RELATIVITY QUOTES

John O'Hanley

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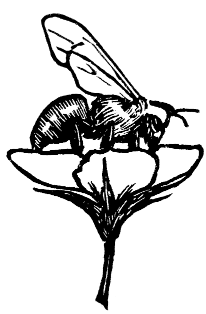

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While Maxwell's theory merely eliminated action at a distance from the realm of electrodynamics, the Lorentz transformation equations ruled out action at a distance from the whole of physics by depriving space and time of their absolute character.

- P. Bergmann, *Introduction to the Theory of Relativity (1942)*

The history of the general theory of relativity has shown that *we have to consider the group of all continuous, differentiable coordinate transformations with non-vanishing Jacobian* in order to develop a relativistic theory of gravitation.

- P. Bergmann, *Introduction to the Theory of Relativity (1942)*

In a Riemannian space, the introduction of a Cartesian coordinate system is impossible.

- P. Bergmann, *Introduction to the Theory of Relativity (1942)*

This non-linearity is characteristic of equations which are covariant with respect to general coordinate transformations; it is, thus, a consequence of the equivalence principle.

- P. Bergmann, *Introduction to the Theory of Relativity (1942)*

We must, therefore, assume that the gravitational fields (that is, the deviations of the actual metric from a flat metric) encountered in celestial mechanics and elsewhere are so weak that the nonlinear character of the field equations leads only to secondary effects.

- P. Bergmann, *Introduction to the Theory of Relativity (1942)*

Although the general theory of relativity has explained a few minor discrepancies between astronomical observations and Newtonian theory, its most important contribution to date has been a complete revolution of our notions of space and time, a revolution that may not yet be completed.

- P. Bergmann, *The Riddle of Gravitation (1968)*

Mathematically, the physicist's search for laws that are the same for all observers is a search for Lorentz-invariant relationships.

- P. Bergmann, *The Riddle of Gravitation (1968)*

These two requirements - that the theory satisfy the requirement of general covariance, and that it approach the laws of Newton under reasonable conditions - are sufficient to remove all ambiguities and to lead to a single possible law of gravitation.

- P. Bergmann, *The Riddle of Gravitation (1968)*

Reliance on the principle of general covariance drastically restricts the range of potential mathematical-geometric relations that might be considered as representing laws of gravitation.

- P. Bergmann, *The Riddle of Gravitation (1968)*

According to Einstein, the geometry of space-time does not lend itself to the selection of a special class of coordinate systems for formulating the laws of nature, but all reasonably continuous and smooth coordinate systems will serve equally well. This formal principle is called the *principle of general covariance*.... In particular, it must be impossible to classify coordinate systems into those in which the axes are straight lines and those in which the axes are curved.

- P. Bergmann, *The Riddle of Gravitation (1968)*

Einstein proposed foregoing all attempts to reinstate the role of inertial frames of reference. Locally, their role would have to be taken over by the free-falling frames of reference, which, however, are not extensible. Among extended frames of reference, there would be no hierarchy, no criteria for selecting a class of special or superior frames. All frames of reference were to be accepted as equally valid.

- P. Bergmann, *The Riddle of Gravitation (1968)*

Only because of the flatness of the Minkowski universe is the state of motion of two physical objects with respect to each other well defined regardless of the distance between them.

- P. Bergmann, *The Riddle of Gravitation (1968)*

The laws of gravitation must somehow restrict the curvature but without ruling out curvature altogether.

- P. Bergmann, *The Riddle of Gravitation (1968)*

The *curvature* of the manifold itself is defined in terms of the change of direction that a vector suffers when transported parallel about a small closed loop.

- P. Bergmann, *The Riddle of Gravitation (1968)*

The analogous formulation of the laws of conservation of energy (mass) and linear momentum requires introduction of the notions of energy density, energy flux density, linear momentum density [proportional to energy flux density], and linear momentum flux density [stress].

- P. Bergmann, *The Riddle of Gravitation (1968)*

Throughout the book, a great deal of attention is paid quite generally to the habitual tendency to regard older modes of thought as inevitable, a tendency that has greatly impeded the development of new ideas in science. This tendency is seen to be based on the tacit assumption that scientific laws constitute absolute truths.

- David Bohm, *The Special Theory of Relativity (1965)*

This leads us to the (hyperbolic) geometry of Minkowski space-time...

- David Bohm, *The Special Theory of Relativity (1965)*

As in the case of our ideas about space, the problem with our common concept of time is evidently not that it is *totally wrong*. If it were, then nobody would be foolish enough to try to hold onto it. The difficulty arises because it is adequate in a limited domain, but inappropriate, as we shall see, when extended beyond this domain. But as a result of the fact that this domain of adequacy includes a tremendous amount of common experience beginning with childhood, and because this experience has been incorporated into the structure of the language, we find it very hard to get out of the habit of regarding our ordinary concept of time as inevitable.

- David Bohm, *The Special Theory of Relativity (1965)*

The interesting point is that the theory of relativity implies that a particle of zero rest mass can have a nonzero energy and momentum... Therefore, a body can have nonzero energy and momentum, even thought its rest mass is zero, *if and only if it is moving at the speed of light*.

- David Bohm, *The Special Theory of Relativity (1965)*

Rather, as in the case of relativity and quantum theory, it shows that, generally speaking, older ideas are on a *completely wrong track* when extended beyond their proper domains, and that radically new ideas are needed, which contradict old ones while at the same time containing them, in some sense, as limiting cases and approximations.

- David Bohm, *The Special Theory of Relativity (1965)*

So although space and time can, to some extent, be transformed into each other in a change of reference frame, there are certain limits on this transformation, in the sense that an interval inside the light cone cannot be transformed into one outside the light cone, or on the light cone.

- David Bohm, *The Special Theory of Relativity (1965)*

So a certain subtle kind of difference between space and time is preserved, in the sense of a lack of *complete* interchangeability under transformation....

- David Bohm, *The Special Theory of Relativity (1965)*

The special theory of relativity is a necessary consequence of any assertion that the unity of physics is essential, for it would be intolerable for all inertial systems to be equivalent from a dynamical point of view yet distinguishable by optical measurements. It now seems almost incredible that the possibility of such a discrimination was taken for granted in the nineteenth century, but at the same time it was not easy to see what was more important - the universal validity of the Newtonian principle of relativity or the absolute nature of time.

- H. Bondi, *Endeavour (1961)*

It is in the nature of transformation laws to change most quantities but to leave some quantities unchanged. These [latter] are called the invariants of the transformation and serve to define its character. A physical statement of what these invariants are is called a principle of relativity, and the fundamental equations of a theory usually define the principle of relativity applicable to it.

- H. Bondi, *Rept. Progr. Phys (1959)*

There is no such thing as absolute simultaneity.

- Max Born, *Einstein's Theory of Relativity (1962)*

The advance made by Einstein's theory is not in the formulation of laws but rather in a fundamental change of viewpoint towards the laws.

- Max Born, *Einstein's Theory of Relativity (1962)*

Thus the contraction is only a consequence of our way of regarding things and is not a change of a physical reality....The view expounded in the preceding paragraph does away with the notorious controversy as to whether the contraction is "real" or only "apparent". If we slice a cucumber, the slices will be larger the more obliquely we cut them. It is meaningless to call the sizes of the various oblique slices "apparent" and call, say, the smallest which we get by slicing perpendicularly to the axis the "real" size.

- Max Born, *Einstein's Theory of Relativity (1962)*

...the mass of the body increases with the velocity approaching that of light.

- Max Born, *Einstein's Theory of Relativity (1962)*

We see that the two theories are really the same. Once one has accepted [the statements] I, II, III, and IV as axioms, the choice between [axiom] V and [the Connectedness Theorem] CT is just a matter of taste. In this sense, the axiom systems I-II-III-IV-V and I-II-III-IV-CT are "equivalent".

- Buskes and van Rooij, *Topological Spaces (1997)*

The intensity of every astonishment gradually wears off; the human mind, by the sheer effect of repetition and habit, gradually becomes accustomed to even the strangest and least familiar ideas.

- M. Capek, *The Philosophical Impact of Contemporary Physics*

It frequently happens that within one and the same mind the true grasp of the mathematical side of the theory coexists with serious misapprehensions of its physical and especially its philosophical meaning....

- M. Capek, *The Philosophical Impact of Contemporary Physics*

This plainly shows that the term "curvature" is essentially metaphorical, for it is based on an analogy with curvature of two-dimensional surfaces.

- M. Capek, *The Philosophical Impact of Contemporary Physics*

...the law of gravitation is a generalized law of inertia.

- M. Capek, *The Philosophical Impact of Contemporary Physics*

What is left of the classical definition of space as "simultaneous juxtaposition of points?" Nothing but a word with misleading connotations.

- M. Capek, *The Philosophical Impact of Contemporary Physics*

Yet the idea of a timeless container, in which all matter is located and in which all motion takes place, has such deep roots in the very structure of human imagination that it is almost impossible to get rid of it.

- M. Capek, *The Philosophical Impact of Contemporary Physics*

The field equations of general relativity determine the geometry of spacetime in terms of the matter content. They do not, in general, determine the topology. The two aren't completely independent; choices of geometry and topology must be compatible, and this places some restrictions on possible spacetimes.

- Steve Carlip

One of the interesting problems in modern cosmology is that of determining the topology of space in the real Universe.

- Steve Carlip

General relativity is the most beautiful physical theory ever invented.

- Sean Carroll, *Spacetime and Geometry (2004)*

The essential idea is perfectly straightforward: while most forces of nature are represented by fields defined on spacetime (such as the electromagnetic field, or the short-range fields characteristic of subnuclear forces), gravity is inherent in spacetime itself. In particular, what we experience as "gravity" is a manifestation of the curvature of spacetime.

- Sean Carroll, *Spacetime and Geometry (2004)*

The interval is defined to be delta-s^2, not the square root of this quantity.

- Sean Carroll, *Spacetime and Geometry (2004)*

The notion of *acceleration* in special relativity has a bad reputation, for no good reason...In particular, there is no truth to the rumor that SR is unable to deal with accelerated trajectories, and general relativity must be invoked.

- Sean Carroll, *Spacetime and Geometry (2004)*

Such a metric, in which all of the eigenvalues are positive, is called Euclidean, while those...which feature a single minus sign are called Lorentzian.

- Sean Carroll, *Spacetime and Geometry (2004)*

...there is no such thing as a cross product between four-vectors.

- Sean Carroll, *Spacetime and Geometry (2004)*

This is why it is rarely necessary to write the basis vectors and dual vectors explicitly; the components do all of the work.

- Sean Carroll, *Spacetime and Geometry (2004)*

A remarkable property of the above tensors - the metric, the inverse metric, the Kronecker delta, and the Levi-Civita symbol - is that, even though they all transform according to the tensor transformation law, their components remain unchanged in any inertial coordinate system in flat spacetime. In fact, these are the only tensors with this property.

- Sean Carroll, *Spacetime and Geometry (2004)*

The metric, as a (0,2) tensor, is a machine that acts on two vectors (or two copies of the same vector) to produce a number.

- Sean Carroll, *Spacetime and Geometry (2004)*

It turns out to be much more convenient to take this as the mass once and for all, rather than thinking of mass depending on velocity.

- Sean Carroll, *Spacetime and Geometry (2004)*

As mentioned at the beginning of this section, general relativity itself is a classical field theory, in which the dynamical field is the metric tensor. It is nevertheless fair to think of GR as somehow different; for the most part other classical field theories rely on the existence of a pre-existing geometry, whereas in GR the geometry is determined by the equations of motion.

- Sean Carroll, *Spacetime and Geometry (2004)*

A general definition of [the energy-momentum tensor] T-mu-nu is "the flux of four-momentum p-mu across a surface of constant x-nu."

- Sean Carroll, *Spacetime and Geometry (2004)*

Gravity is special. In the context of general relativity, we ascribe this specialness to the fact that the dynamical field giving rise to gravitation is the metric tensor describing the curvature of spacetime itself, rather than some additional field propagating through spacetime; this was Einstein's profound insight. The physical principle that led him to this idea was the universality of the gravitational interaction, as formalized by the Principle of Equivalence.

- Sean Carroll, *Spacetime and Geometry (2004)*

The [Weak Equivalence Principle] states that the inertial mass and gravitational mass of any object are equal.

- Sean Carroll, *Spacetime and Geometry (2004)*

This suggests an equivalent formulation of the WEP: there exists a preferred class of trajectories through spacetime, known as inertial (or "freely-falling") trajectories, on which unaccelerated particles travel - where unaccelerated means "subject only to gravity".

- Sean Carroll, *Spacetime and Geometry (2004)*

The WEP can therefore be stated as follows: The [motions] of freely-falling particles are the same in a gravitational field and a uniformly accelerated frame, in small enough regions of spacetime.

- Sean Carroll, *Spacetime and Geometry (2004)*

This reasonable extrapolation became what is now known as the Einstein Equivalence Principle, or EEP: In small enough regions of spacetime, the [nongravitational] laws of physics reduce to those of special relativity; it is impossible to detect the existence of a gravitational field by means of local experiments.

- Sean Carroll, *Spacetime and Geometry (2004)*

Then the Strong Equivalence Principle (SEP) is defined to include all of the laws of physics, gravitational and otherwise.

- Sean Carroll, *Spacetime and Geometry (2004)*

The solution is to retain the notion of inertial frames, but to discard the hope that they can be uniquely extended throughout space and time. Instead we can define locally inertial frames, those that follow the motion of individual freely falling particles in small enough regions of spacetime. (Every time we say "small enough regions", purists should imagine a limiting procedure in which we take the appropriate spacetime volume to zero.)

- Sean Carroll, *Spacetime and Geometry (2004)*

C-infinity maps are sometimes called *smooth*.

- Sean Carroll, *Spacetime and Geometry (2004)*

Roughly speaking, an open set is the interior of some (n-1)-dimensional closed surface (or the union of several such interiors).

- Sean Carroll, *Spacetime and Geometry (2004)*

Most manifolds cannot be covered with just one chart.

- Sean Carroll, *Spacetime and Geometry (2004)*

Just as the partial derivatives along coordinate axes provide a natural basis for the tangent space, the gradients of the coordinate functions x-mu provide a natural basis for the cotangent space.

- Sean Carroll, *Spacetime and Geometry (2004)*

Unfortunately, the partial derivative of a tensor is not, in general, a new tensor. The gradient, which is the partial derivative of a scalar, is an honest (0,1) tensor, as we have seen. But the partial derivative of higher-rank tensors is not tensorial....

- Sean Carroll, *Spacetime and Geometry (2004)*

The metric we use in general relativity cannot be used to define a topology, but it will have other uses.

- Sean Carroll, *Spacetime and Geometry (2004)*

In fact our notation "ds^2" does not refer to the differential of anything, or the square of anything; it's just convenient shorthand for the metric tensor, a multilinear map from two vectors to the real numbers.

- Sean Carroll, *Spacetime and Geometry (2004)*

...it turns out that at any point p there exists a coordinate system in which [the g's] take their canonical form and the first derivatives [of the g's] all vanish...Such coordinates are known as locally inertial coordinates...

- Sean Carroll, *Spacetime and Geometry (2004)*

The usual trick is to take a question of physical interest, answer it in the context of locally inertial coordinates, and then express that answer in a coordinate-independent form.

- Sean Carroll, *Spacetime and Geometry (2004)*

All the ways in which curvature manifests itself rely on something called a "connection", which gives us a way of relating vectors in the tangent spaces of nearby points.

- Sean Carroll, *Spacetime and Geometry (2004)*

...a connection that is symmetric in its lower indices is known as "torsion-free".

- Sean Carroll, *Spacetime and Geometry (2004)*

Our claim is therefore that there is exactly one torsion-free connection on a given manifold that is compatible with some given metric on that manifold.

- Sean Carroll, *Spacetime and Geometry (2004)*

The last thing we need to mention is that converting partial derivatives into covariant derivatives is not always necessary in order to construct well-defined tensors; in particular, the exterior derivative and the vector-field commutator are both well-defined in terms of partials, essentially because both involve an anti-symmetrization that cancels the nontensorial piece of the partial derivative transformation law.

- Sean Carroll, *Spacetime and Geometry (2004)*

...we must simply learn to live with the fact that two vectors can only be compared in a natural way if they are elements of the same tangent space. For example, two particles passing by each other have a well-defined relative velocity, which cannot be greater than the speed of light. But two particles at different points on a curved manifold do not have any well-defined notion of relative velocity - the concept simply makes no sense.

- Sean Carroll, *Spacetime and Geometry (2004)*

Since this phenomenon bears such a close resemblance to the conventional Doppler effect due to relative motion, we are very tempted to say that the galaxies are "receding away from us" at a speed defined by their redshift. At a rigorous level this is nonsense, what Wittgenstein would call a "grammatical mistake" - the galaxies are not receding, since the notion of their velocity with respect to us is not well-defined. What is actually happening is that the metric of spacetime between us and the galaxies has changed (the universe has expanded) along the path of the photon from here to there, leading to an increase in the wavelength of the light.

- Sean Carroll, *Spacetime and Geometry (2004)*

...the demand that the tangent vector be parallel-transported actually constrains the parameterization of the curve, specifically to one related to the proper time...

- Sean Carroll, *Spacetime and Geometry (2004)*

Often the case is that between two points on a manifold there is more than one geodesic.

- Sean Carroll, *Spacetime and Geometry (2004)*

Geodesics provide a convenient way of mapping the tangent space *Tp* of a point *p* to a region of the manifold that contains *p*, called the exponential map. This map in turn defines a set of coordinates for this region that are automatically the locally inertial coordinates...

- Sean Carroll, *Spacetime and Geometry (2004)*

...the best way we have of defining a singularity is as a place where geodesics appear to "end", after we remove trivial cases in which a part of the manifold is artificially excluded by hand.

- Sean Carroll, *Spacetime and Geometry (2004)*

...the cosmological redshift is *not* a Doppler shift...

- Sean Carroll, *Spacetime and Geometry (2004)*

...a convention needs to be chosen for the ordering of the indices [of the Riemann tensor]. There is no agreement at all on what this convention should be, so be careful.

- Sean Carroll, *Spacetime and Geometry (2004)*

We therefore refer to an n-dimensional manifold with (n/2)(n+1) Killing vectors as a maximally symmetric space. [For n=4, the number is 10 Killing vectors.]

- Sean Carroll, *Spacetime and Geometry (2004)*

It follows that we should be able to reconstruct the entire Riemann tensor of such a space from the Ricci scalar R...

- Sean Carroll, *Spacetime and Geometry (2004)*

Locally, then (ignoring questions of global topology), a maximally symmetric space of given dimension and signature is fully specified by R. The basic classification of such spaces is simply whether R is positive, zero, or negative, since the magnitude of R represents an overall scaling of the size of the space.

- Sean Carroll, *Spacetime and Geometry (2004)*

We know that the maximally symmetric spacetime with R=0 is simply Minkowski space. The positively curved maximally symmetric spacetime is called de Sitter space, while that with negative curvature is imaginatively labeled anti-de Sitter space.

- Sean Carroll, *Spacetime and Geometry (2004)*

When people say that GR is diffeomorphism invariant, more likely than not they have one of two (closely related) concepts in mind: the theory is free of "prior geometry", and there is no preferred coordinate system for spacetime.

- Sean Carroll, *Spacetime and Geometry (2004)*

Furthermore, the energy-momentum tensor will generally involve the metric as well.

- Sean Carroll, *Spacetime and Geometry (2004)*

The nonlinearity of general relativity is worth a remark...In GR the gravitational field couples to itself...The nonlinearity of Einstein's equation is a reflection of the back-reaction of gravity on itself.

- Sean Carroll, *Spacetime and Geometry (2004)*

There is nothing unique about [the nonlinearity] of gravity; it is shared by most gauge theories, such as quantum chromodynamics, the theory of the strong interactions. Electromagnetism is actually the exception...

- Sean Carroll, *Spacetime and Geometry (2004)*

A characteristic feature of general relativity is that the source for the gravitational field is the entire energy-momentum tensor. In nongravitational physics, only *changes* in energy from one state to another are measurable; the normalization of the energy is arbitrary...This behavior opens up the possibility of vacuum energy: an energy density characteristic of empty space.

- Sean Carroll, *Spacetime and Geometry (2004)*

The terms "cosmological constant" and "vacuum energy" are essentially interchangeable.

- Sean Carroll, *Spacetime and Geometry (2004)*

The vacuum energy is ultimately a constant of nature in its own right.

- Sean Carroll, *Spacetime and Geometry (2004)*

We know that there are four Killing vectors [in the Schwarzschild spacetime]: three for the spherical symmetry, and one for time translation. [conservation of angular momentum, and energy].

- Sean Carroll, *Spacetime and Geometry (2004)*

The idea of a conformal diagram [Carter-Penrose diagram]...is a crucial tool for analyzing spacetimes in general relativity.

- Sean Carroll, *Spacetime and Geometry (2004)*

Black hole solutions are characterized by a very small number of parameters, rather than the potentially infinite set of parameters characterizing, say, a planet.

- Sean Carroll, *Spacetime and Geometry (2004)*

An event horizon is a hypersurface separating those spacetime points that are connected to infinity by a timelike path from those that are not.

- Sean Carroll, *Spacetime and Geometry (2004)*

The moral of the story seems to be that typical time-dependent solutions in general relativity usually end in singularities. (Or begin in them; some theorems imply the existence of cosmological singularities, such as the Big Bang.) This represents something of a problem for GR, in the sense that the theory doesn't really apply to the singularities themselves, whose existence therefore represents an incompleteness of description. The traditional attitude toward this issue is to hope that a sought-after quantum theory of gravity will somehow resolve the singularities of classical GR.

- Sean Carroll, *Spacetime and Geometry (2004)*

Cosmic censorship conjecture: Naked singularities cannot form in gravitational collapse from generic, initially nonsingular states in an asymptotically flat spacetime obeying the dominant energy condition. A naked singularity is one from which signals can reach I+; that is, one that is not hidden behind an event horizon. Notice that the conjecture refers to the formation of naked singularities, not their existence; there are certainly solutions in which spacelike naked singularities exist in the past (such as the Schwarzschild white hole) or timelike singularities exist for all times (such as in super-extremal charged black holes)...The requirement that the initial data be in some sense "generic" is important, as numerical experiments have shown that finely-tuned initial conditions are able to give rise to naked singularities. A precise proof of some form of the cosmic censorship conjecture remains one of the outstanding problems of classical relativity.

- Sean Carroll, *Spacetime and Geometry (2004)*

The usefulness of homogeneity and isotropy is that they imply that a space is maximally symmetric. Think of isotropy as invariance under rotations, and homogeneity as invariance under translations, suitably generalized. Then homogeneity and isotropy together imply that a space has its maximum possible number of Killing vectors.

- Sean Carroll, *Spacetime and Geometry (2004)*

The conformal diagram of de Sitter space will simply be a representation of the patch of the Einstein static universe to which de Sitter is conformally related.

- Sean Carroll, *Spacetime and Geometry (2004)*

Thus, anti-de Sitter space is conformally related to half of the Einstein static universe.

- Sean Carroll, *Spacetime and Geometry (2004)*

I am sure colleagues know the caricature of the conventional lecture: notes are copied from the lecturer's notebook to the student's notebook without their going through the heads of either - a definition which is perhaps too close for comfort. I was converted at an early stage to the desirability of providing students with printed notes. The main advantage is that it frees up the lecture period from the time-consuming process of note-copying, and the time released can be used more effectively for developing and explaining the course at a rate which the students are able to cope with.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

In the mid-1970s, there were very few undergraduate courses in general relativity in existence in the UK.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

In fact, the essence of the special theory of relativity is contained in the Lorentz transformations.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

We shall call individuals equipped with a clock and a measuring rod or ruler *observers*....However, the approach of the k-calculus is to dispense with the rigid ruler and use radar methods for measuring distances.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

First of all, [the co-ordinate free approach] requires much more of a mathematical background, which in turn takes time to develop. The other disadvantage is that, for all its elegance, when one wants to do a real calculation with tensors, as one frequently needs to, then recourse has to be made to indices.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

...it is only the differentials and not the coordinates themselves which have the tensorial character.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

In short, a tensor equation which holds in one coordinate system necessarily holds in *all* coordinate systems...Put another way, tensorial equations are coordinate-independent. This is something that the index-free or coordinate-free approach makes clear from the outset.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

As in the case with vectors and vector fields in vector analysis, the distinction between a tensor and a tensor field is not always made compeletely clear.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

The key idea is to interpret the vector field as an operator which maps [a] real-valued function into [another] real-valued function.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

This is an invariant division of events which all observers agree upon. This follows because of the invariance of eta-ab under Lorentz transformations, which means that null cones get mapped onto null cones.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

As we have seen, Minkowski space-time admits the Poincaré group as its invariance group.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

For example, the principle of covariance is considered by some authors (e.g. Bondi, Fock) to be empty, whereas there are others (e.g. Anderson) who believe it possible to derive general relativity more or less solely from this principle. There is fairly general agreement that the principle of equivalence is the key principle.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

One school of thought describes them as apparent or fictitious forces which arise in non-inertial frames of reference.... We shall adopt the attitude that if you judge them by their effects then they are very real forces. For, after all, inertial forces cause astronauts to black-out in rocket ships and flywheels to break under centrifugal effects.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

The point of these thought experiments is that the presence of a genuine gravitational field, as distinct from an inertial field, is verified by the observation of the *variation* of the field rather than by the observation of the field itself.

