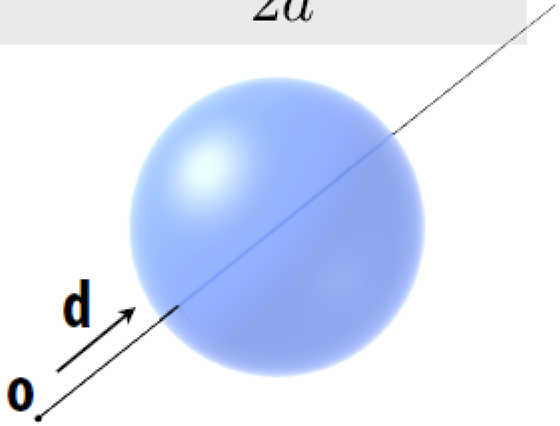
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$f(\mathbf{x}) = |\mathbf{x}|^2 - 1$$

$$\Rightarrow f(\mathbf{r}(t)) = |\mathbf{o} + t\mathbf{d}|^2 - 1$$

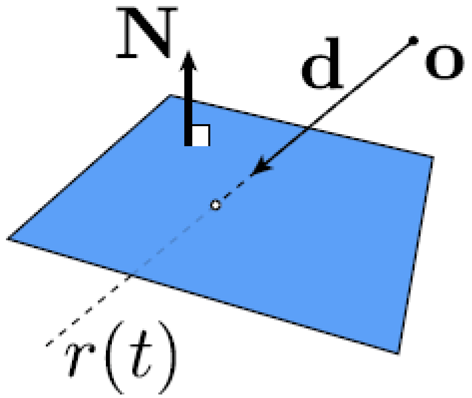
$$\underbrace{|\mathbf{d}|^2}_{a} t^2 + \underbrace{2(\mathbf{o}\cdot\mathbf{d})}_{b} t + \underbrace{|\mathbf{o}|^2 - 1}_{c} = 0$$

$$t = \boxed{-\mathbf{o}\cdot\mathbf{d} \pm \sqrt{(\mathbf{o}\cdot\mathbf{d})^2 - |\mathbf{o}|^2 + 1}}$$




Why two solutions?

# Ray-plane intersection




- **Suppose we have a plane NTx = c**
  - **N - unit normal**
  - **c - offset**
- **How do we find intersection with ray r(t) = o + td?**
- ***Key idea:* again, replace the point x with the ray equation t:**

$$\mathbf{N}^{\mathsf{T}}\mathbf{r}(t) = c$$

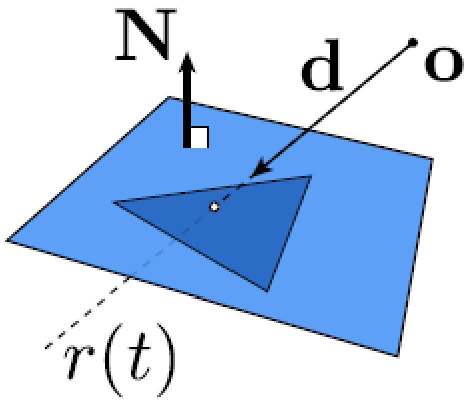
- **Now solve for t:**

$$\mathbf{N}^{\mathsf{T}}(\mathbf{o} + t\mathbf{d}) = c \qquad \Rightarrow t = \frac{c - \mathbf{N}^{\mathsf{T}}\mathbf{o}}{\mathbf{N}^{\mathsf{T}}\mathbf{d}}$$

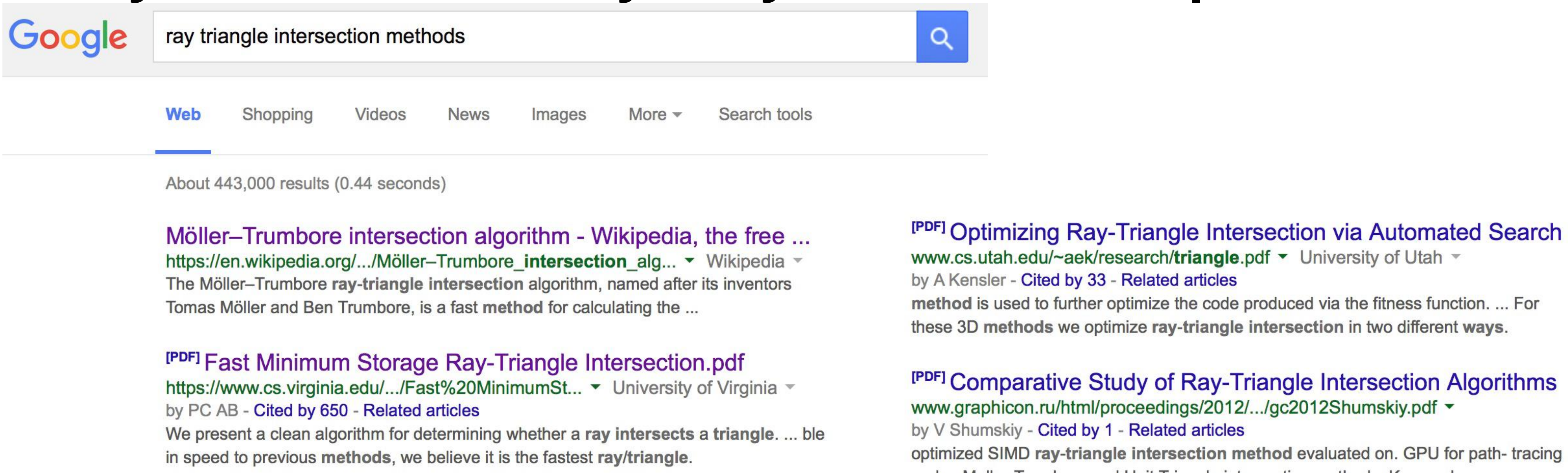
- **And plug t back into ray equation:**

$$r(t) = \mathbf{o} + \frac{c - \mathbf{N}^{\mathsf{T}}\mathbf{o}}{\mathbf{N}^{\mathsf{T}}\mathbf{d}}\mathbf{d}$$

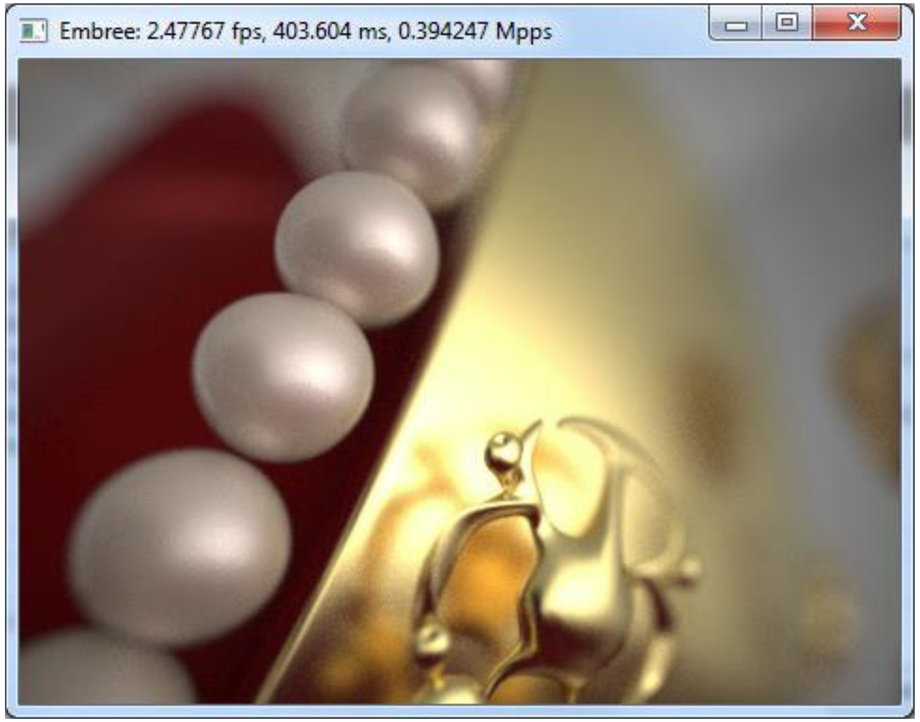
# Ray-triangle intersection




- **Triangle is in a plane...**
- **Not much more to say!**
  - **Compute ray-plane intersection**
  - **Q: What do we do now?**
  - **A: Why not compute barycentric coordinates of hit point?**
  - **If barycentric coordinates are all positive, point in triangle**
- **Actually, a *lot* more to say... if you care about performance!**



# Why care about performance?




Intel Embree

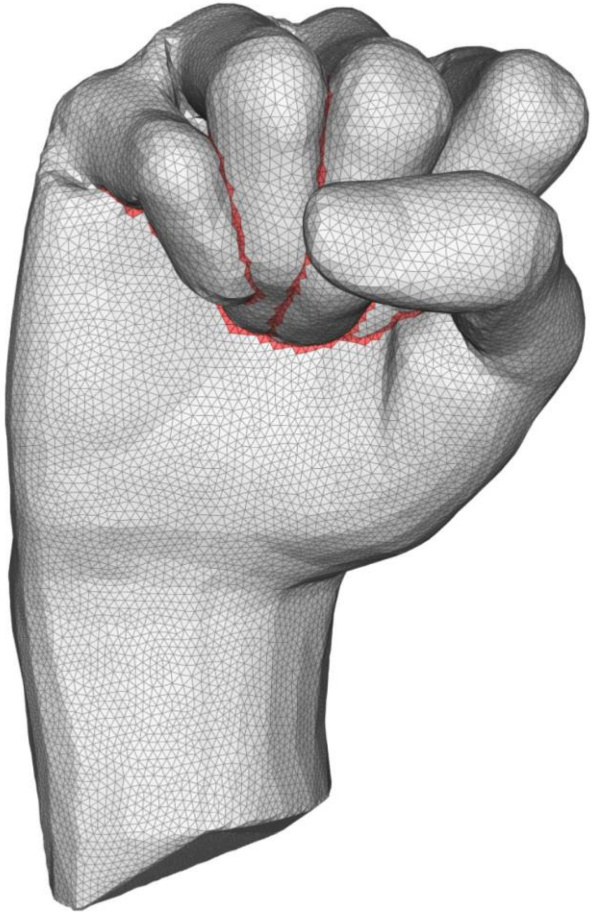
NVIDIA OptiX

# Why care about performance?

“Brigade 3” real time path tracing demo

# One more query: mesh-mesh intersection

- GEOMETRY: How do we know if a mesh intersects itself?
- ANIMATION: How do we know if a collision occurred?



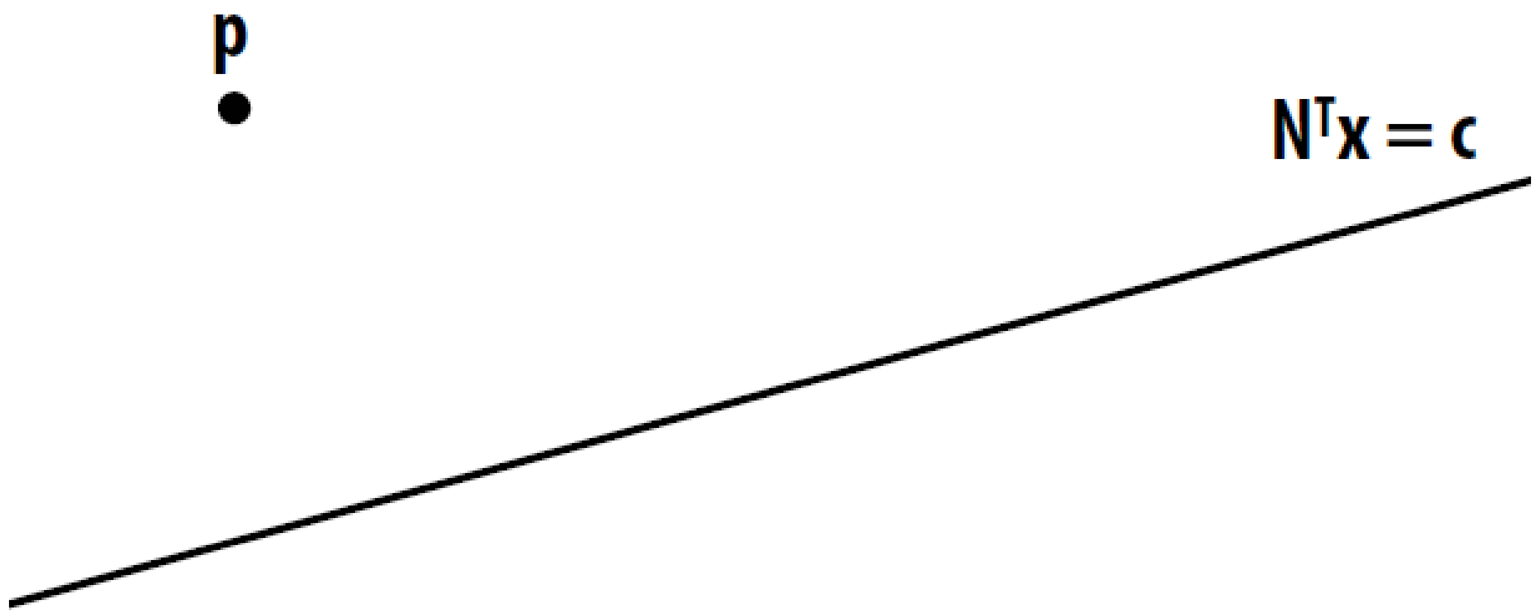
# Warm up: point-point intersection

- Q: How do we know if p intersects a?
- A: ...check if they're the same point!


- Sadly, life is not always so easy.

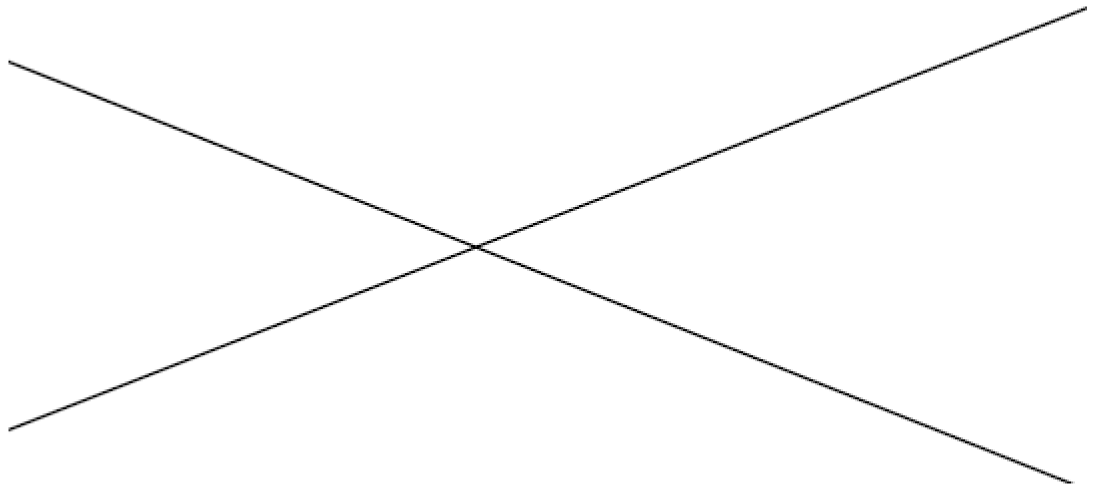
# Slightly harder: point-line intersection

- Q: How do we know if a point intersects a given line?
- A: ...plug it into the line equation!



