Computer Graphics
- Transformations in OpenGL

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http://jjcao.github.io/ComputerGraphics/

Camera Analogy

- OpenGL coordinate system has different origin (lower-left corner) from the window system (upper-left corner)

- The transformation process to produce the desired scene for viewing is analogous to taking a photograph with a camera

- The steps with a camera (or a computer) might be the following:
  - Arrange the scene to be photographed into the desired composition (**modelling** transformation)
  - Set up your tripod and pointing the camera at the scene (**viewing** transformation).
  - Choose a camera lens or adjust the zoom (**projection** transformation)
  - Determine how large you want the final photograph to be (**viewport** transformation)

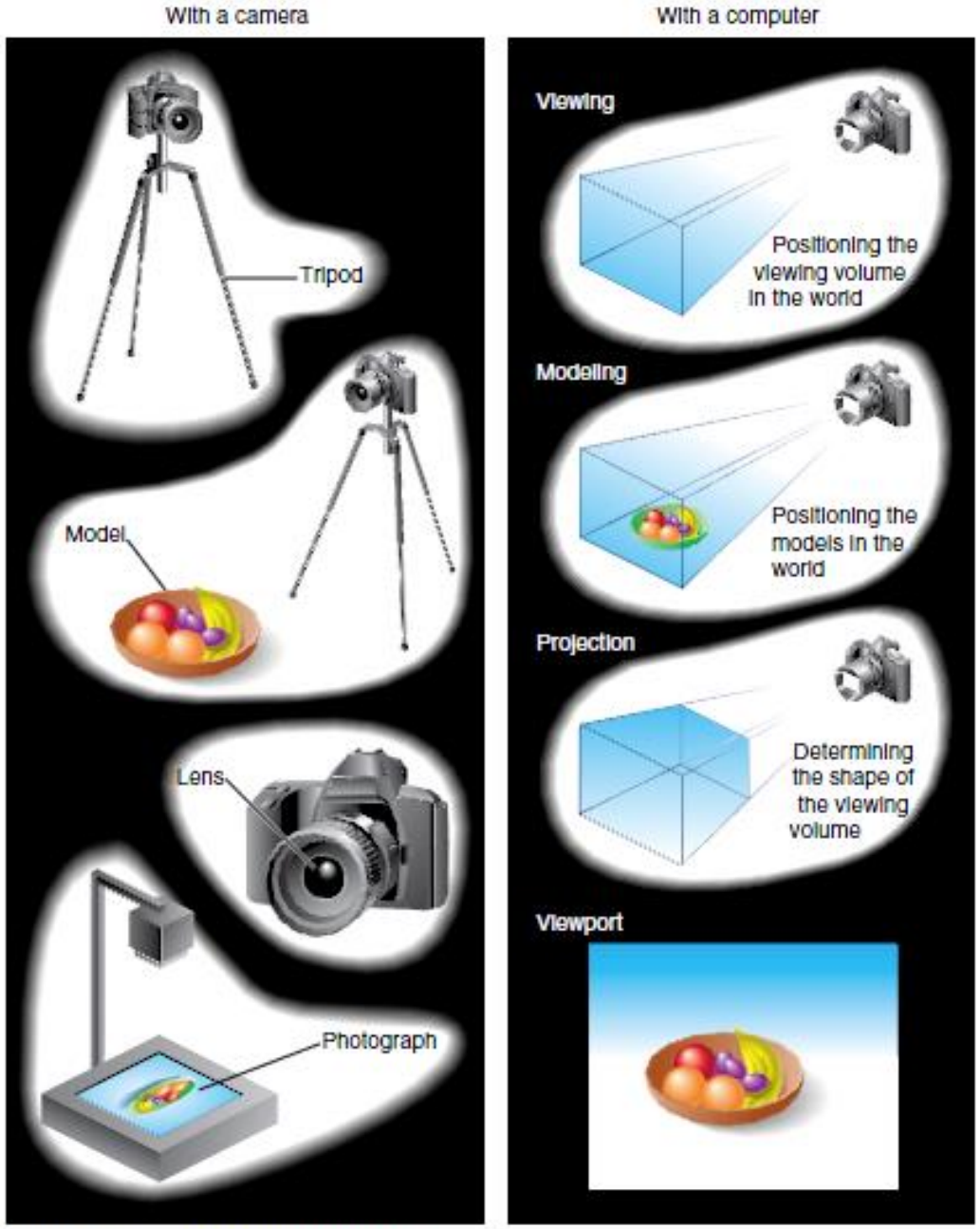
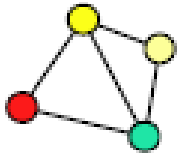


Figure 3-1 The Camera Analogy

Transformation Pipeline



Local (Object) Space

Modeling transformation

World Space

Viewing transformation

Eye Space

Projection transformation

Clip Space

Perspective divide

NDC space

Normalized Device Coordinates

Viewport mapping

Screen space

Recall: Affine Transformations

- Given a point $[x\ y\ z]^\top$
- form homogeneous coordinates $[x\ y\ z\ 1]^\top$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- The transformed point is $[x'\ y'\ z']^\top$

Transformation Matrices in OpenGL

- Transformation matrices in OpenGL are vectors of 16 values (**column-major** matrices)
- in glLoadMatrixf(GLfloat *m); $\mathbf{m}^\top = [m_1, m_2, \ldots, m_{16}]^\top$ represents

$$\begin{bmatrix} m_1 & m_5 & m_9 & m_{13} \\ m_2 & m_6 & m_{10} & m_{14} \\ m_3 & m_7 & m_{11} & m_{15} \\ m_4 & m_8 & m_{12} & m_{16} \end{bmatrix}$$

- OpenGL has 4 different types of matrices
  - **GL_MODELVIEW, GL_PROJECTION**, GL_TEXTURE, and GL_COLOR
  - Switch, e.g. **glMatrixMode(GL_MODELVIEW)**.

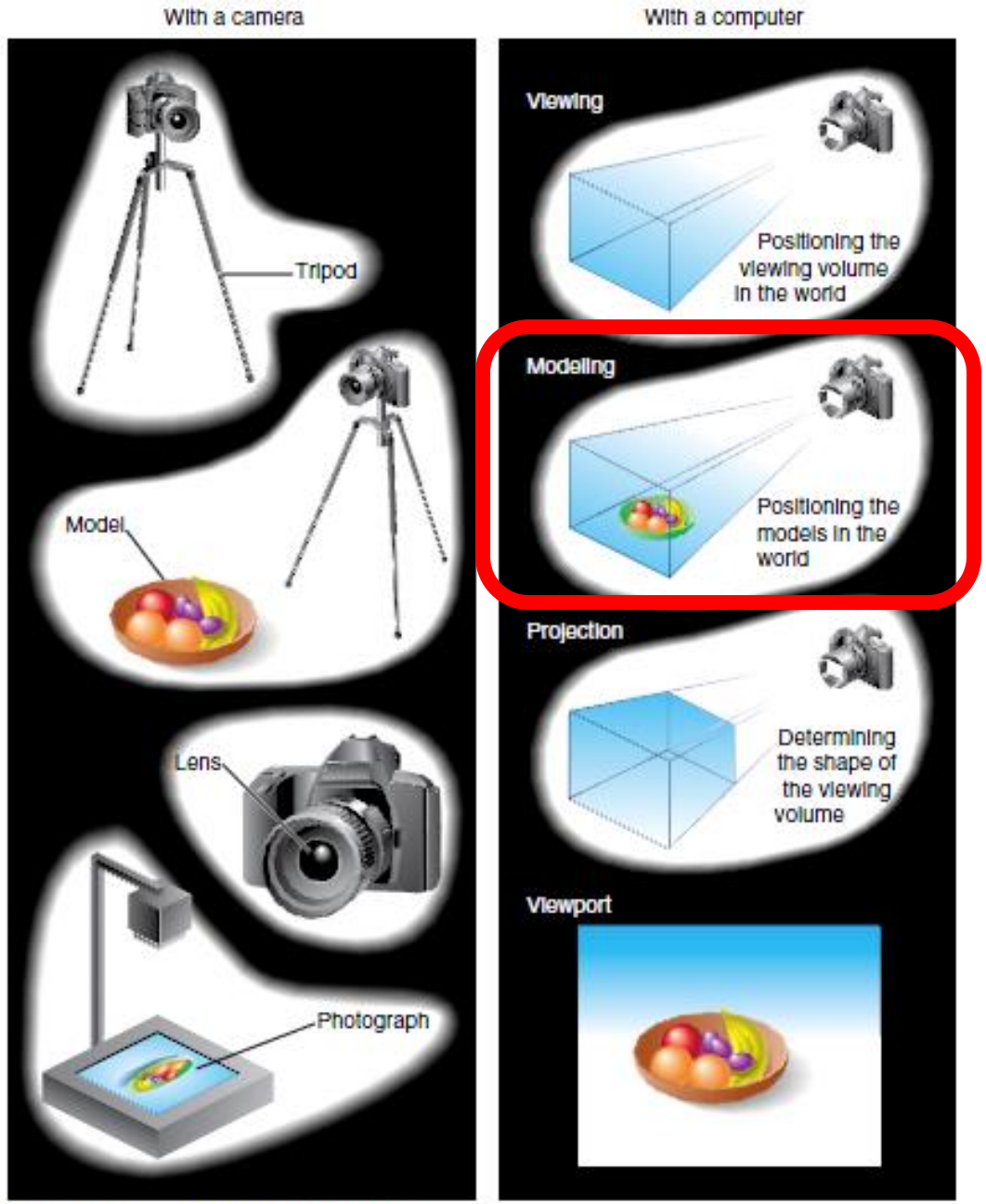
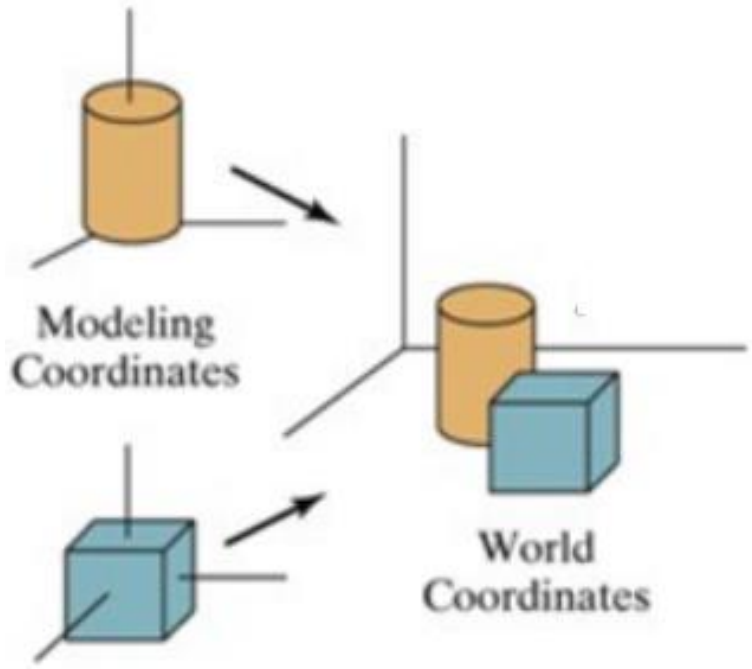


Figure 3-1 The Camera Analogy

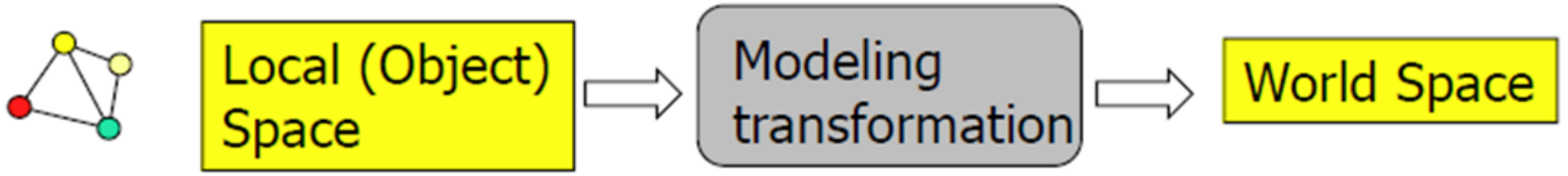
Local Coordinate System

- When you load a file containing a 3d object, its vertices stores coordinates in local CS.
- Assuming obj1, obj2 & obj3 are loaded.
  - Normally, their centers are the origins if they are actually created by code or hand.
  - Sometimes, their centers are not the origins of their local CS respectively if they are results of 3D scanning, etc.
  - Anyway, they are treated as local CS



World Coordinate System

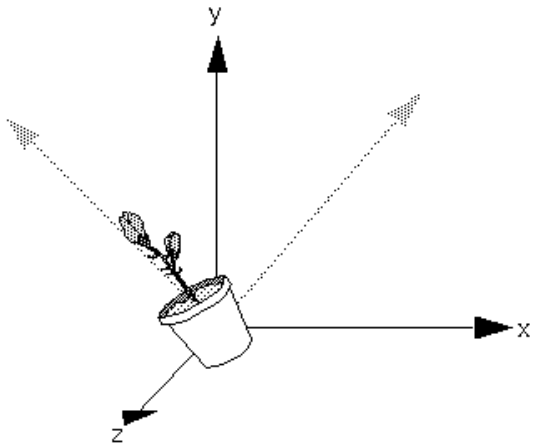
- When the obj is just loaded, its local CS is used as WCS.
- To place multiple objs in your WCS, you need specify position, size, orientation of them
- Transformations need to be performed to position the object in WCS



- A modeling transformation is a sequence of translations, rotations, scalings (in arbitrary order) matrices multiplied together