- R. D'Inverno, *Introducing Einstein's Relativity (1992)*

Now the most dangerous hypotheses are those which are tacit and unconscious.

- A. Eddington, *Space, Time and Gravitation (1920)*

These differences are summed up in the statement that the geometry of space is Euclidean, but the geometry of space-time is semi-Euclidean or "hyperbolic".

- A. Eddington, *Space, Time and Gravitation (1920)*

It is not very profitable to speculate on the implication of the mysterious factor *i*, which seems to have the property of turning time into space. It can scarcely be regarded as more than an analytical device.

- A. Eddington, *Space, Time and Gravitation (1920)*

The division into past and future (a feature of time-order which has no analogy in space-order) is closely associated with our ideas of causation and free-will.

- A. Eddington, *Space, Time and Gravitation (1920)*

But the chief importance of the velocity of light is that no material body can exceed this velocity.

- A. Eddington, *Space, Time and Gravitation (1920)*

He who doubts the reality of the four-dimensional world (for logical, as distinct from experimental, reasons) can only be compared to a man who doubts the reality of the penny, and prefers to regard one of its innumerable appearances as the real object.

- A. Eddington, *Space, Time and Gravitation (1920)*

...gravitation acts not only on the molecules of matter, but on the undulations of light.

- A. Eddington, *Space, Time and Gravitation (1920)*

It will be seen that the sensation of weight is not felt when we are free to respond to the force of gravitation; it is only felt when something interferes to prevent our falling....It seems literally true to say that we never feel the force of the earth's gravitation; what we do feel is the bombardment of the soles of our boots by the molecules of the ground, and the consequent impulses spreading upwards through the body.

- A. Eddington, *Space, Time and Gravitation (1920)*

Thus gravitation is removable locally, but centrifugal force can be removed everywhere.

- A. Eddington, *Space, Time and Gravitation (1920)*

...only certain kinds of space-time can occur in an empty region in nature. The law which determines what kinds can occur is the law of gravitation... a restriction on the possible geometries of the world.

- A. Eddington, *Space, Time and Gravitation (1920)*

It is remarkable that these slender laws [covariance and the limit of flat space-time distant from any matter] are sufficient to indicate almost uniquely a particular law.

- A. Eddington, *Space, Time and Gravitation (1920)*

I prefer to think of matter and energy, not as agents causing the degrees of curvature of the world, but as parts of our perceptions of the existence of the curvature.

- A. Eddington, *Space, Time and Gravitation (1920)*

...a curvature of the world we live in may give an illusion of an attractive force, and indeed can only be discovered through some such effect.

- A. Eddington, *Space, Time and Gravitation (1920)*

Gravity and inertia are one.

- A. Eddington, *Space, Time and Gravitation (1920)*

The conservation of mass, of energy, and of momentum must all be contained implicitly in Einstein's law....It is a great triumph for Einstein's theory that his law gives correctly these experimental principles, which have generally been regarded as unconnected with gravitation.

- A. Eddington, *Space, Time and Gravitation (1920)*

When continuous matter is admitted, *any* kind of space-time becomes possible.

- A. Eddington, *Space, Time and Gravitation (1920)*

Thus the four dimensions of the world bring about a four-fold arbitrariness of choice of mesh-system; this in turn necessitates four identical relations between the G-mu-nu; and finally, in consequence of the law of gravitation, these identities reveal four new facts or laws relating to the density, energy, momentum or stress of matter, summarised in the expressions K-mu-nu. These four laws turn out to be the laws of conservation of momentum and energy.

- A. Eddington, *Space, Time and Gravitation (1920)*

In the relativity theory in particular [action] seems to be the most fundamental thing of all.

- A. Eddington, *Space, Time and Gravitation (1920)*

But even so we have often puzzled ourselves needlessly over paradoxes, which disappear when we realise that the physical quantities are not properties of certain external objects but are relations between these objects and something else.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

...I want to set before you the treasure which has already been unearthed in this field.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

...in our common outlook the idea of position or *location* is now fundamental; and the location of the object is a computational result.... To put the conclusion rather crudely - space is not a lot of points close together; it is a lot of distances interlocked.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

When we encounter *i* in our investigations, we must remember that it has been introduced by our choice of measure-code, and must not think of it as occurring with some mystical physical significance in the external world.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

When the g's are not constants and the fundamental quadratic form is not reducible to [a flat metric], there is still a null-surface, given by $ds=0$... which separates the timelike from the spacelike intervals.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

H. Weyl expresses this specialisation by saying that the world is 3+1 dimensional. Some entertainment may be derived by considering the properties of a 2+2 or a 4+0 dimensional world.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

It may seem strange that we should be able to deduce the contraction of a material rod and the retardation of a material clock from the general geometry of space and time. But it must be remembered that the contraction and retardation do not imply any absolute change in the rod and clock. The "configuration of events" constituting the four-dimensional structure which we call a rod is unaltered; all that happens is that the observer's space and time partitions cross it in a different direction.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

When a number of phenomena are connected together it becomes somewhat arbitrary to decide which is to be regarded as the explanation of the others.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

Covariance and contravariance are a kind of generalised dimension, showing how the measure of one condition of the world is changed when the measure of another condition is changed. The ordinary theory of change of units is merely an elementary case of this.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

Coordinates are the identification-numbers of the points of space-time.

- A. Eddington, *The Mathematical Theory of Relativity (1924)*

Literally hundreds of exact solutions of the full non-linear field equations are now known, despite their complexity. The most important ones are the Schwarzschild and Kerr solutions, determining the geometry of the solar system and of black holes, and the Friedmann-Lemaitre-Robertson-Walker solutions, which are basic to cosmology. Perturbations of these solutions make them the key to astrophysical applications.

- G. F. R Ellis, *100 Years of Relativity (2015)*

This concept of geometry as dynamically determined by its matter content necessarily leads to the non-linearity of both the equations and the physics. This results in the need for new methods of study of these solutions; standard physics methods based on the assumption of linearity will not work in general.

- G. F. R Ellis, *100 Years of Relativity (2015)*

The field equations of classical general relativity do not automatically prevent causality violation: so various causality conditions have been proposed as extra conditions to be imposed in addition to the Einstein equations, the most physically relevant being stable causality (no closed timelike lines exist even if the spacetime is perturbed).

- G. F. R Ellis, *100 Years of Relativity (2015)*

Studying the conformal structure of a spacetime is greatly facilitated by using conformal diagrams. Penrose pioneered this method and showed that one can rescale the conformal coordinates so that the boundary of spacetime at infinite distance is represented at a finite coordinate value, hence one can represent the entire spacetime and its boundary in this way. For example Minkowski space has null infinities I- and I+ for incoming and outgoing null geodesics, an infinity i0 for spacelike geodesics, and past and future infinities i-, i+ for timelike geodesics, and, perhaps surprisingly, the points i0, i- and i+ have to be identified. Penrose diagrams are now a standard tool in general relativity studies, particularly in cosmology, where they make the structure of particle horizons and visual horizons very clear, and in studying black holes.

- G. F. R Ellis, *100 Years of Relativity (2015)*

The situation was totally transformed by a highly innovative paper by Roger Penrose in 1965 that used global methods and causal analysis to prove that singularities will occur in gravitational collapse situations where closed trapped surfaces occur, a causality condition is satisfied, and suitable energy conditions are satisfied by the matter and fields present...Penrose's paper proved that the occurrence of black hole singularities is not due to special symmetries, but is generic.

- G. F. R Ellis, *100 Years of Relativity (2015)*

John Wheeler emphasized that existence of spacetime singularities - an edge to spacetime, where not just space, time, and matter cease to exist, but even the laws of physics themselves no longer apply - is a major crisis for physics. This is of course a prediction of the classical theory. It is still not known if quantum gravity solves this issue or not.

- G. F. R Ellis, *100 Years of Relativity (2015)*

Because the curvature of spacetime allows quite different global properties than in flat spacetime, it is possible for closed timelike lines to occur. Because it allows for a beginning and end to spacetime, where not just matter but even spacetime and the laws of physics cease to exist, it radically alters our views on the nature of existence.

- G. F. R Ellis, *100 Years of Relativity (2015)*

[The Kerr solution] is of considerable importance because most astrophysical objects are rotating. There is one important difference from Schwarzschild: while we can construct exact interior solutions to match the Schwarszchild exterior solution, that is not the case for the Kerr solution. It has a complex and fascinating structure that is still giving new insights.

- G. F. R Ellis, *100 Years of Relativity (2015)*

The [Schwarzschild] singularities are spacelike boundaries to spacetime (one in the future and one in the past), not timelike world lines as one would expect.

- G. F. R Ellis, *100 Years of Relativity (2015)*

The maximally extended Schwarszchild solution is an extraordinary discovery. The very simple looking metric implies the existence of two asymptotically flat spacetime regions connected by a wormhole...and two singularities that are spatially homogeneous in the limit [as r approaches 0]. There is no central worldline, as a point particle picture suggests. Thus just as quantum physics implied a radical revision of the idea of a particle, so does general relativity: there is no general relativity version of the Newtonian idea of a point particle. None of this is obvious. The global topology is not optional; it follows from the way the Einstein equations for this vacuum curve spacetime. And the nature of this solution emphasizes why one should always try to determine exact properties of solutions in general relativity: the global properties of the linearised form of the Schwarzschild solution (which does not exactly satisfy the field equations) will be radically different.

- G. F. R Ellis, *100 Years of Relativity (2015)*

However this diagram is misleading in some ways: it suggests that the central singularity is a timelike world line, which is not the case; it is spacelike because it exists in the part of spacetime corresponding to region II in Figure 2.

- G. F. R Ellis, *100 Years of Relativity (2015)*

Penrose formulated the cosmic censorship hypothesis, that such horizons would indeed form in the generic case. This conjecture is still unresolved...

- G. F. R Ellis, *100 Years of Relativity (2015)*

Much effort has been extended in showing that the Kerr solution is the likely final state of gravitational collapse of a rotating object. Work by Hawking, Carter, Robinson and others shows this indeed seems to be the case.

- G. F. R Ellis, *100 Years of Relativity (2015)*

...the black hole entropy is proportional to the area of its event horizon divided by the Planck area...

- G. F. R Ellis, *100 Years of Relativity (2015)*

[Black holes] were discovered theoretically as an unexpected consequence of the maximal extensions of the Schwarzschild solution.

- G. F. R Ellis, *100 Years of Relativity (2015)*

[Black holes] have no analogue in Newtonian gravitational theory because there is no limit to the speed of propagation of signals in that theory (where the speed of light plays no special role).

- G. F. R Ellis, *100 Years of Relativity (2015)*

These space times [FLRW in cosmology] are conformally flat: Cabcd = 0 (there is no free gravitational field, so no tidal forces or gravitational waves occur in these models).

- G. F. R Ellis, *100 Years of Relativity (2015)*

A key finding is the existence of limits both to causation, represented by particle horizons, and to observations, represented by visual horizons.... Much confusion about their nature was cleared up by Rindler in a classic paper, with further clarity coming from use of Penrose causal diagrams for these models. This showed that particle horizons would occur if and only if the initial singularity was spacelike. There are many statements in the literature that such horizons represent motion of galaxies away from us at the speed of light, but that is not the case; they occur due to the integrated behaviour of light from the start of the universe to the present day, with the visual horizon, determined by the most distant matter we can detect by electromagnetic radiation, lying inside the particle horizon. This is why the visual horizon size can be 42 billion light years in an Einstein de Sitter model with a Hubble scale of 14 billion years.

- G. F. R Ellis, *100 Years of Relativity (2015)*

Cosmological models started off as purely geometrical, but then a major realisation was that standard physics could be applied to the properties of matter in the early universe [hot big bang, nucleosynthesis].

- G. F. R Ellis, *100 Years of Relativity (2015)*

These [cosmological] models are the opposite of the Schwarzschild vacuum solution. Those models represent the dynamics of pure vacuum (there is no matter tensor); these models represent the dynamics of spacetime governed purely by matter (there is no free gravitational field).

- G. F. R Ellis, *100 Years of Relativity (2015)*

It is now broadly agreed that there was indeed such a period of inflation in the very early universe but the details are not clear: there are over 100 different variants...

- G. F. R Ellis, *100 Years of Relativity (2015)*

The real universe is only approximately a Robertson-Walker spacetime. Structure formation in an expanding universe can be studied by using linearly perturbed FLRW models at early times, plus numerical simulations at later times when the inhomogeneities have gone non-linear.

- G. F. R Ellis, *100 Years of Relativity (2015)*

A key realisation was that introduction of a cosmological constant would allow a cold dark matter scenario to match observations of an almost flat universe.

- G. F. R Ellis, *100 Years of Relativity (2015)*

The puzzles are that we do not know the nature of the inflaton, for which there are over 100 models; we do not know the nature of the dark matter that is indicated to exist by dynamical studies, and is far more abundant than baryonic matter; and we do not know the nature of the dark energy causing an acceleration of the expansion of the universe at late times. It is possible that some of these issues may be indicating we need a different theory of gravity than general relativity, for example MOND or a scalar-tensor theory.

- G. F. R Ellis, *100 Years of Relativity (2015)*

Gravitational waves can carry energy, momentum, and information...

- G. F. R Ellis, *100 Years of Relativity (2015)*

The theory of gravitational waves extends the idea of transverse wave propagation from the spin-1 field of electromagnetism (Maxwell's theory) to the spin 2 field of gravitation. They are an essentially relativistic phenomenon: they cannot occur in Newtonian gravitational theory. Gravitational waves can carry energy and arbitrary information, and indeed convey information to us from the earliest history of the universe that we can access. They provide the ultimate limit of our possible access to knowledge about the early universe.

- G. F. R Ellis, *100 Years of Relativity (2015)*

However if one demands only second order equations in four dimensions and with one spacetime metric, general relativity is the unique gravitational theory based in Riemannian geometry, as shown by Lovelock. The theory was derived not because of experiment, but as the result of pure thought; but it has survived all experimental tests...There is no observational or experimental reason to modify or abandon the field equations. However there is a significant problem in terms of the relation of general relativity to quantum field theory calculations that predict [the] existence of a vacuum energy density vastly greater than the observed value of the cosmological constant.

- G. F. R Ellis, *100 Years of Relativity (2015)*

[The] general trend [of the author's views] is to lay stress on the Absolute rather than on the Relative.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

On the other hand, the principles of relativity and equivalence are of limited application and, notwithstanding their heuristic value, they are not unrestrictedly part of Einstein's Theory of Gravitation as expressed by the gravitational equations.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Galilean space is of maximal uniformity. The uniformity of space and time manifests itself in the existence of a group of transformations which leave invariant the four-dimensional distance or interval between two points.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The law of Galileo can be stated in generalized form as the law of the equality of inertial and gravitational mass. It should be stressed that this fundamental law is of a general character whereas the principle of equivalence is strictly local.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The field equations and the boundary conditions are inextricably connected and the latter can in no way be considered less important than the former.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Thus, covariance of equations in itself is in no way the expression of any kind of physical law.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The name *Chronogeometry* suggested by A. D. Fokker would be more appropriate [than "Theory of Relativity"].

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Enough has been said to make clear that the use of the terms "general relativity", "general theory of relativity", or "general principle of relativity" should not be admitted. This usage is inconvenient because it not only leads to misunderstanding, but also reflects an incorrect understanding of the theory itself.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

In addition, it follows that the very notion of a "principle of relativity" becomes well defined only when a definite class of reference frames has been singled out. In the usual theory of relativity this class is that of inertial systems.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Since the greatest possible uniformity is expressed by Lorentz transformations there cannot be a more general principle of relativity than that discussed in ordinary relativity theory. All the more, there cannot be a general principle of relativity, as a physical principle, which would hold with respect to arbitrary frames of reference.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

It is clear that a principle of relativity implies a covariance of equations, but the converse is not true: covariance of differential equations is possible also when no principle of relativity is satisfied.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

... a general principle of relativity, as a physical principle, holding in relation to arbitrary frames of reference, is impossible.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

[The covariance requirement] is a self-evident, purely logical requirement that in all cases in which the coordinate system is not fixed in advance, equations written down in different coordinate systems should be mathematically equivalent. The class of transformations with respect to which the equations must be covariant must correspond to the class of coordinate systems considered. Thus if one deals with inertial systems related by Lorentz transformations and if Galilean coordinates are used, it is sufficient to require covariance with respect to Lorentz transformations. If, however, arbitrary coordinates are employed, it is necessary to demand general covariance.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Thus the principle of equivalence is related to the equality of inertial and gravitational mass, but is not identical with it. The latter is of a general, non-local character while the equivalence of a field of acceleration and a field of gravitation exists only locally, i.e. it refers only to a single point in space...Einstein gave to his principle of equivalence a widened interpretation by taking it to imply the indistinguishability of fields of gravitation and acceleration....However to us such an extended interpretation seems inconsistent. The essence of the principle of equivalence may be seen in the fact that it allows the introduction of an appropriate locally geodesic ("freely falling") frame of reference, by use of which a uniform Galilean space can be defined in the infinitesimal. However this in no way justifies conclusions about the equivalence or indistinguishability of fields of acceleration and gravitation in finite regions of space.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

There exists a maximum speed for the propogation of any action. This is numerically equal to the speed of light in free space.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

This last fact makes it possible to envisage as a model of an inertial reference frame a rigid scaffolding having at each of its junctions a clock, with all clocks synchronous...For all its unwieldliness such a model can be of use and is often discussed in treatments of the Theory of Relativity. However, we prefer the model of the radar station because it allows a continuous determination both of the position of a moving body and of the corresponding time, by the station clock - a determination *based on a single point*, the position of the station.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The radar model is the more flexible one and retains its intuitive value even in cases when the rigid scaffolding is quite inappropriate, for instance, when discussing astronomical distances. In addition, the radar model is one that permits of generalization.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

One could also introduce the notion of clocks nearly or, in the limit, entirely insensitive to impacts and accelerations (for instance atomic systems with large internal frequencies). One could then propose to measure proper time by the readings of such an ideally insensitive clock, this being just the physical meaning of proper time. But one should note that such a proposal, although not in contradiction with the theory of relativity, does not follow from it and represents a special hypothesis.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

It is thus clear that the concepts of dimensions and shape of a moving object are closely related to the concept of simultaneity. We know already that the notion of simultaneity is not something absolute but depends on the reference frame. Therefore we must expect that the shape and size of an object is also not absolute, but must be stated in relation to a definite frame.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

Consequently the volume of the rod decreases proportionally with its longitudinal dimension. The same result is true for a body of any shape.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

If a transformation exists such that the new functions, expressed in terms of the new variables, satisfy equations of the same form as do the old functions in the old variables, the equations are termed *covariant*.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The difference in sign is extremely important for the whole theory because it reflects the existence of a deep distinction between space and time. It would be possible to impose the same sign on all terms of [ds-squared] by introducing imaginary quantities - imaginary space coordinates or imaginary time. This course was adopted by Minkowski and Umov; but we believe that it is not appropriate to introduce a symmetry between space and time into our equations in this manner because it obscures the actually existing distinction between them and does not have any mathematical advantages.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The properties of space-time are objective, they are determined by Nature and do not depend on our choice.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

These were just the lines we followed in developing the Theory of Relativity which is essentially a theory of space and time. The designation "Relativity" has historic reasons and gives only a one-sided idea of the content of the theory.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

...the very notion of a "principle of relativity" becomes well defined only when a definite class of frames of reference has been singled out. In the usual theory of relativity this class is that of inertial systems.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

But light possesses energy and by the law of proportionality of mass and energy all energy is indissolubly connected with mass. Therefore light must possess mass. On the other hand, any mass located in a gravitational field must experience the action of that field and in general its motion will therefore not be rectilinear.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

In short, masses determine the geometrical properties of space and time, and these properties determine the movement of the masses.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

The existence of a finite propagation velocity for gravity removes the contradiction inherent in Newton's theory of gravitation with its admission of instantaneous action at a distance.

- V. Fock, *The Theory of Space, Time and Gravitation (1964)*

By a sport of history, the name relativity theory has been given to a doctrine which claims that the laws of nature are of an absolute invariant character and that the formulation of these laws should preserve their invariancy in spite of any transformation whatever of the coordinates used in the formalism. A great deal of misunderstanding and irrelevant discussion might have been avoided, if from the beginning it had been clear to all, that in this theory one is concerned with *absolute relations of time and space.*

- A. D. Fokker, *Einstein, Inventor of Chronogeometry (1955)*

For example, the Stanford linear accelerator, which accelerates electrons close to the speed of light, is about two miles long and cost 100 million dollars; if Newtonian physics were the correct theory, it need only have been about one inch long.

- Foster/Nightingale, *Short Course in General Relativity (1979)*

Einstein's general theory of relativity postulates that gravitational effects may be explained by the curvature of spacetime, and that gravity should not be regarded as a force in the conventional sense.

- Foster/Nightingale, *Short Course in General Relativity (1979)*

The way in which the dimple around the stalk [of the apple] gives the impression of attraction corresponds to the fact that massive bodies modify the curvature of spacetime in their vicinity, and this modification affects the geodesics in such a way as to give the impression that free particles are acted on by a force, whereas in actual fact they are following the straightest paths in the curved spacetime.

- Foster/Nightingale, *Short Course in General Relativity (1979)*

...we shall adopt a much-used convention, which is to *confuse a tensor with its components.*

- Foster/Nightingale, *Short Course in General Relativity (1979)*

We therefore adopt another characterisation of a straight line, namely its *straightness*, as our guide to defining geodesics.

- Foster/Nightingale, *Short Course in General Relativity (1979)*

...any cartesian tensor equation valid in special relativity may be converted to an equation valid in general relativity in any coordinate system, simply by replacing partial differentiation with respect to coordinates by covariant differentiation, total derivatives along curves by absolute derivatives, and eta by g.

- Foster/Nightingale, *Short Course in General Relativity (1979)*

The last and most treacherous aspect of introducing an observer attached to a given frame of reference is that one may get the impression that this observer has some kind of bird's-eye view of the whole of his reference frame at a given instant. *This is entirely false.* A single observer is not ubiquitous; at a given instant he has awareness only of events occurring at his own location - e.g., a burst of photons striking his retina.... Almost always, these statements [about observers] are simply statements about the space-time coordinates of a particular event as established by measurement in frame S. If one really means to talk about looking or seeing, then an extra feature - the transmission of information from one point to another - is involved.

- A. P. French, *Special Relativity (1966)*

Here is where our intuitions cheifly play us false. There is no such thing as *the* time. "The time" is not a metaphysical abstraction, it is the reading on a clock.

- A. P. French, *Special Relativity (1966)*

We see then that a result upon which observers in all frames agree - what fraction of a group of unstable particles survives between one point event and another - may be attributed to time dilation or to Lorentz contraction, according to one's point of view.

- A. P. French, *Special Relativity (1966)*

The above considerations do not, however, prevent us from having purely geometric velocities greater than *c*. For example, the spot produced on the moon by a laser beam directed from the earth could easily be made to sweep over the moon's surface at a speed much greater than that of light...But no violation of dynamics is involved, for the path of the spot is nothing more than the locus of the points of impact of separate photons, each of which travels from earth to moon with speed *c*.

- A. P. French, *Special Relativity (1966)*

It seems almost incredible in retrospect, but for over 50 years after Einstein's 1905 paper there was an unchallenged belief among physicists that the Lorentz contraction of a moving body could be seen or photographed.... In 1959, J. Terrell showed that the Lorentz contraction is not in general perceived as such by the eye. [Terrell's paper is incorrect, according to later papers.]

- A. P. French, *Special Relativity (1966)*

Unlike in Maxwell's theory, the equations of motion in the gravitational field cannot be postulated independently of the field equations (Einstein-Infeld-Hoffmann).