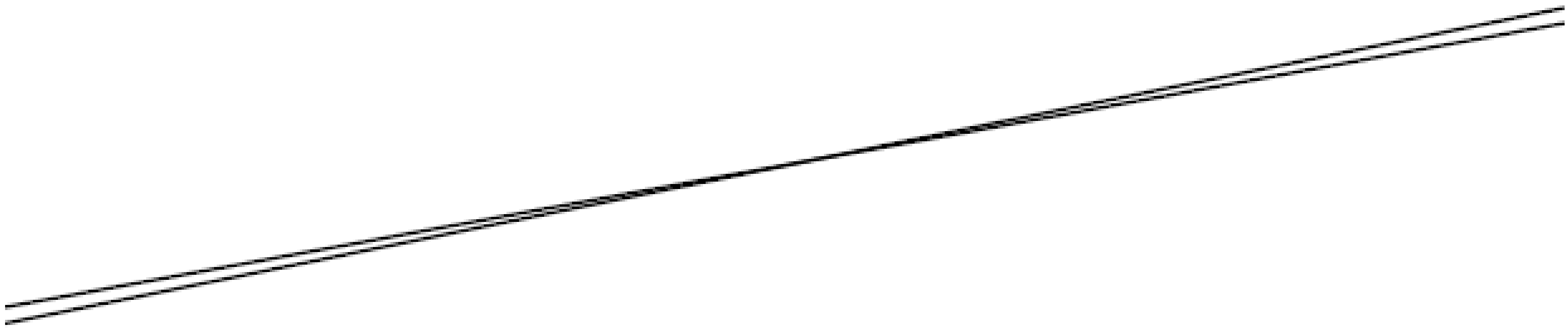
# Finally interesting: line-line intersection

- Two lines: ax=b and cx=d
- Q: How do we find the intersection?
- A: See if there is a simultaneous solution
- Leads to linear system: $$\begin{bmatrix} a_1 & a_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$$

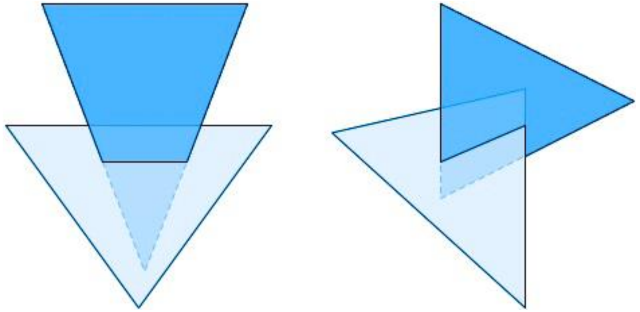


# Degenerate line-line intersection?

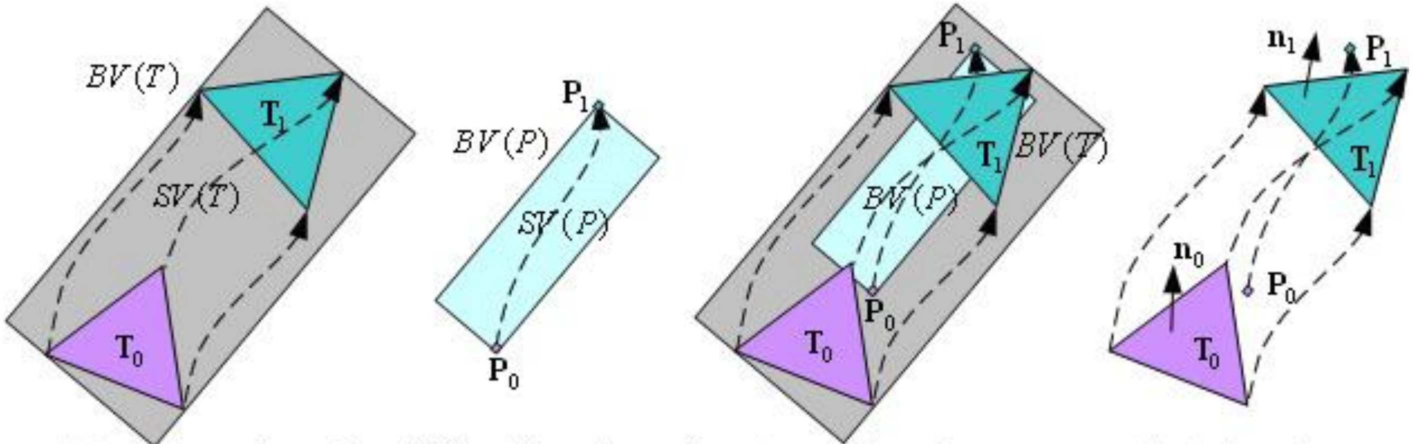
- What if lines are almost parallel?
- Small change in normal can lead to big change in intersection!
- Instability very common, very important with geometric
- predicates. Demands special care (e.g., analysis of matrix).



# Triangle-Triangle Intersection?



- **Lots of ways to do it**
- **Basic idea:**
  - **Q: Any ideas?**
  - **One way: reduce to edge-triangle intersection**
  - **Check if each line passes through plane**
  - **Then do interval test**

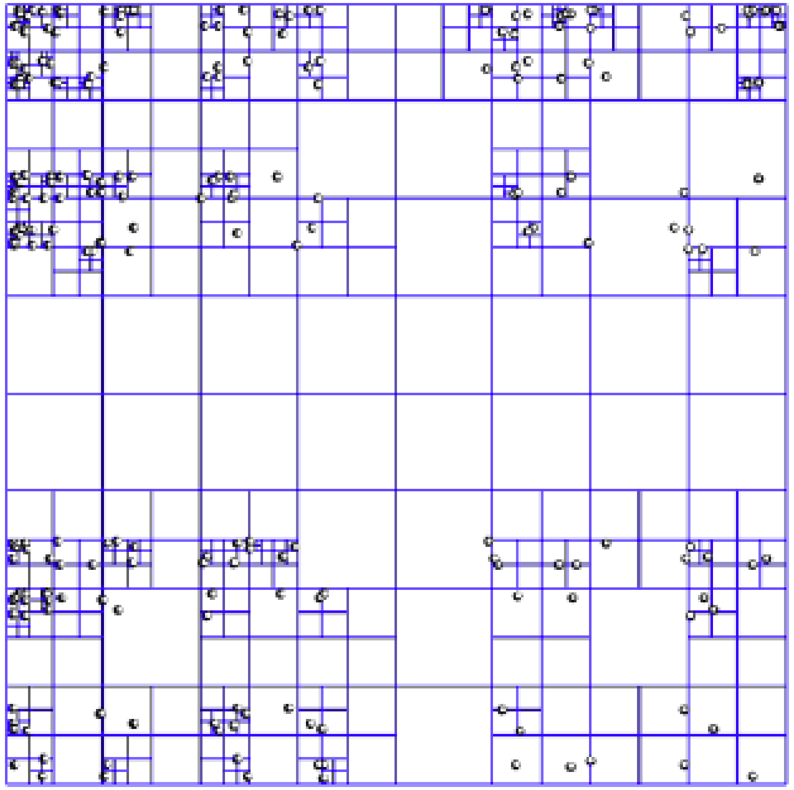


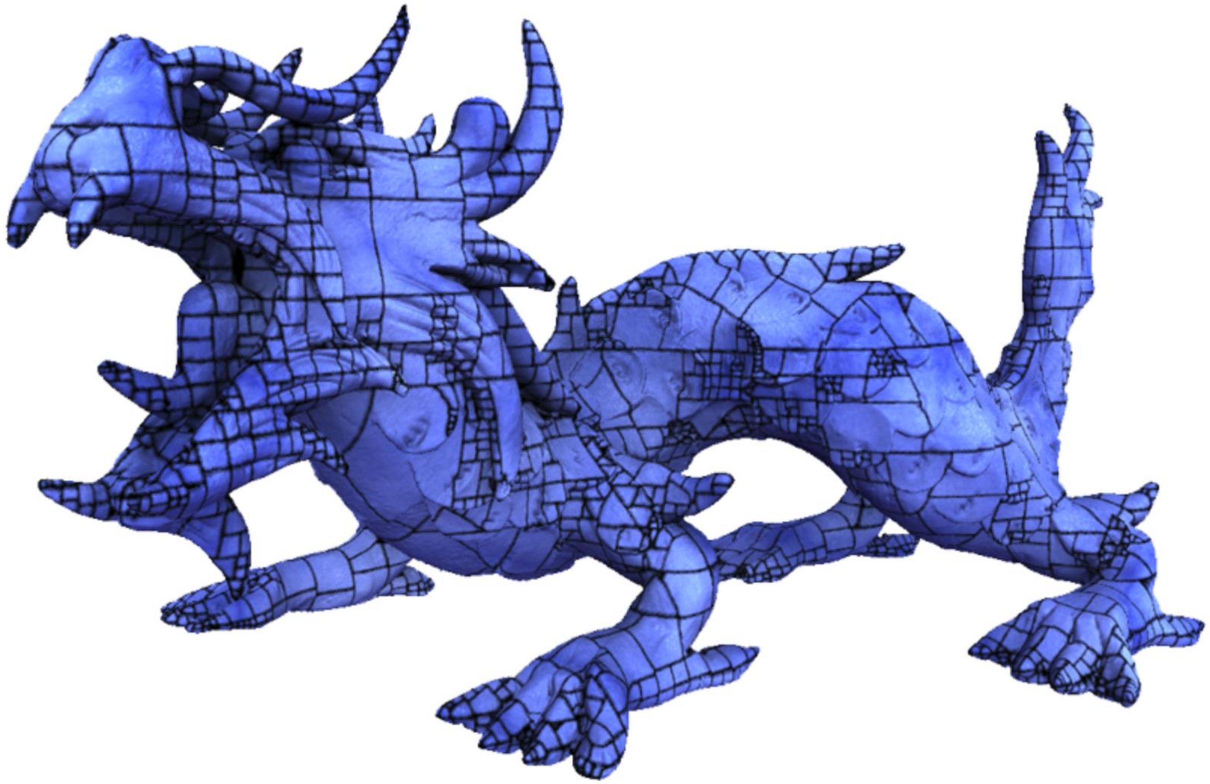

(a) Bounding volume of a deforming triangle (b) Bounding volume of a deforming vertex (c) Bounding volume test (d) Coplanarity test

- **What if triangle is *moving*?**
  - **Important case for animation**
  - **Can think of triangles as *prisms* in time**
  - **Will say more when we talk about animation!**

# Up Next: Spatial Acceleration Data Strucutres

- **Testing every element is *slow!***
- **E.g., linearly scanning through a list vs. binary search**
- **Can apply this same kind of thinking to geometric queries**





# Accelerating Geometric Queries

# Review: ray-triangle intersection

- **Find ray-plane intersection**

Parametric equation of a ray:

$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$$

ray origin

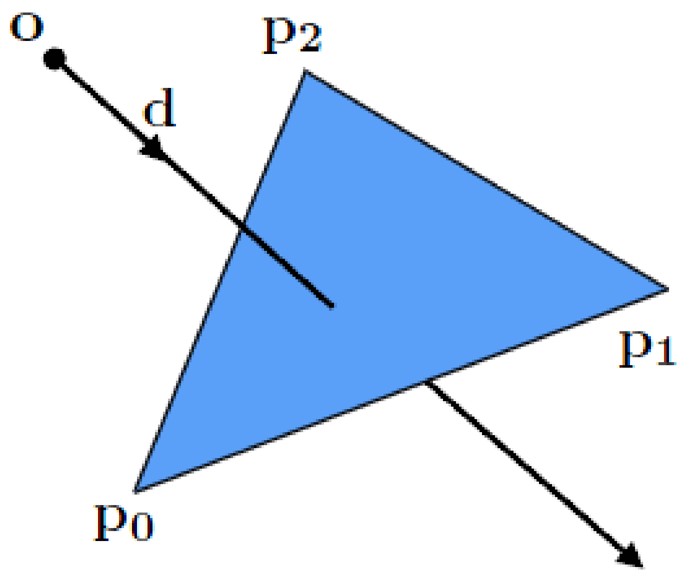
normalized ray direction

Plug equation for ray into implicit plane equation:

$$\mathbf{N}^{\mathbf{T}}\mathbf{x} = c$$
$$\mathbf{N}^{\mathbf{T}}(\mathbf{o} + t\mathbf{d}) = c$$

Solve for *t* corresponding to intersection point:

$$t = \frac{c - \mathbf{N}^{\mathbf{T}}\mathbf{o}}{\mathbf{N}^{\mathbf{T}}\mathbf{d}}$$




- **Determine if point of intersection is within triangle**

# Ray-primitive queries

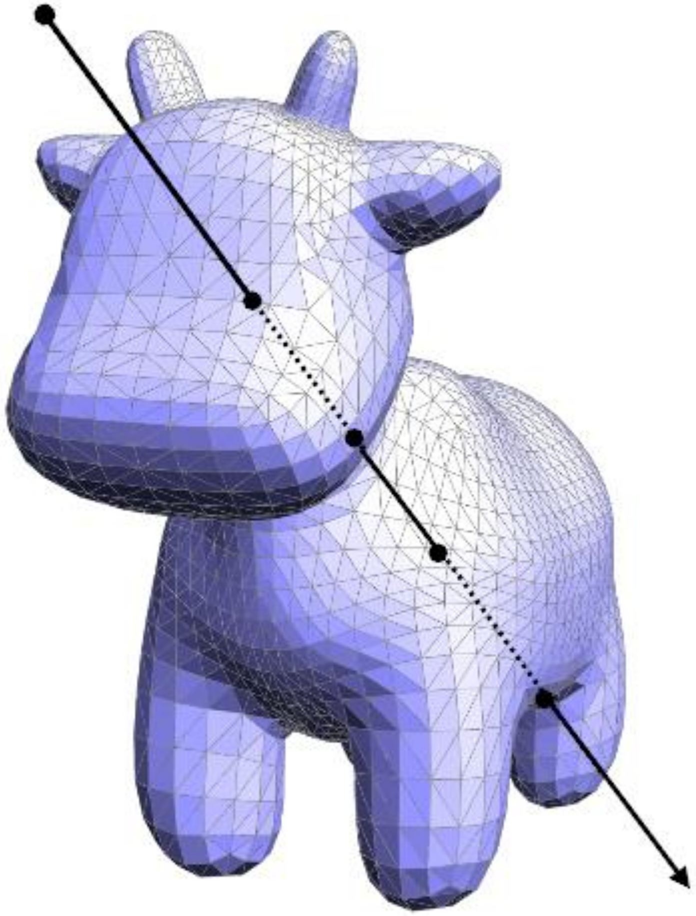
- Given primitive p:
- p.intersect(r) returns value of *t* corresponding to the point of intersection with ray *r*

# Ray-scene intersection

- **Given a scene defined by a set of *N* primitives and a ray *r*, find the closest point of intersection of *r* with the scene**
- **"Find the first primitive the ray hits"**

```
p_closest = NULL
t_closest = inf
for each primitive p in scene:
    t = p.intersect(r)
    if t >= 0 && t < t_closest:
        t_closest = t
        p_closest = p
```