Modeling Transformations

- The three OpenGL routines for modeling transformations are:
  - glTranslate*(),
  - glScale*()
  - void **glRotate**{fd}(TYPE *angle*, TYPE *x*, TYPE *y*, TYPE *z*); **deprecated**
  - **glRotatef(**45.0, 0.0, 0.0, 1.0**)**
- These routines **transform an object (or coordinate system**, if you're thinking of it that way) by moving, rotating, stretching it
- All three commands are **equivalent** to producing an appropriate translation, rotation, or scaling **matrix**, and then calling **glMultMatrix***() with that matrix as the argument
- OpenGL **automatically** computes the matrices for you



Modeling Transformations

- Each of these postmultiplies the *current matrix*
  - E.g., if current matrix is **C**, then **C=CS**
  - E.g., rotate then translate a vector x => T(Rx) = TRx not RTx

- The current matrix is either the **modelview** matrix or the projection matrix (also a texture matrix, won't discuss)
  - Set these with glMatrixMode(), e.g.:

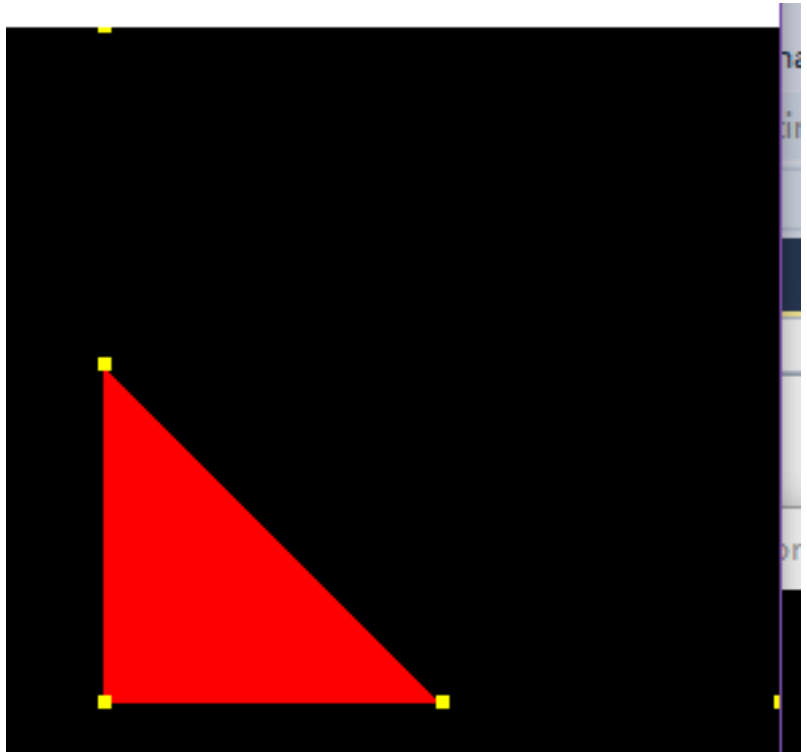
```
glMatrixMode(GL_MODELVIEW);
glMatrixMode(GL_PROJECTION);
```

- **WARNING: common mistake ahead!**
  - Be sure that you are in **GL_MODELVIEW** mode before making modeling or viewing calls!
  - Ugly mistake because it can appear to work, at least for a while…, see https://sjbaker.org/steve/omniv/projection_abuse.html

Example for Modeling Transformation 1

```
void display() {
glClear(GL_COLOR_BUFFER_BIT);
glColor4f(1,1,0,1); //glColor* have been deprecated in OpenGL 3

// draw triangle 1
glBegin(GL_TRIANGLES);
glColor4f(1.0,0.0,0.0,1.0);glVertex3f(0.0, 0.0, -10.0);
glVertex3f(1.0, 0.0, -10.0); glVertex3f(0.0, 1.0, -10.0);
glEnd();
...
```



Example for Modeling Transformation 2

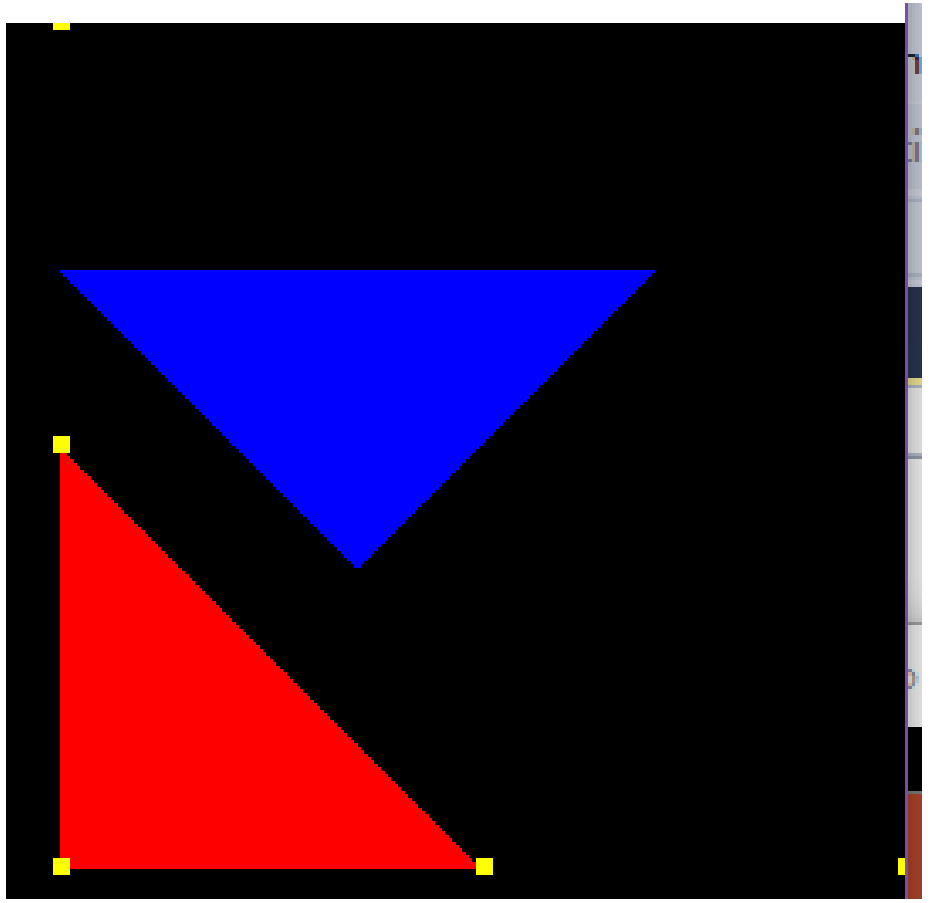
```
// draw triangle 3
glMatrixMode(GL_MODELVIEW);
glPushMatrix(); glLoadIdentity(); //More details will be explained
glRotatef(45, 0, 0, 1);
glTranslatef(1, 0, 0);
glBegin(GL_TRIANGLES);
glColor4f(0.0, 1.0, 0.0, 1.0);glVertex3f(0.0, 0.0, -10.0);
glVertex3f(1.0, 0.0, -10.0);glVertex3f(0.0, 1.0, -10.0);
glEnd();
glPopMatrix();


glutSwapBuffers();
}
```

Could you draw the two triangles on some paper?

Example for Modeling Transformation 2

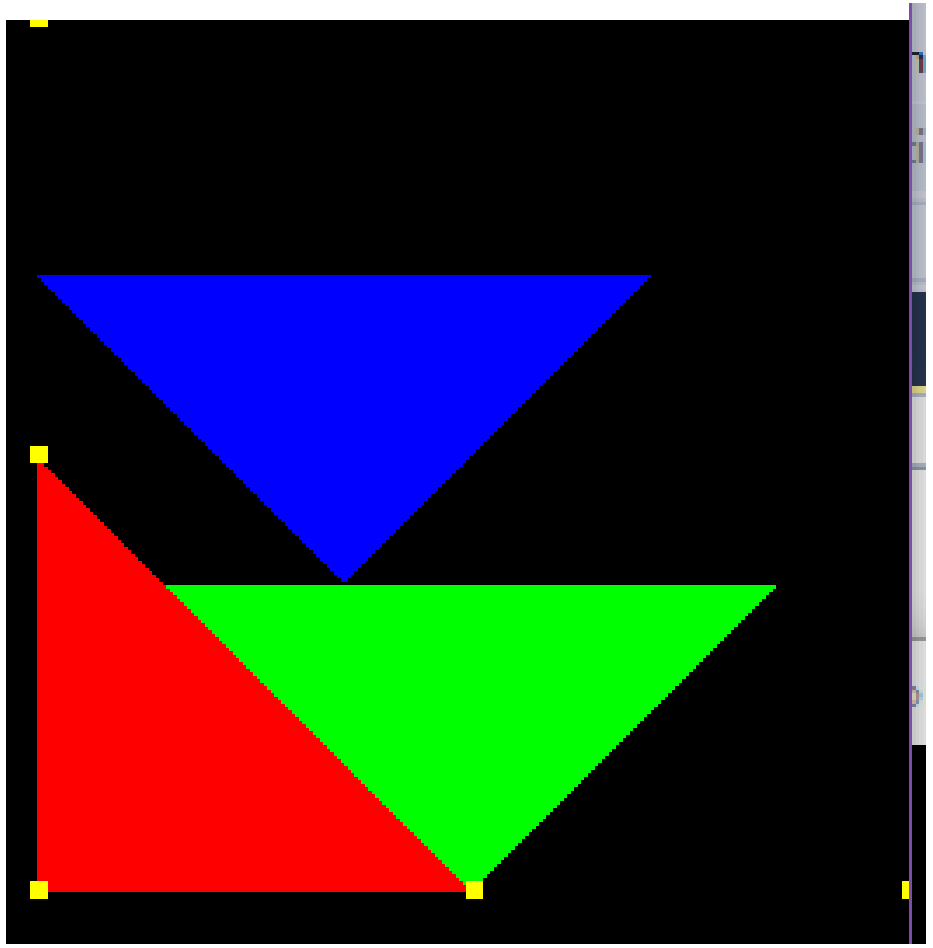
```
// draw triangle 3
glMatrixMode(GL_MODELVIEW);
glPushMatrix(); glLoadIdentity();
glRotatef(45, 0, 0, 1);
glTranslatef(1, 0, 0);
glBegin(GL_TRIANGLES);
glColor4f(0.0, 1.0, 0.0, 1.0);
glVertex3f(0.0, 0.0, -10.0);
glVertex3f(1.0, 0.0, -10.0);
glVertex3f(0.0, 1.0, -10.0);
glEnd();
glPopMatrix();
glutSwapBuffers();
}
```



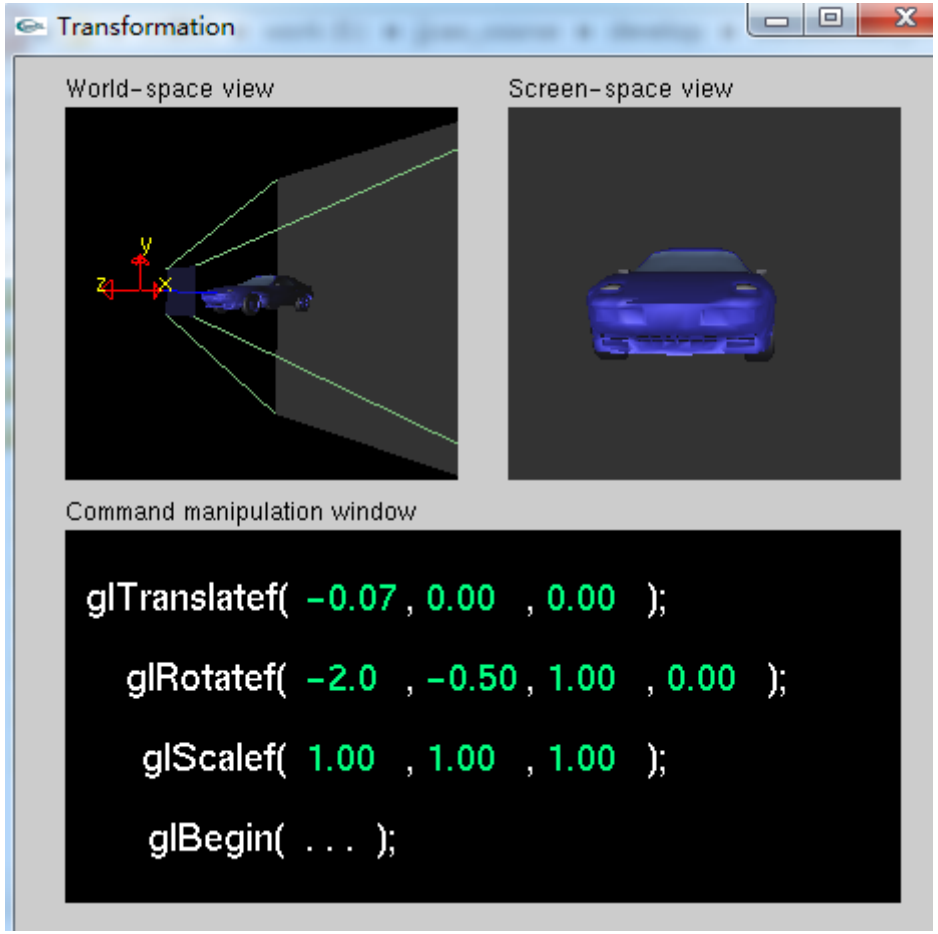
Example for Modeling Transformation 2

```
// draw triangle 2
glPushMatrix(); glLoadIdentity();
glTranslatef(1, 0, 0);
glRotatef(45, 0, 0, 1);
glColor4f(0.0, 1.0, 0.0, 1.0);
glBegin(GL_TRIANGLES); … glEnd();
glPopMatrix();

// draw triangle 3
glPushMatrix(); glLoadIdentity();
glRotatef(45, 0, 0, 1);
glTranslatef(1, 0, 0);
glColor4f(0.0, 1.0, 0.0, 1.0);
glBegin(GL_TRIANGLES);  … glEnd();
glPopMatrix();
```