- Hubert Goenner, *A Golden Age of General Relativity? (2016)*

The history of general relativity before the 1920s is not taken into account by many narrations. Besides Einstein and his entourage, and a few researchers in Berlin, Leiden and Vienna, very few studied general relativity, particularly in France, England and the United States.

- Hubert Goenner, *A Golden Age of General Relativity? (2016)*

Like Molière's hero who was delighted to learn he had been talking prose all his life without knowing it, we have really been talking about the restricted Lorentz transformation from the beginning of this section without realizing it.

- Herbert Golstein, *Classical Mechanics (1980)*

A restricted Lorentz transformation that describes the relation between two systems with parallel axes moving uniformly relatively to each other, i.e., without any spatial rotation, is called a *pure* Lorentz transformation or, in jargon, a "boost".

- Herbert Golstein, *Classical Mechanics (1980)*

In their massive treatise on gravity, Misner, Thorne, and Wheeler call for the extinction of the complex Minkowski space: "One sometime participant in special relativity will have to be put to the sword: 'x4=ict'". It is claimed that the use of a Cartesian space hides the basic indefinite nature of the metric - that the square of the magnitude of vectors can be positive, zero, or negative. Further, the artifice of a complex Cartesian space is feasible only in special relativity; in the theory of general relativity the space is curved and the use of a non-Cartesian metric tensor is inescapable. It is also pointed out that in quantum mechanics where wave functions or state vectors are complex, the use of an imaginary coordinate will complicate the operation of complex conjugation....It is difficult to oppose such distinguished authority. Nonetheless, we shall here make a stand against the euthanasia of Minkowski space.

- Herbert Golstein, *Classical Mechanics (1980)*

Yet, paradoxically, general relativity - so well established, so important for several branches of physics, and so simple in its basic conception - is often not represented anywhere in the undergraduate physics curriculum.

- J. B. Hartle, *General Relativity in the Undergraduate Physics Curriculum (2008)*

It is probably fruitless to speculate on why a subject as basic, accessible, and important as general relativity is not taught more widely as part of the undergraduate physics curriculum. Limited time, limited resources, inertia, tradition, and misconceptions may all play a role.

- J. B. Hartle, *General Relativity in the Undergraduate Physics Curriculum (2008)*

To exhibit a spacetime geometry, the only 'new math' ideas required are the metric and its relation to distances in space and time.

- J. B. Hartle, *General Relativity in the Undergraduate Physics Curriculum (2008)*

The simplest spacetimes are the most physically relevant.

- J. B. Hartle, *General Relativity in the Undergraduate Physics Curriculum (2008)*

This theory leads to two remarkable predictions about the universe: first, that the final fate of massive starts is to collapse behind an event horizon to form a 'black hole' which will contain a singularity; and secondly, that there is a singularity in our past which constitutes, in some sense, a beginning to the universe.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The view of physics that is most generally accepted at the moment is that one can divide the discussion of the universe into two parts. First, there is the question of the local laws satisfied by the various physical fields. These are usually expressed in the form of differential equations. Secondly, there is the problem of the boundary conditions for these equations, and the global nature of their solutions. This involves thinking about the edge of space-time in some sense. These two parts may not be independent. Indeed it has been held that the local laws are determined by the large-scale structure of the universe. This view is generally connected with the name of Mach....

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In fact most of our results will be independent of the detailed nature of the physical laws, but will merely involve certain general properties such as description of space-time by a pseudo-Riemannian geometry and the positive definiteness of energy density.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

One can think of a singularity as a place where our present laws of physics break down. Alternatively, one can think of it as representing part of the edge of space-time, but a part which is at a finite distance instead of at infinity. On this view, singularities are not so bad, but one still has the problem of the boundary conditions. In other words, one does not know what will come out of the singularity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In chapter 3 a formulation of the General Theory of Relativity is given in terms of three postulates about a mathematical model for space-time. This model is a manifold M with a metric g of Lorentz signature. The physical significance of the metric is given by the first two postulates: those of local causality and of local conservation of energy-momentum.... The third postulate, the field equations for the metric....

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The curvature can be interpreted as a differential or tidal force which induces relative accelerations between neighbouring geodesics. If the energy-momentum tensor satisfies certain positive definite conditions, this differential force always has a net converging effect on non-rotating families of geodesics. One can show by use of Raychaudhuri's equation that this then leads to focal or conjugate points where neighbouring geodesics intersect.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

For a long time it was thought that these singularities might simply be a result of the high degree of symmetry, and would not be present in more realistic models. It will be one of our main objects to show that this is not the case.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The causal structure of space-time can be used to define a boundary or edge to space-time. This boundary represents both infinity and the part of the edge of space-time which is at a finite distance, i.e. the singular points.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The space-time structure discussed in the next chapter, and assumed through the rest of this book, is that of a manifold with a Lorentz metric and associated affine connection.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

A manifold is essentially a space which is locally similar to Euclidean space in that it can be covered by coordinate patches. This structure permits differentiation to be defined, but does not distinguish intrinsically between different coordinate systems. Thus the only concepts defined by the manifold structure are those which are independent of the choice of a coordinate system.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The definition of a manifold given so far is very general. For most purposes one will impose two further conditions, that M is Hausdorff and that M is paracompact, which will ensure reasonable local behaviour.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Unless otherwise stated, all manifolds considered will be paracompact, connected C-infinite Hausdorff manifolds without boundary. It will turn out later that when we have imposed some additional structure on M (the existence of an affine connection) the requirement of paracompactness will be automatically satisfied because of the other restrictions.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The mathematical model we shall use for space-time, i.e. the collection of all events, is a pair (M,g) where M is a connected four-dimensional Hausdorff C-infinite manifold and g is a Lorentz metric (i.e. metric of signature +2) on M.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Strictly speaking then, the model for space-time is not just one pair (M,g) but a whole equivalence class of all pairs (M', g') which are equivalent to (M,g).

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The manifold M is taken to be connected since we would have no knowledge of any disconnected component. It is taken to be Hausdorff since this seems to accord with normal experience. However in chapter 5 we shall consider an example in which one might dispense with this condition. Together with the existence of a Lorentz metric, the Hausdorff condition implies that M is paracompact.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In fact, the order of differentiability of the metric is probably not physically significant. Since one can never measure the metric exactly, but only with some margin of error, one could never determine that there was an actual discontinuity in its derivatives of any order. Thus one can always represent one's measurements by a C-infinite metric.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

There will be various fields on M, such as the electromagnetic field, the neutrino field, etc., which describe the matter content of space-time. These fields will obey equations which can be expressed as relations between tensors on M in which all derivatives with respect to position are covariant derivatives with respect to the symmetric connection defined by the metric g. This is so because the only relations defined on a manifold structure are tensor relations, and the only connection defined so far is that given by the metric.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

[Postulate of local causality:] The equations governing the matter fields must be such that if U is a convex normal neighborhood and p and q are points in U then a signal can be sent in U between p and q if and only if p and q can be joined by a C1 curve lying entirely in U, whose tangent vector is everywhere non-zero and is either timelike or null; we call such a curve, *non-spacelike*.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus by observing which points can communicate with p, one can determine the null cone Np in Tp [the tangent space at p]. Once Np is known, the metric at p may be determined up to a conformal factor.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The conformal factor in the metric may be determined using postulate (b) below [local conservation of energy and momentum]; thus all the elements of the theory will be physically observable.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

One might think that one could include gravitation by keeping the metric flat and by introducing an extra field on space-time. However, experiments have shown that light rays travelling near the sun are deflected. Since light rays are geodesics, this shows that the space-time metric cannot be flat or even conformal to a flat metric.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

These equations should be tensor equations involving the matter only through its energy-momentum tensor, i.e. should not distinguish between two different matter fields which have the same distribution of energy and momentum.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

...the pressure contributes to the total mass. This means that in some circumstances it can actually assist rather than prevent gravitational collapse.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus the field equations really provide only six independent differential equations for the metric [16->10->6]. This is in fact the correct number of equations to determine the space-time, since four of the ten components of the metric can be given arbitrary values by use of the four degrees of freedom to make coordinate transformations....The field equations should define the metric only up to an equivalence class under diffeomorphisms, and there are four degrees of freedom to make diffeomorphisms.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

This theory in which the metric is restricted to be conformally flat is known as the Nordstrom theory. It can be reformulated as a theory in which the metric is the flat metric and in which the gravitational interaction is represented by an additional scalar field. As mentioned before, this sort of theory would be inconsistent with the observed deflection of light by massive objects, and it would not account for the measured advance in the perihelion of Mercury.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

It is [the postulate of local causality] which sets the metric g apart from other fields on M and gives it its distinctive geometrical character.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Physically it would seem reasonable to suppose that there is a local thermodynamic arrow of time defined continuously at every point of space-time, but we shall only require that it should be possible to define continuously a division of non-spacelike vectors into two classes, which we arbitrarily label future- and past-directed. If this is the case, we shall say that space-time is time-orientable. In some space-times it is not possible to define such a time-orientation [e.g. de Sitter space]. If a space-time (M,g) is not time-orientable, then it has a double covering space which is.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

One may also ask whether space-time is *space-orientable*, that is whether it is possible to divide bases of three spacelike axes into right handed and left handed bases in a continuous manner.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In particular if one assumes that space-time is time-orientable then it must also be space-orientable. (This in fact follows on using the experimental evidence alone without appealing to the CPT theorem.)

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

This example, incidentally, illustrates a useful technique for constructing space-times with given causal properties: one starts with some simple space-time (unless otherwise indicated this will be Minkowski space), cuts out any closed set and, if desired, pastes it together in an appropriate way (i.e. one makes identifications of points of M). The result is still a manifold with a Lorentz metric and therefore still a space-time even though it may look rather incomplete where points have been cut out. As mentioned above, however, this incompleteness can be cured by an appropriate conformal transformation which sends the cut out points to infinity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

of space-time at which this condition does not hold. The set of all such points will be called the *chronology violating* set of M...

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

If M is compact, the chronology-violating set of M is non-empty....From this result it would seem reasonable to assume that space-time is non-compact. Another argument against compactness is that any compact, four-dimensional manifold on which there is a Lorentz metric cannot be simply connected.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

We shall say that the *causality condition* holds if there are no closed non-spacelike curves.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The *strong causality condition* is said to hold at p if every neighborhood of p contains a neighborhood of p which no non-spacelike curve intersects more than once.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

A set N is said to be globally hyperbolic if the strong causality assumption holds on N and if for any two points p,q in N, J+(p)intersectJ-(q) is compact and contained in N. In a sense this can be thought of as saying that J+(p)intersectJ-(q) does not contain any points on the edge of space-time, i.e. at infinity or a singularity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus if the whole of space-time were globally hyperbolic, i.e. if there were a global Cauchy surface, its topology would be very dull.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The importance of global hyperbolicity for chapter 8 lies in the following result [the existence of geodesics]: Let p and q lie in a globally hyperbolic set N with q in J+(p). Then there is a non-spacelike geodesic from p to q whose length is greater than or equal to that of any other non-spacelike curve from p to q.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The Einstein equations are *non-linear*. Actually in this respect they are not so different from other fields, for while the electromagnetic field, the scalar field, etc., *by themselves* obey linear equations in a given space-time, they form a non-linear system when their mutual interactions are taken into account. The distinctive feature of the gravitational field is that it is *self-interacting*: it is non-linear even in the absence of other fields. This is because it defines the space-time over which it propagates. To obtain a solution of the non-linear equations one employs an iterative method on approximate linear equations whose solutions are shown to converge in a certain neighbourhood of the initial surface.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus the solutions of the field equations can be unique only up to a diffeomorphism. In order to obtain a definite member of the equivalence class of metrics which represents a space-time, one introduces a fixed 'background' metric and imposes four 'gauge conditions' on the covariant derivatives of the physical metric with respect to the background metric. These conditions remove the four degrees of freedom to make diffeomorphisms and lead to a unique solution for the metric components. They are analogous to the Lorentz condition which is imposed to remove the gauge freedom for the electromagnetic field.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

We adopt b-incompleteness, a generalization of the idea of geodesic incompleteness, as an indication that singular points have been cut out of space-time, and characterize two possible ways in which b-incompleteness can be associated with some form of curvature singularity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

We also show that there is not only one incomplete geodesic, but a three-parameter family of them. In [section] 8.5 we discuss the situation in which the incomplete curves are totally or partially imprisoned in a compact region of space-time. This is shown to be related to non-Hausdorff behaviour of the b-boundary. We show that in a generic space-time, an observer travelling on one of these incomplete curves would experience infinite curvature forces.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

a finite interval of proper time. This would appear to be an even more objectionable feature than infinite curvature and so it seems appropriate to regard such a space as singular....one should probably also regard a null geodesically incomplete space-time as singular.... As nothing moves on spacelike curves, the significance of spacelike geodesic incompleteness is not so clear. We shall therefore adopt the view that *timelike and null geodesic completeness are minimum conditions for space-time to be considered singularity-free*. Therefore if a space-time is timelike or null geodesically complete, we shall say that it has a singularity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

However, the class of timelike and/or null incomplete space-times does not include all those one might wish to consider as singular in some sense. For example Geroch (1968b) has constructed a space-time which is geodesically complete but which contains an inextendible timelike curve of bounded acceleration and finite length. An observer with a suitable rocketship and a finite amount of fuel could traverse this curve. After a finite interval of time he would no longer be represented by a point of the space-time manifold. If one is going to say that there is a singularity in a space-time in which a freely falling observer comes to an untimely end, one should presumably do the same for an observer in a rocketship.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

One feels intuitively that a singularity ought to involve the curvature becoming unboundedly large near a singular point. However since we have excluded singular points from our definition of space-time, difficulty arises in defining both 'near' and 'unboundedly large'.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Such results, although suggestive, do not necessarily have any physical significance because they depend on the symmetry being exact and clearly in any physical situation this will not be the case. It was therefore suggested by a number of authors that singularities were simply the result of symmetries and that they would not occur in general solutions.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

What is needed is a prescription for attaching some sort of boundary [del] to M which is uniquely determined by measurements at non-singular points, i.e. by the structure of (M,g).

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

...we interpret the preceding theorems as indicating not that geodesic incompleteness necessarily occurs, but that General Relativity breaks down in very strong gravitational fields.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In this chapter we consider the effect of space-time curvature on families of timelike and null curves. These could represent flow lines of fluids or the histories of photons.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In [section] 4.3 we discuss the general inequalities on the energy-momentum tensor which imply that the gravitational effect of matter is always to tend to cause convergence of timelike and null curves.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

However, in general, space-time will not have any Killing vectors. Thus one will not have any special frame against which to measure acceleration; the best one can do is to take two bodies close together and measure their relative acceleration. This will enable one to measure the gradient of the gravitational field. If one thinks of the metric as being analogous to the Newtonian potential, the gradient of the Newtonian field would correspond to the second derivative of the metric. These are described by the Riemann tensor.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The effect of this 'tidal force' term can be seen, for example, by considering a sphere of particles falling freely towards the earth. Each particle moves on a straight line through the centre of the earth but those nearer the earth fall faster than those further away. This means that the sphere does not remain a sphere but is distorted into an ellipsoid with the same volume.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus the Weyl tensor is that part of the curvature which is not determined locally by the matter distribution.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In fact, one has little idea of the behaviour of matter under extreme conditions of density and pressure. Thus it might seem that one has little hope of predicting the occurrence of singularities in the universe from the Einstein equations as one does not know the right-hand side of the equations. However there are certain inequalities which it is physically reasonable to assume for the energy-momentum tensor. ...It turns out that in many circumstances these are sufficient to prove the occurrence of singularities, independent of the exact form of the energy-momentum tensor.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

[The weak energy condition] is equivalent to saying that the energy density as measured by any observer is non-negative. This would seem very reasonable physically.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

[The dominant energy condition] may be interpreted as saying that to any observer the local energy density appears non-negative and the local energy flow vector is non-spacelike....In other words, the dominant energy condition is the weak energy condition with the additional requirement that the pressure should not exceed the energy density. This holds for all known forms of matter and there is in fact good reason for believing that this should be the case in all situations.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Since any Cauchy surface intersects all timelike and null geodesics...

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

We shall mean by an exact solution of Einstein's field equations, a space-time (M,g) in which the field equations are satisfied with Tab the energy-momentum tensor of some specified form of matter which obeys postulate (a) ('local causality') of chapter 3, and one of the energy conditions of chapter [section] 4.3. In particular, one may look for exact solutions for empty space..., for an electromagnetic field..., for a perfect fluid..., or for a space containing an electromagnetic field and a perfect fluid. Because of the complexity of the field equations, one cannot find exact solutions except in spaces of rather high symmetry. Exact solutions are also idealized in that any region of space-time is likely to contain many forms of matter, while one can obtain exact solutions only for rather simple matter content. Nevertheless, exact solutions give an idea of the qualitative features that can arise in General Relativity, and so of possible properties of realistic solutions of the field equations.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

...the set of all space-times, that is, all non-compact four-dimensional manifolds and all Lorentz metrics on them.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In this chapter, we shall show that stars of more than about 1.5 times the solar mass should collapse when they have exhausted their nuclear fuel. If the initial conditions are not too asymmetric, the conditions of theorem 2 should be satisfied and so there should be a singularity. This singularity is however probably hidden from the view of an external observer who sees only a 'black hole' where the star once was. We derive a number of properties of such black holes, and show that they probably settle down finally to a Kerr solution.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

In [section] 9.1 we discuss stellar collapse, showing how one would expect a closed trapped surface to form around any sufficiently large spherical star at a late stage in its evolution. In [section] 9.2 we discuss the event horizon which seems likely to form around such a collapsing body. In [section] 9.3 we consider the final stationary state to which the solution outside the horizon settles down. This seems to be likely to be one of the Kerr family of solutions. Assuming that this is the case, one can place certain limits on the amount of energy which can be extracted from such solutions.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

However the point is that even if the star is not exactly spherically symmetric, a closed trapped surface will still occur providing the departures from spherical symmetry are not too great. This follows from the stability of the Cauchy development proved in [section] 7.5....

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Another way of seeing this is that if L=m^2 and anguluar momentum is conserved during the collapse, then the velocity of the surface of the star would be about the velocity of light when the star was at its Schwarzschild radius. Now many stars have an angular momentum greater than the square of their mass (for the sun, L~m^2). However it seems reasonable to expect some loss of angular momentum during the collapse because of braking by magnetic fields and because of gravitational radiation. The situation is therefore that in some stars, and probably most, angular momentum would not prevent occurrence of closed trapped surfaces, and hence a singularity.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Thus a star containing somewhat more than [so many] nucleons will not reach nuclear densities until it is already inside its Schwarzschild radius.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

This example also shows that the conditions when a body passes through its Schwarzschild radius need not be in any way extreme.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

The most striking feature of spherically symmetric collapse is that the singularity occurs within the region r<2m, from which no light can escape to infinity. Thus if one remained outside r=2m one would never see the singularity predicted by theorem 2. Further the breakdown of physical theory which occurs at the singularity cannot affect one's ability to predict the future in the asymptotically flat region of space-time.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

By [section] 6.3 the event horizon is an achronal boundary which is generated by null geodesic segments which may have past endpoints but which can have no future endpoints.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

...the following result shows that black holes can never bifurcate.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Once the surface of the star has passed inside the event horizon, the metric of the exterior region is that of the Schwarzschild solution, and is unaffected by the fate of the star.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

That is, a partial Cauchy surface is a spacelike hypersurface which no non-spacelike curve intersects more than once.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

That is, a Cauchy surface is a spacelike hypersurface which every non-spacelike curve intersects exactly once.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

Being a Cauchy surface is a property not only of the surface itself but also of the whole space-time in which it is imbedded. For example, if one cuts a single point out of Minkowski space, the resultant space-time admits no Cauchy surface at all.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

If there were a Cauchy surface for M, one could predict the state of the universe at any time in the past or future if one knew the relevant data on the surface. However one could not know the data unless one was to the future of every point on the surface, which would be impossible in most cases. There does not seem to be any physically compelling reason for believing tha the universe admits a Cauchy surface; in fact there are a number of known exact solutions of the Einstein field equations which do not, among them the anti-de Sitter space, plane waves, Taub-NUT space and Reissner-Nordstrom solution, all described in chapter 5.

- Hawking and Ellis, *The Large-Scale Structure of Space-Time*

[Penrose] was the first to show that one could discover general properties without solving the [field] equations exactly.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

General relativity is a beautiful theory that agrees with every observation that has been made.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

But I believe [gravity] is distinctively different, because it shapes the arena in which it acts, unlike other fields which act in a fixed spacetime background. It is this that leads to the possibility of time having a beginning. It also leads to regions of the universe that one can't observe, which in turn gives rise to the concept of gravitational entropy as a measure of what we can't know.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The crucial technique for investigating singularities and black holes that was first introduced by Roger, and which I helped develop, was the study of the global causal structure of spacetime.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The reason one gets conjugate points in spacetime is that gravity is an attractive force. It therefore curves spacetime in such a way that neighbouring geodesics are bent toward each other rather than away.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

Between 1965 and 1970 Penrose and I used the techniques I have described to prove a number of singularity theorems. These theorems had three kinds of conditions. First there was an energy condition such as the weak, strong, or generic energy conditions. Then there was some global condition on the causal structure such as that there shouldn't be any closed timelike curves. And finally, there was some condition that gravity was so strong in some region that nothing could escape.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The prediction of singularities means that classical general relativity is not a complete theory. Because the singular points have to be cut out of the spacetime manifold, one cannot define the field equations there and cannot predict what will come out of a singularity.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

As the Einstein equations are generally hard to solve, one looks instead for global properties that imply the existence of singularities.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

In fact, cosmic censorship is very important, as the whole theory depends on it, and without it we might see dreadful things instead of a black hole.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

I would have liked to draw you a four-dimensional picture. However, government cuts have meant that Cambridge University can afford only two-dimensional screens.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The no-hair theorem, proved by the combined work of Israel, Carter, Robinson, and myself, shows that the only stationary black holes in the absence of matter fields are the Kerr solutions. These are characterized by two parameters, the mass M and the angular momentum J. The no-hair theorem was extended by Robinson to the case where there was an electromagnetic field. This added a third parameter Q, the electric charge.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

What the no-hair theorem shows is that a large amount of information is lost when a body collapses to form a black hole.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The reason is gravity allows different topologies for the spacetime manifold.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

Gravity is the only [interaction] which affects causality, with profound implications with regard to black holes and information loss.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

One can paraphrase this [the no-boundary proposal] as "The Boundary Condition of the Universe Is That It Has No Boundary."

- Hawking and Penrose, *The Nature of Space and Time (1996)*

The no-boundary proposal implies that the universe is spatially closed. A closed universe will collapse again before an observer has time to see all the universe.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

I have emphasized what I consider the two most remarkable features that I have learned in my research on space and time: (1) that gravity curls up spacetime so that it has a beginning and an end; (2) that there is a deep connection between gravity and thermodynamics that arises because gravity itself determines the topology of the manifold on which it acts.

- Hawking and Penrose, *The Nature of Space and Time (1996)*

[Most physicists] distrusted general relativity because of its classical and geometrical nature. This dislike of geometry is shown, for example, by the particle physicist Steven Weinberg in the introduction to his otherwise excellent book *Gravitation and Cosmology*... "too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics... I believe that the geometrical approach has driven a wedge between general relativity and the theory of elementary particles."

- Stephen Hawking, *Nature 249, review of MTW*

With prophetic vision Riemann wrote: "The basis of metrical determination must be sought outside the manifold in the binding forces which act on it."

- Max Jammer, *Concepts of Space (1969)*

Moreover, already in 1870 Clifford saw in Riemann's conception of space the possibility for a fusion of geometry with physics...Clifford conceived matter and its motion as a manifestation of the varying curvature.

- Max Jammer, *Concepts of Space (1969)*

But marching along with truth is error.

- Max Jammer, *Concepts of Space (1969)*

Gravitational waves are simply the vibration of space-time itself.

- I. R. Kenyon, *General Relativity (1990)*

A number of experiences familiar to modern man indicate that there is a close resemblance between the gravitational force and the effects of acceleration. High-speed centrifuges generate large inertial forces which are used to separate materials from liquid suspensions that would sediment only slowly, if at all, under gravity. Pilots of jet aircraft making tight turns feel forces that are labelled 'g forces', and in the realm of space exploration proposals exist to build giant wheels that would rotate to provide an artificial gravity. These are all examples of centrifugal acceleration. Linear acceleration is less readily sustained and has fewer familiar applications. However, the parallel between a linear acceleration and gravitation is conceptually simpler. Einstein realized that this parallel is a principle of nature, the equivalence principle, which states that a region of uniform gravitational field and a uniformly accelerating frame are equivalent. That is to say, there is no way to distinguish between them provided that measurements do not extend beyond the region of uniformity.