Complexity: O(N)



# A simpler problem

- Imagine I have a set of integers S
- Given a new integer *k*, find the element in S that is closest to *k:*

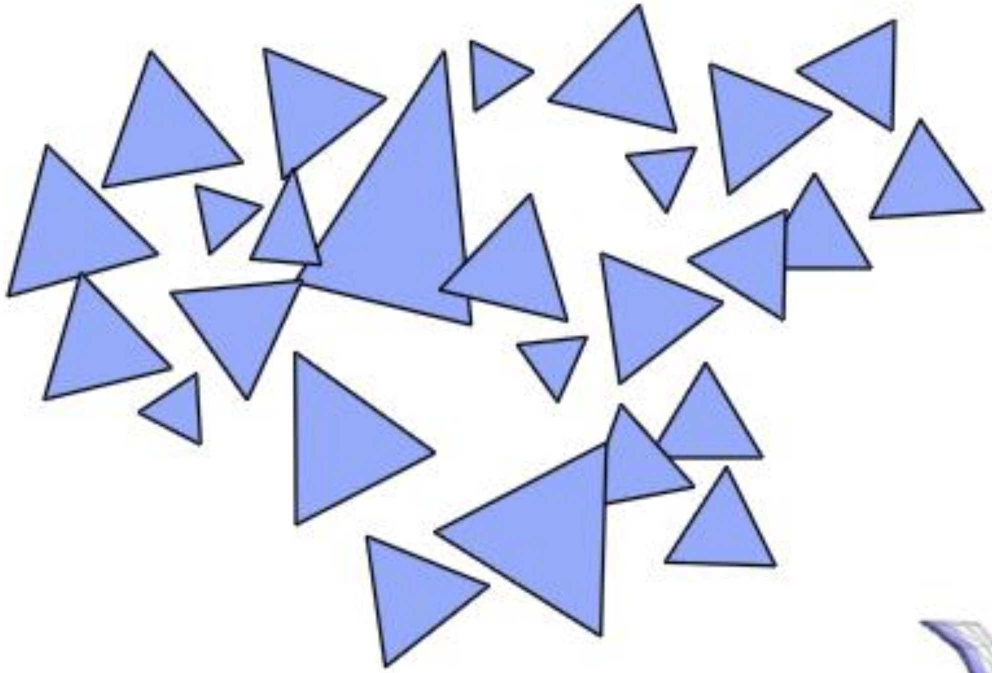
10 123 20 100 6 25 64 11 200 30

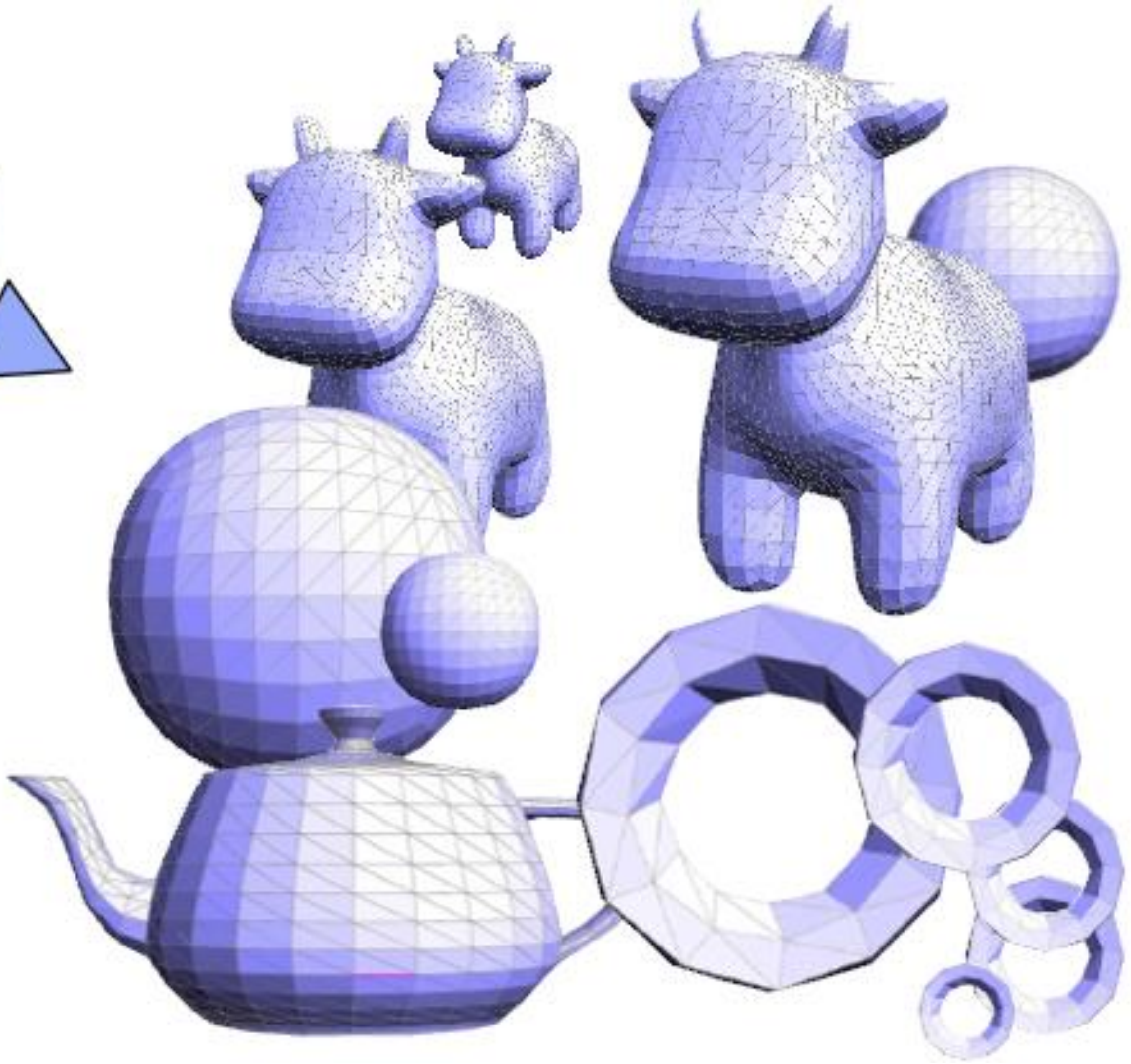
- Example: *k*=18
- Sort integers:

6 10 11 20 25 30 64 100 123 200

- How would you perform a modified binary search?

# How do we organize scene primitives to enable fast ray-scene intersection queries?





# Ray-axis-aligned-box intersection

- **What is ray's closest/farthest intersection with axis-aligned box?**

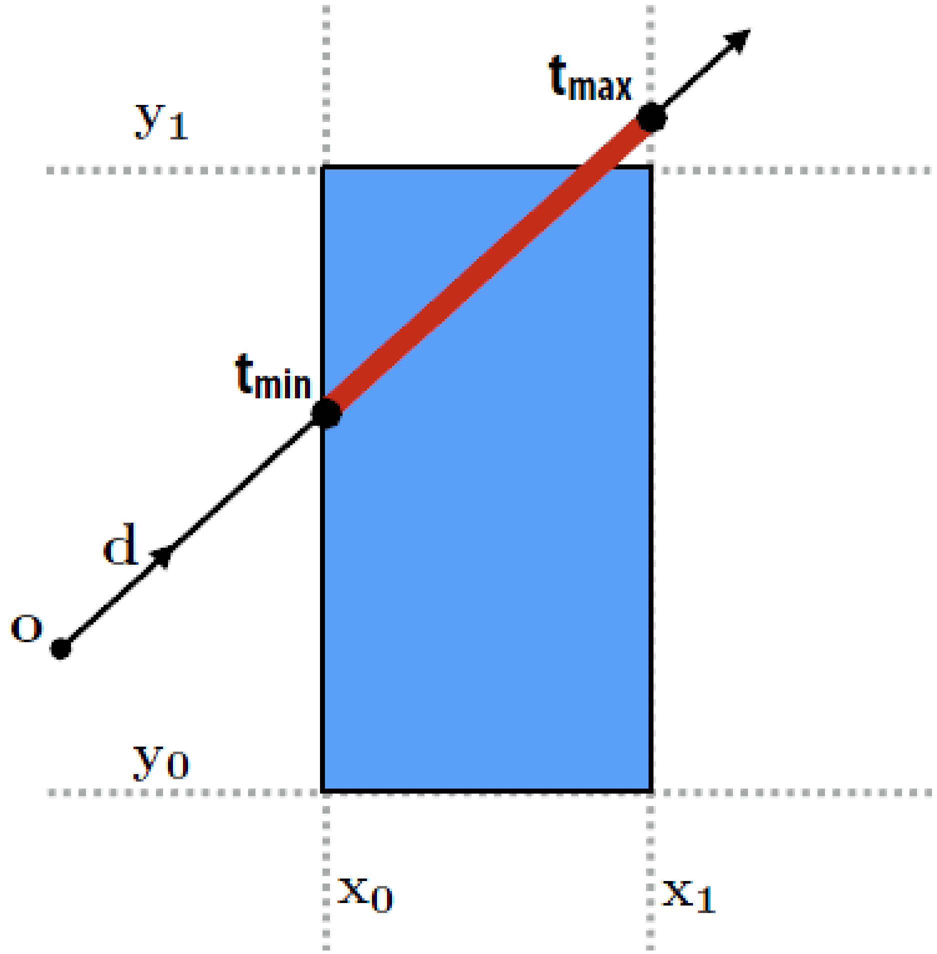



**Figure shows intersections with $x$=$x$0 and $x$=$x$1 planes.**

**Find intersection of ray with all planes of box:**

$$\mathbf{N}^{\mathbf{T}}(\mathbf{o} + t\mathbf{d}) = c$$

**Math simplifies greatly since plane is axis aligned (consider $x$=$x_0$ plane in 2D):**

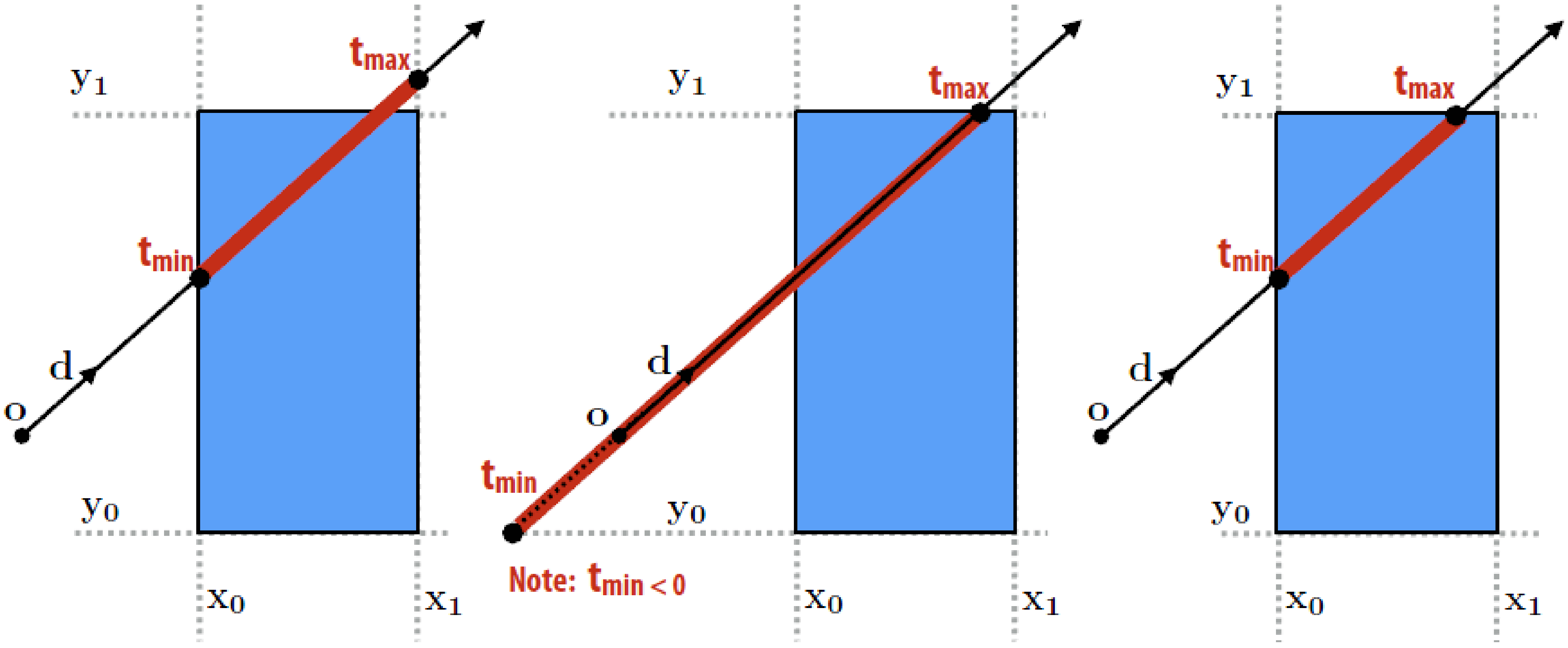
$$\mathbf{N}^{\mathbf{T}} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

$$c = x_0$$

$$t = \frac{x_0 - \mathbf{o_x}}{\mathbf{d_x}}$$

# Ray-axis-aligned-box intersection

- Compute intersections with all planes, take intersection of tmin/tmax intervals



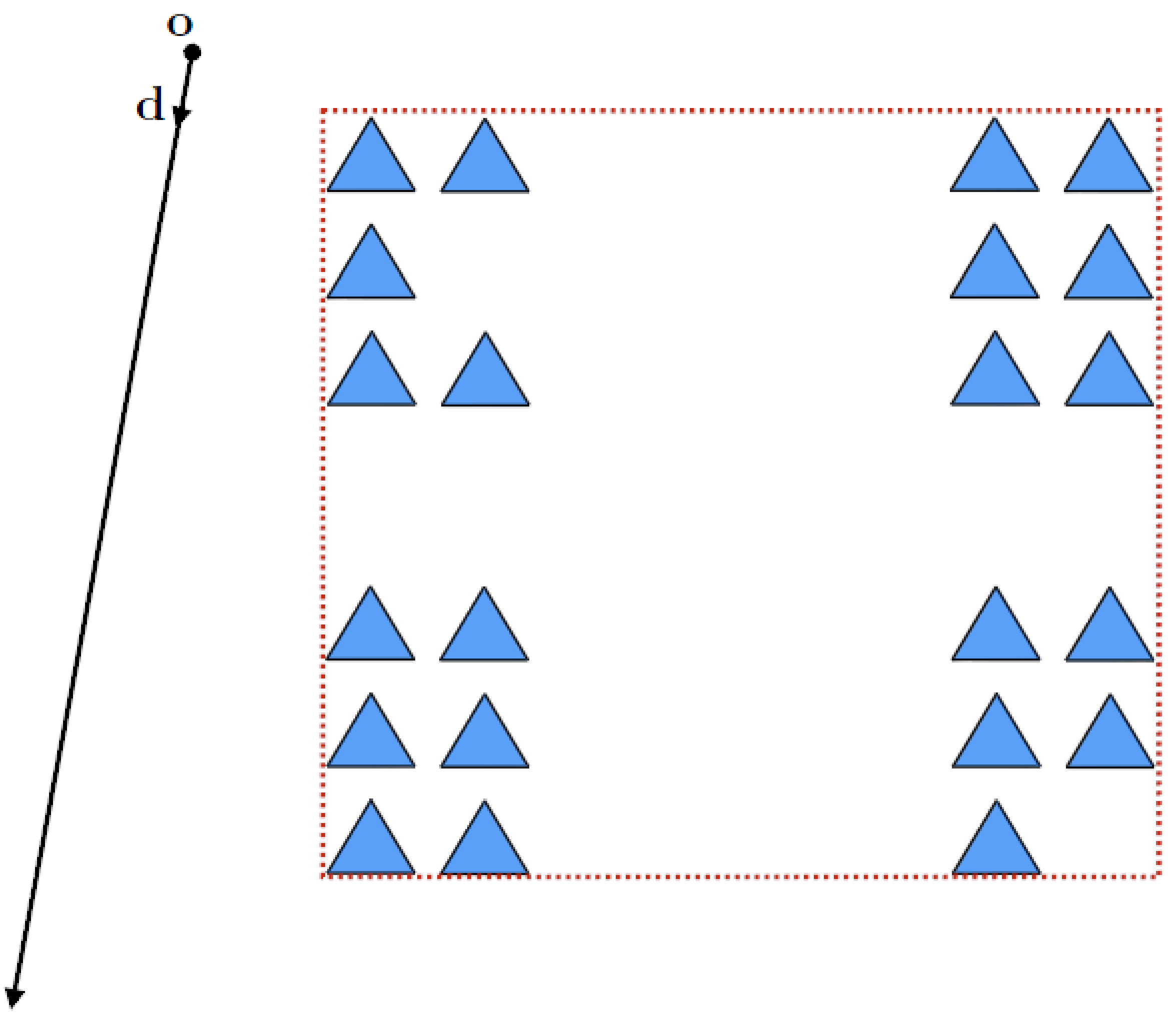

Intersections with $x$ planes

Intersections with $y$ planes

Final intersection result

How do we know when the ray misses the box?