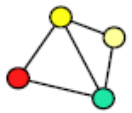


Modeling Transformations (cont)



Nate_Robins_tutorials: Transformation

Viewing transformation



Local (Object) Space → Modeling transformation → World Space → Viewing transformation → Eye Space → Projection transformation → Clip Space → Perspective divide → NDC space → Viewport mapping → Screen space

Normalized **D**evice **C**oordinates

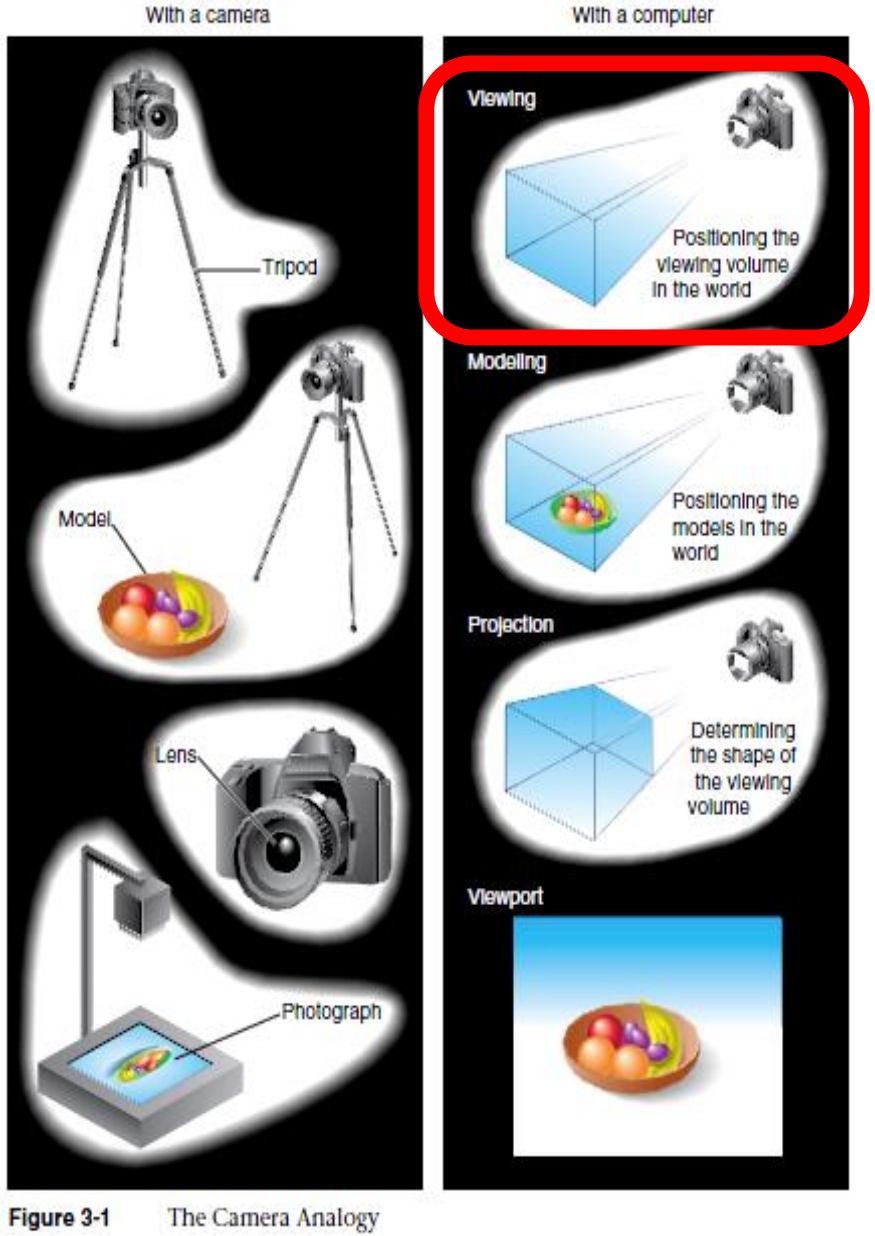


Figure 3-1 The Camera Analogy

Viewing Transformation

- Convert from WCS to the camera (eye) coordinate sys

- The camera position is the origin initially.
- The objs are also in the origin mostly. Or have been placed well in WCS

- Anyway, we need move the camera to see what we wish to see (may see nothing using default camera/viewing transformation)

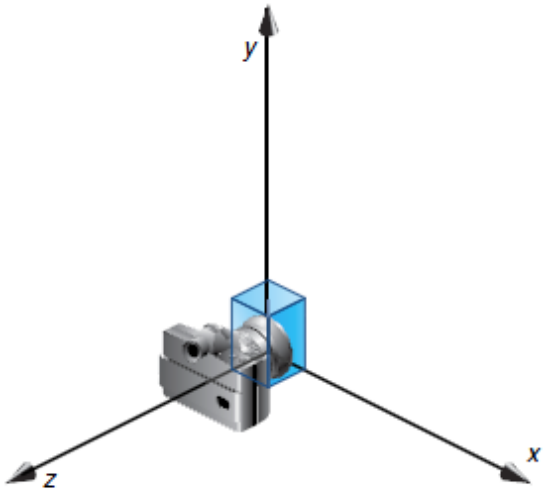
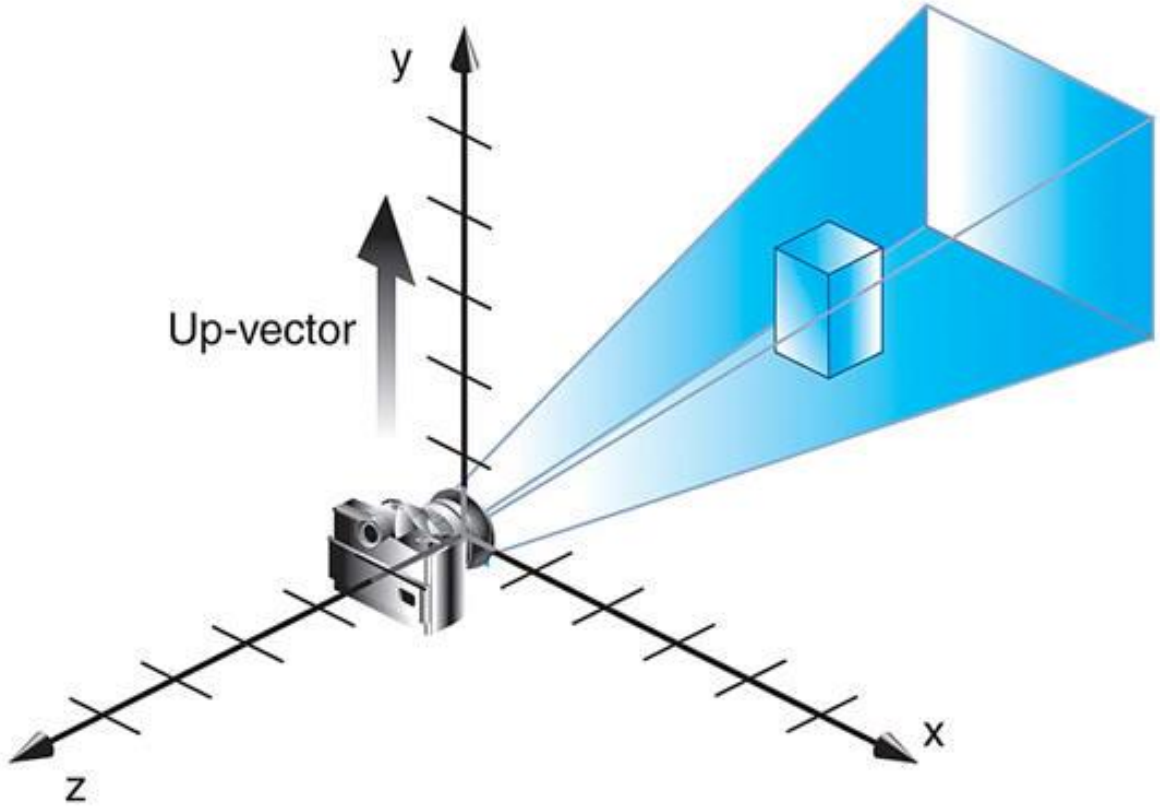
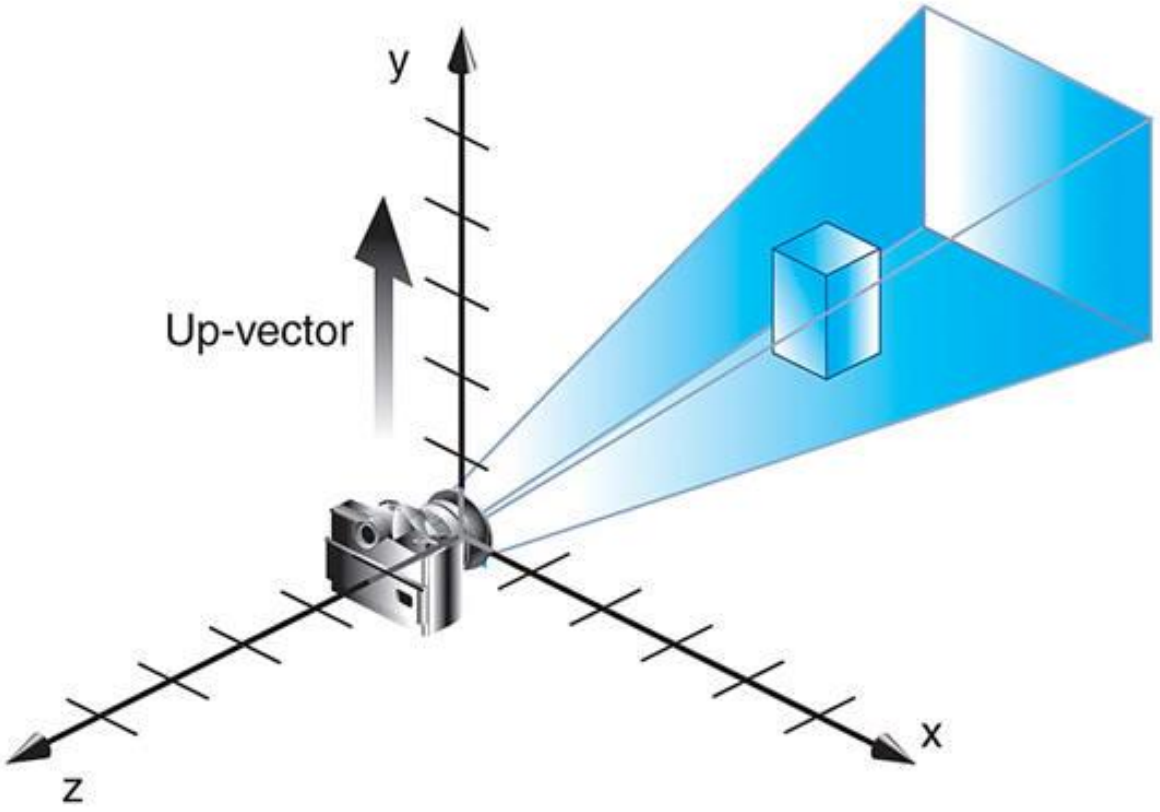


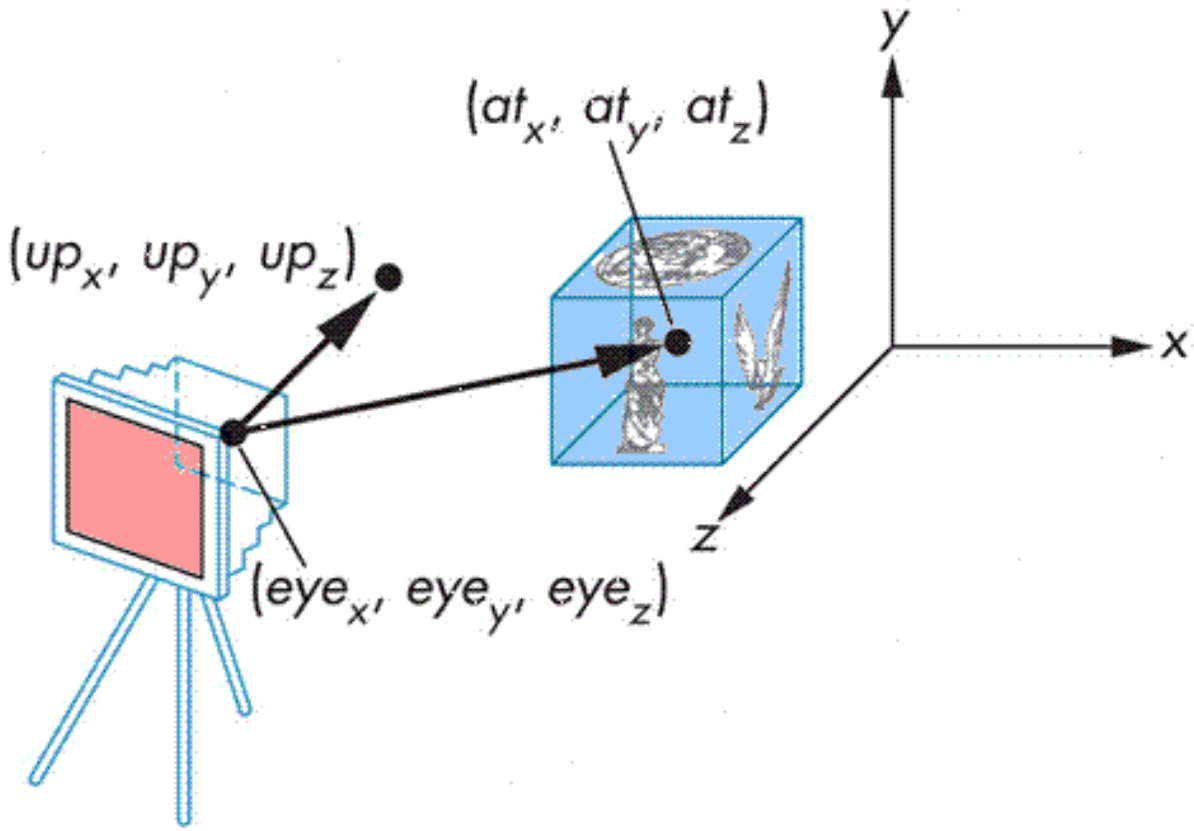
Figure 3-9 Object and Viewpoint at the Origin



Viewing Transformation

- void **gluLookAt**(*eyeX*, *eyeY*, *eyeZ*, *centerX*, *centerY*, *centerZ*, *upX*, *upY*, *upZ*);





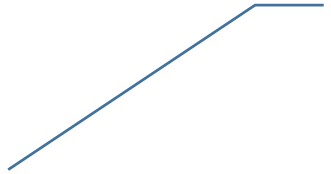
Example: modeling + viewing transformation

- With all this, we can give an outline for a typical display routine for drawing an image of a 3D scene with OpenGL 1.1:

```
// possibly set clear color here, if not set elsewhere
glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );

// possibly set up the projection here, if not done elsewhere
glMatrixMode( GL_MODELVIEW ); glLoadIdentity();
gluLookAt( eyeX,eyeY,eyeZ, refX,refY,refZ, upX,upY,upZ ); // Viewing transform

glRotatef(45, 0, 0, 1);
glTranslatef(1, 0, 0);
glBegin(GL_TRIANGLES);
glColor4f(0.0, 1.0, 0.0, 1.0);glVertex3f(0.0, 0.0, -10.0);
glVertex3f(1.0, 0.0, -10.0);glVertex3f(0.0, 1.0, -10.0);
glEnd();
…
```



Where are we drawing actually?