- I. R. Kenyon, *General Relativity (1990)*

...the motion of a neutral test body released at a given point in space-time is independent of its composition. This statement is known as the *weak equivalence principle.*

- I. R. Kenyon, *General Relativity (1990)*

Einstein generalized the [weak equivalence principle] to cover both electromagnetic and mechanical experiments.... In a nutshell, physics is the same in all freely falling frames.

- I. R. Kenyon, *General Relativity (1990)*

For most purposes we can ignore the distinction and include the pseudo-Riemannian spaces with the Riemannian spaces.

- I. R. Kenyon, *General Relativity (1990)*

With rectangular Cartesian coordinates in Euclidean space the distinction between vector and covector components disappears, which explains why covector components are not usually used in Newtonian mechanics.

- I. R. Kenyon, *General Relativity (1990)*

T-mu-nu [the energy-momentum tensor] is the flow of the mu-th component of the four-momentum along the nu-direction.

- I. R. Kenyon, *General Relativity (1990)*

It is well known that the pressure inside a star resists the gravitational collapse, and yet it now looks as if pressure can, by virtue of contributing to a component of the stress-energy tensor, hasten gravitational collapse. This is indeed the case, and pressure contributes to the contraction of sufficiently massive stars to black holes.

- I. R. Kenyon, *General Relativity (1990)*

Curiously, the cosmological constant is once again in vogue. Quantum theory has taught us that the vacuum is not a featureless void but is continuously disturbed by the particle-anti-particle pair creation and annihilation and that this endows the vacuum with energy. A non-zero cosmological constant no longer seems at all unreasonable.

- I. R. Kenyon, *General Relativity (1990)*

For the description of processes taking place in nature, one must have a *system of reference*. By a system of reference we understand a system of coordinates serving to indicate the position of a particle in space, as well as clocks fixed in this system serving to indicate the time. [First sentence.]

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Experiment shows that the so-called *principle of relativity* is valid. According to this principle all the laws of nature are identical in all inertial systems of reference. In other words, the equations expressing the laws of nature are invariant with respect to transformations of coordinates and time from one inertial system to another. This means that the equation describing any law of nature, when written in terms of coordinates and time in different inertial reference systems, has one and the same form.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

The combination of the principle of relativity with the finiteness of the velocity of propagation of interactions is called the *principle of relativity of Einstein* in contrast to the principle of relativity of Galileo, which was based on an infinite velocity of propagation of interactions.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Thus the principle of relativity of Einstein introduces very drastic and fundamental changes in basic physical concepts. The notions of space and time derived by us from our daily experiences are only approximations linked to the fact that in daily life we happen to deal only with velocities which are very small compared to the velocity of light.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

It is frequently useful for reasons of presentation to use a fictitious four-dimensional space, on the axes of which are marked three space coordinates and the time.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

The form of expressions (2.3) and (2.4) permits us to regard the interval, from the formal point of view, as the distance between two points in a fictitious four-dimensional space....

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Thus we arrive at a very important result: the interval between two events is the same in all inertial frames of reference, i.e. it is invariant under transformation from one inertial system to another. This invariance is the mathematical expression of the constancy of the velocity of light.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

We see that to compare the rates of clocks in two reference frames we require several clocks in one frame and one in the other, and that therefore this process is not symmetric with respect to the two systems. The clock that appears to lag is always the one which is being compared with different clocks in the other system.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Suppose that in a certain inertial reference system we observe clocks which are moving relative to us in an arbitrary manner. At each different moment of time this motion can be considered as uniform. Thus at each moment of time we can introduce a coordinate system rigidly linked to the moving clocks, which with the clocks constitutes an inertial frame of reference.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

On the other hand, the result of two Lorentz transformations does depend, in general, on their order....The sole exception is the case of transformations with parallel vectors V1 and V2...

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

We call attention to the fact that in relativistic mechanics the energy of a free body (i.e. the energy of any closed system) is a completely definite quantity which is always positive and is directly related to the mass of the body. In this connection we recall that in classical mechanics the energy of a body is defined only to within an arbitrary constant, and can be either positive or negative.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Thus in relativistic mechanics the law of conservation of mass does not hold: the mass of a composite body is not equal to the sum of the masses of its parts. Instead only the law of conservation of energy, in which the rest energies of the particles are included, is valid.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

We cannot speak of a direct interaction of particles located at a distance from one another. Interactions can occur at any one moment only between neighboring points in space (contact interaction). Therefore we must speak of the interaction of one particle with the field, and of the subsequent interaction of the field with the second particle.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

In the previous section it was shown that a noninertial system of reference is equivalent to a certain field of force. We now see that in relativistic mechanics, these fields are determined by the quantities g-ik. The same also applies to "actual" gravitational fields. Any gravitational field is just a change in the metric of space-time, as determined by the quantities g-ik. This important fact means that the geometrical properties of space-time (its metric) are determined by physical phenomena, and are not fixed properties of space and time. The theory of gravitational fields, constructed on the basis of the theory of relativity, is called the general theory of relativity. It was established by Einstein (and finally formulated by him in 1916), and represents probably the most beautiful of all existing physical theories. It was remarkable that it was developed by Einstein in a purely deductive manner and only later was substantiated by astronomical observations.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

An "actual" gravitational field cannot be eliminated by any transformation of coordinates. In other words, in the presence of a gravitational field space-time is such that the quantities g-ik determining the metric cannot, by any coordinate transformation, be brought to their galilean values over all space. Such a space-time is said to be *curved*, in contrast to *flat* space-time, where such a reduction is possible.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

A change in the metric of space-time also means a change in the purely spatial metric. To a galilean g-ik in flat space-time, there corresponds a euclidean geometry of space. In a gravitational field, the geometry of space becomes non-euclidean. This applies both to "true" gravitational fields, in which space-time is "curved", as well as to fields resulting from the fact that the reference system is non-inertial, which leave the space-time flat.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

This result essentially changes the very concept of a system of reference in the general theory of relativity, as compared to its meaning in the special theory. In the latter we meant by a reference system a set of bodies at rest relative to one another in unchanging relative positions. Such systems of bodies do not exist in the presence of a variable gravitational field, and for the exact determination of the position of a particle in space we must, strictly speaking, have an infinite number of bodies which fill all the space like some sort of "medium". Such a system of bodies with arbitrarily running clocks fixed on them constitutes a reference system in the general theory of relativity.

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

In connection with the arbitrariness of the choice of reference system, the laws of nature must be written in the general theory of relativity in a form which is appropriate to any four-dimensional system of coordinates (or, as one says, in covariant form).

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Thus, generally speaking, in the general theory of relativity the concept of a definite distance between bodies loses its meaning, remaining valid only for infinitesimal distances. The only case where the distance can be defined also over a finite domain is that in which the g-ik do not depend on the time....

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

Thus we arrive at the important conclusion that the requirements of relativistic invariance, as expressed by the tensor character of the quantities Tik, automatically lead to a definite connection between the energy flux and the [momentum density]: the energy flux density is equal to the [momentum density] multiplied by c-squared [typos in the text are corrected here].

- Landau and Lifshitz, *The Classical Theory of Fields (1979)*

...the [triangle] inequality d(x,z)d(x,y)+d(y,z) may be thought of as asserting the transitivity of [the] closeness [relationship]; that is, if x is close to y and y is close to z, then z is close to x.

- Bert Mendelson, *Topology*

Relativity is perfect for the high school curriculum not only because it offers an astonishing application of elementary high school mathematics, but also because everybody is intimately acquainted with the subject of relativity. Relativity is about time.

- David Mermin, *It's About Time (2005)*

That no inherent meaning can be assigned to the simultaneity of distant events is the single most important lesson to be learned from relativity....

- David Mermin, *It's About Time (2005)*

....I realized that the axes [in Minkowski space-time] are an unnecessary and potentially confusing distraction and that all of the sometimes cumbersome trigonometry of my earlier exposition can be replaced by some very simple plane geometry.... Space-time diagrams, as I present them here, are to conventionally presented space-time diagrams as the plane geometry of Euclid is to the analytic geometry of Descartes. Analytic geometry is the more powerful tool for professional calculations; Euclid's approach is essential for a deeper understanding.

- David Mermin, *It's About Time (2005)*

An important lesson of relativity is that there is less that is intrinsic in things than we once believed.... But that which we have learned is inherent - the interval between two events, for example - turns out to be strange and unfamiliar, while what we once thought was inherent - the time between two events, for example - has turned out to be merely conventional.

- David Mermin, *It's About Time (2005)*

Invariants - quantities that everybody agrees on regardless of their frame of reference - play a more important role in our understanding of the world than quantities that vary from one frame of reference to another. The theory of relativity identifies such invariant quantities, and in this sense "theory of relativity" is a terrible name. "Theory of invariance" would have been better, since the most important content of the theory is its identification of quantities that do not change from frame to frame.

- David Mermin, *It's About Time (2005)*

The important lesson of the pole-in-barn paradox is that even so innocent sounding a sentence as "The pole is shut up in the barn" can involve an implicit judgment about the simultaneity of events in different places...

- David Mermin, *It's About Time (2005)*

...teaching relativity is rather like conducting psychotherapy. It is not enough simply to state what is going on, for there is an enormous amount of resistance to be broken down.

- N. David Mermin, *Space and Time in Special Relativity (1968)*

...misunderstanding is continually generated by commonplace, incorrect notions that are often implicit in the very language we use (a pre-relativistic structure) and are therefore particularly difficult to recognize.

- N. David Mermin, *Space and Time in Special Relativity (1968)*

The historical approach, though fascinating, provides an indirect and frequently confusing route to a clear statement of what the modern theory says.

- N. David Mermin, *Space and Time in Special Relativity (1968)*

Thus although light appears in a discussion of special relativity as the best-known example of something that moves with the speed *c*, the interesting thing about the speed *c* is *not* that it is the speed of light.

- N. David Mermin, *Space and Time in Special Relativity (1968)*

Language can be enormously confusing when it is used to discuss relativity, because words and even grammar often introduce physical assumptions into what we say with such subtlety that we fail to realize that the assumptions are present.

- N. David Mermin, *Space and Time in Special Relativity (1968)*

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

- Hermann Minkowski, *Address in Cologne (1908)*

Physics is simple only when viewed locally: that is Einstein's great lesson.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

In christening events with coordinates, one demands smoothness but forgoes every thought of mensuration.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Free fall is synonymous with weightlessness: absence of any force to drive the object away from its normal track through spacetime.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Time is defined so that motion looks simple.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

To avoid distraction by the nonlocal element (the Earth) in the situation, conduct the study in the interior of a spaceship, also in orbit about the Earth. But this region has already been counted as a local inertial frame! What gravitational field is to be seen there? None. Relative to the spaceship and therefore relative to each other, the two test particales move in a straight line with uniform velocity, to the precision of measurement that is contemplated. [Expressed in a misleading way.]

- Misner/Thorne/Wheeler, *Gravitation (1972)*

To look at local physics, however, means to compare one geodesic of one test particle with geodesics of other test particles traveling (1) nearby with (2) nearly the same directions and (3) nearly the same speeds. Then one can "look at the separation between the nearby test particles and from the second time-rate of change of these separations and the 'equations of geodesic deviation' read out the curvature of spacetime."

- Misner/Thorne/Wheeler, *Gravitation (1972)*

More generally, a certain piece of the Riemann tensor, called the *Einstein tensor* and denoted ... G, is always generated directly by the local distribution of matter.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

The field equation even contains within itself the equations of motion for the matter whose stress-energy generates the curvature.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Everything that goes on in spacetime has its geometric description, and almost every one of these descriptions lends itself to ready generalization from flat spacetime to curved spacetime.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

In Einstein's geometric theory of gravity, this equation of geodesic deviation summarizes the entire effect of geometry on matter. It does for gravitation physics what the Lorentz force equation does for electromagnetism.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

One sometime participant in special relativity will have to be put to the sword: "*x4=ict*".

- Misner/Thorne/Wheeler, *Gravitation (1972)*

This, in fact, is the mathematical definition of a 1-form: *a 1-form is a linear, real valued function of vectors...*

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Here and elsewhere in science, as stressed not least by Henri Poincaré, that view is out af date which used to say, "Define your terms before you proceed." All the laws and theories of physics, including the Lorentz force law, have this deep and subtle character, that they both define the concepts they use .... and make statements about these concepts.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

It helps in analyzing gravitation to consider a situation where gravity is mocked up by acceleration. [gravitational redshift, deflection of light]

- Misner/Thorne/Wheeler, *Gravitation (1972)*

When spacetime is flat, move however one will, special relativity can handle the job.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

A tourist in a powered interplanetary rocket feels "gravity". Can a physicist by local effects convince him that his "gravity" is bogus? Never, says Einstein's principle of the local equivalence of gravity and accelerations. [Wrong: see Ohanian-Ruffini.]

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Thus the [uniformly accelerated] observer will attain relativistic velocities after maintaining this acceleration for something like one year of his own proper time. He can outrun a photon if he has a head start on it of one light-year or more.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Einstein elevated the idea of the universality of gravitational interactions to the status of a fundamental principle of equivalence, that all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

This nonmeshing of local Lorentz frames, like the nonmeshing of local Cartesian coordinates on a curved 2-surface, is a clear manifestation of spacetime curvature.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Gravitation is a manifestation of spacetime curvature, and that curvature shows up in the deviation of one geodesic from a nearby geodesic.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Nothing is more wonderful about the relation between Einstein's theory of gravity and Newton's theory of gravity than this, as discovered by Elie Cartan (1923, 1924): that both theories lend themselves to description in terms of curvature; that in both this curvature is governed by the density of mass-energy; and that this curvature allows itself to be defined and measured without any use of or reference to any concept of metric.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

But full power this will be only if it can be exercised in three ways: in pictures, in abstract notation, and in component notation.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Each local geometric object has its own official place of residence (event P); it can interact with other objects residing there (tensor algebra); but it cannot interact with any object at another event Q, until it has been carefully transported from P to Q.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

The differentiability of a manifold (i.e., the possibility of defining differentiable functions on it) permits one to introduce coordinate systems locally, if not globally, and also curves, tangent spaces, tangent vectors, 1-forms and tensors, just as is done for spacetime. But the mere fact that a manifold is differentiable does not that such concepts such as geodesics, parallel transport, curvature, metric, or length exist in it. These are additional layers of structure possessed by some manifolds, but not by all. Roughly speaking, every manifold has smoothness properties and topology, but without additional structure it is shapeless and sizeless. That branch of geometry which adds geodesics, parallel transport, and curvature (shape) to a manifold is called affine geometry; that branch which adds a metric is called Riemannian geometry.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Notice that the word "maximum" in [the integral for proper time along a history] has been replaced by "extremum"... When [events] A and B are widely separated, they may be connected by several geodesics with differing lapses of proper time. Each timelike geodesic extremizes [proper time] with respect to nearby deformations of itself, but the extremum need not be a maximum. When several distinct geodesics connect two events, the typical one is not a local maximum ("mountain peak") but a saddle point ("mountain pass")...

- Misner/Thorne/Wheeler, *Gravitation (1972)*

But on occasion the reader will encounter a pedantic-sounding paragraph written in mathematics-sect jargon. Such paragraphs deal with concepts and relationships so complex that standard physics usage would lead to extreme confusion. They also should prevent the reader from becoming so conditioned to physics usage that he is allergic to the mathematical literature, where great advantages of clarity and economy of thought are achieved by consistent reliance on wholly unambiguous notation.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Included [in the energy-momentum tensor] are energy, momentum, and stress associated with all forms of matter and all nongravitational fields.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

Any forward step in human knowledge is truly creative in this sense: that theory, concept, law, and method of measurement - forever inseparable - are born into the world in union.

- Misner/Thorne/Wheeler, *Gravitation (1972)*

However, in a textbook of today I think it is useful to stress again the fundamental physical difference between space and time, which was somewhat concealed by the purely formal four-dimensional representation.

- C. Moller, *The Theory of Relativity (1952)*

Therefore, according to relativistic conceptions, the notion of the length of a stick has an unambiguous meaning only in relation to a given inertial frame, this length being different for the different systems of inertia. This means, however, that the concept of length has lost its absolute meaning. We can only speak of an absolute length in the approximation where the velocity of light can be regarded as infinitely large.

- C. Moller, *The Theory of Relativity (1952)*

It expresses, however, not so much a quality of the moving stick itself as rather a reciprocal relation between measuring-sticks in motion relative to each other. In this case it is natural to ask for the cause of the contraction. According to the principle of relativity, the answer must be that such a question is just as delusive as if, after the discovery of the law of inertia, the question were put why a body left to itself will continue to move straight forward with uniform velocity. While such a question was well justified in Aristotelian physics it must be rejected as meaningless after Galileo's discovery. According to Galilean and Newtonian mechanics only the deviations from uniform translatory motions require a cause.

- C. Moller, *The Theory of Relativity (1952)*

Instead of considering the contraction to be a phenomenon which has to be *explained* on the basis of an atomistic theory of material bodies, it should rather be regarded as something *elementary* which cannot be traced back to simpler phenomena.

- C. Moller, *The Theory of Relativity (1952)*

In principle it is permissible, however, to use coordinate clocks of an arbitrary rate, provided that the time variable t defined by these coordinate clocks gives a reasonable chronological ordering of the physical events. In accelerated systems of reference the spatial and temporal coordinates thus lose every physical significance; they simply represent a certain arbitrary, but unambiguous, numbering of the physical events.

- C. Moller, *The Theory of Relativity (1952)*

...I frame no hypothesis; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy.

- I. Newton, *Mathematical Principles of Natural Philosophy (1729)*

And to us it is enough that gravity does really exist, and acts according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

- I. Newton, *Mathematical Principles of Natural Philosophy (1729)*

Einstein discovered his theory of gravitation in 1916. By rights, this theory should not have been discovered until twenty years later, when physicists acquired a clear understanding of relativistic field theory and of gauge invariance. Einstein's profound and premature insights into the nature of gravitation had more to do with intuition than with logic. In contrast to the admirably precise and clear operational foundations on which he based his theory of special relativity, the foundations on which he based general relativity were vague and obscure. As has been emphasized by Synge and Fock, even the very name of the theory indicates a misconception: there is no such thing as a relativity more general than special relativity. But, whatever murky roads he may have taken, in the end Einstein's intuition led him to create a theory of dazzling beauty.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

It is the objective of this book to develop gravitational theory in the most logical and straightforward way - in the way it probably would have developed without Einstein's intervention. This means that we will begin with the linear approximation and regard gravitation as a field theory, entirely analogous to electrodynamics.... One advantage of this approach is that it gives a clearer insight as to why gravitation is geometry.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

This shows that the shape of the tidal ellipsoid is independent of its size.... We therefore regard the prolateness of the tidal ellipsoid as a *local* measure of the gravitational force.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Although some tidal effects do vanish near the origin of a freely falling reference frame, some other effects remain finite, and it is these that ultimately count.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

A more familiar graphical representation of gravitational fields uses the lines of force of the ordinary gravitational force, rather than the tidal force. This is, of course, very useful to an outside observer who wishes to study the motion of a spacecraft, or whatever, through the field. But this representation fails to describe what happens locally. If we want a description of what the astronauts experience inside their spacecraft, then the picture of tidal forces is much more relevant than that of ordinary force.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

We can therefore use the angular acceleration to detect the tidal force. If the [non-spherical] body is originally spinning, then the tidal force will cause a precession of the spin. A well-know example is the equinoctial precession of the spin of the Earth, which is caused by the lunar and solar tidal forces acting on the equatorial bulge of the Earth.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

From this survey of actual experiments and *Gedanken*-experiments, we see that there exist several methods for measuring the tidal field *locally*, in an arbitrarily small neighborhood of a given point.... The tidal field is no less a local quantity than, say, the electric field.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Local experiments can distinguish between a reference frame in free fall in a gravitational field and a truly inertial reference frame placed far away from all gravitational fields. *Local* experiments can distinguish between a reference frame at rest in a gravitational field and an accelerated reference frame far away from all gravitational fields. Gravitational effects are *not equivalent* to the effects arising from an observer's acceleration.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

In order to avoid confusion, we will base our further development of gravitional theory on the very precise and unambiguous equality mi=mg. This equality is necessary and, to a large extent, sufficient for the construction of the relativistic theory.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Unfortunately, because the velocity of light is so large, everyday experience leads us to acquire a certain number of misconceptions about the structure of spacetime. This set of misconceptions serves as the foundation of Newtonian, or Galilean, spacetime. The true structure of spacetime was discovered by Einstein in a study of electrodynamics (1905).

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

In Newtonian physics, as well as in special-relativistic physics, this parameterization of all points of spacetime by four coordinates can be carried out globally; if we use rectangular coordinates x, y, z, t, each of which ranges from minus infinity to plus infinity, then these coordinates span the entire spacetime. In general-relativistic physics, where spacetime is curved, such a global parameterization is not possible; attempts at extending the coordinates in all directions usually result in singularities in the coordinates. Such coordinate singularities are similar to what a mapmaker finds when he attempts to use the longitude and latitude angles as coordinates for the curved two-dimensional surface of the Earth; these coordinates develop singularities at the poles of the Earth, where the longitude fails to be unique. Nevertheless, even for the curved spacetime of general relativity, it is still possbile to construct well-behaved four-dimensional coordinate patches locally, for a finite neigborhood of a given point.... Mathematically, a space that can be covered by coordinate patches with real numbers as coordinates is called a *manifold*. Thus, spacetime is a four-dimensional manifold.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Unfortunately, some books follow the abominable custom of introducing a factor of *i* into the time component of four-vectors. This simplifies the notation somewhat, because it eliminates the the metric tensor from all equations; but since the metric is precisely the most important feature of special relativity it makes little sense to pretend that it does not exist.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Why do we prefer fields to action-at-a-distance? The answer is simple: we need fields in order to uphold the laws of conservation of energy and momentum.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Newtonian spacetime has *two distances* [distances and durations], relativistic spacetime *only one* [interval].[How is distance an invariant, even in Newton's world?]

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Although it is true that the most spectacular results of gravitational theory depend in a crucial way on the nonlinearity of the field equations, almost all of the results that have been the subject of experimental investigation can be described by the linear approximation.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The reason why we believe in curved space can best be understood by beginning with flat spacetime and seeking the field equations that describe the gravitational field (in linear approximation) in this flat spacetime. As we will see, the behavior of "clocks" and "meter sticks" are then such that measurements of spacetime indicate that the real geometry is curved. Thus, the flat spacetime with which we begin turns out to be an unobservable, fictitious geometry.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Since the behavior of freely falling clocks is completely predictable from the principle of equivalence, we will use freely falling clocks for all our measurements in the gravitational field, even measurements at a fixed position, for instance, a measurement at a fixed position on the surface of the Earth. For this purpose, we use a freely falling clock, instantaneously at rest at the fixed position. As soon as this clock has fallen too far from our fixed position and acquired too much speed, we must replace it by a new clock, instantaneously at rest. Whenever we speak of the time as measured by "a clock located at a fixed position" in a gravitational field, this phrase must be understood as shorthand for a complicated measurement procedure, involving many freely falling, disposable clocks, used in succession.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Atomic clocks are not usually found in free fall. They are usually at rest in the gravitational field (for instance, at rest on the surface of the Earth or a star), and this means that they are accelerated relative to freely falling clocks. We might worry about the disturbances caused by this acceleration. These disturbances can be estimated, and they are negligible.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

However, there are three interesting observable effects that we can calculate from the linear approximation: the gravitational time dilation, the deflection of light by the gravitational field of the Sun, and the retardation of light signals.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The experiments on gravitational time dilation may be regarded as direct measurements of the g00 component of the metric tensor.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Evidently, the observations of the light deflection are afflicted with large experimental errors, and in spite of fifty years of efforts, the errors in the most recent observations are about as large as those in the first observations.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

More precise results [on the deflection of light] have been obtained by the use of radio waves rather than optical light. In this case, it is not necessary to wait for an eclipse; rather one must wait for the Sun's limb to approach some radio source in the sky.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Another observable effect that we can obtain from the linear approximation is the time delay suffered by a radar signal sent from the Earth to a planet and reflected back to Earth. As first recognized by Shapiro, the gravitational field of the Sun contributes a measurable increment to this time delay, because it reduces the speed of propagation of light signals.... A comment on just what is meant by the "slowing down" of the speed of propagation of light may be helpful. Although a light signal that passes close to the Sun will be delayed, this does not mean that an observer who measures the speed of the signal at a point near the Sun will obtain a different result from the usual speed of light. The speed given by Eq. [47] is not a locally measured speed but an effective speed measured with standards of time and length that are far away, outside of the gravitational field.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

There are three different aspects to the structure of spacetime: the differential structure, the topological structure, and the geometric structure. Crudely speaking, the differential structure tells us how smooth spacetime is, and how many dimensions it has. The topological structure tells us how the different parts of spacetime are connected, that is, which points are in the neighborhood of which....Finally, the geometric structure tells us how to construct parallel lines, and it tells us the distances between points in spacetime.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

However, such "lenses" do not have the optical properties of genuine lenses. The magnitude of the deflection angle of a ray passing through a genuine lens increases in direct proportion to the impact paramater... But the magnitude of the deflection angle of a ray passing through the gravitational field of a star *decreases* with impact parameter.... Hence such a gravitational lens has no well-defined focal length, and it cannot produce genuine images, real or virtual.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

From the analysis of the amount of distortion [in a gravitational lens] as a function of radial distance from the center of the cluster, it is possible to calculate the cluster mass. The mass deduced in this way is larger than the sum of the masses of the luminous galaxies in the cluster; thus, there must be dark matter in the cluster.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

A gravitational lens can be simulated optically by a disk of glass with its surface shaped in such a way that light rays are deflected by the same angle as in a gravitational field.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The energy, pulse shape, and polarization of bursts of gravitational radiation could reveal to us the astrophysical processes in which these bursts are generated.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The deformations shown in all these figures [of distortions made by gravity waves] are essentially tidal effects...but time dependent rather than static.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Mathematically, a Riemannian space is a differentiable manifold endowed with a topological structure and a geometric structure. In the discussion of the geometric structure of a curved space we can make a distinction between the *affine* geometry and the *metric* geometry. These two kinds of geometry correspond to two different ways in which we can detect the curvature of a space. One way is by examination of the behavior of parallel line segments, or parallel vectors....Another way in which we can detect curvature is by measurement of lengths and areas.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Thus, to carry out the comparison of theory and experiment, the physicist cannot ultimately avoid the language of components; only a pure mathematician can adhere exclusively to the abstract, coordinate-free language of differential forms.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

In flat spacetime, rectangular coordinates are preferred because both the metric and the Lorentz transformations take their simplest form when expressed in these coordinates.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Unfortunately, in a curved spacetime the coordinates that lead to the simplest mathematics often lack any obvious physical interpretation...