# Simple case



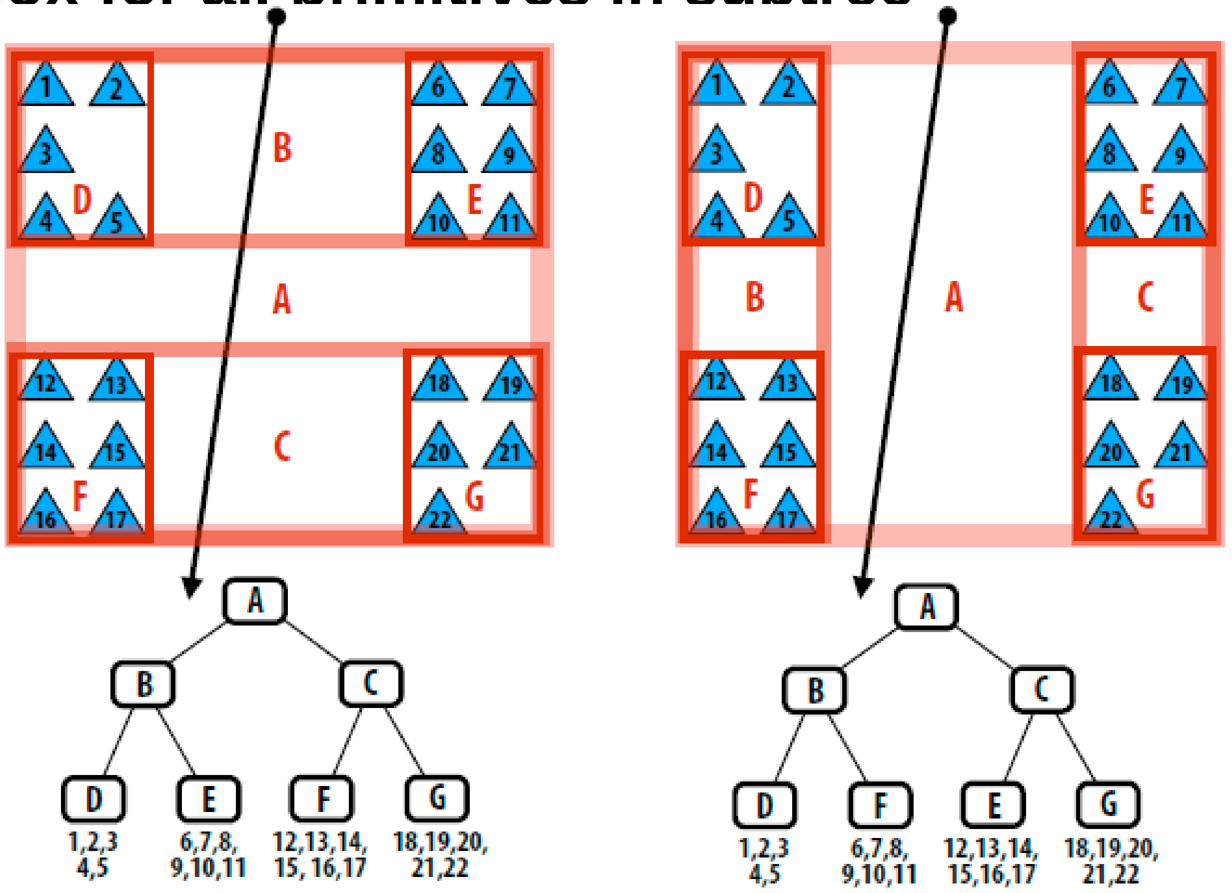

Ray misses bounding box of all primitives in scene
O(1) cost: requires 1 ray-box test

# Bounding volume hierarchy (BVH)

- **Interior nodes:**
  - **Represents subset of primitives in scene**
  - **Stores aggregate bounding box for all primitives in subtree**
- **Leaf nodes:**
  - **Contain list of primitives**

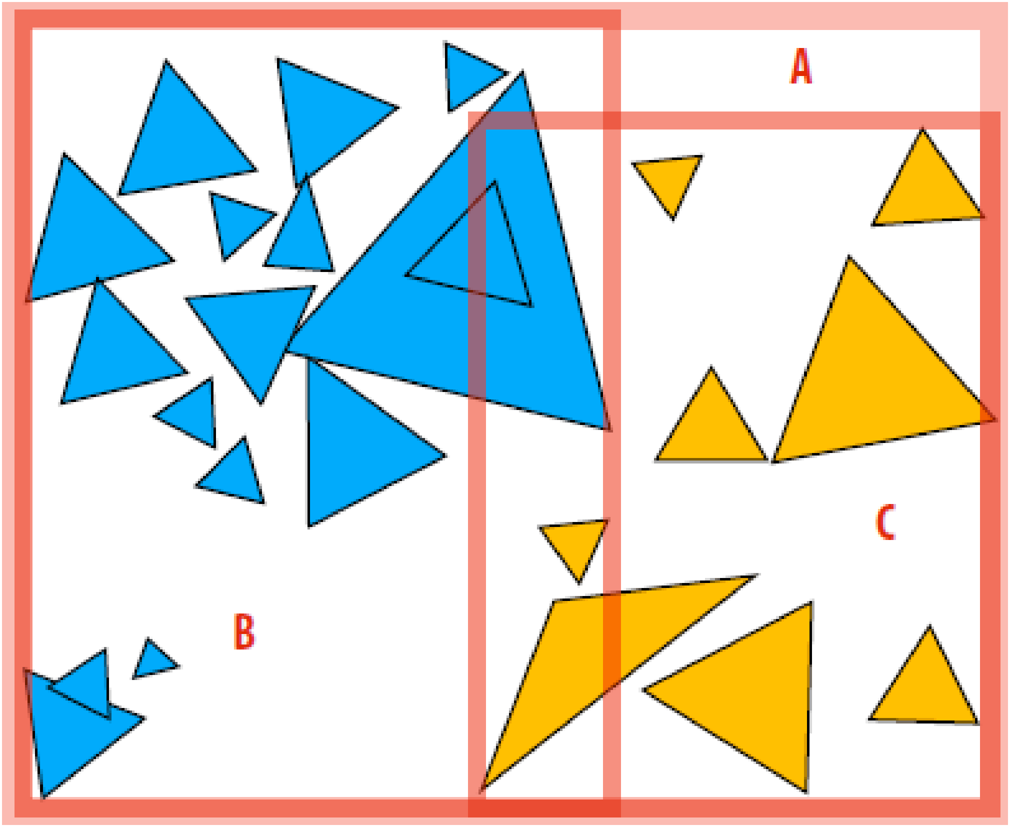
Left: two different BVH organizations of the same scene containing 22 primitives.

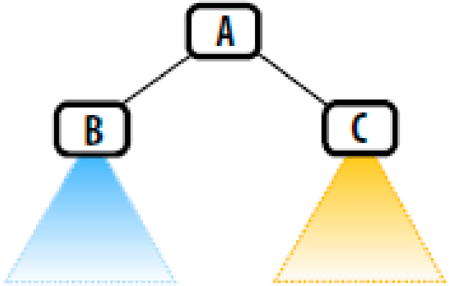
Is one BVH better than the other?



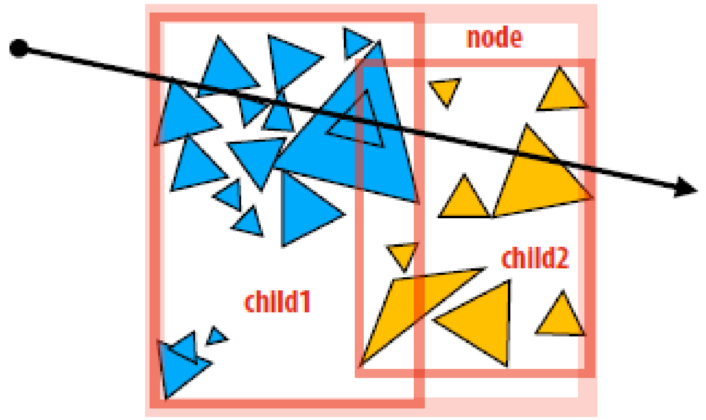
# Another BVH example

- BVH partitions each node's primitives into disjoints sets
  - Note: The sets can still be overlapping in space (below: child bounding boxes may overlap in space)



# Ray-scene intersection using a BVH

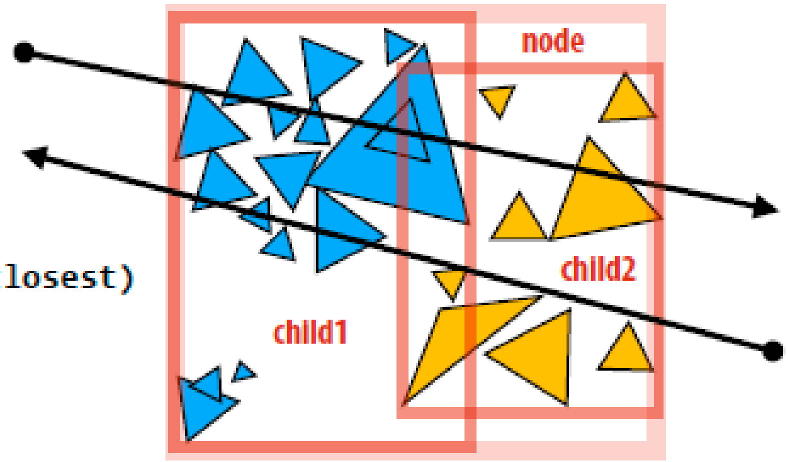



```
struct BVHNode {
   bool leaf;
   BBox bbox;
   BVHNode* child1;
   BVHNode* child2;
   Primitive* primList;
};

struct ClosestHitInfo {
   Primitive prim;
   float min_t;
};

void find_closest_hit(Ray* ray, BVHNode* node, ClosestHitInfo* closest) {

   if (!intersect(ray, node->bbox) || (closest point on box is farther than closest.min_t))
      return;

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit && t < closest.min_t) {
            closest.prim = p;
            closest.min_t = t;
         }
      }
   } else {
      find_closest_hit(ray, node->child1, closest);
      find_closest_hit(ray, node->child2, closest);
   }
}
```

How could this occur?

# Improvement: “front-to-back” traversal

Invariant: only call find_closest_hit() if ray intersects bbox of node.




```
void find_closest_hit(Ray* ray, BVHNode* node, ClosestHitInfo* closest)
{

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit && t < closest.min_t) {
            closest.prim = p;
            closest.min_t = t;
         }
      }
   } else {
      (hit1, min_t1) = intersect(ray, node->child1->bbox);
      (hit2, min_t2) = intersect(ray, node->child2->bbox);

      NVHNode* first = (min_t1 <= min_t2) ? child1 : child2;
      NVHNode* second = (min_t1 <= min_t2) ? child2 : child1;

      find_closest_hit(ray, first, closest);
      if (second child's min_t is closer than closest.min_t)
         find_closest_hit(ray, second, closest);
   }
}
```

“Front to back” traversal. Traverse to closest child node first. Why?

# Another type of query: any hit

- **Sometimes it's useful to know if the ray hits ANY primitive in the scene at all (don't care about distance to first hit)**

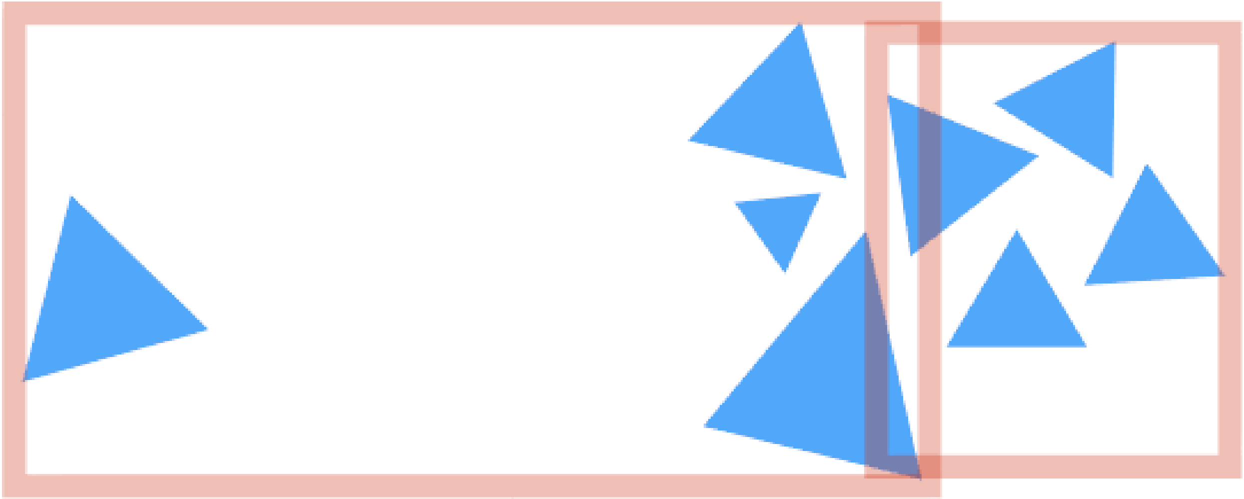
```
bool find_any_hit(Ray* ray, BVHNode* node) {

   if (!intersect(ray, node->bbox))
      return false;

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit)
            return true;
   } else {
     return ( find_closest_hit(ray, node->child1, closest) ||
              find_closest_hit(ray, node->child2, closest) );
   }
}
```

Interesting question of which child to enter first. How might you make a good decision?