Where are we drawing actually?

- We are drawing in the eye coord

$$\begin{pmatrix} x_{eye} \\ y_{eye} \\ z_{eye} \\ w_{eye} \end{pmatrix} = M_{modelView} \cdot \begin{pmatrix} x_{obj} \\ y_{obj} \\ z_{obj} \\ w_{obj} \end{pmatrix} = M_{view} \cdot M_{model} \cdot \begin{pmatrix} x_{obj} \\ y_{obj} \\ z_{obj} \\ w_{obj} \end{pmatrix}$$

```
glMatrixMode( GL_MODELVIEW ); glLoadIdentity();
gluLookAt( eyeX,eyeY,eyeZ, refX,refY,refZ, upX,upY,upZ );
// Viewing transform

glRotatef(45, 0, 0, 1);
glTranslatef(1, 0, 0);
glBegin(GL_TRIANGLES);
…
```

Implementing the Look-At Function

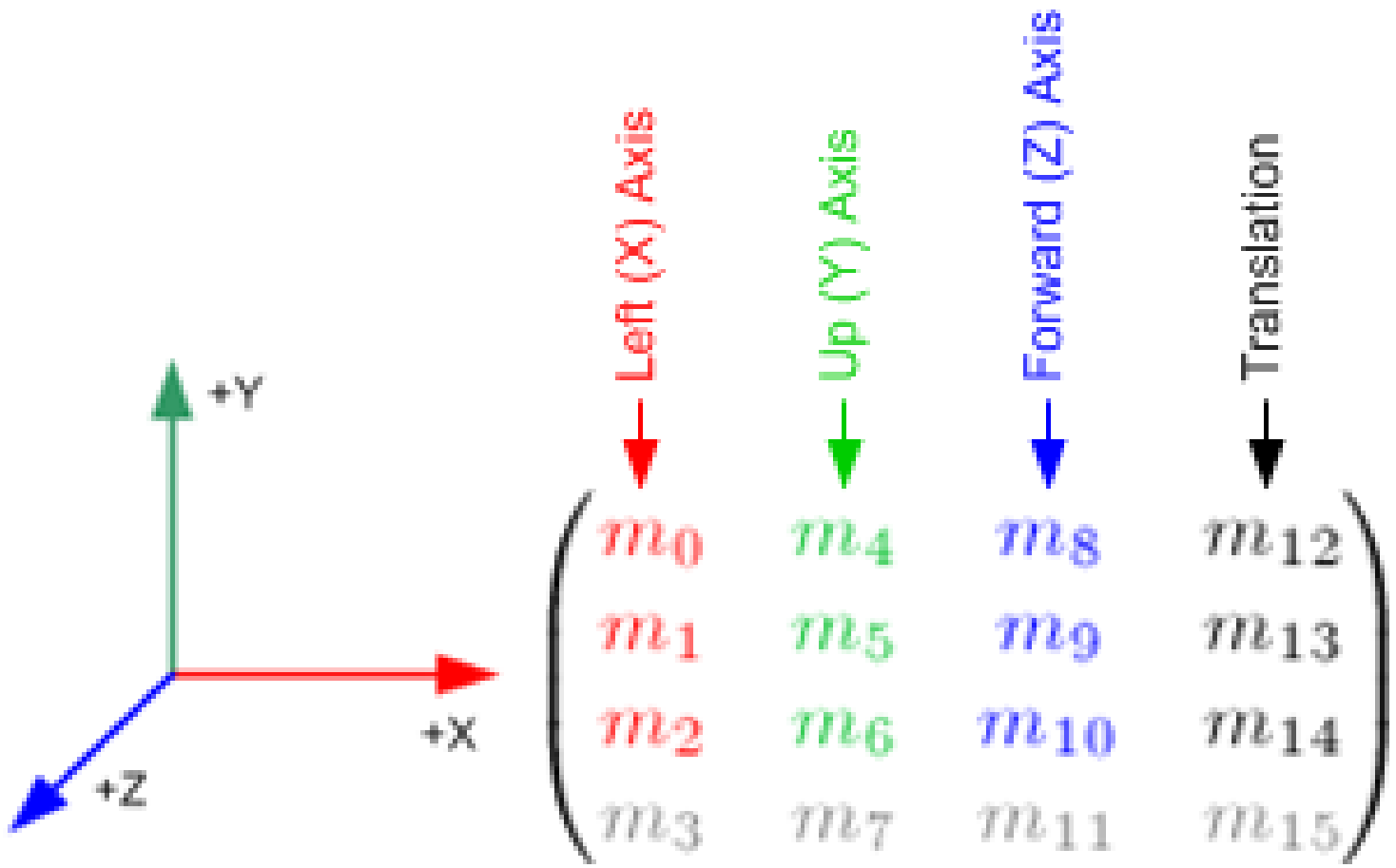
1. Transform world frame to camera frame
   - Compose a rotation $\mathbf{R}$ with translation $\mathbf{T}$
   - $\mathbf{W} = \mathbf{TR}$

2. Invert $\mathbf{W}$ to obtain viewing transformation $\mathbf{V}$
   - $\mathbf{V} = \mathbf{W}^{-1} = (\mathbf{TR})^{-1} = \mathbf{R}^{-1}\mathbf{T}^{-1}$
   - Derive $\mathbf{R}$, then $\mathbf{T}$, then $\mathbf{R}^{-1}\mathbf{T}^{-1}$

Implementing the Look-At Function

- gluLookAt(0.0, 0.0, 125.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0);对应的M

1, 0, 0, 0;
0, 1, 0, 0;
0, 0, 1, 0;
0, 0, -125, 1;



4 columns of GL_MODELVIEW matrix

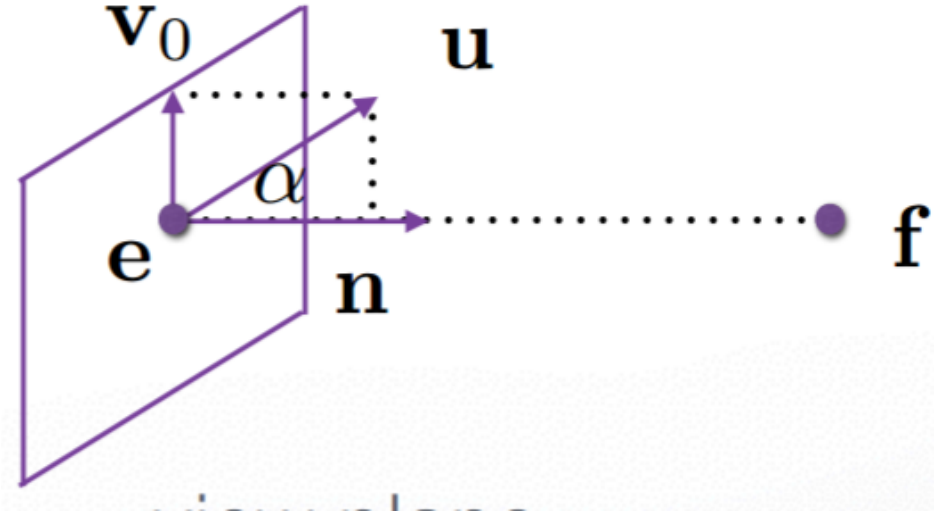
Camera Frame: n v w

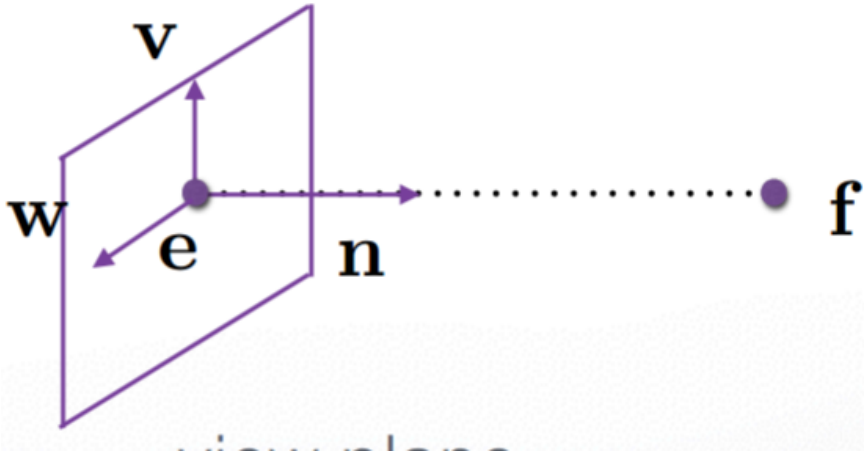
- gluLookAt(ex, ey, ez, fx, fy, fz, ux, uy, uz);
- n = (f-e)/||f-e|| is unit normal to view plane

$$\alpha = \mathbf{u}^\top \mathbf{n} / \|\mathbf{n}\| = \mathbf{u}^\top \mathbf{n}$$

$$\mathbf{v}_0 = \mathbf{u} - \alpha \mathbf{n}$$

$$\mathbf{v} = \mathbf{v}_0 / \|\mathbf{v}_0\|$$





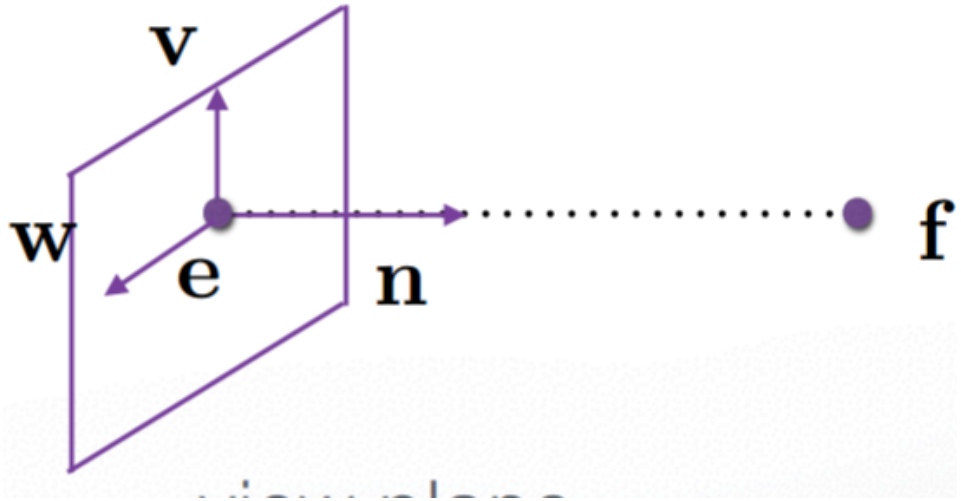
- w = cross(n,v); w = w/||w||

- w v –n is right-handed

gluLookAt does not require that the up vector you provide be perpendicular to the direction you're looking.