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

One way to label spacetime points is to imagine all of space to be filled with small clocks, each of which carries a dog tag with the values of the spatial coordinates printed on it. There is no need that these clocks be "at rest"; a continuous streaming will keep the coordinate values smooth and acceptable. The coordinates of spacetime points in the empty space between adjacent clocks can be obtained by interpolation, using meter-sticks or a radar-ranging procedure (the interpolation can be performed as in a flat spacetime, since locally any curved spacetime is approximately flat).

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

A somewhat more elegant proposal for labeling spacetime points is the following: Suppose that at some distance from the region where we want to introduce coordinates, there are three "fixed" stars (red, white, and blue), which are not collinear. For the spatial coordinates of a given point in space we can then adopt the three angles between the stars as seen from the given point. For the time coordinate we can adopt the angle between, say, the red star and a fourth star (yellow) which is in motion with respect to red, white, and blue. Note that it is not really necessary that the red, white, and blue stars be fixed; even if they are in motion, the coordinates will be well defined.... We might call the numbers obtained in this construction astrogator coordinates....

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Note that *x is not a vector* with respect to general coordinate transformations...

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

[Tangent vectors and differential forms] give us more concise equations, and also permit us to derive some identities in a more elegant manner.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The solutions of this equation, called *Killing vectors*, give us the symmetry transformations of the metric, both the infinitesimal transformations and the finite transformations.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

...with a suitable choice of coordinates, any symmetry transformation of the metric can be regarded as a translation of one of the coordinates, and the metric is then independent of this one coordinate. Checking for the independence of the metric from some coordinate is, of course, a familiar method for discovering symmetries "by inspection".

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The Killing equation gives us a straightforward, systematic method for finding the symmetries.... Each linearly independent Killing vector corresponds to a separate symmetry of the spacetime.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Apart from flat spacetime, the only maximally symmetric spacetimes are those of constant curvature, such as the isotropic and homogeneous models of the universe....

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Since Einstein's theory of "general relativity" is no more relativistic than special relativity, it would be preferable to adopt the name *geometrodynamics* for this theory. This name, coined by Wheeler, puts the emphasis where it belongs - on the dynamical geometry that acts on and reacts to matter.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Principle of general covariance: All laws of physics shall be stated as equations covariant with respect to general coordinate transformations. We will obey this commandment for the best of all reasons - it costs us nothing to do so. As we have seen, an equation that is not covariant can be easily transformed into an equivalent equation that is covariant. From a mathematical point of view, the covariance principle is therefore seen to be a triviality. That the principle of covariance imposes no restrictions on the content of the physical laws, but only on the form in which they are written, was recognized by Kretschmann (1917) in a critical examination of Einstein's theory.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Unfortunately, the [field equations are] very complicated... No general procedure exists for solving the equations analytically; one guesses solutions as best one can.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The tensor T appearing on the right side [of the field equations] is the energy-momentum tensor of matter. The energy-momentum of the gravitational field is already included (implicitly) on the left side of the equation - all the nonlinear terms on the left side represent the energy-momentum tensor of the gravitational field.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The Einstein equations do not determine the unknown functions [the g's] entirely.... the ambiguity in the solution for g arises from the ambiguity in the choice of coordinates. That the Einstein field equations determine the g's only up to a general coordinate transformation is actually highly desirable: it would be absurd that the field equations should not only determine the geometry, but also prescribe what coordinates we must use to describe the geometry.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Hilbert adopted the physical basis that Einstein had gradually laid for the theory of gravitation in work extended over several years, and he actually discovered and published the field equations a few days before Einstein.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The cosmological term can be interpreted as an energy-momentum of the vacuum.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

It must be kept in mind that the mass M [in the Schwarzschild metric] is the total mass of the system; the mass-energy contributed by the gravitational fields is included in M.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

...it can be shown that any spherically symmetric vacuum solution of Einstein's equations must be static and must agree with the Schwarzschild solution. This is *Birkhoff's theorem*.... The field that a spherically symmetric mass distribution produces in the surrounding region is always the static Schwarzschild field, regardless of whether the mass is static, collapsing, expanding, or pulsating.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Fock...strongly condemns Einstein for describing the theory of gravitation as "general relativity". Although Fock's criticisms are excessive, they do help to clear the air.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The dimensionless quantity GM/rc^2 may be regarded as a measure of the strength of the gravitational field. This quantity enters into the formulas for light deflection, light retardation, gravitational redshift, perihelion precession, etc. The small magnitude of the relativistic gravitational effects in the Solar System is related to the small magnitude of this quantity; even at the solar surface, GM/rc^2 is only 2x10^-6. Large relativistic effects are found in the gravitational field of an extremely compact mass, where GM/rc^2 can attain values of the order of magnitude of 1.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

In the region r less than the Schwarzschild radius, t is a *spacelike* coordinate, and r is a *timelike* coordinate.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The surface r = [the Schwarzschild radius] acts as a "one-way membrane", through which signals can be sent in, but not out. This is a global (or nonlocal) property because in order to test it, we must examine the propagation of light signals and other signals and check what happens to them in the long run.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Such a hovering light signal on the horizon of a black hole should not be confused with a signal in a circular orbit around the black hole; the hovering signal has no circular motion, and it is completely stationary.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

For the case of the Schwarzschild geometry, the coordinates that make things easy are the Kruskal coordinates....

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The white-hole portion of the complete Kruskal solution is probably as irrelevant to our universe as are the advanced solutions of Maxwell's equations.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

From a practical point of view, the complete Kerr geometry is as irrelevant to physics as the complete Schwarzschild geometry.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Finally, note that in the case of collapse of a spherical, nonrotating star, the Schwarzschild solution fails to hold in the interior, but the Birkhoff theorem guarantees that it will at least hold in the exterior. In the case of collapse of a rotating star, there is no such theorem, and the exterior solution need not be that of Kerr. Only a long time after the collapse, when everything becomes stationary, will the exterior solution tend toward those of the Kerr solution.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Essentially, a degenerate Fermi gas of electrons supplies the equilibrium pressure in a white dwarf and a Fermi gas of neutrons that in a neutron star.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

At the white-dwarf densities...the electrons are detached from their nuclei and move quite freely throughout the volume of the star. The star consists of interpenetrating gases of electrons and nuclei.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The density of a neutron star is comparable with nuclear densities...and hence the star may be described as a single giant nucleus.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

The reason for this is that stiffness of a material is directly related to the speed of sound waves in the material - the stiffer the material, the higher the speed of sound. The requirement that the speed of sound be no more than the speed of light then sets a limit on the stiffness of the material [in the core of a collapsing neutron star]…. This leads to the conclusion that the mass of a neutron star can never exceed 3.2 solar masses, independent of any assumptions about the details of the equation of state at high densities.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

However, the "frozen star" very soon becomes practically indistinguishable from a black hole. Calculations of the intensity of the light emitted by the surface of a collapsing star show that the brightness seen by an outside observer decreases sharply while the redshift increases (the brightness decreases exponentially with time, with a characteristic "half-life" of the order GM…)

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

It is known that trapped surfaces will form in all cases of gravitational collapse that are reasonably close to spherical collapse, and it is believed that trapped surfaces will form even if collapse is very different from spherical collapse. The Hawking-Penrose theorem then tells us that singularities will form.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

It is tempting to conjecture that the Hawking-Penrose theorem of classical geometrodynamics is as irrelevant to the real world as the Earnshaw theorem of classical electrodynamics...according to which atoms are unstable and must collapse to a singularity. It may well be that when quantum effects are taken into account, the collapsed matter attains some nonsingular final state.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Accretion by a Schwarzschild black hole can convert up to 5.7% of the rest-mass energy of the infalling material into radiation; accretion by a Kerr black hole can convert up to 42%. This is to be compared with the maximum energy released in nuclear reactions in stars, which is only about 0.7% of the rest-mass energy.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Thus, on a large scale, the most remarkable feature of the universe is its lack of distinguishing features. The assertion that all positions in the universe are essentially equivalent, except for local irregularities, is know as the *Cosmological Principle.*

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

Thus, a count of numbers of galaxies as a function of brightness can be used as a direct observational test of homogeneity.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

...the counts indicate that the density of quasars must have been much higher in the past than it is now.

- Ohanian and Ruffini, *Gravitation and Spacetime (1994)*

...the argument has been put forward that it is only an "apparent" contraction, in other words, that it is only when it can be simulated by our space-time measurements....But we do not consider such a point of view as appropriate, and in any case the Lorentz contraction is in principle observable....It therefore follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between two such rods moving relatively to each other, and this relation is in principle observable.

- W. Pauli, *Theory of Relativity (1921)*

While this consequence of the Lorentz transformation [time dilation] was already implicitly contained in Lorentz's and Poincaré's results, it received its first clear statement only by Einstein.

- W. Pauli, *Theory of Relativity (1921)*

Should one, then, in view of the above remarks, completely abandon any attempt to explain the Lorentz contraction atomistically? We think that the answer to this question should be No. The contraction of a measuring rod is not an elementary but a very complicated process.

- W. Pauli, *Theory of Relativity (1921)*

It is clear that this "transforming away" is only possible because the gravitational field has the fundamental property that it imparts the same acceleration to all bodies; or, stated differently, because the gravitational mass is always equal to the inertial mass.

- W. Pauli, *Theory of Relativity (1921)*

We thus see that the ten tensor components [the g's] in Einstein's theory take the place of the scalar Newtonian potential; the [Christoffel symbols], formed from their derivatives, determine the magnitude of the gravitational force.

- W. Pauli, *Theory of Relativity (1921)*

This fusion of two previously quite disconnected subjects - metric and gravitation - must be considered as the most beautiful achievement of the general theory of relativity....This fusion results conclusively from the principle of equivalence and from the validity of special relativity in the infinitely small.

- W. Pauli, *Theory of Relativity (1921)*

The motion of a particle is *force-free.*

- W. Pauli, *Theory of Relativity (1921)*

That the non-Euclidean character of the space-time world should show up so little in the behaviour of measuring rods and clocks, but very strongly in the deviation from rectilinear uniform motion of particles (i.e. for the case of gravity), is due to the magnitude of the velocity of light.

- W. Pauli, *Theory of Relativity (1921)*

For this reason Kretschmann took the view that the postulate of general covariance does not make any assertions about the physical *content* of the physical laws, but only about the mathematical *formulation*; and Einstein entirely concurred with this view.

- W. Pauli, *Theory of Relativity (1921)*

...observations now reach gravitationally lensed objects, and objects at redshifts so high that they are viewed through the gravitational lens of the universe itself in a way that is not trivially computed by symmetry arguments.

- P. J. E. Peebles, *Principles of Physical Cosmology (1993)*

Since the components of a vector thus depend on the coordinate labelling, they have no absolute significance; observables are scalars constructed out of vectors and tensors.

- P. J. E. Peebles, *Principles of Physical Cosmology (1993)*

Now we will define two classes of vectors, called covariant and contravariant (for reasons that are lost in the mists of time).

- P. J. E. Peebles, *Principles of Physical Cosmology (1993)*

...the definition of [ds-squared] is that if [it] is positive, ds is the time interval measured by an observer on a world line that passes between the events; and if [ds-squared] is negative, [the absolute value of ds] is the distance between the events measured by an observer who is moving such that they are seen to happen simultaneously.

- P. J. E. Peebles, *Principles of Physical Cosmology (1993)*

The conclusion is that we can choose the coordinate labels so the ordinary line element of special relativity applies in the neighborhood of a chosen point, with the metric tensor equal to the Minkowski form, and the first derivatives of the metric tensor equal to zero. We will see in the next section that this represents the locally orthogonal coordinate system that a freely falling observer would set up using proper physical rods and clocks. If all the second derivatives of the metric tensor cannot be eliminated, they describe the tidal field that causes the relative gravitational acceleration of neighboring freely moving test particles, and one says spacetime is curved.

- P. J. E. Peebles, *Principles of Physical Cosmology (1993)*

However, a vector space is itself a very special type of space, and something much more general is needed for the mathematics of much of modern physics. Even Euclid's ancient geometry is not a vector space, because a vector space has to have a particular distinguished point, namely the *origin*, whereas in Euclidean geometry every point is on an equal footing. In fact, Euclidean space is an example of what is called an *affine* space. An affine space is much like a vector space but we 'forget' the origin....

- Roger Penrose, *The Road to Reality (2004)*

In certain respects, the null cones are indeed more fundamental than the metric.

- Roger Penrose, *The Road to Reality (2004)*

Many workers in the field prefer to concentrate on the (+++-)-signature pseudometric ... since this is convenient when considering spatial geometry, the quantity represented above as 'dl^2' being positive for spacelike displacements.... But the quantity 'ds^2' defined by the (+---)-signature ... is more directly physical, because it is positive along the timelike curves that are the allowable worldlines of massive particles, the integral of ds being interpretable as the actual physical time measured by an ideal clock with this as its world line.

- Roger Penrose, *The Road to Reality (2004)*

Groups that possess no non-trivial normal subgroups at all are called simple groups. The group SO(3) is an example of a simple group. Simple groups are, in a clear sense, the basic building blocks of group theory.

- Roger Penrose, *The Road to Reality (2004)*

It is thus an important achievement of the 19th and 20th centuries in mathematics that all the finite groups and all the continuous groups are now known.

- Roger Penrose, *The Road to Reality (2004)*

Many of the groups that feature in particle physics are in fact product groups of simple groups (or elementary modifications of such).

- Roger Penrose, *The Road to Reality (2004)*

Those linear transformations which map the entire vector space down to a region (subspace) of smaller dimension within that space are called singular.

- Roger Penrose, *The Road to Reality (2004)*

Any (adequate) system of coordinates will do. They need not be 'Cartesian' or of any other standard kind. This is the beauty of the Lagrangian (and also Hamiltonian) approach. The choice of coordinates is governed merely by convenience.

- Roger Penrose, *The Road to Reality (2004)*

Moreover, the quantum versions of Hamiltonians provide essential ingredients for the standard [non-relativistic] quantum formalism. In the case of relativistic quantum theory, however, it is the Lagrangian framework that has generally been found to provide the more natural leaping-off point.

- Roger Penrose, *The Road to Reality (2004)*

To exemplify the limitations of Noether's theorem in the case of gravitational theory, it should be pointed out that a significant question mark still hangs over the issue of angular momentum in general relativity, even in the case of asymptotically flat spacetimes.

- Roger Penrose, *The Road to Reality (2004)*

Lagrangians for fields are undoubtedly extremely useful as mathematical devices, and they enable us to write down large numbers of suggestions for physical theories. But I remain uneasy about relying on them too strongly in our searches for improved fundamental physical theories.

- Roger Penrose, *The Road to Reality (2004)*

Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons (quoting G. H. Hardy).

- Roger Penrose, *The Road to Reality (2004)*

It would, no doubt, have come as a great surprise to all those who had voiced their suspicion of complex numbers to find that, according to the physics of the latter three-quarters of the 20th century, the laws governing the behaviour of the world, at the tiniest scales, is fundamentally governed by the complex-number system.

- Roger Penrose, *The Road to Reality (2004)*

But this is just the very beginning. We can ask about cube roots, fifth roots, 999th roots, pi-th roots - or event i-th roots. We find, miraculously, that whatever complex root we choose and whatever complex number we apply to it (excluding 0), there is always a complex-number solution to this problem.

- Roger Penrose, *The Road to Reality (2004)*

For comparison, we may recall that *i* was introduced, in effect, simply to provide a solution to the one particular equation z^2 = -1. We get all the rest [solutions to polynomial equations] for free!

- Roger Penrose, *The Road to Reality (2004)*

The remarkable general answer, for any power series whatsoever, is that there is some circle in the complex plane, centred at 0, called the *circle of convergence*, with the property that if the complex number z lies strictly inside the circle then the series converges for that value of z, whereas if z lies strictly outside the circle then the series diverges for that value of z... Thus, complex numbers supply us with deep insights into the behaviour of power series that are simply not available from the consideration of their real-variable structure.

- Roger Penrose, *The Road to Reality (2004)*

Einstein's general relativity stands out, in my opinion, as that century's greatest single achievement.

- Roger Penrose, *The Road to Reality (2004)*

In my view, general relativity is probably here to stay as a description of spacetime in the large-scale limit...

- Roger Penrose, *The Road to Reality (2004)*

Recall that Einstein's general relativity was crucially based on his insight (the principle of equivalence) which had been implicit in observational data that had been around since (and before) the time of Galileo, but not fully appreciated.

- Roger Penrose, *The Road to Reality (2004)*

...it is undoubtedly the case that the more deeply we probe Nature's secrets, the more profoundly we are driven into Plato's world of mathematical ideals as we seek our understanding. Why is this so? At present, we can only see that as a mystery.

- Roger Penrose, *The Road to Reality (2004)*

The most important single insight that has emerged from our journey, of more than two and one-half millenia, is that there is a deep unity between certain areas of mathematics and the workings of the physical world...

- Roger Penrose, *The Road to Reality (2004)*

It is perhaps even more striking that the most violent explosions seen in the universe are caused by the weakest force of all - if it is fair to call it a force - namely gravitation...where black holes fuel the unbelievably powerful energy sources of quasars.

- Roger Penrose, *The Road to Reality (2004)*

Let us return to the question of mass/energy in the gravitational field itself. Although there is no room for such a thing in the energy-momentum tensor T, it is clear that there are situations where a 'disembodied' gravitational energy is actually playing a physical role.

- Roger Penrose, *The Road to Reality (2004)*

To take the light speed as fundamental is, in spacetime terms, to take the light cones as fundamental. In fact, from the point of view that is appropriate for the geometry of manifolds, it is often better to think of the 'light cone' as a structure in the tangent-space...Frequently, the term *null cone* is used for this tangent-space structure...the term 'light cone' being reserved for the actual locus in spacetime that is swept out by the light rays passing through a point p.

- Roger Penrose, *The Road to Reality (2004)*

Galilean spacetime is not a product space E1 x E3, it is a *fibre bundle* with a base E1 and fibre E3!

- Roger Penrose, *The Road to Reality (2004)*

The agreed-upon procedure for synchronization then is that A will turn on his light source when his clock reads t=0 and observer B will set his clock to t=L/c the instant he receives the signal. This accounts for the transmission time and synchronizes the clocks in a consistent way.

- Robert Resnick, *Introduction to Special Relativity (1968)*

Although clocks in a moving frame all appear to go at the same slow rate when observed from a stationary frame with respect to which the clocks move, the *moving clocks appear to differ from one another in their readings by a phase constant which depends on their location*, that is, *they appear to be unsynchronized.*

- Robert Resnick, *Introduction to Special Relativity (1968)*

There are many shorthand expressions in relativity which can easily be misunderstood by the uninitiated. Thus the phrase "moving clocks run slow" means that a clock moving at a constant velocity relative to an inertial frame containing synchronized clocks will be found to run slow *when timed by those clocks.* We compare *one moving clock* with *two synchronized stationary clocks.* Those who assume the phrase means anything else often encounter difficulties.

- Robert Resnick, *Introduction to Special Relativity (1968)*

Similarly, we often refer to "an observer". The meaning of this term also is quite definite, but it can be misinterpreted. An observer is really an infinite set of recording clocks distributed throughout space, at rest and synchronized with respect to one another.... Measurements thus recorded throughout space-time (we might call them local measurements) are then available to be picked up and analyzed by an experimenter. Thus, the observer can also be thought of as the experimenter who collects the measurements made in this way.

- Robert Resnick, *Introduction to Special Relativity (1968)*

It is interesting and instructive to note that there are speeds in excess of c. Although matter or energy (i.e., signals) cannot have speeds greater than c, certain kinematical processes can have super-light speeds. For example, the succession of points of intersection of the blades of giant scissors, as the scissors are rapidly closed, may be generated at a speed greater than c. Here geometrical points are involved, the motion being an illusion, whereas the material objects involved (atoms in the scissors blades, e.g.) always move at speeds less than c. Other similar examples are the succession of points on a fluorescent screen as an electron beam sweeps across the screen, or the light of a searchlight beam sweeping across the cloud cover in the sky. The electrons, or the light photons, which carry the energy, move at speeds not exceeding c.

- Robert Resnick, *Introduction to Special Relativity (1968)*

It is instructive to note that the transverse Doppler effect has a simple time-dilation interpretation. The transverse Doppler effect is another physical example confirming the relativistic time dilation.

- Robert Resnick, *Introduction to Special Relativity (1968)*

Viewed in this way, the speed of electromagnetic waves in vacuum assumes a role wider than the travel rate of a particular physical entity. It becomes instead a limiting speed for the motion of anything in nature.

- Robert Resnick, *Introduction to Special Relativity (1968)*

The theory of relativity could have been called, instead, the theory of absolutism with some justification.

- Robert Resnick, *Introduction to Special Relativity (1968)*

Is the length contraction "real" or "apparent"? We might answer this by posing a similar question. Is the frequency, or wavelength, shift in the Doppler effect real or apparent?

- Robert Resnick, *Introduction to Special Relativity (1968)*

We do not speak about theories of matter to explain the contraction but, instead, we invoke the measurement process itself....Hence length contraction is due to the relativity of simultaneity. Since length measurements involve a comparison of two lengths (moving rod and measuring rod, e.g.) we can see that the Lorentz length contraction is really not a property of a single rod by itself but instead is a relation between two such rods in relative motion.

- Robert Resnick, *Introduction to Special Relativity (1968)*

...at bottom, time measurements are primary and space measurements secondary.

- Robert Resnick, *Introduction to Special Relativity (1968)*

However, we can choose to regard the mass as an invariant scalar quantity which gives the inertial property of a body....Nevertheless, there are advantages of a pedagogic nature to using the concept of relativistic mass.

- Robert Resnick, *Introduction to Special Relativity (1968)*

An event-horizon [or a visual horizon, according to Wald], for a given fundamental observer A, is a (hyper) surface in space-time which divides all events into two non-empty classes: those that have been, are, or will be observable by A, and those that are forever outside A's possible powers of observation.

- W. Rindler, *Mon Not Roy Ast Soc, 116, 662 (1956)*

The other type of horizon, which I shall call a *particle-horizon*, is exemplified by the Einstein-de Sitter model-universe. It may be defined as follows: A particle-horizon, for any given fundamental observer A and cosmic instant t0 is a surface in the instantaneous 3-space t=t0, which divides all fundamental particles into two non-empty classes : those that have already been observable by A at time t0 and those that have not.