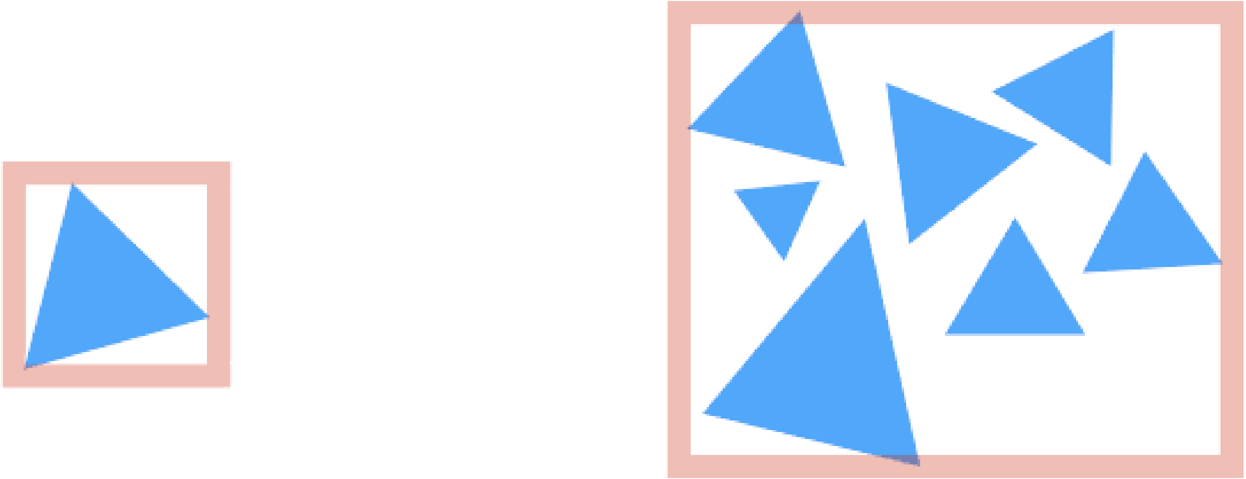
# For a given set of primitives, there are many possible BVHs

- (2N-2 ways to partition N primitives into two groups)
- How do we build a high-quality BVH?

# How would you partition these triangles into two groups?



Partition into child nodes with equal numbers of primitives



Better partition

Intuition: want small bounding boxes (minimize overlap between children, avoid empty space)

# What are we really trying to do?

- A good partitioning minimizes the cost of finding the closest intersection of a ray with primitives in the node.

- If a node is a leaf node (no partitioning):

$$C = \sum_{i=1}^{N} C_{\text{isect}}(i)$$

Where $C_{\text{isect}}(i)$ is the cost of ray-primitive intersection for primitive *i* in the node.

$$= N C_{\text{isect}}$$

(Common to assume all primitives have the same cost)

# Cost of making a partition

- **The expected cost of ray-node intersection, given that the node's primitives are partitioned into child sets A and B is:**

$$C = C_{\text{trav}} + p_A C_A + p_B C_B$$

$C_{\text{trav}}$ **is the cost of traversing an interior node (e.g., load data, bbox check)**

$C_A$ **and** $C_B$ **are the costs of intersection with the resultant child subtrees**

$p_A$ **and** $p_B$ **are the probability a ray intersects the bbox of the child nodes A and B**

**Primitive count is common approximation for child node costs:**

$$C = C_{\text{trav}} + p_A N_A C_{\text{isect}} + p_B N_B C_{\text{isect}}$$

**Where:** $N_A = |A|, N_B = |B|$

# Estimating probabilities

- **For convex object A inside convex object B, the probability that a random ray that hits B also hits A is given by the ratio of the surface areas SA and SB of these objects.**

$$P(\mathrm{hit}A|\mathrm{hit}B) = \frac{S_A}{S_B}$$

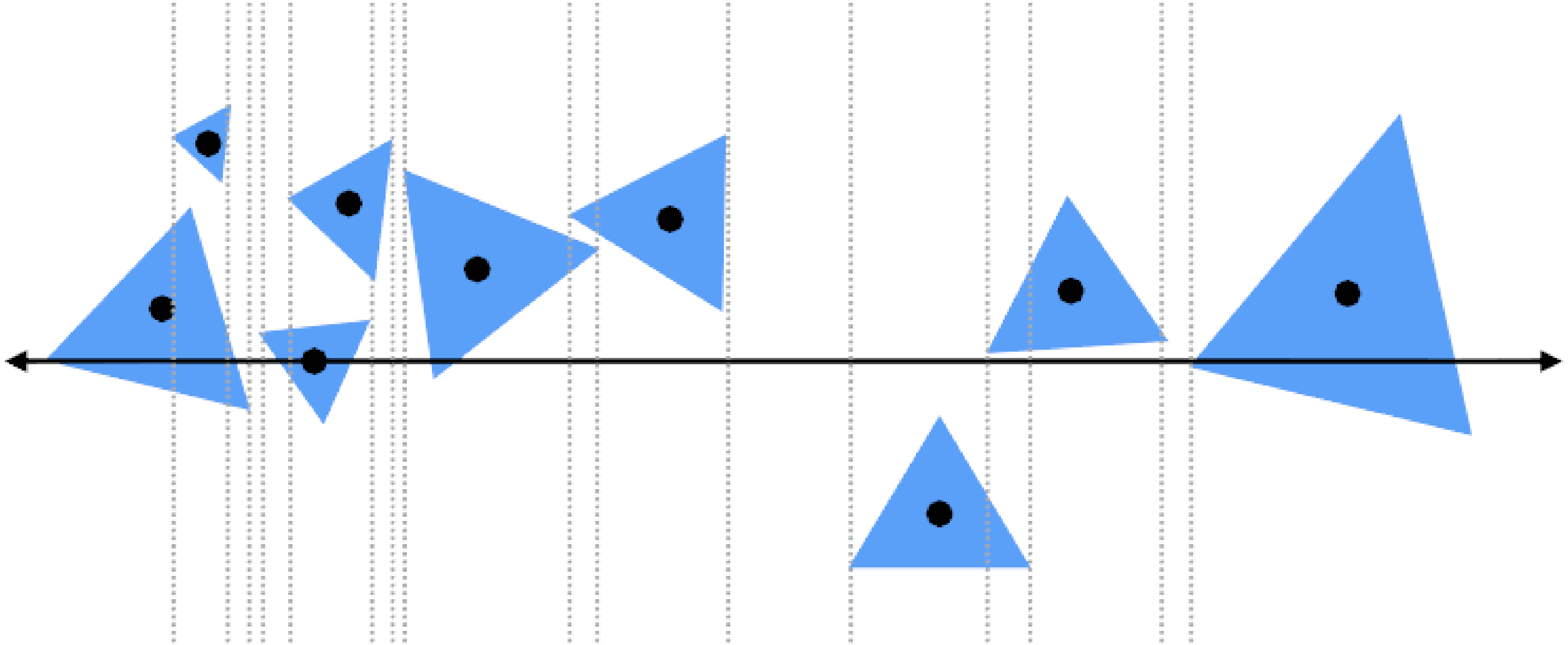
- **Surface area heuristic (SAH):**

$$C = C_{\mathrm{trav}} + \frac{S_A}{S_N} N_A C_{\mathrm{isect}} + \frac{S_B}{S_N} N_B C_{\mathrm{isect}}$$

- **Assumptions of the SAH (may not hold in practice):**
  - **Rays are randomly distributed**
  - **Rays are not occluded**

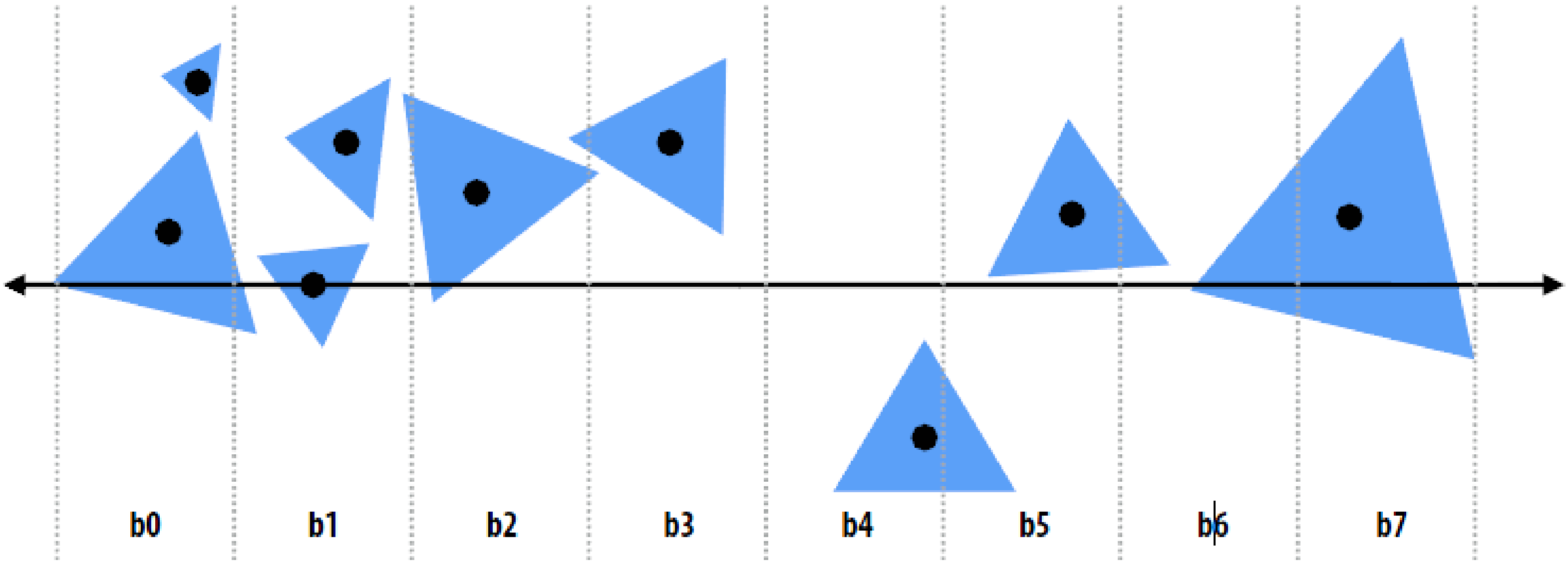
# Implementing partitions

- Constrain search for good partitions to axis-aligned spatial partitions
  - Choose an axis
  - Choose a split plane on that axis
  - Partition primitives by the side of splitting plane their centroid lies
  - 2N-2 possible splitting positions for node with N primitives. (Why?)



# Efficiently implementing partitioning

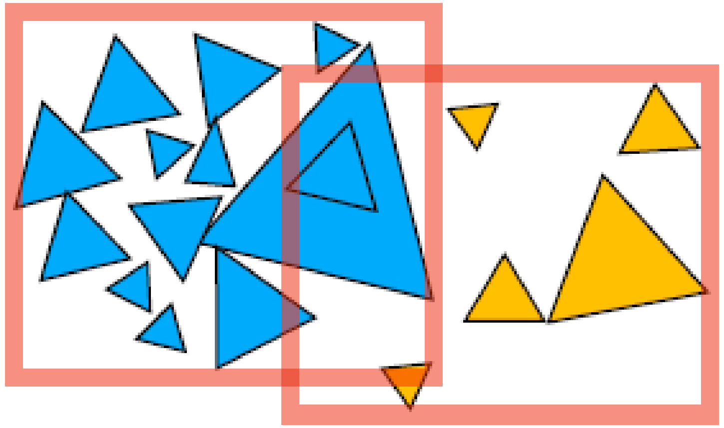
- **Efficient modern approximation: split spatial extent of primitives into B buckets (B is typically small: B < 32)**



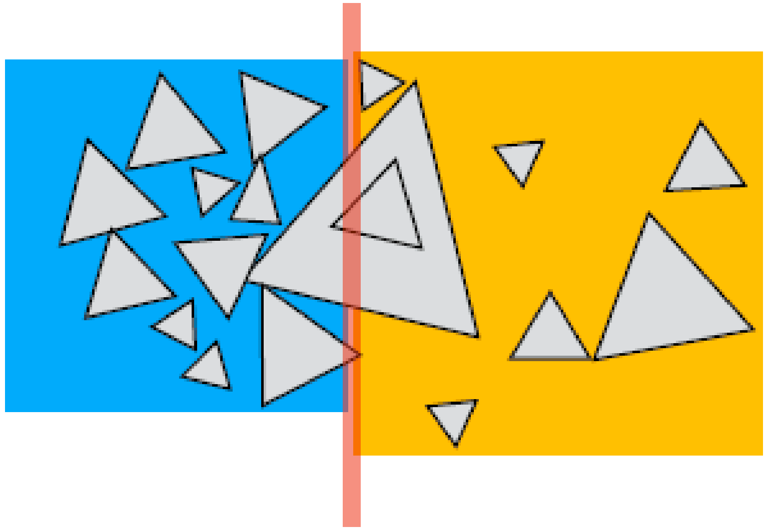

```
For each axis: x,y,z:
   initialize buckets
   For each primitive p in node:
      b = compute_bucket(p.centroid)
      b.bbox.union(p.bbox);
      b.prim_count++;
   For each of the B-1 possible partitioning planes evaluate SAH
Execute lowest cost partitioning found (or make node a leaf)
```

# Primitive-partitioning acceleration structures vs. space-partitioning structures

- Primitive partitioning (bounding volume hierarchy): partitions node's primitives into disjoint sets (but sets may overlap in space)