Summary of Rotation of M_view from gluLookat()

- w v –n is right-handed



- gluLookAt(e_x, e_y, e_z, f_x, f_y, f_z, u_x, u_y, u_z);
- $\mathbf{n} = (\mathbf{f} - \mathbf{e})/\|\mathbf{f} - \mathbf{e}\|$,
- $\mathbf{v} = (\mathbf{u} - (\mathbf{u}^\top \mathbf{n})\mathbf{n})/\|\mathbf{u} - (\mathbf{u}^\top \mathbf{n})\mathbf{n}\|$,
- $\mathbf{w} = \mathbf{n} \times \mathbf{v}$.
- Rotation must map:
  - $[1\ 0\ 0]$ to $\mathbf{w}$
  - $[0\ 1\ 0]$ to $\mathbf{v}$
  - $[0\ 0\ -1]$ to $\mathbf{n}$

$$\begin{bmatrix} w_x & v_x & -n_x & 0 \\ w_y & v_y & -n_y & 0 \\ w_z & v_z & -n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Implementing the Look-At Function

- Translation of origin to $\mathbf{e}^\top = [e_x\ e_y\ e_z\ 1]^\top$

$$T = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

M_view from gluLookat()

- $\mathbf{V} = \mathbf{W}^{-1} = (\mathbf{TR})^{-1} = \mathbf{R}^{-1}\mathbf{T}^{-1}$,
- $\mathbf{R}$ is rotation, so $\mathbf{R}^{-1} = \mathbf{R}^{\top}$

$$R^{-1} = \begin{bmatrix} w_x & w_y & w_z & 0 \\ v_x & v_y & v_z & 0 \\ -n_x & -n_y & -n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $\mathbf{T}$ is translation, so $\mathbf{T}^{-1}$ negates displacement

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting it Together

- Calculate $\mathbf{V} = \mathbf{R}^{-1}\mathbf{T}^{-1}$

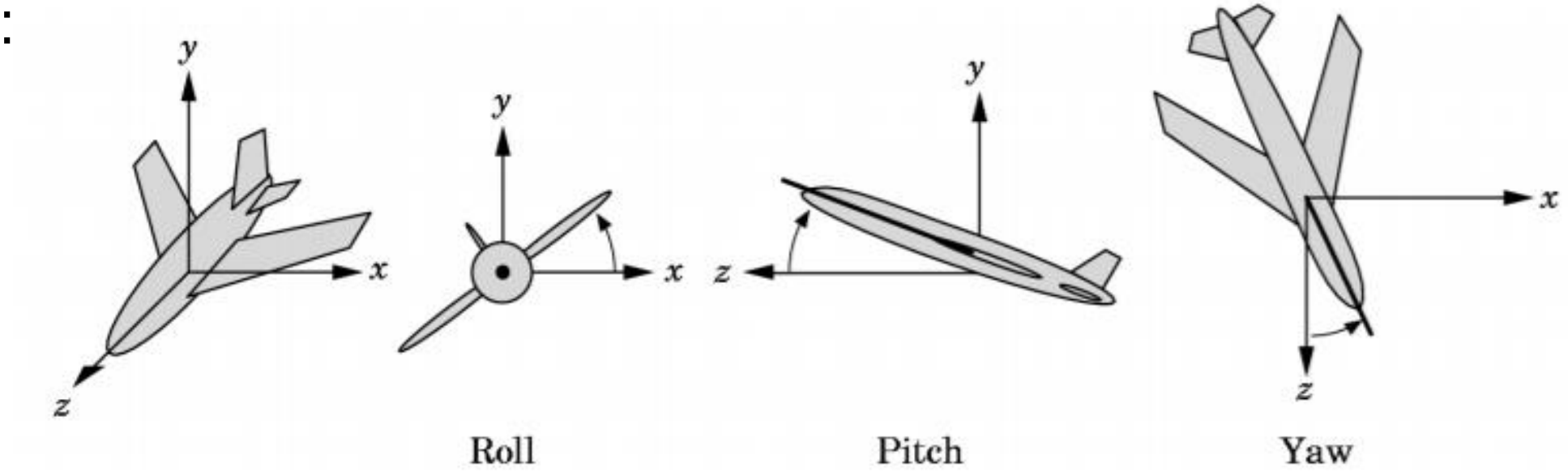
$$V = \begin{bmatrix} w_x & w_y & w_z & -w_x e_x - w_y e_y - w_z e_z \\ v_x & v_y & v_z & -v_x e_x - v_y e_y - v_z e_z \\ -n_x & -n_y & -n_z & n_x e_x + n_y e_y + n_z e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- This is different from book [Angel, Ch. 5.3.2]

- There, $\mathbf{u}, \mathbf{v}, \mathbf{n}$ are right-handed (here: $\mathbf{u}, \mathbf{v}, -\mathbf{n}$)

Other Viewing Functions

- A pilot wants:



- Polar coord

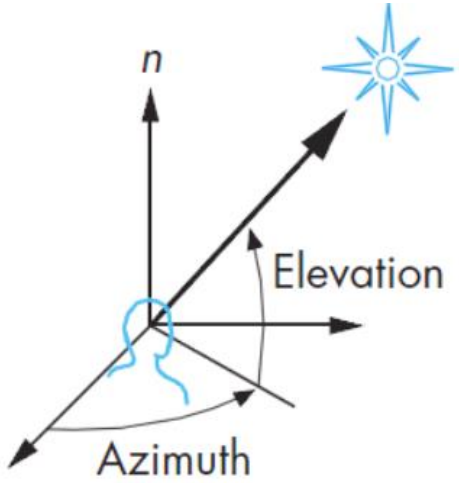
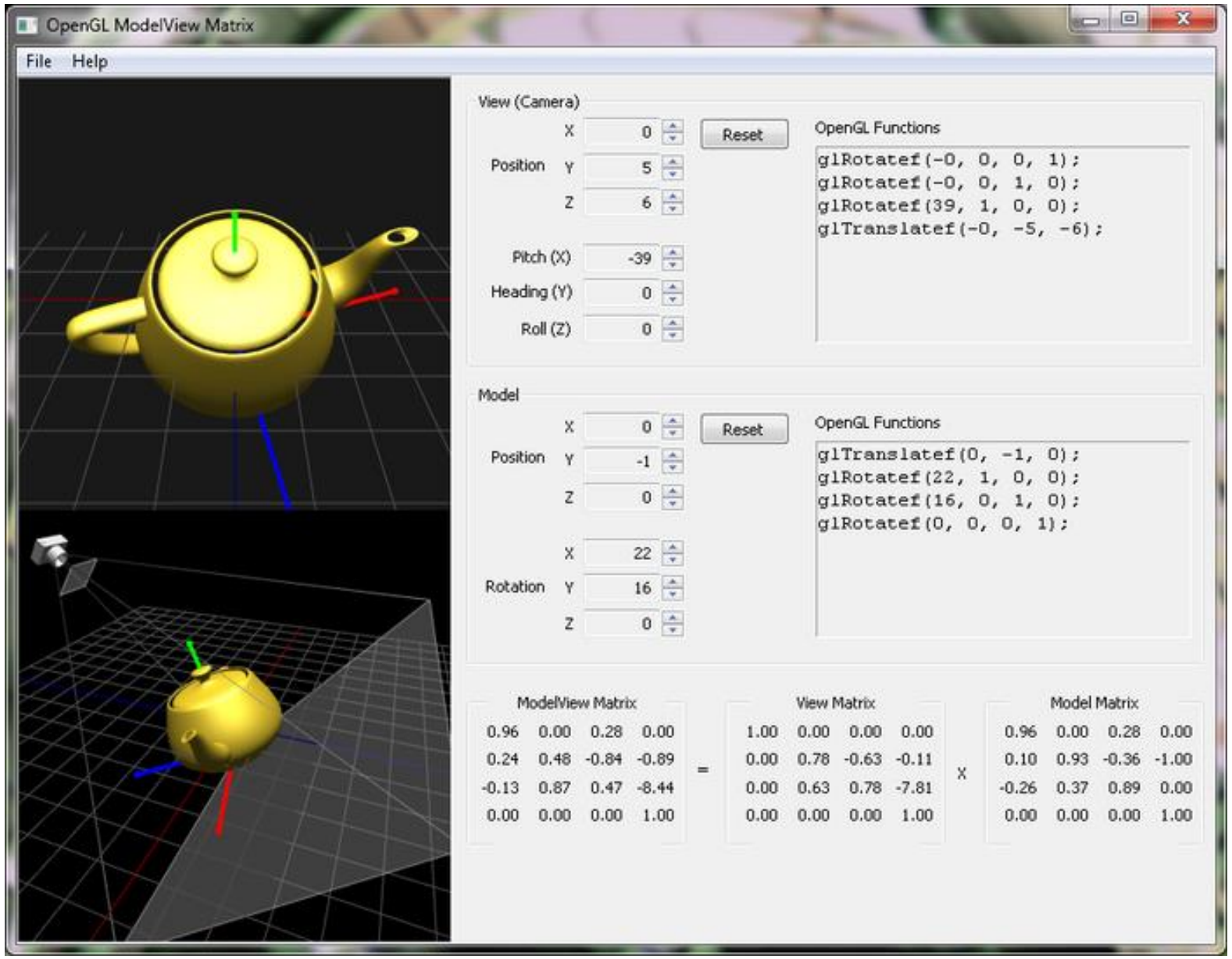


FIGURE 4.20 Elevation and azimuth.

Example: ModelView Matrix



- http://www.songho.ca/opengl/gl_transform.html#example1

Transformation Pipeline

Local (Object) Space → Modeling transformation → World Space → Viewing transformation → Eye Space → Projection transformation → Clip Space → Perspective divide → NDC space → Viewport mapping → Screen space