- W. Rindler, *Mon Not Roy Ast Soc, 116, 662 (1956)*

We note, finally, that any effect whose speed of propagation *in vacuo* is finite and constant could have been used, as light was, in the derivation of the Lorentz equations. Since only one transformation can be valid, it follows that all such effects must be propagated with the speed of light.

- W. Rindler, *Special Relativity (1966)*

It must be stressed that the phenomena is not to be regarded as illusory, due perhaps to some peculiarity in our methods of measurement: relative to a given frame it is real in every possible sense.

- W. Rindler, *Special Relativity (1966)*

If an *ideal* clock moves *non-uniformly* through an inertial frame, we shall assume that acceleration as such has no effect on the rate of the clock, i.e. that its instantaneous rate depends only on its instantaneous speed *v* in accordance with the [formula for time-dilation]. This we shall call the clock hypothesis. Alternatively it can be regarded as the definition of an ideal clock.

- W. Rindler, *Special Relativity (1966)*

Care must be taken, however, not to regard Minkowski's world as a straightforward generalization of ordinary Euclidean 3-space to four dimensions, with time as just one more dimension. Owing to the distribution of signs in the metric, the time-coordinate x4 is not on the same footing as the three space-coordinates, and the world, regarded as a Riemannian 4-space, has non-isotropic properties quite unlike Euclidean space with its positive-definite metric.

- W. Rindler, *Special Relativity (1966)*

The elegant calculus developed by Minkowski (though anticipated by Poincaré) thus enables us, among other things, to recognize by its form alone whether a given or proposed law is Lorentz invariant without having to apply a transformation. This has great heuristic value. Moreover, by automatically combining such entities as space and time, momentum and energy, electric and magnetic field, etc., the formalism illuminates some profound physical inter-connections.

- W. Rindler, *Special Relativity (1966)*

The vector PR is said to be timelike. It has, however, one important property without analogue among spacelike vectors, namely that the temporal sequence of P and R is absolute, i.e. the same in all frames.

- W. Rindler, *Special Relativity (1966)*

But theories of gravitation within the framework of special relativity have also been devised, one of the earliest being Nordstrom's (1913) and one of the latest G. D. Birkhoff's (1942).

- W. Rindler, *Special Relativity (1966)*

Thus the inertial mass increases as the velocity increases....The inertial mass is least, namely m0, when the particle is at rest.

- W. Rindler, *Special Relativity (1966)*

In accordance with Einstein's hypothesis, *every* form of energy has a mass equivalent. Thus associated with even an electromagnetic field there is a certain amount of mass. Anticipating that every mass exerts and suffers gravitational influence, we shall therefore expect even an electromagnetic field to exert a certain gravitational pull, and light to bend under gravity.

- W. Rindler, *Special Relativity (1966)*

Any transfer of energy, being equivalent to a transfer of mass, will necessarily involve momentum. Thus, for example, all forms of radiation must exert pressure...

- W. Rindler, *Special Relativity (1966)*

Thus in relativistic mechanics the acceleration is not, in general, in the same direction as the force. The only cases in which these vectors are in the same direction are when the velocity is momentarily zero, or when the force is either parallel or perpendicular to the velocity.

- W. Rindler, *Special Relativity (1966)*

In this way there arose the concepts of longitudinal mass... and transverse mass...Nevertheless, it is preferable not to think of the mass of the particle in this ambivalent way connected with the 3-acceleration, but rather to regard it as the unique coefficient of u in the expression for the 3-momentum.

- W. Rindler, *Special Relativity (1966)*

It must be stressed that the theory of special relativity does not say that one cannot have velocities exceeding the velocity of light *in vacuo*, but simply says that energy and momentum cannot be transmitted with a velocity exceeding the velocity of light *in vacuo*...Thus the point of intersection of the rulers can move with a velocity exceeding the velocity of light.

- W.G.V Rosser, *Introductory Relativity (1967)*

For example, in the example of a game of tennis on board a ship going out to sea...it was reasonable within the context of Newtonian mechanics to find that the velocity of the tennis ball was different relative to the ship than relative to the beach...According to the theory of special relativity, not only the measures of the velocity of the ball relative to the ship and seashore will be different, but the measures of the dimensions of the tennis court parallel to the direction of relative motion and the measures of the time of events will also be different.

- W.G.V Rosser, *Introductory Relativity (1967)*

[Cosmic ray] protons of [high energy] have a value of gamma of approximately 10 billion. If they were not deflected by galactic magnetic fields, such protons would reach the edge of the galaxy in about 5 min, measured in the rest frame of the proton....Relative to [the same protons] the dimension of the galaxy are Lorentz contracted by [the same factor of 10 billion] and, in the direction of relative motion of the proton and the galaxy, relative to the proton the galaxy would have roughly the dimensions of the solar system.

- W.G.V Rosser, *Introductory Relativity (1967)*

The use of the term observers in such contexts led some people to conclude that the theory of special relativity required the intervention of observers in measurements in a way different from classical Newtonian mechanics, and for this reason the theory of relativity was assumed by many people to be more 'idealistic' and less 'materialistic' than Newtonian mechanics. In both Newtonian mechanics and in the theory of special relativity the measurements can be performed by instruments, and the information transmitted to a base where it can be fed into a calculating machine, and after constructing and setting up the apparatus the 'observer' need only read and interpret the results when necessary.

- W.G.V Rosser, *Introductory Relativity (1967)*

For purposes of discussion it was found convenient to think of the co-ordinate system sigma and sigma-prime as consisting of a series of 'rigid' rulers parallel to the axes, with a series of synchronized clocks distributed throughout space so that the co-ordinates and time of an event could be measured when and where it happened....One could perform the measurements in practice using the radar techniques described in....

- W.G.V Rosser, *Introductory Relativity (1967)*

Now [beta cos theta] is the component of the velocity of the source in the direction of the observer at the instant when the light was emitted by the source... [Doppler has two parts - time dilation plus radial motion].

- W.G.V Rosser, *Introductory Relativity (1967)*

The light from a moving object undergoes both aberration and the Doppler shift in frequency.

- W.G.V Rosser, *Introductory Relativity (1967)*

Thus, according to the theory of special relativity, the visual appearance of the moving cube is the same as that of a stationary cube rotated through an angle [arcsin v/c]. [Transverse case, subtending a small angle].

- W.G.V Rosser, *Introductory Relativity (1967)*

It was shown by Terrell that, in the general case [not transverse], when the moving object is viewed at any angle then, according to the theory of special relativity, provided the angle subtended by the moving object at the point of observation is very small, the visual appearance of a moving object is undistorted in shape but rotated by the angle of aberration, where the angle of aberration can be calculated from eqn (4.35). If the object subtends a finite angle at the point of observation, then the different parts of the object appear rotated by different amounts and this leads to a distortion of the shape.

- W.G.V Rosser, *Introductory Relativity (1967)*

Some readers may find it simpler to visualize the measurement of the times and co-ordinates of distant events, using the radar methods described above, rather than use all the paraphernalia of rulers and clocks distributed through space, with observers distributed throughout space to record events when and where they occur.

- W.G.V Rosser, *Introductory Relativity (1967)*

In Newtonian mechanics it was possible to make statements such as: 'the length of the rod is 30 cm.' Since, in the theory of special relativity, the length of a body is not absolute, such a statement has no meaning unless the length is specified relative to some standard of rest.

- W.G.V Rosser, *Introductory Relativity (1967)*

From the requirement that the quantities appearing in the laws of mechanics must transform according to the transformations of the theory of special relativity, it was necessary to redefine the mass of a moving particle in the laboratory system.

- W.G.V Rosser, *Introductory Relativity (1967)*

According to the Lorentz transformations the *x* and *m* co-ordinates of [B-prime] are [gamma * beta] and [gamma] respectively. [Simply corresponds to how matrices work: the columns are the result of mapping unit vectors.]

- W.G.V Rosser, *Introductory Relativity (1967)*

Deep down in our Newtonian bones we understand by an inertial frame a frame of reference that is at rest or in uniform motion with respect to absolute space. Absolute space is the imagined framework in which bodies move.

- R. D. Sard, *Relativistic Mechanics (1970)*

The name "relativity" is justified historically by the fact that the basic concern of the theory is with how the description of phenomena changes when one shifts from one reference frame to another....the original, somewhat negative, name has meanwhile become established by usage.

- R. D. Sard, *Relativistic Mechanics (1970)*

If the rod's orientation is neither longitudinal nor transverse to the direction of relative motion, both the length and the inclination are different in the rod's rest frame and the laboratory frame.

- R. D. Sard, *Relativistic Mechanics (1970)*

The inequality of theta and h [angles of a stick with respect to its direction of relative motion] is as much of an affront to pre-Einsteinian habit as is the length contraction and the slowing down of clocks. A weathervane pointing northeast on a house flying east with speed such that gamma=2.4, points north-northeast in the ground frame!

- R. D. Sard, *Relativistic Mechanics (1970)*

Space-time is a four-dimensional topological manifold with a smooth atlas, carrying a torsion-free connection compatible with a Lorentzian metric and a time-orientation satisfying the Einstein equations.

- Frederic Schuller

One can only wish that an earlier generation of physicists had chosen more appropriate names for theses theories!

- Bernard Schutz, *A First Course in General Relativity (1985)*

It is important to realize that an 'observer' is in fact a huge information-gathering system, not simply one man with binoculars.

- Bernard Schutz, *A First Course in General Relativity (1985)*

One can envision this coordinate system, rather fancifully, as a lattice of rigid rods filling space, with a clock at every intersection of the rods.

- Bernard Schutz, *A First Course in General Relativity (1985)*

The student to whom this is new should probably regard the notation delta-s-squared as a single symbol, not as the square of a quantity delta-s. Since delta-s-squared can be either positive or negative, it is not convenient to takes its square root. Some authors do, however, call delta-s-squared the 'squared interval', reserving the name 'interval' for its square root.

- Bernard Schutz, *A First Course in General Relativity (1985)*

This is what's known as a 'paradox', but like all 'paradoxes' in SR, this comes from not having reasoned correctly.

- Bernard Schutz, *A First Course in General Relativity (1985)*

Students should realize that all 'paradoxes' are really mathematically ill-posed problems, that SR is a perfectly consistent picture of spacetime which has been experimentally verified countless times....

- Bernard Schutz, *A First Course in General Relativity (1985)*

One of the best methods for developing a modern intuition is to be completely familiar with the geometrical picture of SR: Minkowski space, the effect of Lorentz transformations on axes, and the 'picture' of such things as time dilation and Lorentz contraction.

- Bernard Schutz, *A First Course in General Relativity (1985)*

...there are of course an infinity of freely falling frames at any point. They differ in their velocities and in the orientation of their spatial axes, but they all accelerate relative to Earth at the same rate.

- Bernard Schutz, *A First Course in General Relativity (1985)*

Since *any* nonuniformity is, in principle, detectable, a frame can only be regarded mathematically as inertial in a vanishingly small region.

- Bernard Schutz, *A First Course in General Relativity (1985)*

Einstein's important advance was to see the similarity between Riemannian spaces and gravitational physics. He identified the trajectories of freely falling particles with the geodesics of a curved geometry: they are locally straight since spacetime admits local inertial frames in which those trajectories are straight lines, but globally they do not remain parallel.

- Bernard Schutz, *A First Course in General Relativity (1985)*

Basically, a manifold is any set that can be continuously parameterized. The number of independent parameters is the *dimension* of the manifold, and the parameters themselves are the *coordinates* of the manifold.

- Bernard Schutz, *A First Course in General Relativity (1985)*

This is the way to think of a manifold: it is a space with coordinates, that locally looks Euclidean but that globally can warp, bend, and do almost anything (as long as it stays continuous).

- Bernard Schutz, *A First Course in General Relativity (1985)*

A differentiable manifold in which a symmetric tensor field g has been singled out to act as the metric at each point is called a Riemannian manifold...The differential manifold is 'primitive': an amorphous collection of points, arranged locally like the points of Euclidean space, but not having any distance relation or shape specified. Giving the metric g gives it a specific shape, as we shall see.

- Bernard Schutz, *A First Course in General Relativity (1985)*

As remarked above, it is customary to refer to [the field equations] as Einstein's equations, a custom which we follow in this book, but it might be fairer to call them the Einstein-Hilbert equations, because the mathematician D. Hilbert derived them independently of Einstein in the same year. Hilbert was motivated by Einstein's own physical arguments, which were known to him, but his mathematical approach to deriving the equations was far more elegant than Einstein's...

- Bernard Schutz, *A First Course in General Relativity (1985)*

The dragging of orbits has become so strong that this photon *cannot* move in the directon opposite the rotation. Clearly, any particle, which must move slower than a photon, will therefore have to rotate with the hole, even if it has an angular momentum arbitrarily large in the opposite sense to the hole's!

- Bernard Schutz, *A First Course in General Relativity (1985)*

The Kerr metric has less symmetry than the Schwarzschild metric, so it might be expected that particle orbits would have fewer conserved quantities and therefore be harder to calculate. This is, quite remarkably, false: even orbits out of the equator have three conserved quantities: energy, angular momentum, and a difficult-to-interpret quantity associated with the theta motion. The same remarkable property carries over to the wave equations that govern electromagnetic fields and gravitational waves in the Kerr metric: these equations separate completely in certain coordinate systems.

- Bernard Schutz, *A First Course in General Relativity (1985)*

In the modern view, the *real* signature of gravity, the part of gravity that can't be removed by going into free fall, is the *tidal force*, whose most spectacular effect on Earth is to raise the ocean tides.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Indeed, many astronomical systems transmit tidal forces as signals right across the Universe, signals that we call gravitational waves.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

This [relative acceleration] is entirely due to the fact that the Earth's gravitational field is *non-uniform*: it pulls with different accelerations in different places.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Tidal effects have this characteristic behaviour under rotations: places where the tidal effect is similar are separated by a rotation of only 180 degrees. We will encounter this symmetry again when we discuss gravitational waves....

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The fact that the Sun and Moon happen to exert similar tidal forces on the Earth is deeply related to another exceptional "accidental" fact that might at first seem to be completely unconnected, namely that eclipses occur, i.e. that the Moon and the Sun are of similar angular size on the sky.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Unfortunately, special and general relativity are probably the worst-named theories of modern physics. Their names convey little meaning, and this sometimes causes confusion right from the start.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Because it deals with the general properties of measurements, special relativity is not really a theory about any particular physical system. Rather, it is a set of general principles that all the other theories of physics have to obey to deal correctly with fast-moving bodies.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

This produces a blueshift in relativity where there is none in the Newtonian Doppler formula. This is called the transverse Doppler shift. [Error: it's a redshift.]

- Bernard Schutz, *Gravity From The Ground Up (2003)*

...the more pressure a gas has, the more difficult it is to accelerate.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

We will have to steer a careful course between the rocky shoals of too much mathematical complexity and the becalmed waters of over-simplification.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The geometry of a space, like the Earth's surface, is described by the distances between places, not the coordinates of the places.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Notice that this is written as the square of a number *s*. The spacetime-interval is the quantity *s-squared*, not *s*. In fact, we will not often deal with *s* itself. The reason is that *s-squared* is not always positive, unlike distance in space....You should just regard *s-squared* as a single symbol, rather than as the square of something.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Gravity itself creates gravity...Thus, gravitational fields have energy and this feeds back into the gravitational field.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Since his cosmological negative pressure was to be fundamental, not tied to any accidental matter field or configuration, he needed the pressure to be constant in space and time, so that an observer could not pick out any special place or time by measuring the pressure. It had to be a fundamental *constant* of nature. Einstein actually introduced, instead of a fundamental pressure, a fundamental constant lambda, which he called the cosmological constant.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Put graphically, if the nucleus were magnified to the size of an apple, then its electrons would be 1.6km (1 mile) away! All the space between the nucleus and its electrons is empty.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

...lambda itself has the dimensions of a frequency squared.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Gravitational waves produce tidal accelerations only in directions *perpendicular* to the direction they are traveling in.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

This is a key concept: whatever action a gravitational wave has on matter is also the motion by which matter produces gravitational waves.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

However, the deformation produced by the Moon is partly directed towards the Moon (the longitudinal direction), whereas gravitational waves are transverse.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The deformation ellipse that is produced by a wave has the same area as the original circle, so we say that gravitational waves in general relativity are *area-preserving*. Only two polarizations are illustrated, because only two are needed. The second is obtained by rotating the first by 45 degrees. Any other action of a gravitational wave in the same plane can be described by combining these two.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

If the particles are part of a solid body, however, then the resulting deformation will be a result of all the forces, the tidal accelerations and the internal stresses of the material.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

When gravitational waves move through a region they do not induce differences between the rates of nearby clocks. Instead, they deform proper distances...

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Many relativists, including at times Einstein himself, believed that gravitational waves were a mathematical illusion...

- Bernard Schutz, *Gravity From The Ground Up (2003)*

A source must deform in some kind of irregular way to emit radiation. In particular, a spherical star that collapses but remains spherical only deforms circles into smaller circles, and this motion will emit *no* internal motions onto the "plane of the sky", which means onto a plane perpendicular to the line-of-sight to the source. Then only the motions in that plane that are some combinations of the ['+' and 'x' motions] will generate gravitational waves. Moreover, the detector will respond with exactly the same combination of motions: detectors simply mimic the tidal distortions of the source.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The frequency of a gravitational wave is determined by the typical time-scale for the things to happen in its source. The upper bound on expected frequencies is about 10,000 Hz, because it is difficult to get large astronomical bodies, with masses comparable to the Sun or larger, to do anything on time-scales shorter than a tenth of a millisecond or so....There is no lower bound, and in fact scientists are planning detectors that reach down below [1 millihertz].

- Bernard Schutz, *Gravity From The Ground Up (2003)*

So when two [black] holes merge, the result is independent of how they originally formed, and indeed it does not involve any matter at all. It is pure gravity in a vacuum. The merger is pure dynamical geometry.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Because gravity penetrates everywhere, the Universe is transparent to gravitational radiation, and has been so from the first moment.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

A telescope maker on Earth spends enormous effort to make the mirrors and lenses of a modern telescope smooth and perfectly shaped to small fractions of the wavelength of light, and then the telescope is used to observe light that has come to us through a bumpy, astigmatic, partly absorbing natural gravitational lens!

- Bernard Schutz, *Gravity From The Ground Up (2003)*

...in this chapter we will concentrate on understanding simple lenses. We will discover the peculiar nature of the gravitational lens, divergent in some regions and convergent in others; we will see why lenses magnify objects and make them brighter; we will see why there are in principle always an odd number of images in any lensed object, although not all of them will necessarily be bright enough to be detected; and we will see how lensing can be used to measure the mass of the lens itself, and, possibly, the expansion rate of the Universe.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

We have shown that the surface brightness of an object is unchanged by the lens....The star occupies a larger angular size on the sky. Since it has the same surface brightness, we get more light in total from it.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

What normally happens is that there is only one image, a little brighter. Other rays, that pass on the other side of the lensing galaxy, do not deflect enough to reach the astronomer...It is important to realize that for gravitational lenses, as for any other kinds of optics, the location of the observer is as important for determining what is seen as are the locations of the source and the lens.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

...there is a general mathematical theorem that the number of images must be odd, and that, after the first, they come in pairs: one direct, the other inverted. The only exception to this theorem is if there is a gravitational field without a smooth galaxy or other smooth mass distribution....in relativity, black holes create a field with no smooth center. Lensing by a black hole does not need to create on odd number of images...

- Bernard Schutz, *Gravity From The Ground Up (2003)*

However, our simple and apparently obvious assumption that gravity is attractive has recently been called into question by astronomical observations....providing a strong indication that the expansion of the Universe is actually accelerating today...If this is the case, then a singularity at the beginning of time is not an inevitable consequence of physical laws.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Even if the acceleration proves in the end to be an illusion, the stimulus it has given to physicists and astronomers to come up with new ideas and justify old ones has been a positive result.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Their temperature, about 2.7K, is a factor of about 1000 lower than that of a plasma that can ionize hydrogen....This implies that the Universe has expanded by the same factor of 1000 since decoupling.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

In fact, we don't need to look in opposite directions: because the radiation originates so much closer to the time of the Big Bang than does the light given off by galaxies, the microwave radiation coming to us from directions separated by only a couple of degrees on the sky is coming from regions that would have had no knowledge of one another at the time the radiation was emitted.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

But what is intriguing about this is that there is only one Universe, so the Universe really does have a preferred rest frame. This is the frame in which the mass in the Universe is at rest, on average....This rest frame is easily the best for describing the physics of the Universe.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The expansion of the Universe is an observable fact precisely *because* ordinary matter does not expand with it.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

...the clumpiness of dark matter helps accelerate the formation of structure in the evolving Universe.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

In particular, there seems to be single length-scale on which missing gravity takes over from Newtonian or Einsteinian gravity. This is not surprising if one accepts that the missing gravity is created by missing mass: each galaxy or cluster condensed around an individual clump of dark matter, and since these clumps are random, the gravity they create will be different from galaxy to galaxy and cluster to cluster.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

Notice first that a space that is fully isotropic has to be homogeneous.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The general rule is that if the universe model is closed then angular diameters decrease less rapidly with distance than in a flat universe, and if the model is open they decrease more rapidly.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

The origin, and even the nature, of these [ultra-high energy] cosmic-ray particles is a complete mystery. Maybe their sources are dark and represent new physics, or maybe the particles themselves are new.

- Bernard Schutz, *Gravity From The Ground Up (2003)*

As you see, the war treated me kindly enough, in spite of the heavy gunfire, to allow me to get away from it all and take this walk in the land of your ideas.

- Karl Schwarzschild, *Letter to Einstein*

The name 'metric tensor' refers to the fact that by its use the quantities of length and angle which are fundamental to geometrical measurement can be defined and calculated.

- Hans Stephani, *General Relativity (1982)*

...inside an Earth satellite or a falling box bodies move force-free for the co-moving observer. [Ohanian-Ruffini contradicts.]

- Hans Stephani, *General Relativity (1982)*

It says that the tangent vector...of a geodesic remains parallel to itself. The geodesic is thus not only the shortest curve between two points, but also the straightest. The straight line in Euclidean space also has these two properties.

- Hans Stephani, *General Relativity (1982)*

To summarize, we can thus make the following completely equivalent statements. The curvature tensor...vanishes if and only if: (a) the space is flat, that is, Cartesian coordinates...can be introduced throughout the space; or (b) the parallel transport of vectors is independent of path; or (c) covariant derivatives commute; or (d) the geodesic deviation (the relative acceleration) of two arbitrary particles moving force-free vanishes.

- Hans Stephani, *General Relativity (1982)*

Many criticism have been raised against this principle [of general covariance], their aim being to assert that neither is it a *physical* principle, nor does it guarantee the correctness of the equations thus obtained.

- Hans Stephani, *General Relativity (1982)*

In other words, a freely falling observer in a gravitational field cannot detect the gravitational field by physical experiments in his immediate neighbourhood; for him *all* events occur as in an inertial system. [Ohanian-Ruffini contradicts.]

- Hans Stephani, *General Relativity (1982)*

...one formulates the physical laws in a Lorentz-invariant manner in an inertial system and substitutes covariant for partial derivatives. This prescription ensures simultaneously the covariance of the resulting equations and their validity upon using curvilinear coordinates in Minkowski space.

- Hans Stephani, *General Relativity (1982)*

As we have already indicated more than once, the basic idea of Einstein's theory of gravitation consists in geometrizing the gravitational force, that is, mapping all the properties of the gravitational force and its influence upon physical processes onto the properties of Riemannian space.

- Hans Stephani, *General Relativity (1982)*

It is one of its particular merits that, in the Einstein theory, the equations of motion are a consequence of the field equations....Even after the Einstein field equations had been set up it was thought that one had to demand in addition that the geodesic equation be the equation of motion of a test particle, but eventually it was realized that this can be deduced [as a] consequence of local energy-momentum conservation.

- Hans Stephani, *General Relativity (1982)*

An extended body, for example, will in general not move exactly along a geodesic. This is due not so much to the gravitational field caused by the body itself as to the action of 'tidal forces.'

- Hans Stephani, *General Relativity (1982)*

Since a choice of coordinates always leads to requirements on the metric functions, we must proceed carefully in order not to lose solutions by making the restrictions too strong.

- Hans Stephani, *General Relativity (1982)*

In the discussion of physical properties of the Schwarzschild metric one must always remember that *r* and *t* in particular are only coordinates and have no immediate physical significance. We therefore call *t* the *coordinate time*, to distinguish it, for example, from the proper time tau of an observer at rest in the gravitational field....The radial coordinate *r* is so defined that the surface area of a sphere r=constant, t=constant has the value [4 pi r-squared]. The infinitesimal displacement in the radial direction is...always greater than the difference in the radial coordinates.