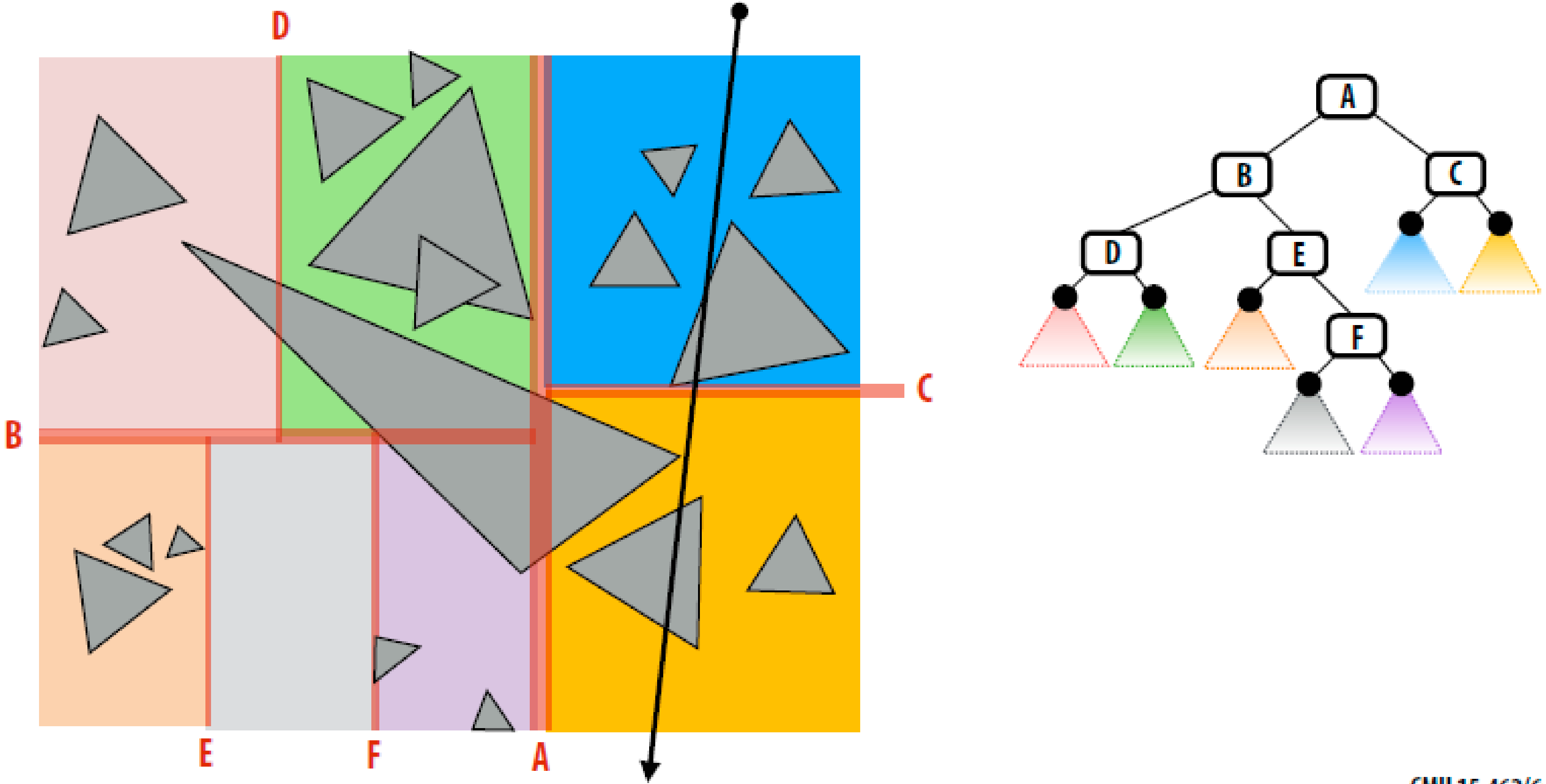


- Space-partitioning (grid, K-D tree) partitions space into disjoint regions (primitives may be contained in multiple regions of space)



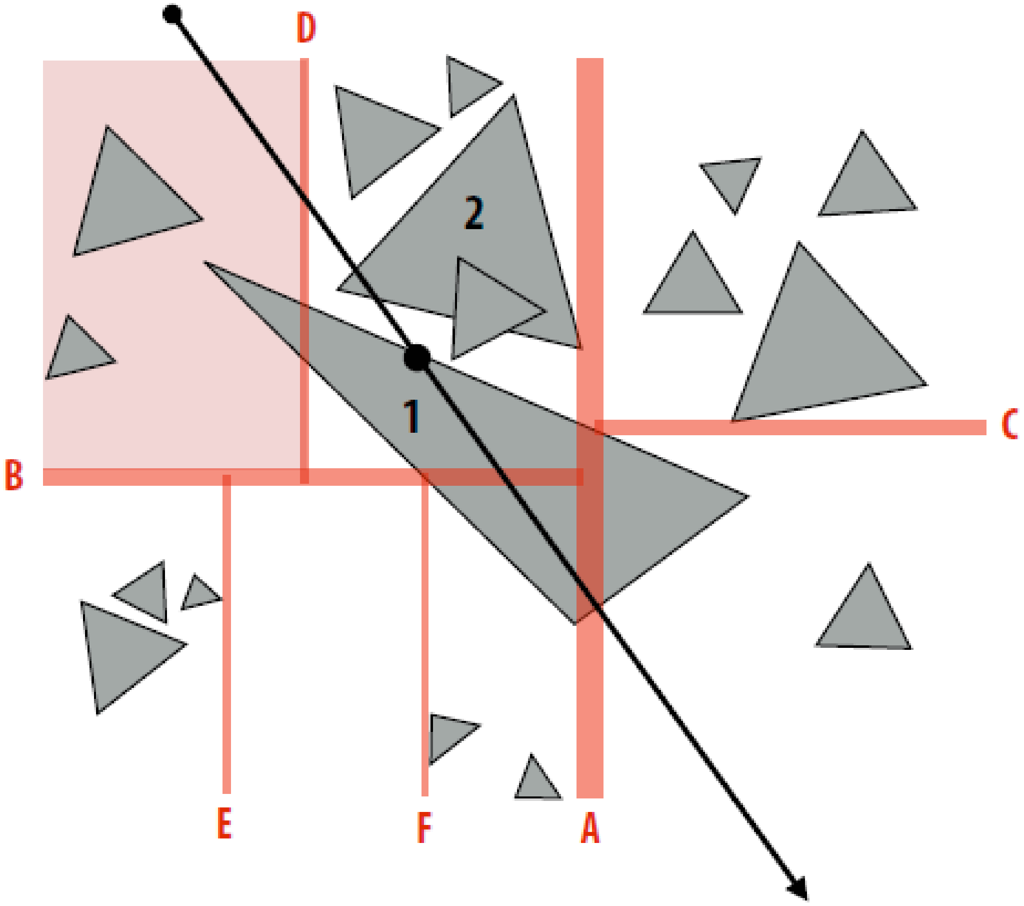
# K-D tree

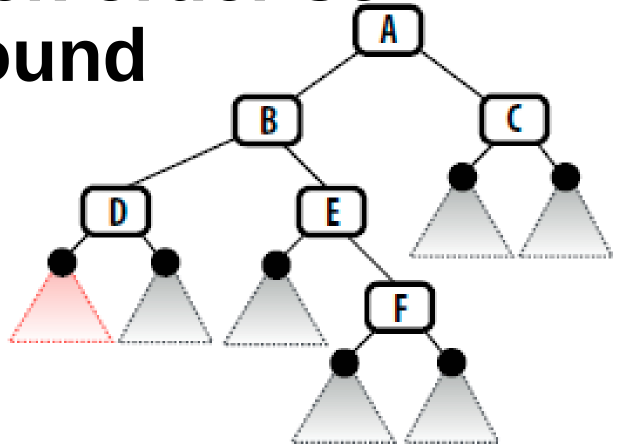
- Recursively partition space via axis-aligned partitioning planes
  - Interior nodes correspond to spatial splits (still correspond to spatial volume)
  - Node traversal can proceed in front-to-back order (unlike BVH, can terminate search after first hit is found).
  - Intuition: partitions curve out empty space (construction of K-D tree may produce more tree nodes than primitives depending on ratio of C_trav & C_isect



# Challenge: objects overlap multiple nodes

- **Want node traversal to proceed in front-to-back order so traversal can terminate search after first hit found**




Triangle 1 overlaps multiple nodes.

Ray hits triangle 1 when in highlighted leaf cell.

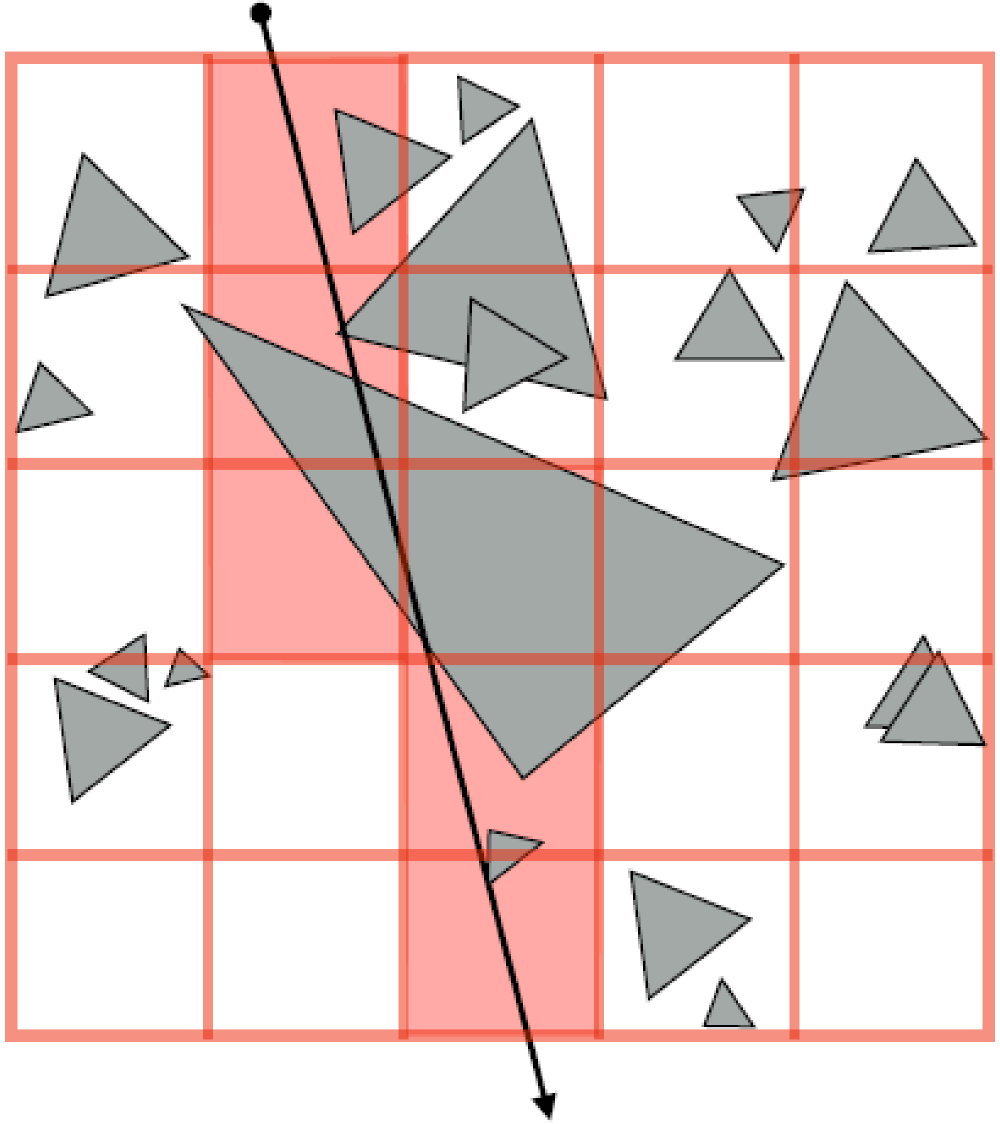
But intersection with triangle 2 is closer! (Haven't traversed to that node yet)

Solution: require primitive intersection point to be within current leaf node.

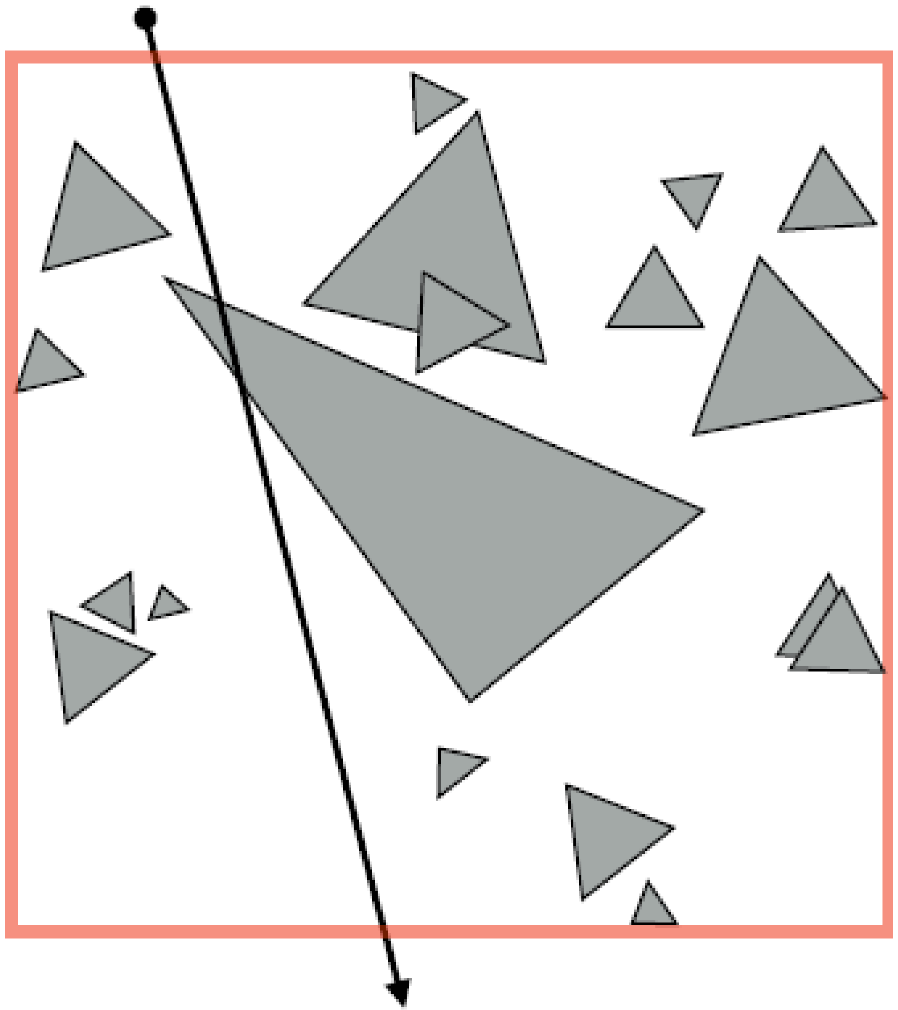
(primitives may be intersected multiple times by same ray *)