Normalized **D**evice **C**oordinates

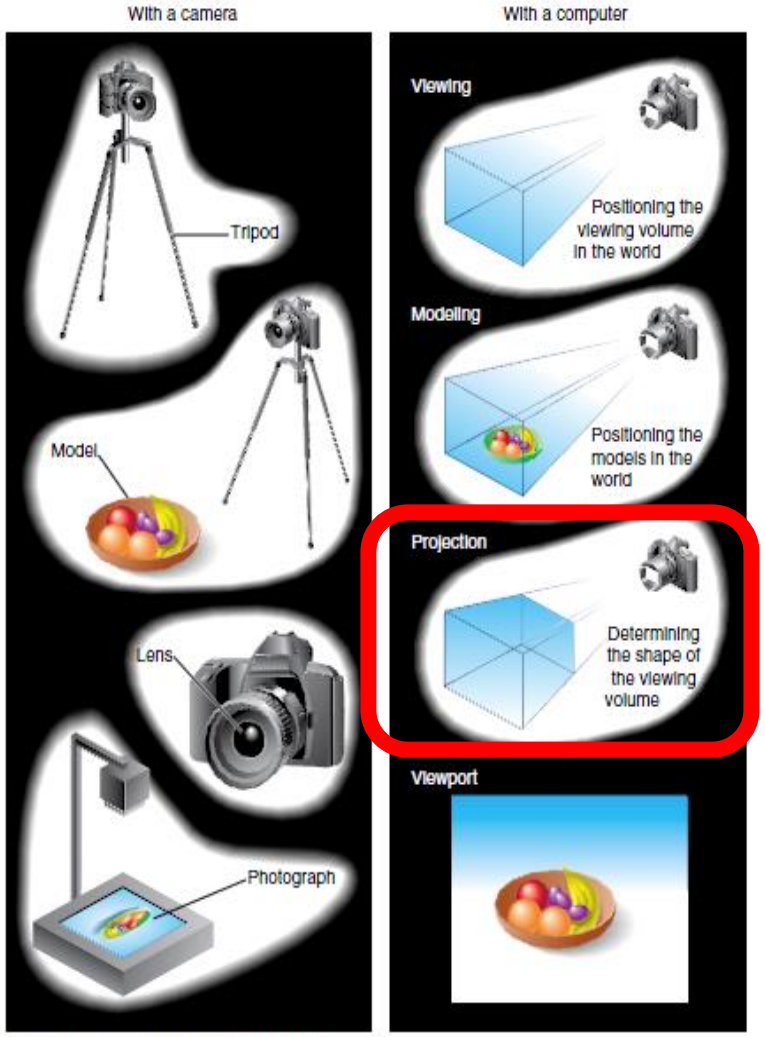


Figure 3-1 The Camera Analogy

Perspective projection

Rudimentary perspective in cave drawings

Lascaux, France source: Wikipedia

Painting in middle ages: incorrect perspective

- Art in the service of religion
- Perspective abandoned or forgotten

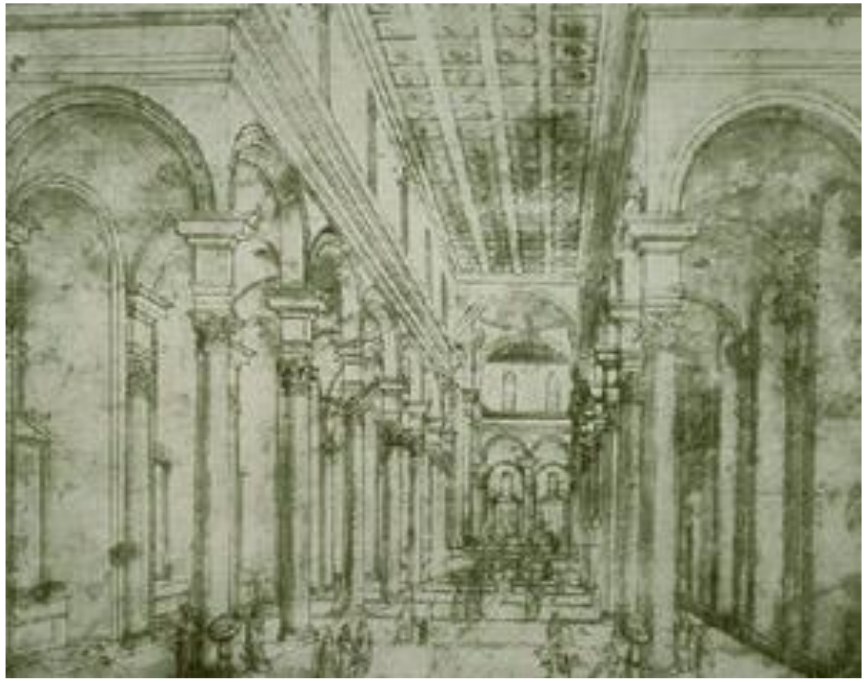
Ottonian manuscript, ca. 1000

8-9th century painting

Renaissance

- Rediscovery, systematic study of perspective

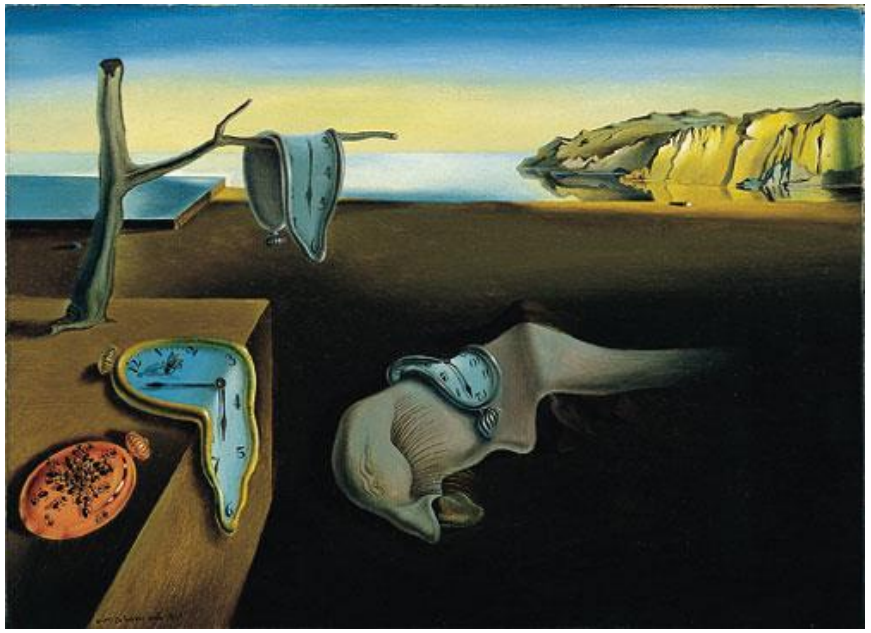
Filippo Brunelleschi Florence, 1415

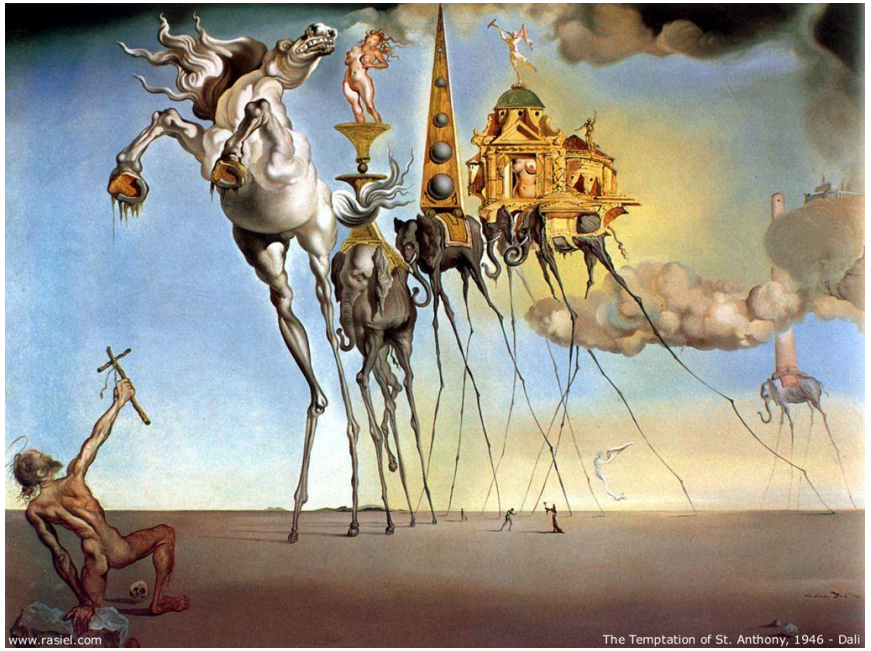


Brunelleschi, elevation of Santo Spirito, 1434-83, Florence

Masaccio - The Tribute Money
c. 1426-27
Fresco, The Brancacci Chapel, Florence

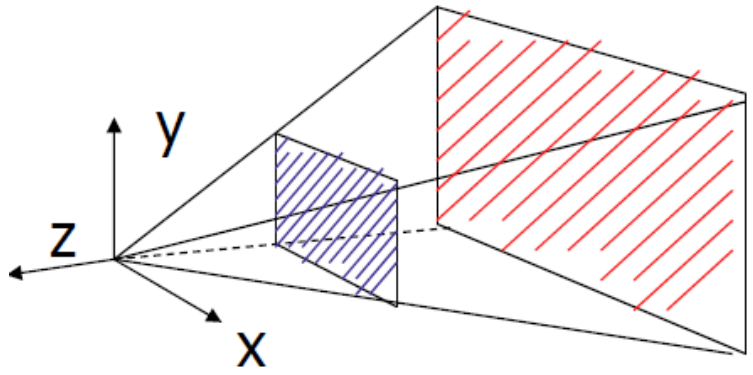
Later… rejection of proper perspective projection



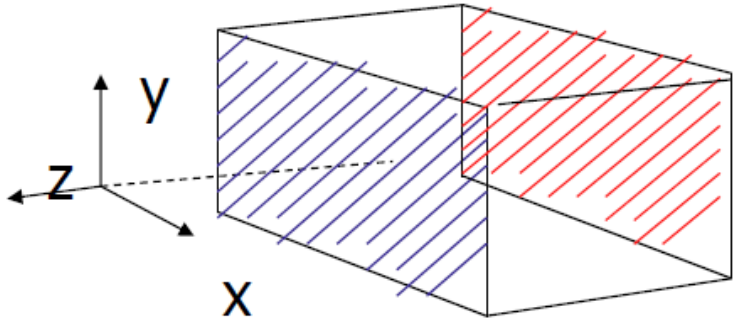


Projection Transformation

- Specifying PT is like choosing a **lens for a camera**
- The purpose of PT is to define a **viewing volume**, which is used in two ways.
  - The viewing volume determines **how an object is projected** onto the screen (that is, by using a perspective or an orthographic projection), and
  - Defines **which objects or portions of objects are clipped out** of the final image
- Need to establish the appropriate mode for constructing the viewing transformation, or in other words select the projection mode
  - **glMatrixMode(GL_PROJECTION);**
- This designates the projection matrix as the current matrix, which is originally set to the identity matrix



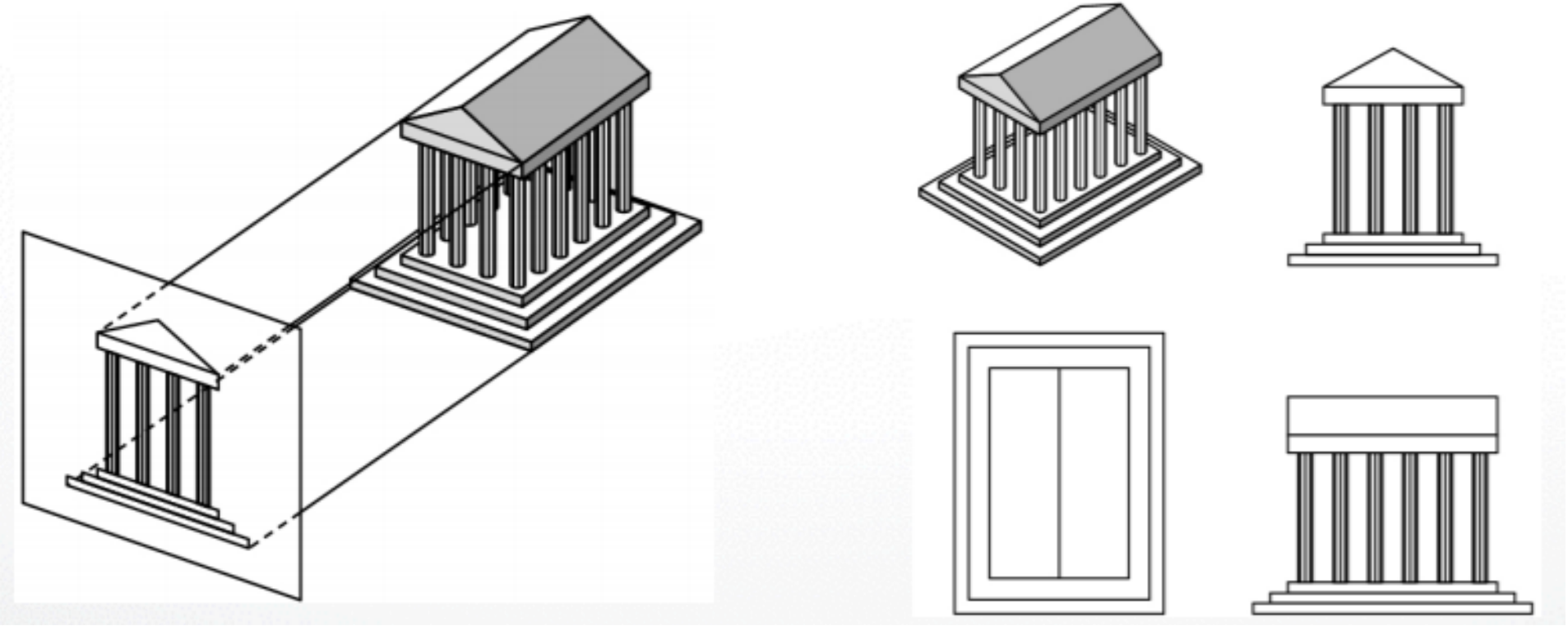
Perspective: **gluPerspective()**



Parallel: **glOrtho()**

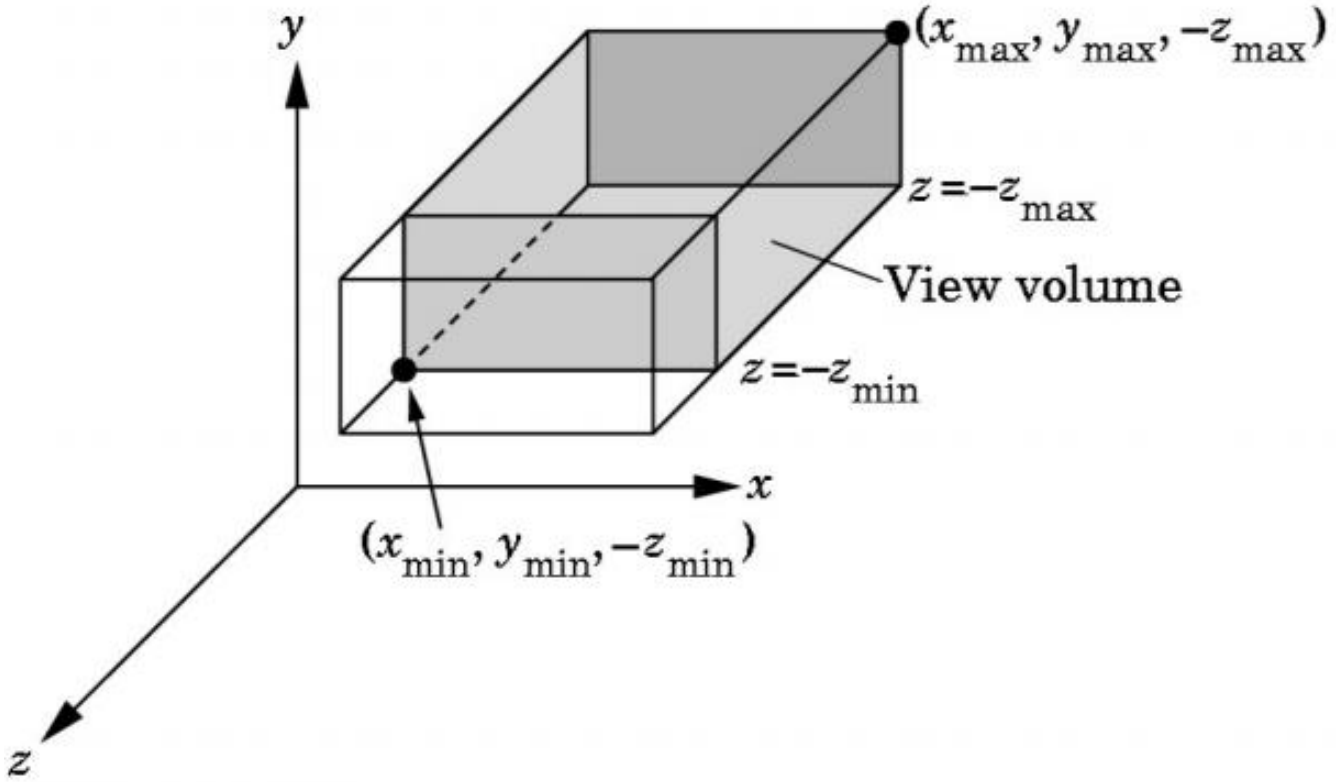
Orthographic Projection

- A special kind of parallel projection:
- projectors perpendicular to projection plane
- Simple, but not realistic
- Used in blueprints (multiview projections)



Orthographic Projection

- void **glOrtho** (left, right, bottom, top, near, far);