- Hans Stephani, *General Relativity (1982)*

Predictions about the numerical value of the velocity of light have little value in general relativity; the only essential thing is that light propagates along null geodesics...

- Hans Stephani, *General Relativity (1982)*

In non-linear theories the situation is more complicated, because the singularity need not occur at the position of the source.

- Hans Stephani, *General Relativity (1982)*

Therefore if the metric is singular at a point, one investigates whether this singularity can be removed by introducing a new coordinate system. Or, appealing more to physical intuition, one asks whether a freely falling observer can reach this point and can use a local Minkowski system there. If both are possible, then the observer notices no peculiarities of the physical laws and phenomena locally, and hence there is no singularity present.

- Hans Stephani, *General Relativity (1982)*

But first a warning: the reader has perhaps assumed an exact definition of a singularity that includes all mathematical and physical properties, whereas research in this area has not yet advanced so far that such a definition can be given.

- Hans Stephani, *General Relativity (1982)*

To avoid misunderstanding we emphasize that the Kerr solution is not the gravitational field of an arbitrary axisymmetric rotating star, but rather only the exterior field of a very special source.

- Hans Stephani, *General Relativity (1982)*

Black holes are objects which belong to both regimes [the small and the large]. They're big, and they're heavy, but they have quantum properties. In fact, they are very quantum mechanical objects.

- Leonard Susskind, *Youtube, Closer To Truth*

But it seems to me that the spirit of this book is best described by the word *irony*, used the sense of ... Socrates, and that sense is not easy to explain.... The lust for calculation must be tempered by periods of inaction, in which the mechanism is completely unscrewed and then put together again.... It would perhaps seem that irony of this sort belongs to evaluation rather than to creation, but in the whole history of science there is no greater example of irony than when Einstein said he did not know what absolute time was, a thing which everyone knew.

- J. L. Synge, *Relativity: The General Theory (1960)*

I have never been able to understand this principle... Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with an observer's worldline... The Principle of Equivalence performed the essential office of midwife at the birth of general relativity... I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced.

- J. L. Synge, *Relativity: The General Theory (1960)*

To the ironical mind there is little distinction between the mundane and the exalted, and that is no doubt why Socrates had to drink the hemlock cup.

- J. L. Synge, *Relativity: The General Theory (1960)*

It is to support Minkowski's way of looking at relativity that I find myself pursuing the hard path of the missionary. When, in a relativistic discussion, I try to make things clearer by a space-time diagram, the other participants look at it with polite detachment and, after a pause of embarrassment as if some childish indecency has been exhibited, resume the debate on their own terms. Perphaps they speak of the Principle of Equivalence. If so, it is my turn to have a blank mind, for I have never been able to understand this Principle....Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world-line....The Principle of Equivalence performed the essential office of midwife at the birth of general relativity, but, as Einstein remarked, the infant would never have got beyond its long-clothes had it not been for Minkowski's concept. I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

- J. L. Synge, *Relativity: The General Theory (1960)*

In some respects it is unimportant that the metric form of space-time is indefinite. The fact does not obtrude itself in calculating the Riemann tensor, for example. But in other respects it is fundamental, and it is most unwise to mislead students of relativity by writing down, as a definition of the metric of flat space-time, an equation [using ds-squared]. This is an undigested heritage from positive-definite days. [These expressions imply that] ds is real for some displacements and imaginary for others. No serious worker in relativity will be confused by this, for he recognizes [such expressions for ds-squared] as a sort of physicist's slang. But why perpetuate such nonsense? What we are concerned with is a quadratic form [phi, not ds-squared]... and [we take the absolute value of phi before taking the square root], always real.

- J. L. Synge, *Relativity: The General Theory (1960)*

To understand a subject, one must tear it apart and reconstruct it in a form intellectually satisfying to oneself, and that (in view of the differences between individual minds) is likely to be different from the original form.

- J. L. Synge, *Relativity: The Special Theory (1955)*

Axioms, as we now realise, are not the self-evident truths they were long supposed to be, but rather the rules of a game, the pieces of which are elements or concepts which remain and must ever remain undefined because there is nothing in terms of which to define them.

- J. L. Synge, *Relativity: The Special Theory (1955)*

Let us now get to work. We start with a tabula rasa, a clean sheet, a mind in a state of intellectual nudity.

- J. L. Synge, *Relativity: The Special Theory (1955)*

For my own part (and this is a matter involving one's personal mode of thought) relativity without space-time diagrams -relativity embedded in formulae which are to be interpreted for physics solely in terms of modified Newtonian imagery - such relativity is as dull and confusing as an attempt to contemplate the five regular solids without using three-dimensional spatial intuition. Intuition about four-dimensional space-time is difficult and sometimes deceptive on account of false Euclidean analogies.

- J. L. Synge, *Relativity: The Special Theory (1955)*

These transformations are basic in the special theory of relativity, which might almost be called "the theory of the Lorentz transformations."

- J. L. Synge, *Relativity: The Special Theory (1955)*

The first approach (axes fixed and events in space-time changed under the transformation) has been used extensively by E. Cartan and his school. The second approach (events fixed and coordinates changed) is that of the Princeton school of tensor calculus (Eisenhart and Veblen).

- J. L. Synge, *Relativity: The Special Theory (1955)*

The results we are going to establish seem very queer to most people at first sight. That is because it is so hard for us, reared on the absolute time of Newton, to break away from that concept and recognise (as we must in relativity) that there is no absolute simultaneity.

- J. L. Synge, *Relativity: The Special Theory (1955)*

...on the whole [a space-time diagram's] chief function is as a source of suggestion and a rough guide to precise formal calculations.

- J. L. Synge, *Relativity: The Special Theory (1955)*

... of all forms of communication, [communication of scientific matters] is the most exciting when it succeeds, and the most depressing when it fails. But it does not take place through the formal channels precscribed for it, at least not nearly to the extent that many people suppose.

- J. L. Synge, *Talking About Relativity (1970)*

One of course does not decry the elaborate educational systems existent in all civilised countries. They exist for this very purpose, but with overtones (economic, political, national and so on) which tend to mask the simple human fact that what is sought is not really economic prosperity or prestige (national or personal) but understanding, pure and simple - the religion of modern man.

- J. L. Synge, *Talking About Relativity (1970)*

So I called this book 'Talking About Relativity', each word pregnant, at least to me. 'Relativity' - obvious, this execrable word for which we can find no alternative.

- J. L. Synge, *Talking About Relativity (1970)*

Nevertheless it may be asserted that mathematical symbolism is merely a very efficient shorthand for recording what might be said or written out at length in words.

- J. L. Synge, *Talking About Relativity (1970)*

It is only in mathematics that we find clear-cut concepts, and it is probably this inhuman characteristic that makes the subject repellent to many people.

- J. L. Synge, *Talking About Relativity (1970)*

There are two important forks in the road, but they lie so close together that they are hardly distinguishable. One fork is at the concept of *simultaneity* or absolute time... and the other at the concept of *rigidity*.

- J. L. Synge, *Talking About Relativity (1970)*

Signals are so important in relativity that the subject might well be called the 'theory of signalling' - it would be less confusing than the accepted title.

- J. L. Synge, *Talking About Relativity (1970)*

...absolute time, or equivalently, instantaneous signalling.

- J. L. Synge, *Talking About Relativity (1970)*

It is a relief to know that we can fall back on atomic clocks.... We may say that an atomic clock ticks with a definite frequency - it keeps time - and this agrees with remarkable accuracy with the usual astronomical time. Indeed it is becoming the vogue to *define* the unit of time in terms of an atomic clock, and to regard astronomical time as secondary.

- J. L. Synge, *Talking About Relativity (1970)*

Let us then frankly admit that thinking about relativity puts one in a state of mental tension. It is so easy for some Newtonian idea to leak out from the subconscious and vitiate an otherwise (relativistically) impeccable discussion. Only under two conditions do I find myself secure against that leak: when I make a space-time diagram... and secondly when immersed in a mathematical argument in which the various symbols are natural to relativity. In the domain of mere words, I feel insecure and liable to slip into Newtonian ways of thinking unless I exercise great care.

- J. L. Synge, *Talking About Relativity (1970)*

But a curious thing happened. In my first draft of these chapter headings, I put *a surface* in the earlier heading, but *space-time* without the indefinite article in this one, and I began to wonder why I did that. It is worth talking about. Why do I now insert *a* in this heading?

- J. L. Synge, *Talking About Relativity (1970)*

What we seem to be saying is this: a very small part of a surface may be regarded as a plane... It is clear that [this] calls for some caution. It is sometimes stated thus: a surface is elementarily flat. But this is nonsense, because, no matter how we choose the coordinates, the curvature of the surface remains - it is invariant.

- J. L. Synge, *Talking About Relativity (1970)*

What is gravitation? It is the curvature tensor.... If the curvature tensor vanishes, space-time is flat. It is the curvature tensor that constitutes the gravitational field.

- J. L. Synge, *Talking About Relativity (1970)*

I now have to stop harping on the fact that there are no privileged coordinates in space-time, because in special relativity there is indeed a privileged class of coordinates, namely those for which the line-element takes the simple Minkowskian form....

- J. L. Synge, *Talking About Relativity (1970)*

So [in special relativity] we have a cloud of clocks moving rigidly and they are synchronised in the above manner.

- J. L. Synge, *Talking About Relativity (1970)*

You might think that it would be possible to effect this transformation by playing with a lattice of rods pinned together, but this would not do, because although each square is deformed into a parallelogram, the (Euclidean) lengths of the sides are increased.

- J. L. Synge, *Talking About Relativity (1970)*

Concepts useful in exploring the very fast help us to examine spacetime near very massive objects.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

In relativity every invariant quantity is a diamond, to be treasured.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Principle of Extremal Aging: The path a free object takes between two events in spacetime is the path for which the time lapse between events, recorded on the object's wristwatch, is an extremum. [Some cases are a saddle point. He refers to maxima and saddle points as extremal.]

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Particle mass *m* is an *invariant*, independent of reference frame....

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Both space and time enter into the specification of the limiting dimensions of a free-float frame. Therefore - for a given sensitivity of the measuring devices - a reference frame is free-float only within a limited region of *spacetime*.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

There are many valid ways to synchronize clocks. Here is one: Pick one clock as the standard, the reference clock. At midnight the reference clock sends out a synchronizing flash of light in all directions. Prior to emission of the synchronizing flash, every other clock in the lattice has been stopped and set to a time...later than midnight equal to the straight-line distance...of that clock from the reference clock. Each clock is then started when it receives the reference flash.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The lattice clocks, when installed by a foresighted experimenter, will be recording clocks. Each clock is able to detect the occurrence of an event (collision, passage of light flash or particle). Each reads into its memory the nature of the event, the time of the event, and the location of the clock. The memory of all the clocks can then be read out and analyzed later at some command center.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

For spacetime near nonrotating Earth or black hole, the task of binding together individual localized free-float frames is carried out by the Schwarzschild metric....

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Describe an object with a table of distances between points. Describe spacetime with a table of intervals between events.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Spacetime geometry exists - and can be described - even when no frame of reference is used at all!

- Taylor and Wheeler, *Exploring Black Holes (2000)*

People who use general relativity as a tool change reference frames more often than they change clothes.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The constant, ever-present "force of gravity" that we experience on Earth is gone, eliminated as we step into a free-float frame. What remains of "gravity"? Only curvature of spacetime remains. What is this curvature? Nothing but tidal acceleration. Curvature is tidal acceleration and tidal acceleration is curvature.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

When, instead, the separation between the two events is spacelike, that is, when the space part of the separation predominates over the time part, we reverse the signs of the terms [of tau-squared, the square of the timelike spacetime interval] to keep the combination positive.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The r-coordinate is *defined* so that 2pi*r is the measured distance around a circle centered on the attracting mass; hence its name, *reduced circumference.*

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The time t is called *far-away time* and is measured on clocks far away from the center of attraction...

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The wonderful thing about a black hole is that it has no surface, no structure, and no matter with which one will collide. A test particle can explore *all* of spacetime around a black hole without bumping into the surface - since there is no surface at all.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

A singularity is a *nobody knows* phenomenon....The truth is, nobody has figured it out yet! No one has developed a theory of quantum gravity that combines quantum mechanics with general relativity.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

In general relativity, every coordinate system is partial and limited, correctly representing one or another feature of curved spacetime and misrepresenting other features. Figures and diagrams that display these coordinate systems embody the same combination of clarity and distortion.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

These figures embed curved-space geometry in the flat Euclidean three-space geometry perspective shown on the printed page. Therefore these figures are called embedding diagrams. But flat Euclidean geometry is *not* curved space geometry. Therefore we expect embedding diagrams to misrepresent cuved space in some ways. They lie!

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The metric is a "four-dimensional micrometer" for measuring the small spacetime separation between a chosen pair of events. You *own* the metric. You *choose* the events whose separation you wish to measure with the metric. The "metric micrometer" translates bookkeeper coordinate increments ... into proper time or proper distance between the pairs of events *you* choose.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Moreover, the shell observer and a passing free-float observer can use the Lorentz transformation of special relativity to exchange data on local events that are close together in spacetime. We shall use this ability to exchange data many times...

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The strength of bookkeeper coordinates is universality; their weakness is isolation of most data entries from direct experience. In contrast, people can, in principle, live and work in free-float frames and on spherical shells, taking and analyzing data as if they were in flat spacetime, but unfortunately they can do so only for limited patches of spacetime.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The Schwarzschild metric is central in the translation of coordinates back and forth between direct observers (shell observers and free-float observers) and between each of these observers and the Schwarzschild bookkeeper.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

"Go straight!" spacetime shouts at the stone. The stone's wristwatch verifies that its path is straight.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Spacetime shouts, "Go straight!" The free stone obeys. What does "straight" mean? Straight with respect to what? We know the answer: The path of the stone is straight in a free-float frame.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Nature's command to the stone in its general form is "*Follow a straight worldline in a local inertial frame.*" No description could be simpler.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

On the theory side, Einstein says that you can do away entirely with Newton's gravitational force, substituting instead the idea of geodesic: A free test particle moves along a worldline *straight* in *spacetime* as described with respect to every *local* frame through which it passes. But the result may be a path *curved* in *space* as described by *global* Schwarzschild coordinates.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

General relativity stitches together the quilt squares of local free-float frames into a full quilt that covers wide regions of spacetime.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Einstein's account of motion in curved spacetime recognizes no such distinction [between potential energy and kinetic energy]. This new outlook presents us with a new unity. Energy associated with speed, energy associated with location, and energy associated with mass are stirred together inextricably into a greater and simpler whole.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

In summary, for a radically plunging object that starts from rest at infinity, the principle of constancy of energy tells us that the inward speed measured by shell observers increases steadily for smaller values of r, rising to the speed of light at the horizon. In contrast, the inward speed of the object drops to zero at the horizon when reckoned from the accounts of the Schwarzschild bookkeeper.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Unless we get quite close to it, a black hole will no more grab us than Sun grabs Earth.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

...for orbits that stay at radii greater than about [150 times the Schwarzschild radius], Newtonian mechanics predicts the motion to a good approximation.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The same irresistible collapse forbids any stationary structure or object inside the horizon.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Inside [the event horizon] there is an interchange of the character of the t-coordinate and the r-coordinate.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

A maximum of 15 microseconds does not seem long to live when we fall into a black hole [i.e., from the event horizon to the singularity].

- Taylor and Wheeler, *Exploring Black Holes (2000)*

The selected black hole is an old one, so we presume that the sea of gravitational waves trapped inside the horizon at its formation will have died away.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

According to their theory, as a plunging observer approaches the center point, spacetime oscillates chaotically, squeezing and stretching the poor traveler in random directions like an electric mixer...

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Extremal aging implies that energy and angular momentum are constants of the motion.

- Taylor and Wheeler, *Exploring Black Holes (2000)*

Any difference in the direction [of motion of a particle] in one local frame and that in a nearby frame is described in terms of the "curvature of spacetime", Einstein tells us. The existence of this curvature destroys the possibility of describing motions with respect to a single ideal Euclidean reference frame that pervades all space. What is simple is only the geometry in a region small enough to look flat.

- Taylor and Wheeler, *Spacetime Physics (1996)*

Where does mass get its moving orders? Locally, answers Einstein. From a distance, answers Newton.

- Taylor and Wheeler, *Spacetime Physics (1996)*

To try to set up an all-encompassing Euclidean reference frame and attempt to refer motion to it is the wrong way to do physics. Don't try to describe motion relative to far away objects. *Physics is simple only when analyzed locally.*

- Taylor and Wheeler, *Spacetime Physics (1996)*

What could be simpler than the moving orders for mass: "Follow a straight line in the local inertial reference frame." Does a satellite have to know the location of the earth and the moon and the sun before it knows how to move? Not at all... it only has to sense the local structure of spacetime - right where it is - in order to follow the correct track.

- Taylor and Wheeler, *Spacetime Physics (1996)*

More sensitive apparatus detects the "tide-producing action" of gravity - the accelerating shortening of separations parallel to the earth's surface, the accelerating lengthening of vertical separations.

- Taylor and Wheeler, *Spacetime Physics (1996)*

These relative accelerations are doubled when the separations are doubled.

- Taylor and Wheeler, *Spacetime Physics (1996)*

All this talk about the identity of "gravitational mass" and "inertial mass" completely obscures the truth. Curvature and nothing more is all that is required to describe the increasing rate at which A and B approach each other.

- Taylor and Wheeler, *Spacetime Physics (1996)*

This effect is better described by the phrase "tidal field" than by the ambiguous phrase "gravitational field": "tidal" because the *relative* acceleration of water particles on opposite sides of the earth, caused by the moon, shows itself in tides.

- Taylor and Wheeler, *Spacetime Physics (1996)*

Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but simply because man finds it difficult to outgrow established ways of looking at nature.

- Taylor and Wheeler, *Spacetime Physics (1996)*

The definition of an inertial frame requires that *no gravitational fields will be felt in it.*

- Taylor and Wheeler, *Spacetime Physics (1996)*

Inertial frames of reference are always *local* ones, that is, inertial in a limited region of spacetime.

- Taylor and Wheeler, *Spacetime Physics (1996)*

As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed one-tenth meter from it and initially at rest will, in less than three minutes, undergo a displacement of 1 millimeter.

- Taylor and Wheeler, *Spacetime Physics (1996)*

All the laws of physics are the same in every inertial reference frame. We call this statement the *principle of relativity*.... Both the *form* of the laws of physics and the *numerical values of the physical constants* that these laws contain are the same in every inertial reference frame.... Expressed in negative terms, the principle of relativity says that *the laws of physics cannot provide a way to distinguish one inertial reference frame from another*....

- Taylor and Wheeler, *Spacetime Physics (1996)*

Physical values differ in *value* between the two frames but fulfill identical *laws*.

- Taylor and Wheeler, *Spacetime Physics (1996)*

Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the "jungle gym" seen on playgrounds. At every intersection of this latticework fix a clock.

- Taylor and Wheeler, *Spacetime Physics (1996)*

In relativity we often speak about "the observer". Where is this observer? At one place or all over the place? *The word "observer" is a shorthand way of speaking about the whole collection of recording clocks* associated with one inertial frame of reference. No one real observer could easily do what we ask of the "ideal observer" in our analysis of relativity. So it is best to think of the observer as the man who goes around picking up the punched cards turned out by all the recording clocks in his employ. This is the sophisticated sense in which we will be hereafter using the phrase "the observer finds such and such."

- Taylor and Wheeler, *Spacetime Physics (1996)*

Proper time is to a world line in Lorentz geometry what length is to a path in Euclidean geometry.

- Taylor and Wheeler, *Spacetime Physics (1996)*

Today we have learned not to overstate Minkowski's argument. It is right to say that time and space are inseparable parts of a larger unity. It is wrong to say that time is identical in quality with space.

- Taylor and Wheeler, *Spacetime Physics (1996)*

The portion "variant" of the adjective "covariant" indicates that the coordinates vary from one reference system to another. The prefix "co" implies a coordinated variation of the coordinates of all events according to the same law.

- Taylor and Wheeler, *Spacetime Physics (1996)*

The *4-vector* that leads from A to B is the unifying idea. The components of this 4-vector, the displacements dx, dy, dz, and dt....

- Taylor and Wheeler, *Spacetime Physics (1996)*

Does Einstein's statement that mass and energy are equivalent mean that energy is the *same* as mass? No.... The distinction between mass and energy is this: mass measures the magnitude of a 4-vector and energy measures the time component of the *same* 4-vector.

- Taylor and Wheeler, *Spacetime Physics (1996)*

The concept of "relativistic mass" is subject to misunderstanding and is not used here.

- Taylor and Wheeler, *Spacetime Physics (1996)*

It is an empirical fact that the 'missing mass problem', usually interpreted as observational evidence for dark matter, only occurs when the gravitational acceleration falls below a certain critical value that is of the order of a0 [the Hubble acceleration scale].

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

The appearance of the cosmological acceleration scale a0 in galactic dynamics is striking and gives a strong hint towards an explanation in terms of emergent gravity, as envisaged in [38].

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

Our goal is to give a theoretical explanation for why the emergent laws of gravity differ from those of general relativity precisely when the inequality (1.4) is obeyed. We will find that this criterion is directly related to the presence of the volume law contribution to the entanglement entropy. At scales much smaller than the Hubble radius gravity is in most situations well described by general relativity, because the entanglement entropy is still dominated by the area law of the vacuum. But at large distances and long time scales the enormous de Sitter entropy in combination with the extremely slow thermal dynamics lead to modifications to these familiar laws. We will determine these modifications and show that precisely when the surface mass density falls below the value (1.3) the reaction force due to the thermal contribution takes over from the 'normal' gravity force caused by the area law.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

The central idea of this paper is that the volume law contribution to the entanglement entropy, associated with the positive dark energy, turns the otherwise "stiff" geometry of spacetime into an elastic medium.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

Hence, although we derived the same relation as modified Newtonian dynamics, the physics is very different.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

For these reasons proponents of modified newtonian dynamics still have to assume a form of particle dark matter at the cluster scale. These discrepancies can be significantly reduced and perhaps completely explained away in our theoretical description.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

Most of the baryonic mass in clusters is contained in X-ray emitting gas, which extends all the way to the outer parts of the cluster.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

This means that the 'missing mass problem' in clusters is significantly reduced and given the uncertainty about the amount of baryons, possibly entirely removed.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

Since up to now there appeared to be no evidence that general relativity or Newtonian gravity could be wrong at the scales in question, the most generally accepted point of view is that these observations indicate that our universe contains an enormous amount of a yet unknown form of dark matter particle. However, the discrepancy between the observed gravitational force and the one caused by the visible baryonic matter is so enormous that it is hard to claim that these observations provide evidence for the validity of general relativity or Newtonian gravity in these situations. Purely based on the observations it is more appropriate to say that these familiar gravitational theories can only be saved by assuming the presence of dark matter. Therefore, without further knowledge, the evidence in favour of dark matter is just as much evidence for the possible breakdown of the currently known laws of gravity.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

The real reason why most physicists believe in the existence of particle dark matter is not the observations, but because there was no theoretical evidence nor a conceptual argument for the breakdown of these laws at the scales where the new phenomena are being observed. It has been the aim of this paper to provide a theoretical and conceptual basis for the claim that this situation changes when one regards gravity as an emergent phenomena. We have shown that the emergent laws of gravity, when one takes into account the volume law contribution to the entropy, start to deviate from the familiar gravitational laws precisely in those situations where the observations tell us they do. We have only made use of the natural constants of nature, and provided reasonably straightforward arguments and calculations to derive the scales and the behavior of the observed phenomena. Especially the natural appearance of the acceleration scale a0 should in our view be seen as a particularly convincing aspect of our approach.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

Our main conclusion therefore is: The observed phenomena that are currently attributed to dark matter are the conesquence of the emergent nature of gravity and are caused by an elastic response due to the volume law contribution to the entanglement entropy in our universe.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

In order to explain the observed phenomena we did not postulate the existence of a dark matter particle, nor did we modify the gravitational laws in an ad hoc way. Instead we have to tried to understand their origin and their mutual relation by taking seriously the theoretical indications coming from string theory and black hole physics that spacetime and gravity are emergent. We believe this approach and the results we obtained tell us that the phenomena associated with dark matter are an unavoidable and logical consequence of the emergent nature of spacetime itself. The net effect should be that in our conventional framework one has to add a dark component to the stress energy tensor, which behaves very much like the cold dark matter needed to explain structure formation, but which in its true origin is an intrinsic property of spacetime rather than being caused by some unknown particle. Indeed, we have argued that the observed dark matter phenomena are a remnant, a memory effect, of the emergence of spacetime together with the ordinary matter in it.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

In particular, we have made clear why the apparent dark matter behaves exactly in the right way to explain the phenomenological success of modified Newtonian dynamics, as well as its failures, without the introduction of any freely adjustable parameters. We have found that in many, but not all, aspects the apparent dark matter behaves similar to as one would expect from particle dark matter.