# Uniform grid

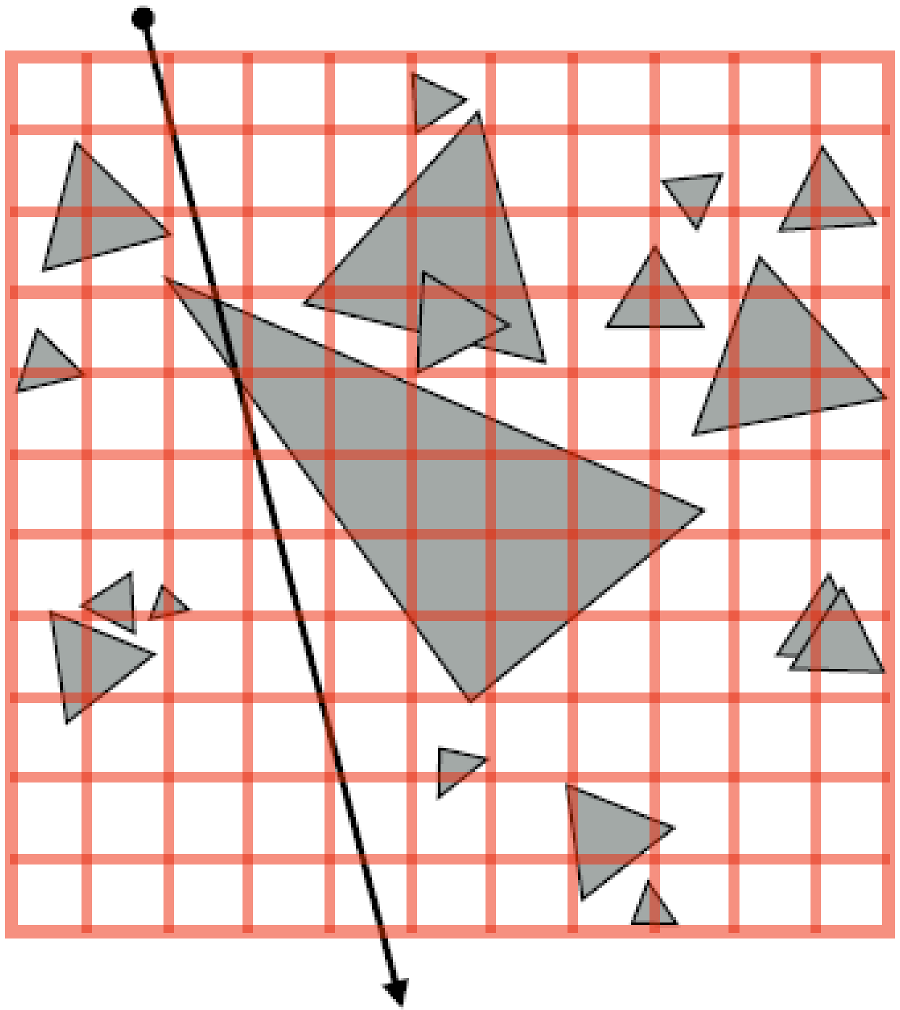
- Partition space into equal sized volumes ("voxels")
- Each grid cell contains primitives that overlap voxel. (very cheap to construct acceleration structure)
- Walk ray through volume in order
  - Very efficient implementation possible (think: 3D line rasterization)
  - Only consider intersection with primitives in voxels the ray intersects



# What should the grid resolution be?



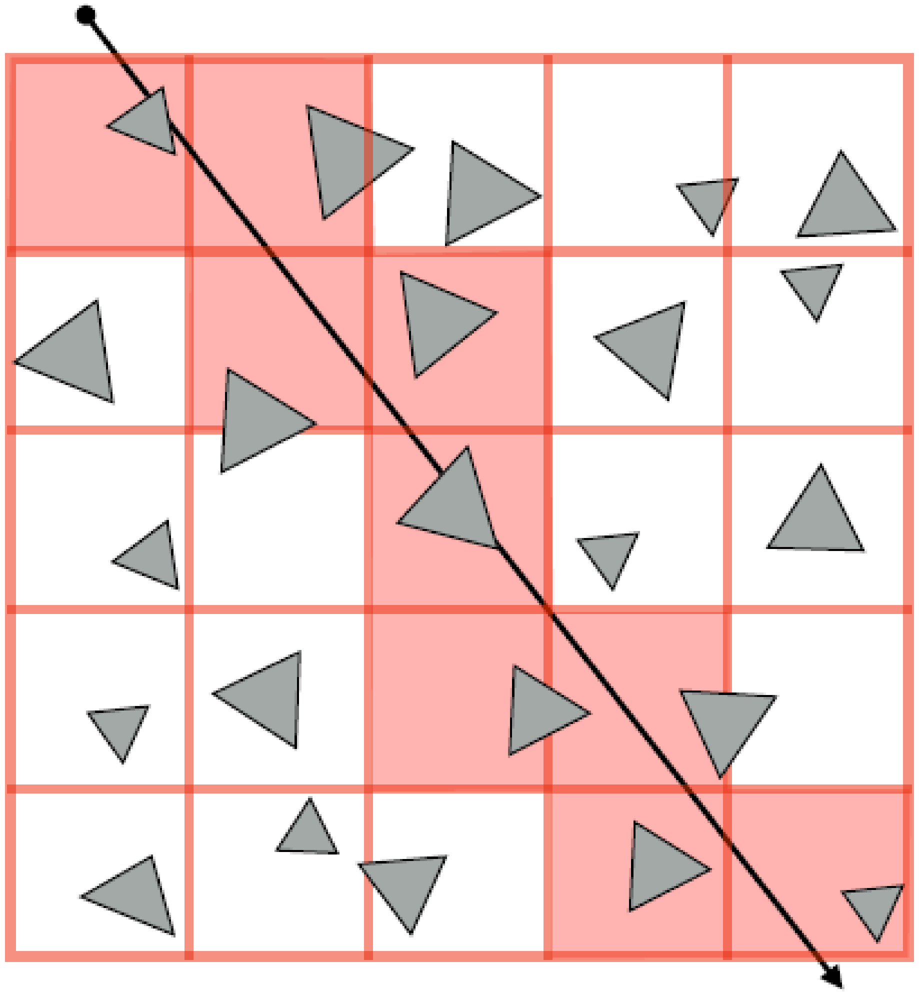
Too few grids cell: degenerates to brute-force approach



Too many grid cells: incur significant cost traversing through cells with empty space

# Heuristic

- **Choose number of voxels ~ total number of primitives (constant prims per voxel — assuming uniform distribution of primitives)**



Intersection cost: $O(\sqrt[3]{N})$

# Uniform distribution of primitives

Terrain / height fields:

[Image credit: Misuba Renderer]

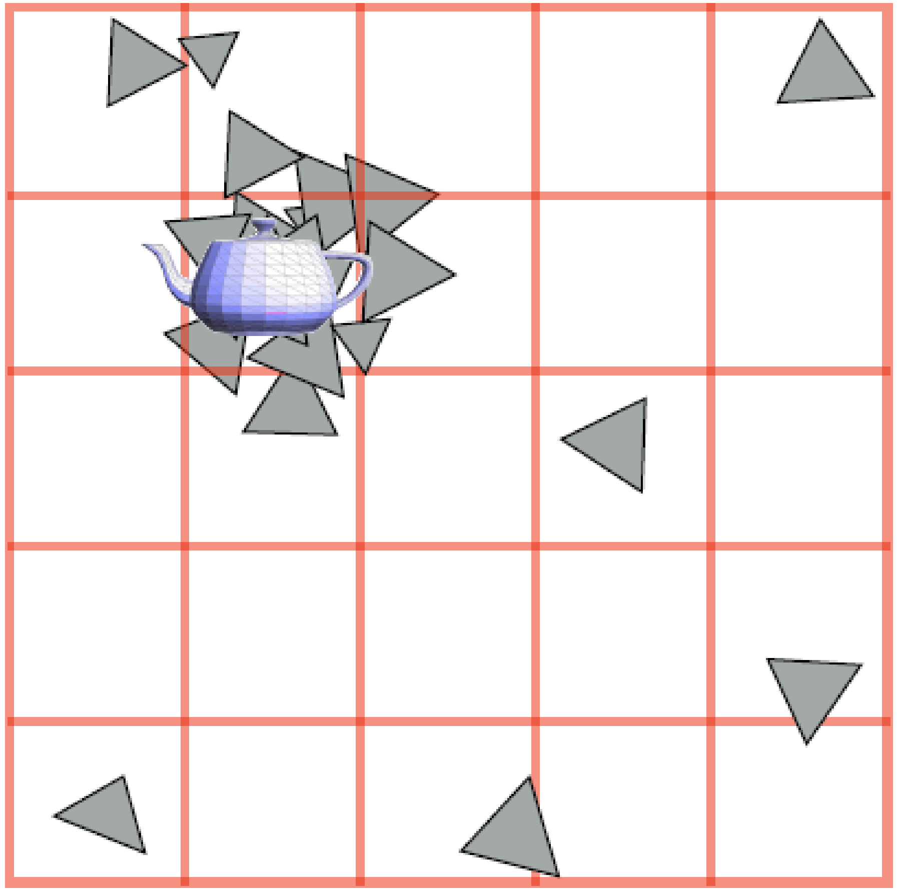
Grass:

[Image credit: www.kevinboulanger.net/grass.html]

# Uniform grid cannot adapt to non-uniform distribution of geometry in scene

- (Unlike K-D tree, location of spatial partitions is not dependent on scene geometry)

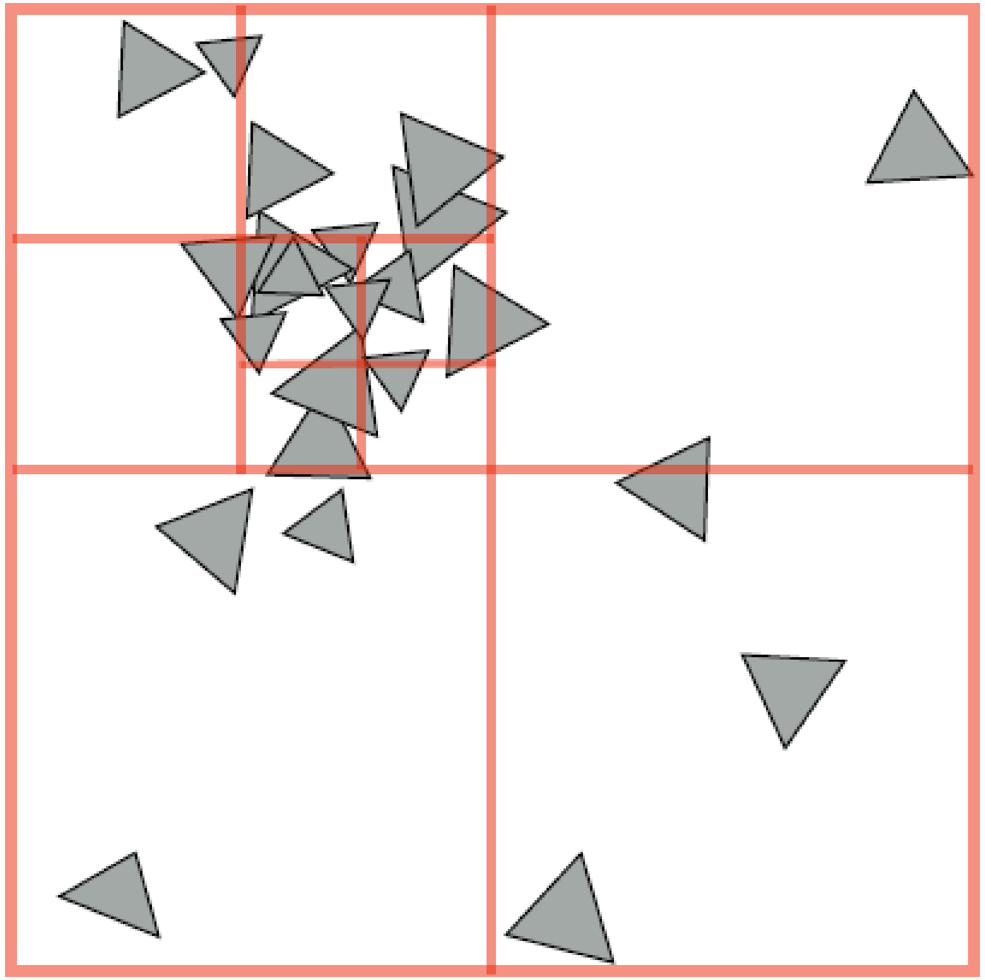
- “Teapot in a stadium problem”
- Scene has large spatial extent.
- Contains a high-resolution object that
- has small spatial extent (ends up in one
- grid cell)



# Non-uniform distribution of geometric detail

# Quad-tree / octree

- Like uniform grid: easy to build (don't have to choose partition planes)
- Has greater ability to adapt to location of scene geometry than uniform grid.
- But lower intersection performance than K-D tree (only limited ability to adapt)



Quad-tree: nodes have 4 children (partitions 2D space)

Octree: nodes have 8 children (partitions 3D space)

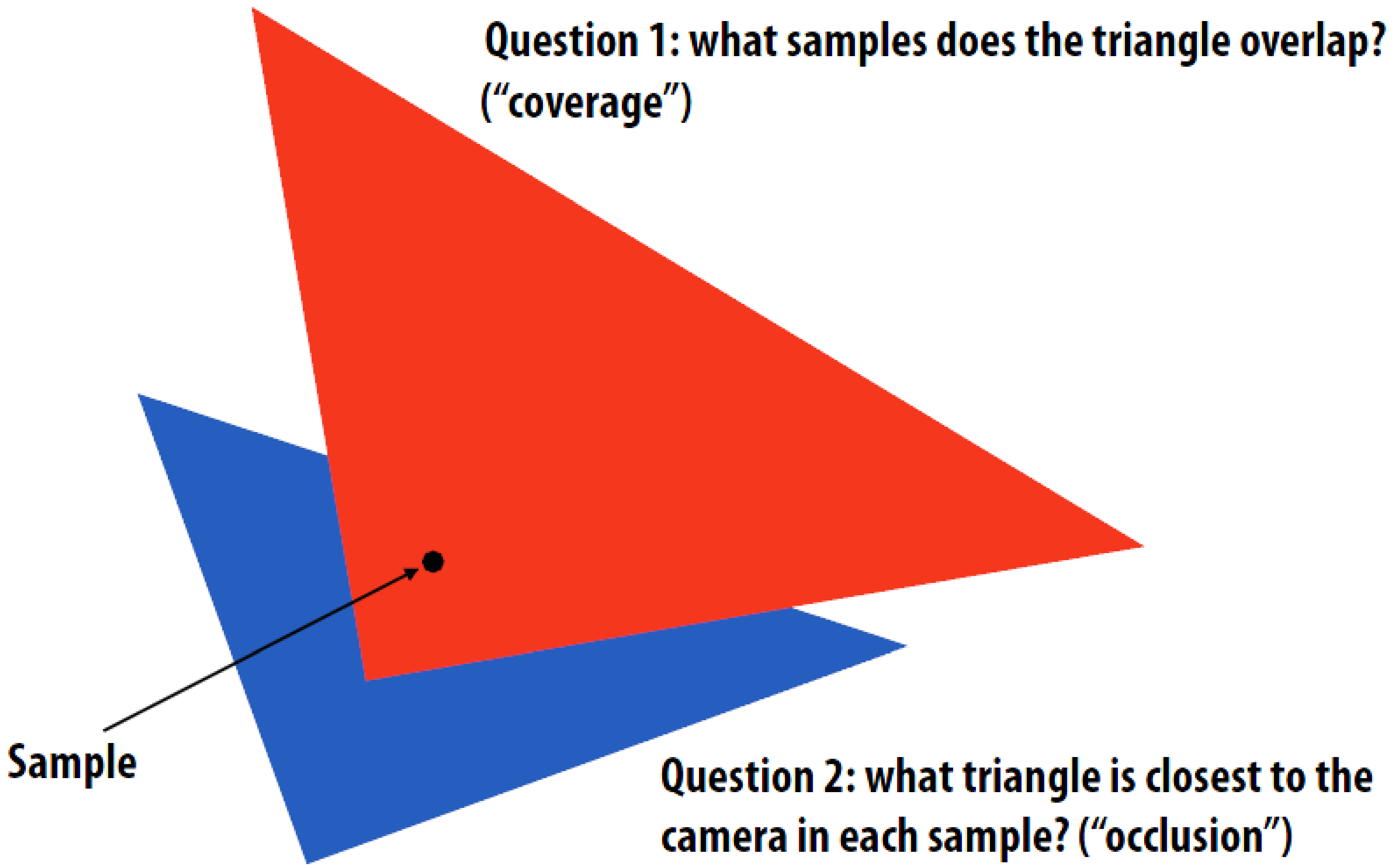
# Summary of accelerating geometric queries: choose the right structure for the job