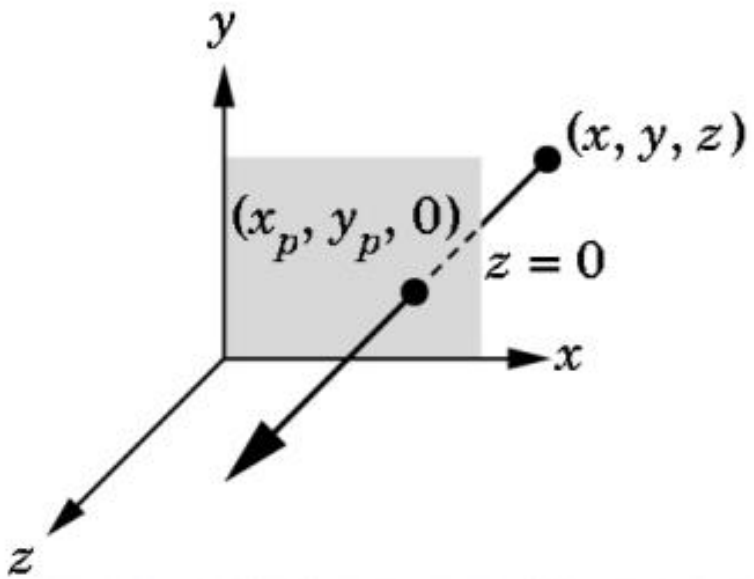


$$z_{min} = \text{near}, z_{max} = \text{far}$$

Maps (projects) everything in the visible volume into a canonical view volume

Orthographic Projection Matrix

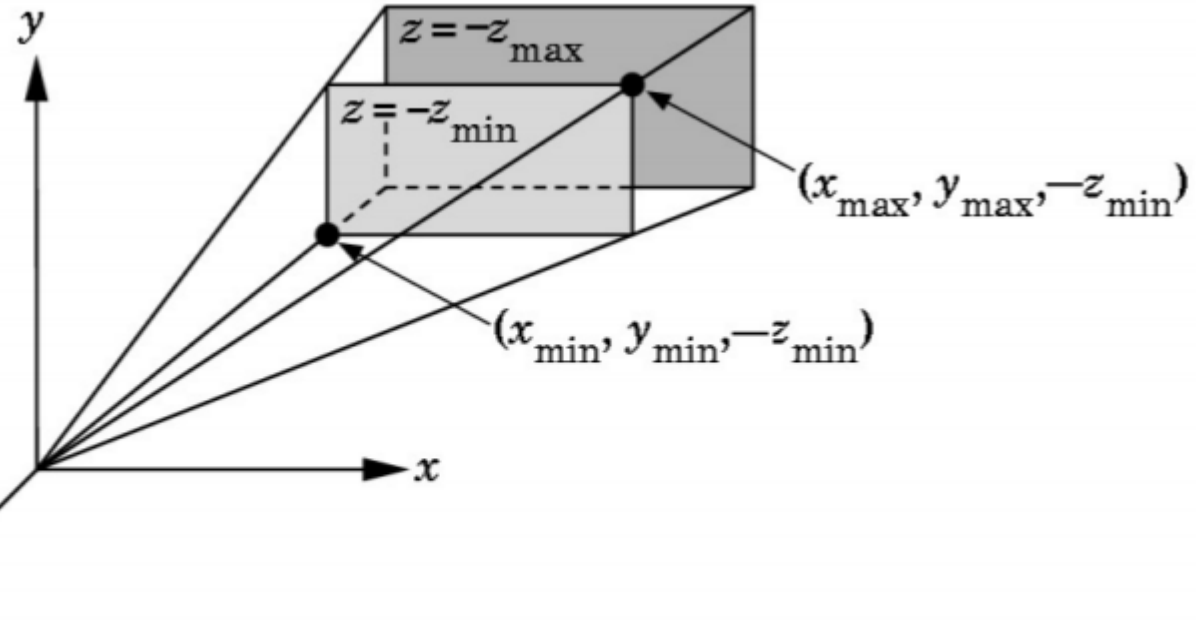
- Project onto $z = 0$
- $x_p = x$, $y_p = y$, $z_p = 0$
- In homogenous coordinates



$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Projection

glFrustum(xmin, xmax, ymin, ymax, N, F); N = near plane, F = far plane

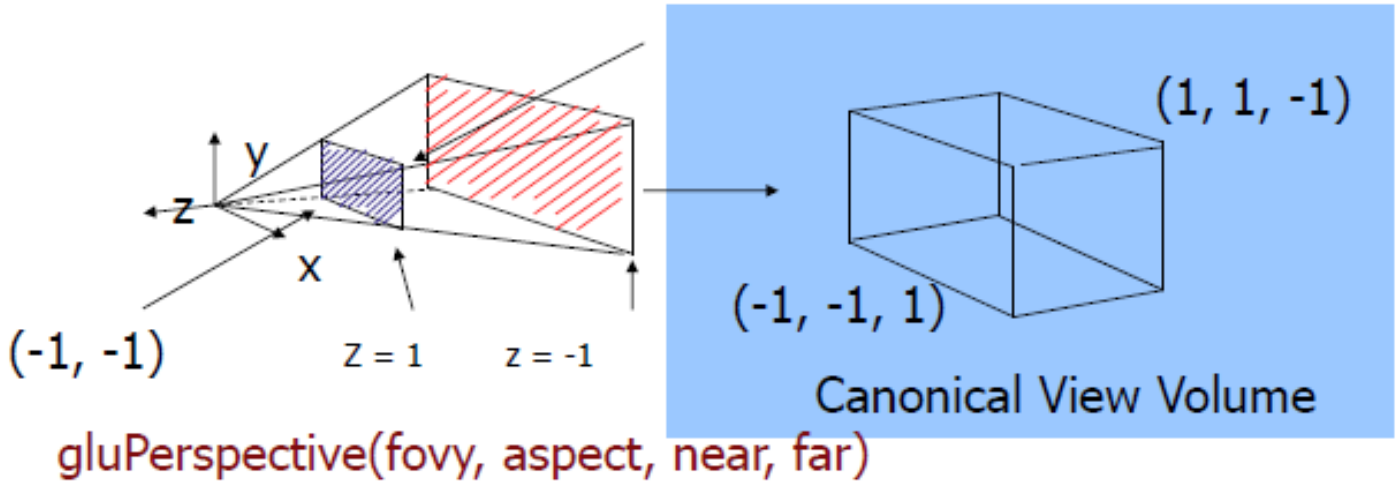


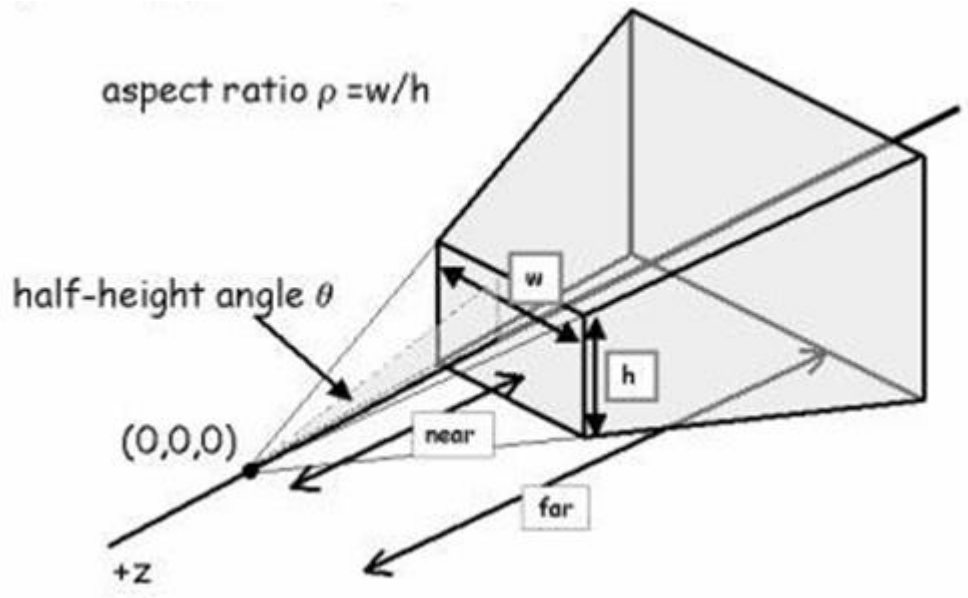
Projection Matrix

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \frac{2N}{xmax-xmin} & 0 & \frac{xmax+xmin}{xmax-xmin} & 0 \\ 0 & \frac{2N}{ymax-ymin} & \frac{ymax+ymin}{ymax-ymin} & 0 \\ 0 & 0 & -\frac{F+N}{F-N} & -\frac{2F*N}{F-N} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

gluPerspective

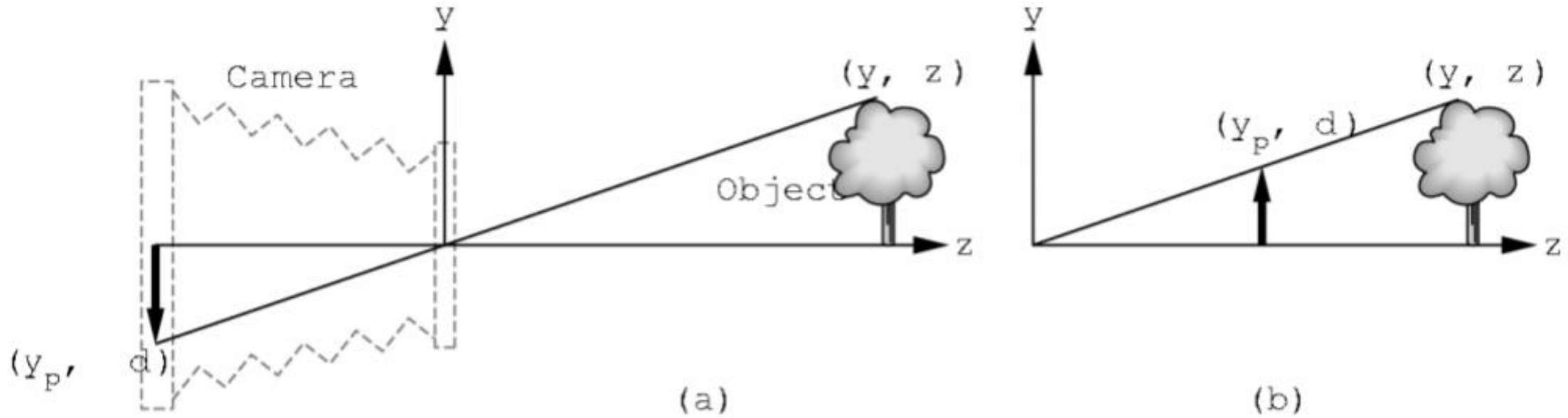
- glFrustum() isn't intuitive to use so can use **gluPerspective** to specify
  - Fovy: the angle of the field of view in the y direction
  - Aspect: the aspect ratio of the width to height (x/y)
  - Near & far: distance between the viewpoint and the near and far clipping planes
- Note that gluPerspective() is limited to creating frustums that are symmetric in both the x- and y-axes along the line of sight





Maps (projects) everything in the visible volume into a canonical view volume

Perspective Viewing Mathematically



- d = focal length
- $y/z = y_p/d$ so $y_p = y/(z/d) = yd/z$
- Note that y_p is non-linear in the depth z!

homogeneous coordinates

Perspective projection is not affine:

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

has no solution for M

Idea: exploit homogeneous coordinates

$$p = w \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

for arbitrary $w \neq 0$

Perspective Projection Matrix

- Use multiple of point

$$(z/d)\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix}$$

$$M\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

No solution

- Solve

$$M\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix} \quad \text{with} \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix}$$

Projection Algorithm

- **Input**: 3D point $[x\ y\ z]^\top$ to project
- Form $[x\ y\ z\ 1]^\top$
- Multiply M with $[x\ y\ z\ 1]^\top$; obtaining $[X\ Y\ Z\ W]^\top$
- Perform perspective division:

  X/W , Y/W , Z/W
- **Output**: $[X/W, Y/W, Z/W]^\top$
- (last coordinate will be d)

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix} \quad \text{with} \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

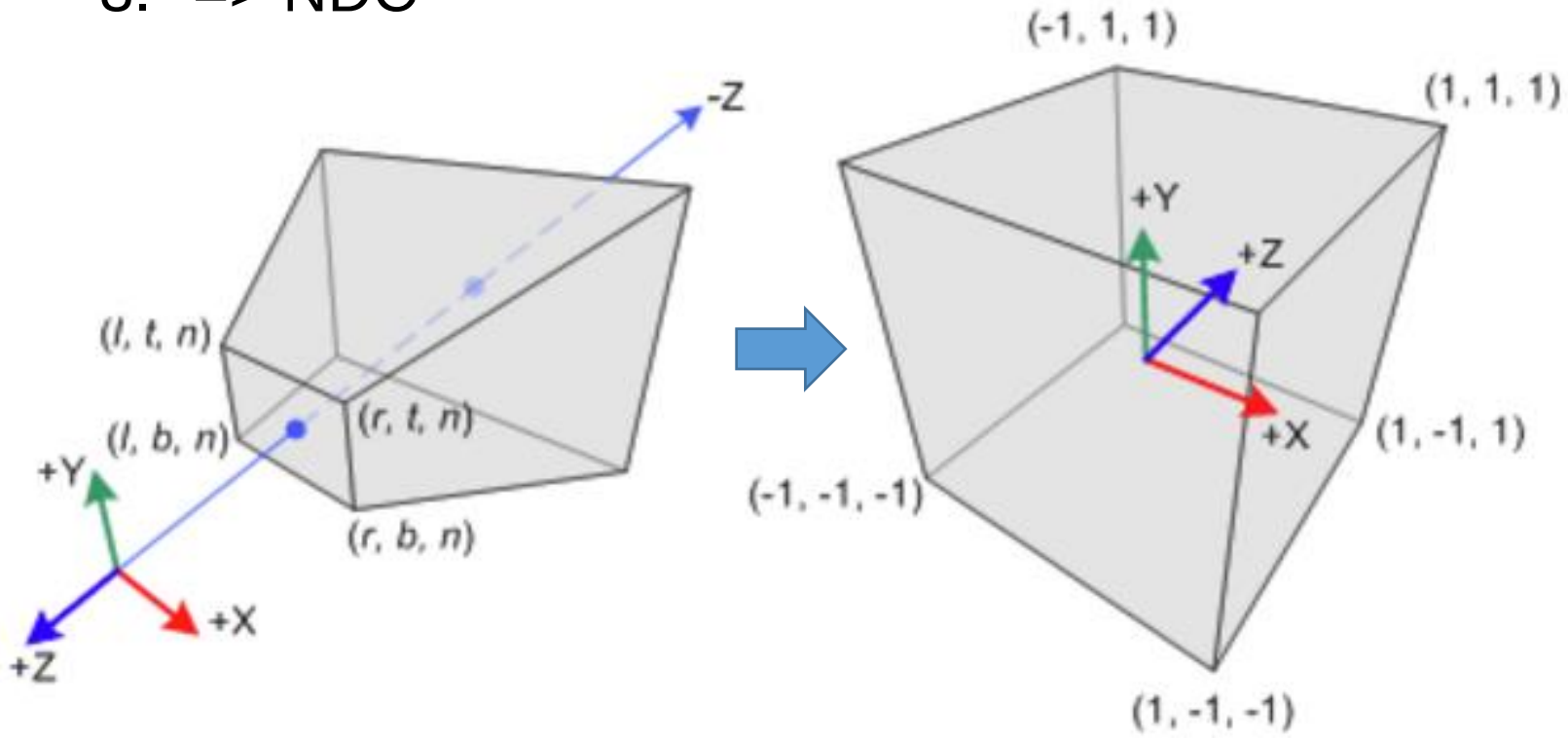
Perspective Division => Normalized Device Coordinates

- Normalize $[X\ Y\ Z\ W]^\top$ to $[X/W,\ Y/W,\ Z/W,\ 1]^\top$
- Perform perspective division after projection

- Projection in OpenGL is more complex (includes clipping)

Normalized Device Coordinates (NDC)

- Eye space => Clip space => NDC (a cube)
  1. Matrix multiplication: Mx
  2. Perspective division: Mx/w
  3. => NDC



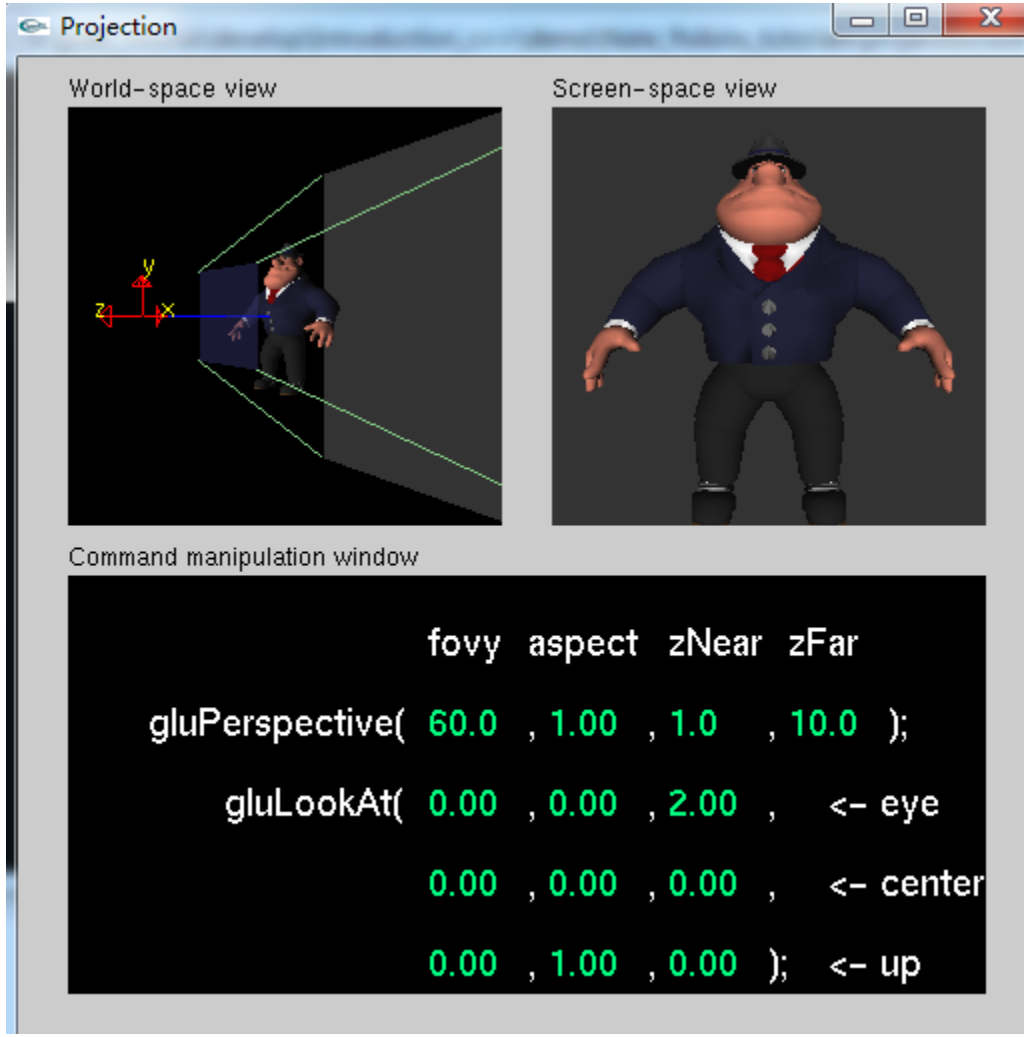
$$\begin{bmatrix} x \\ y \\ z \\ \frac{z}{d} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

What we derived, but not really happened in GL

Both clipping (frustum culling) and NDC transformations are integrated into GL_PROJECTION matrix.

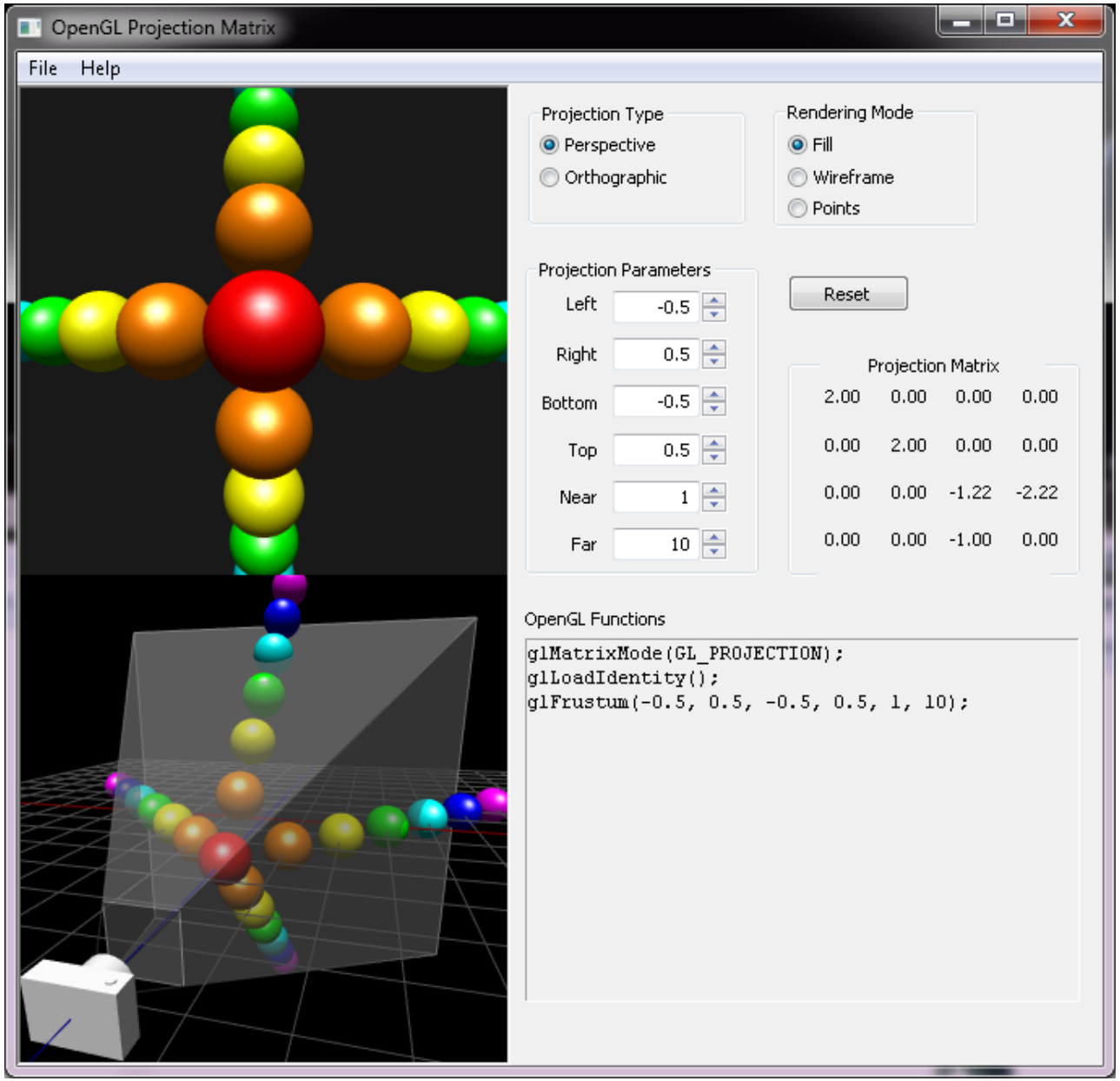
http://www.songho.ca/opengl/gl_projectionmatrix.html

Projection & Viewpoint (cont)



Nate_Robins_tutorials: Projection

Example: Projection Matrix



- http://www.songho.ca/opengl/gl_transform.html#projection

The Golden Rule

- Modeling transformation
  - glMatrixMode(GL_MODELVIEW); glRotate3f?
- Viewing transformation
  - glMatrixMode(GL_MODELVIEW); gluLookAt()
- Projection transformation
  - glMatrixMode(GL_PROJECTION);
  - glLoadIdentity - to initialize current matrix.
  - **gluPerspective/glFrustum/glOrtho/gluOrtho2 - to set the appropriate projection onto the stack.**
  - You *could* use glLoadMatrix to set up your own projection matrix (if you understand the restrictions and consequences) - but I'm told that this can cause problems for some OpenGL implementations which rely on data passed to glFrustum, etc to determine the near and far clip planes.

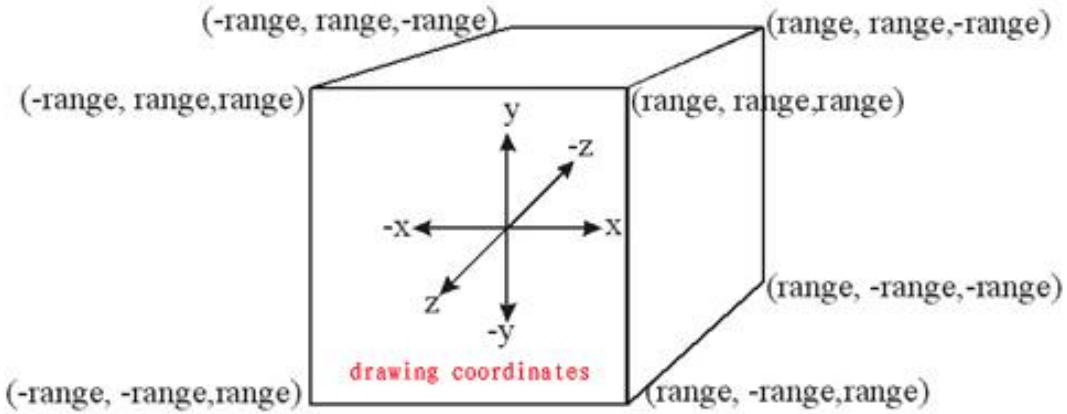
Transformation Pipeline

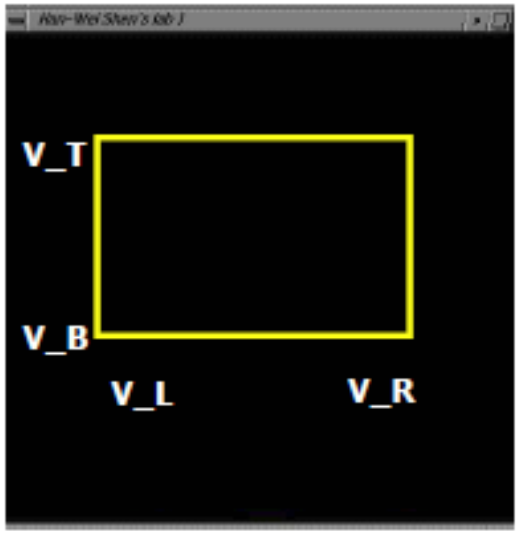
Local (Object) Space → Modeling transformation → World Space → Viewing transformation → Eye Space → Projection transformation → Clip Space → Perspective divide → NDC space → Viewport mapping → Screen space

Normalized **D**evice **C**oordinates

Viewpoint transformation

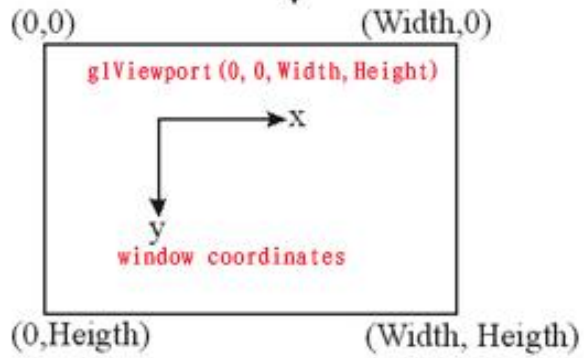
- The **viewport** maps the **drawing coordinates** to **window coordinates** and therefore defines the region of the scene, which can be seen. If the user resizes the window, we have to adjust the **viewport** and correct the aspect ratio.
-






```
glViewport(int  left, int  bottom,
           int  (right-left),
           int (top-bottom));
```

call this function before drawing (calling glBegin() and glEnd())



Thanks