- Primitive vs. spatial partitioning:
  - Primitive partitioning: partition sets of objects
    - Bounded number of BVH nodes, simpler to update if primitives in scene change position
  - Spatial partitioning: partition space
    - Traverse space in order (first intersection is closest intersection), may intersect primitive multiple times
- Adaptive structures (BVH, K-D tree)
  - More costly to construct (must be able to amortize construction over many geometric queries)
  - Better intersection performance under non-uniform distribution of primitives
- Non-adaptive accelerations structures (uniform grids)
  - Simple, cheap to construct
  - Good intersection performance if scene primitives are uniformly distributed
- Many, many combinations thereof

# Rendering via ray casting:
# one common use of ray-scene intersection tests *

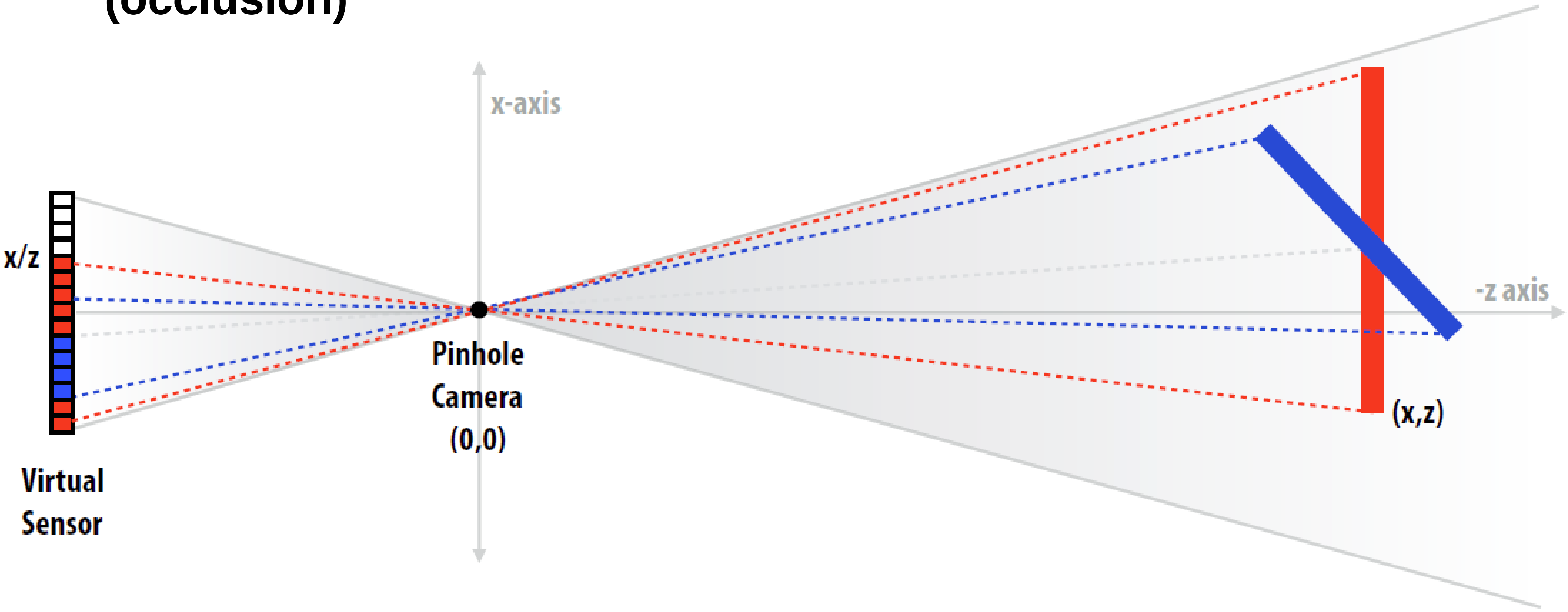
**Rasterization and ray casting are two algorithms for solving the same problem: determining “visibility from a camera”**

# triangle visibility:



# The visibility problem

- What scene geometry is visible at each screen sample?
  - What scene geometry projects into a screen pixel? (coverage)
  - Which geometry is visible from the camera at that pixel? (occlusion)



# Basic rasterization algorithm

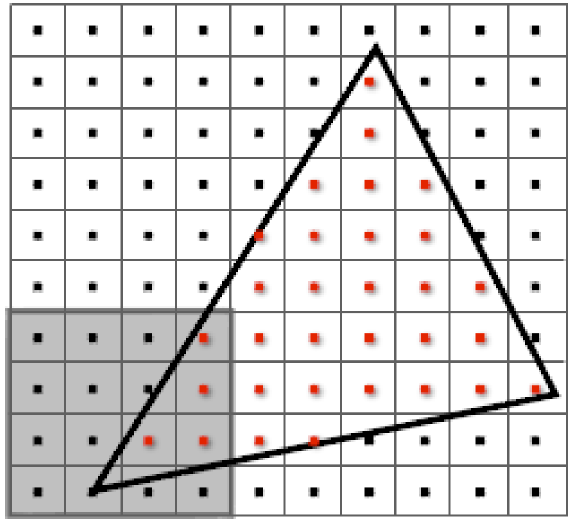
Sample = 2D point

Coverage: 2D triangle/sample tests (does projected triangle cover 2D sample point)

Occlusion: depth buffer

```
initialize z_closest[] to INFINITY                  // store closest-surface-so-far for all samples
initialize color[]                                  // store scene color for all samples
for each triangle t in scene:                       // loop 1: triangles
    t_proj = project_triangle(t)
    for each 2D sample s in frame buffer:           // loop 2: visibility samples
        if (t_proj covers s)
            compute color of triangle at sample
            if (depth of t at s is closer than z_closest[s])
                update z_closest[s] and color[s]
```

"Given a triangle, find the samples it covers"
(finding the samples is relatively easy since they are distributed uniformly on screen)

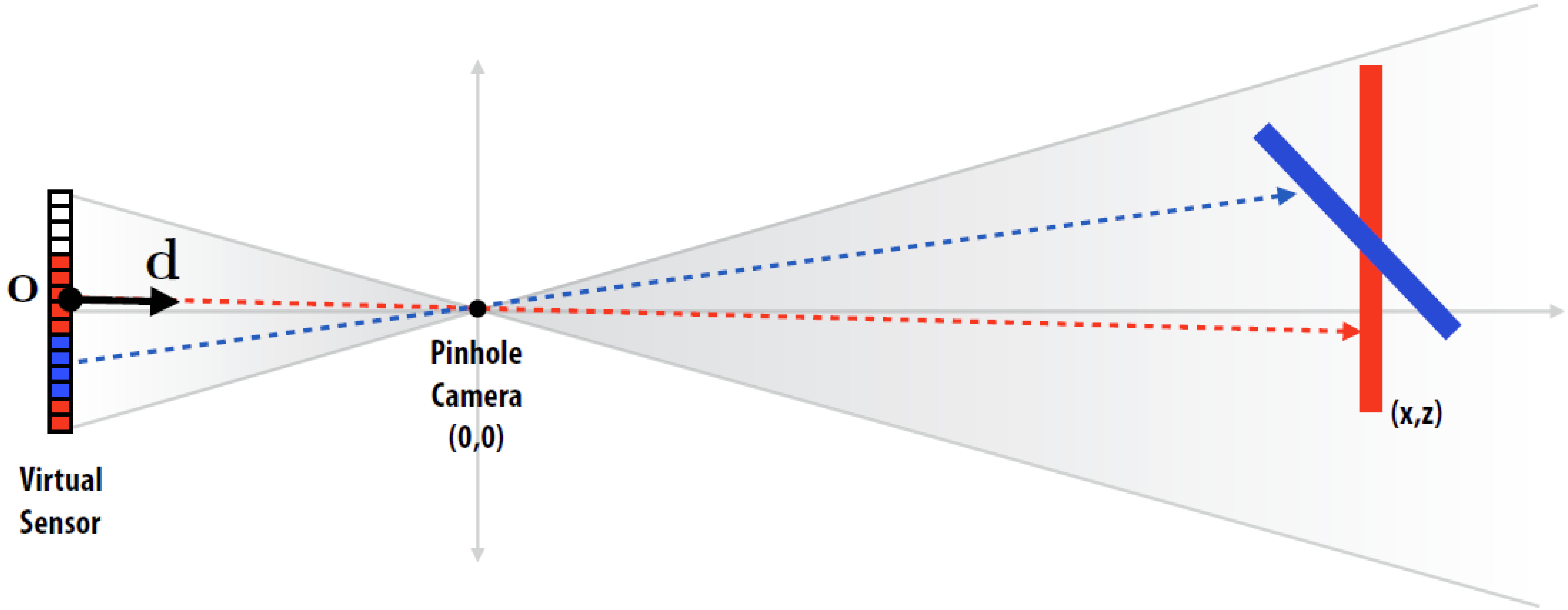


But what from this lecture do modern hierarchical rasterization algorithms remind you of?

(for each tile of image, if triangle overlaps tile, check all samples in tile)

# The visibility problem (described differently)

- In terms of casting rays from the camera:
  - What scene primitive is hit by a ray originating from a point on the virtual sensor and traveling through the aperture of the pinhole camera? (coverage)



# Basic ray casting algorithm

Sample = a ray in 3D

Coverage: 3D ray-triangle intersection tests (does ray "hit" triangle)

Occlusion: closest intersection along ray

```
initialize color[]                                   // store scene color for all samples
for each sample s in frame buffer:                   // loop 1: visibility samples (rays)
    r = ray from s on sensor through pinhole aperture
    r.min_t = INFINITY                               // only store closest-so-far for current ray
    r.tri = NULL;
    for each triangle tri in scene:                   // loop 2: triangles
        if (intersects(r, tri)) {                     // 3D ray-triangle intersection test
            if (intersection distance along ray is closer than r.min_t)
                update r.min_t and r.tri = tri;
        }
    color[s] = compute surface color of triangle r.tri at hit point
```

Compared to rasterization approach: just a reordering of the loops! (+ math in 3D)

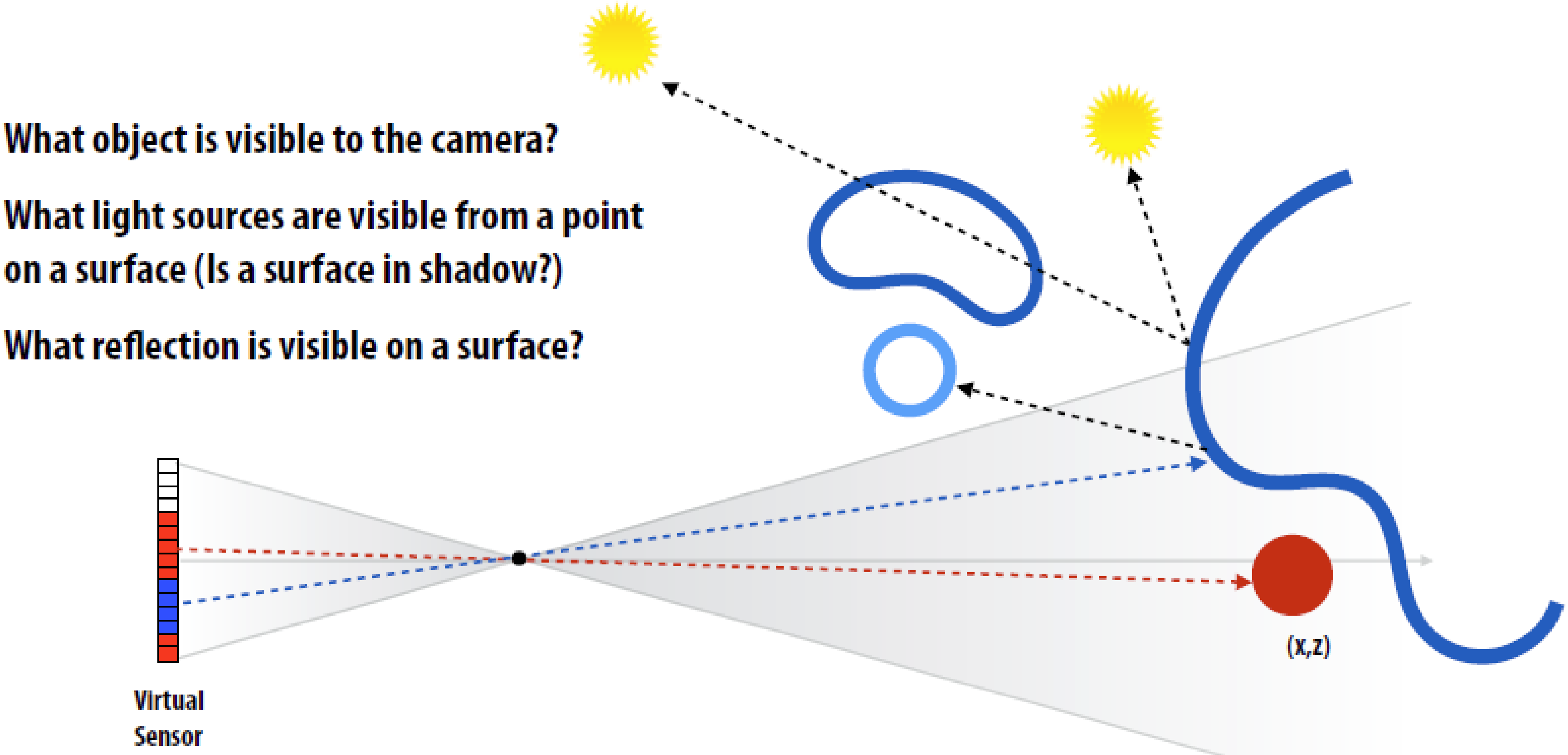
"Given a ray, find the closest triangle it hits"

As we saw today, the brute force "for each triangle" loop is typically accelerated using an acceleration structure. (A rasterizer's "for each sample" inner loop is not just a loop over all screen samples either.)

# Basic rasterization vs. ray casting

- Rasterization:
  - Proceeds in triangle order (never have to store in entire scene, naturally supports unbounded size scenes)
  - Store depth buffer (random access to regular structure of fixed size)
- Ray casting:
  - Proceeds in screen sample order
    - Never have to store closest depth so far for the entire screen (just current ray)
    - Natural order for rendering transparent surfaces (process surfaces in the order the are encountered along the ray: front-to-back or back-to-front)
  - Must store entire scene (random access to irregular structure of variable size: depends on complexity and distribution of scene)
- Modern high-performance implementations of rasterization and ray-casting embody very similar techniques
  - Hierarchies of rays/samples
  - Hierarchies of geometry

# Ray-scene intersection is a general visibility primitive: What object is visible along this ray?




(In contrast, rasterization is a highly-specialized solution for computing visibility for a set of uniformly distributed rays originating from the same point.)

# Thank you