- Eric Verlinde, *Emergent Gravity and the Dark Universe (2016)*

After learning the theory, one cannot help feeling that one has gained some deep insights into how nature works.

- Robert Wald, *General Relativity (1984)*

The notion that there is absolute simulaneity is a deeply ingrained one. The fact that there is no such notion is one of the most difficult ideas to adjust to in the theory of special relativity.

- Robert Wald, *General Relativity (1984)*

An inertial observer can label the events of spacetime in the following manner. He can build himself a rigid frame and label the grid points of the frame with the Cartesian coordinates x, y, z of the (assumed Euclidean) geometry of the frame. He can then have a clock placed at each grid point and synchronize each clock with his by a symmetrical procedure, e.g., by making sure that a given clock and his give the same reading when they receive a signal sent out in a symmetrical manner by an observer stationed halfway between the two.... The observer may carry the grid, complete with synchronized clocks, in a nonrotating manner. Each event in spacetime can now be labeled with the coordinates x, y, z of the grid point at which the event occurred and the reading t of the (synchronized) clock at that event.

- Robert Wald, *General Relativity (1984)*

Thus, the theory of special relativity asserts that *spacetime is the manifold R^4 with a flat metric of Lorentz signature defined on it.*

- Robert Wald, *General Relativity (1984)*

In the context of special relativity, the principle of general covariance states that the spacetime metric is the only quantity pertaining to spacetime structure which can appear in any physical law. This principle is believed to apply to the laws of physics in special relativity with one important modification. Experiments demonstrating parity violation and (indirectly) demonstrating the failure of time reversal symmetry have shown that two further aspects of spacetime structure can appear in physical laws: the time orientation and space orientation of spacetime.

- Robert Wald, *General Relativity (1984)*

Different timelike curves connecting the same pair of events may have different elapsed times (the "twin paradox"), just as different paths between two points of space can have different lengths. As discussed in section 3.3, the maximum elapsed time between two events is given by geodesic (i.e., inertial) motion.

- Robert Wald, *General Relativity (1984)*

As a result, we have no meaningful way of describing a gravitational force field; rather, we are forced to view gravity as an aspect of spacetime structure. Although absolute gravitational force has no meaning, the relative gravitational force (i.e. tidal force) between two nearby points still has meaning and can be measured by observing the relative acceleration of two freely falling bodies. This relative acceleration is directly related to the curvature of spacetime by the geodesic deviation equation.

- Robert Wald, *General Relativity (1984)*

The framework of general relativity permits the Lorentz metric *g* of spacetime to be curved. Indeed, it asserts that spacetime must be curved in all situations where, physically, a gravitational field is present. Since we are allowing curved geometries, it is also much more natural to allow spacetime to have a manifold structure M other than R^4. Hence, in general relativity we place no *a priori* restriction on the spacetime manifold.

- Robert Wald, *General Relativity (1984)*

Perhaps, then, the paths of freely falling bodies are always geodesics, but the spacetime metric is not always that given by special relativity. What we think of as a gravitational field would then not be a new field at all, but rather would correspond to a deviation of the spacetime geometry from the flat geometry of special relativity.

- Robert Wald, *General Relativity (1984)*

The laws of physics in general relativity are governed by two basic principles: (1) the principle of general covariance... which states that the metric g and quantities derivable from it are the only spacetime quantities that can appear in the equations of physics; (2) the requirement that equations must reduce to the equations satisfied in special relativity in the case where g is flat. As mentioned previously, the first principle is imprecise because the term "spacetime quantity" is not well defined. As will be illustrated below, these two principles alone do not uniquely determine the laws of physics in general relativity. However, together with simplicity and aesthetics, they serve as general guides which, in many cases, lead directly to natural candidates for physical laws.

- Robert Wald, *General Relativity (1984)*

The entire content of general relativity may be summarized as follows: Spacetime is a manifold M on which there is defined a Lorentz metric *g*. The curvature of *g* is related to the matter distribution in spacetime by Einstein's equation.

- Robert Wald, *General Relativity (1984)*

Thus, in general relativity, one must solve simultaneously for the spacetime metric and the matter distribution.

- Robert Wald, *General Relativity (1984)*

Note, however, that bodies which are "large" enough to feel the tidal forces of the gravitational field will deviate from geodesic motion.

- Robert Wald, *General Relativity (1984)*

The issue of energy in general relativity is a rather delicate one. In general relativity there is no known meaningful notion of local energy density of the gravitational field.

- Robert Wald, *General Relativity (1984)*

After Hubble's redshift observations in 1929 demonstrated the expansion of the universe, the original motivation for the introduction of lambda was lost. Nevertheless, lambda has been reintroduced on numerous occasions when discrepancies have arisen between theory and observations, only to be abandoned again when these discrepancies have been resolved.

- Robert Wald, *General Relativity (1984)*

For many years it was generally believed that the prediction of a singular origin of the universe was due merely to the assumptions of exact homogeneity and isotropy, that if these assumptions were relaxed one would get a non-singular "bounce" at a small *a* rather than a singularity. However, the singularity theorems of general relativity show that singularities are generic features of cosmological solutions; they have ruled out the possibility of "bounce" models close to the homogeneous, isotropic models. Of course, at the extreme conditions very near the big bang singularity one expects that quantum effects will become important, and the predictions of classical general relativity are expected to break down.

- Robert Wald, *General Relativity (1984)*

In general relativity, the causal structure of spacetime is *locally* of the same qualitative nature as in the flat spacetime of special relativity. However, significant differences can occur globally because of nontrivial topology, spacetime singularities, or the "twisting" of the direction of light cones as one moves from point to point.

- Robert Wald, *General Relativity (1984)*

These [singularity] theorems prove that singularities *are* true, generic features of cosmological and collapse solutions. Although they give very little information concerning the detailed structure of these singularities, they show that models such as the nonsingular "bouncing universe" are incompatible with general relativity provided only that certain energy conditions are satisfied by matter and several other conditions hold in the spacetime.

- Robert Wald, *General Relativity (1984)*

Although the singularity theorems do not prove that the singularities of classical general relativity must involve unboundedly large curvature, they strongly suggest the occurrence in cosmology and gravitational collapse of conditions in which quantum or other effects which invalidate classical general relativity will play a dominant role.

- Robert Wald, *General Relativity (1984)*

Intuitively, a spacetime singularity is a "place" where the curvature "blows up" or other "pathological behavior" of the metric takes place. The difficulty in making this notion into a satisfactory, precise definition of a singularity stems from the above terms placed in quotes.

- Robert Wald, *General Relativity (1984)*

Thus, the "big bang" singularity of the Robertson-Walker solution is not considered to be part of the spacetime manifold; it is not a "place" or a "time." Similarly, only the region r>0 is incorporated into the Schwarzschild spacetime... the singularity at r=0 is not a "place."

- Robert Wald, *General Relativity (1984)*

Remarkably, it has been possible to show that the two-parameter family of solutions - characterized by total mass M, and total angular momentum J - found by Kerr (1963) are the only vacuum solutions of Einstein's equations describing stationary black holes.

- Robert Wald, *General Relativity (1984)*

A surprising development in the theory of black holes was the discovery that energy can be extracted from a "rotating" black hole, i.e., a Kerr black hole with non-zero J. Although nothing can escape from a black hole, it is possible to make a black hole "swallow" a particle or wave with negative energy.

- Robert Wald, *General Relativity (1984)*

Finally, one of the most intriguing aspects of the theory of black holes is the analogy between the laws of black hole physics and the ordinary laws of thermodynamics. At first sight, it would appear that the nature of these laws hardly could be more different. The laws of black hole physics are rigourous theorems in differential geometry, while the laws of thermodynamics are only macroscopic approximations to complicated, exact microscopic laws of physics. Nevertheless, a remarkably close mathematical analogy exists between these laws, highlighted by the analogy between the law of area increase for black holes and the law of entropy increase for thermodynamic systems.

- Robert Wald, *General Relativity (1984)*

Cosmic Censorship Conjecture (version 1; physical formulation). The complete gravitational collapse of a body always results in a black hole rather than a naked singularity; i.e., all singularities of gravitational collapse are "hidden" within black holes, where they cannot be "seen" by distant observers.

- Robert Wald, *General Relativity (1984)*

The issue of whether the cosmic censorship hypothesis is correct remains the key unresolved issue in the theory of gravitational collapse. The physical relevance of black holes depends in large measure on the validity of this conjecture.

- Robert Wald, *General Relativity (1984)*

Cosmic Censorship Conjecture (version 2; physical formulation). All physically reasonable spacetimes are globally hyperbolic, i.e., apart from a possible initial singularity (such as the "big bang" singularity) no singularity is ever "visible" to any observer.

- Robert Wald, *General Relativity (1984)*

Indeed, except for the singularity theorems, very little is known about the general, global properties of solutions of Einstein's equations.

- Robert Wald, *General Relativity (1984)*

It should be noted that black holes formed by stellar collapse must lie in the relatively narrow mass range of of 2-100 solar masses since stars with lower mass should not collapse, while ordinary stars with higher mass do not exist on account of pulsational instabilities.

- Robert Wald, *General Relativity (1984)*

In chapter 9, we proved that if a trapped surface is present in a spacetime, appropriate energy conditions are satisfied by matter, and a number of further hypotheses hold, then a singularity must occur. Thus, trapped surfaces are associated with singularities. ... All trapped surfaces must be entirely contained within black holes.

- Robert Wald, *General Relativity (1984)*

Hence, in astrophysically reasonable situations it appears that e<<M, so we may neglect the effects of the electromagnetic field on the spacetime geometry and consider only the Kerr family of black holes.

- Robert Wald, *General Relativity (1984)*

...we interpret the true singularity at sigma=0 as a ring singularity, i.e., that we define the charged Kerr metrics on a manifold whose structure in a neighbourhood of this singularity has the topology of R4 with the set S1xR - that is, a ring, S1, cross "time", R - removed.

- Robert Wald, *General Relativity (1984)*

Thus, closed timelike curves exist in a neighbourhood of the ring singularity... In this case, the ring singularity is "naked", i.e., the charged Kerr metrics fail to be strongly asymptotically predictable, and thus they do *not* describe black holes. Furthermore, one may make use of the causality violation occurring near the ring singularity to go "backwards in time" by an arbitrarily large amount as measured by the *t* coordinate... and thereby produce closed timelike curves passing through any point in the spacetime.

- Robert Wald, *General Relativity (1984)*

In other words, all observers in the ergosphere are forced to rotate in the direction of rotation of the black hole. This may be viewed as an extreme case of how some aspects of Mach's principle are incorporated into general relativity.

- Robert Wald, *General Relativity (1984)*

Remarkably, as a result of theorems of Israel, Carter, Hawking, and Robinson obtained between 1967 and 1975, a virtually complete proof of has been given that the Kerr black holes are the only possible stationary vacuum black holes. Thus, if the first cosmic censorship hypothesis is correct and if the spacetime resulting from gravitational collapse always "settles down" to a stationary, vacuum final state, the end product of collapse must always be a Kerr black hole. The complete gravitational collapse of two bodies differing greatly from each other in composition, shape, and structure will produce indistinguishable final states provided only that their end products have the same total mass and total angular momentum.

- Robert Wald, *General Relativity (1984)*

The area theorem states that in any physically allowed process, the total area of all black holes in the universe cannot decrease.

- Robert Wald, *General Relativity (1984)*

Similarly, in a quantum theory of gravitation based on general relativity, one would expect that the fundamental scale at which the classical description becomes wholly inadequate should be set by h-bar, c, and the gravitational constant G. There is a unique combination of these constants which has the dimensions of length... called the *Planck length.* As might be expected, the Planck length arises naturally in attempts to formulate a quantum theory of gravity. Thus, dimensional arguments suggest that a classical description of spacetime structure should break down at scales of the order of the Planck length and smaller.

- Robert Wald, *General Relativity (1984)*

In cgs units the Planck length is only 10^-33 cm. The smallness of [the Planck length] compared with typical length scales occurring in atomic, nuclear, and elementary particle physics is directly related to the weakness of the gravitational force between two elementary particles as compared with other fundamental forces...

- Robert Wald, *General Relativity (1984)*

Thus, it is quite possible that a quantum theory of gravitation may even play an important role in the unification of the strong and electroweak interactions. A unified theory of all forces undoubtedly would yield many new predictions of phenomena at presently observable scales.

- Robert Wald, *General Relativity (1984)*

The essential difference between general relativity and other classical theories appears to be the dual role played by the field g as both the quantity which describes the dynamical aspects of gravity and the quantity which describes the background spacetime structure.

- Robert Wald, *General Relativity (1984)*

Nevertheless, the existence of the laws of black hole thermodynamics indicates the likelihood of a deep connection between gravitation, quantum theory, and statistical physics. It remains for future investigations to explore this connection further.

- Robert Wald, *General Relativity (1984)*

If one takes the time to teach the mathematical material properly, one runs the risk of turning the course into a course on differential geometry and doing very little physics. On the other hand, if one does not teach it properly, then one is greatly handicapped in one's ability to explain the major conceptual differences between general relativity and the pre-relativitistic and special relativistic notions of spacetime structure.

- Robert Wald, *Teaching General Relativity (2008)*

Prior to 1905, it was taken for granted that the causal structure of spacetime defines a notion of *simultaneity*...It is important to emphasize to students the key role of this assumption in pre-relativistic notions of spacetime structure.

- Robert Wald, *Teaching General Relativity (2008)*

The major revolution introduced by special relativity is largely premised on the fact that the assertions of the previous paragraph concerning the causal structure of spacetime are wrong. Most strikingly, the set of events that fail to be causally connected to an event A comprise much more than a 3-dimensional region.

- Robert Wald, *Teaching General Relativity (2008)*

The geometry required for an understanding of general relativity is simply the generalization of Riemannian geometry to metrics that are not positive-definite. Fortunately, there are few significant mathematical changes that result from this generalization. Consequently, much of the intuition that most people have for understanding the Riemannian geometry of two-dimensional surfaces encountered in everyday life - such as the surface of a potato - can usually be extended to general relativity in a reliable manner.

- Robert Wald, *Teaching General Relativity (2008)*

This situation changes dramatically in general relativity, since the vector space character of space and/or spacetime depends crucially on having a flat geometry. In general relativity, it does not make any more sense to "add" two events in spacetime than it would make sense to try to define a notion of addition of points on the surface of a potato.

- Robert Wald, *Teaching General Relativity (2008)*

The appropriate notion is that of a manifold, which is a set that locally "looks like" Rn with respect to differentiability properties, but has no metrical or other structure. The points of an n-dimensional manifold can thereby be labeled locally by coordinates [x] but these coordinate labels are arbitrary and could equally well be replaced by any other coordinate labels [x'] that are related to [x] in a smooth, nonsingular manner. A precise definition of an n-dimensional manifold can be given as a set that can be covered by local coordinate systems that satisfy suitable compatibility conditions in the overlap regions.

- Robert Wald, *Teaching General Relativity (2008)*

Many treatments of general relativity effectively bypass the above treatment of tensors by working only with the components of tensors in bases associated with coordinate systems....It has the advantage that one can then quickly move on to other topics without spending much time talking about tensors. However, it has the obvious disadvantage that although students may still be trained to use tensors correctly in calculations, they usually end up having absolutely no understanding of what they are.

- Robert Wald, *Teaching General Relativity (2008)*

It is worth noting that the same relation between symmetries and conservation laws that one has in Lagrangian mechanics (namely, Noether's theorem) then automatically applies to geodesics, so in a spacetime with a sufficiently high degree of symmetry, one can actually solve the geodesic equation (or, more precisely, "reduce it to quadratures") using only constants of motion.

- Robert Wald, *Teaching General Relativity (2008)*

The main price paid by presenting the mathematical material in this way is a sacrifice of clarity in explaining the fundamental conceptual basis of general relativity - particularly its difference from all prior theories with regard to the nonexistence of any non-dynamical background structure of spacetime - since this conceptual basis is very difficult to understand if one does not formulate the theory in a coordinate independent way. In addition, students will not have the necessary mathematical tools to advance their study of general relativity to topics involving "global methods" - such as the singularity theorems and the general theory of black holes - where it is essential that the concepts be formulated in a coordinate independent way.

- Robert Wald, *Teaching General Relativity (2008)*

It is widely held that in 1929 Edwin Hubble discovered the expanding universe and that his discovery was based on his extended observations of redshifts in spiral nebulae. Both statements are incorrect... There is a great irony in these falsehoods still being promoted today. Hubble himself never came out in favor of an expanding universe; on the contrary, he doubted it to the end of his days.

- Way and Nussbaumer, *Letter to Physics Today (2011)*

It should be stressed that general covariance is empty of physical content. Any equation can be *made* generally covariant by writing it in any one coordinate system, and then working out what it looks like in other arbitrary coordinate systems...The significance of the Principle of General Covariance lies in its statement about the effects of gravitation, that a physical equation by virtue of its general covariance will be true in a gravitational field if it is true in the absence of gravitation.

- Steven Weinberg, *Gravitation and Cosmology (1972)*

The meaning of general covariance can be brought forward by comparing it with Lorentz invariance. Just as any equation can be made generally covariant, so any equation can be made Lorentz-invariant, by writing it in one coordinate system and then working out what it looks like after a Lorentz transformation. However, if we do this with a nonrelativistic equation like Newton's second law, we find that after making it Lorentz-invariant that a new quantity has entered the equation, which of course is the velocity of the coordinate frame with respect to the original reference frame. The requirement that this velocity *not* appear in the transformed equation is what we call the Principle of Special Relativity, or "Lorentz invariance" for short, and this requirement places a very powerful restriction on the original equation. Similarly, when we make an equation generally covariant, new ingredients will enter, that is, the metric tensor and the affine connection. The difference is that we do not require that these quantities drop out at the end, and hence we do not obtain any restrictions on the equation we start with; rather, we exploit the presence of the [metric tensor and affine connection] to represent gravitational fields. To put this briefly: The Principle of General Covariance is *not* an invariance principle, like the Principle of Galilean or Special Relativity, but is instead a statement about the effects of gravitation, and about nothing else. In particular, general covariance does not imply Lorentz invariance....

- Steven Weinberg, *Gravitation and Cosmology (1972)*

However, the reader should be warned that not everything is a tensor; in particular, the affine connection, despite its appearance, is *not* a tensor.

- Steven Weinberg, *Gravitation and Cosmology (1972)*

We must first write the equations as they hold in special relativity, then decide how each quantity in the equations is to transform under general coordinate transformations, and then replace eta with g and all derivates with covariant derivatives. The resulting equations will be generally covariant and true in the absence of gravitation, and hence true in arbitrary gravitational fields, provided that the system in question is small enough compared with the scale of the fields.

- Steven Weinberg, *Gravitation and Cosmology (1972)*

We next prove that [the curvature tensor] is the *only* tensor that can be constructed from the metric tensor and its first and second derivatives, and is linear in the second derivatives.

- Steven Weinberg, *Gravitation and Cosmology (1972)*

Maxwell's equations are linear because the electromagnetic field does not itself carry charge, whereas gravitational fields do carry energy and momentum and must therefore contribute to their own source. That is, the gravitational field equations will have to be nonlinear partial differential equations, the nonlinearity representing the effect of gravitation on itself.

- Steven Weinberg, *Gravitation and Cosmology (1972)*

The scope and power of this century's view of gravity and spacetime is seen nowhere more dramatically than in its prediction of the expansion of the universe. To have predicted, and predicted successfully against all expectation, a phenomenon so fantastic is the greatest token yet of our power to understand this strange and beautiful universe.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Gauss had found that the curvature in the neighborhood of a given point of a specified two-dimensional space geometry is given by a single number. Riemann found that six numbers are needed to describe the curvature of a three-dimensional space at a given point, and that 20 numbers are required for a four-dimensional geometry.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Why then did he [Riemann] not, half a century before Einstein, arrive at a geometric account of gravity? No obstacle in his way was greater than this: he thought only of space and the curvature of space, whereas Einstein, as we will soon learn, discovered that he had to deal with spacetime and the curvature of spacetime.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Two ideas are central to Einstein's conception of gravity. The first is free-float. The second is spacetime curvature.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

General relativity is the name he gave to the later idea that gravitational fall is really free float. Today it is more natural to describe his teaching as Einstein's geometric theory of gravity.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

That innocent but ambiguous term [observer] has created endless confusion for more than half a century. It fails to distinguish between two totally different concepts of observer. One is right. The other is wrong. The camera epitomizes the wrong idea - wrong because it is not located where the action is.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Felix Klein (1849-1925) enunciated in his 1872 lectures at the University of Erlangen what was to prove one of the most fruitful theses in all of mathematics: one kind of geometry is distinguished from another by the kinds of quantities that are invariant in it.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The right order of ideas is not first frames of reference, then spacetime. The right order is first spacetime, then frames of reference.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Null interval is a feature of the geometry of spacetime totally different from anything in the geometry of space.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

...a straight line makes the total distance between two points the shortest. Spacetime has quite a different geometry: a straight worldline makes the total interval between two events the longest.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

But when a piece of iron is heated, the zigzag speed of the atoms increases; their worldlines kink more sharply. So the atoms in a heated piece of iron rack up fewer clicks in a second of laboratory time than do the ones in a colder piece. And that's not talk. That's fact. Warming iron by just 1 degree Celcius cuts down by 7700 the number of clicks ticked off by each internal iron-atom clock during a laboratory second....Pound and Rebka were able to measure that difference.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

With speed close enough to the speed of light, we can make the aging factor so small that while the Earth and stars age 50 billion years, we age 40 years.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The interval is the building strut of flat spacetime, the local spacetime of a local free-float frame. It is also the building strut of curved spacetime, spacetime in a region where gravity is strong enough to make itself felt.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

And of curvature in a given locale, we need not a hundred but only six views. Those views - each of them a two-dimensional slice of the full four-dimensional spacetime - I call *plaquettes.* Why are there six views, six natural choices of plaquette? Because from a spacetime of four dimensions, there are six ways to select out two of those four dimensions for special attention.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Like a great new sculpture, so curvature preserves its unity, strength, and beauty as we detect and measure it from three quite different standpoints, called *bending*, *transport*, and *distance.*

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The difference in spacetime curvature inside and outside the Earth is equally striking. Inside, contractile. Outside, noncontractile. Noncontractile but *tide-driving*!

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

In brief, where there's mass, there curvature is contractile....Contractility there is proportional to mass density there.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The tide-driving curvature of spacetime grows ever weaker the more distant from the central mass. Why? It grows weaker by *dilution*...Twice the distance from the center of the mass? Then the volume over which the influence is felt is doubled in extent in all three space dimensions; the volume is enlarged by a factor of 8. At that distance, spacetime bending will be diluted to one-eigth of what it is at the Earth's surface. More generally, and in keeping with this reasoning, the curvature falls off inversely as the cube of the distance.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Of Kepler Galileo even said, "More than other people he was a person of independent genius... [but he] later pricked up his ears and became interested in the action of the moon on the water, and in other occult phenomena, and similar childishness."

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Words promote insight.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Like the time-like interval separating the positions of two events in spacetime, the length of the momenergy arrow is invariant, that is, independent of the reference frame. This length measures the object's mass.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

However, the increase in the internal energy of agitation [heat] - and therefore in mass - is too small even for today's best instruments to detect.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

An important principle teaches us something new every time we learn to state it in a new way.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Conservation of momenergy is automatic because the forty-eight 2-D face-boundaries of the eight 3-D cube-boundaries of any 4-D spacetime region self-annihilate. Within a 4-D cube or block of spacetime - a region of space examined for an interval of time - momenergy is automatically conserved.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The French mathematician and geometer Élie Cartan (1969-1951) used his calculus of "exterior differential forms" to derive the simple Einstein-Cartan equation from Einstein's much more complicated geometrodynamic field equation. Cartan's boundary of a boundary geometric approach to Einstein's theory provides a wonderful simplicity.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Einstein's influence law thus displays not only the grip of mass where it is on spacetime where it is. It also tells us how bent spacetime here grabs adjacent spacetime. Even beyond the grip of mass on spacetime, the moment-of-rotation equation reveals and rules the grip of spacetime on mass. Grip *on* mass? Yes, reaction to the grip *of* mass! That's the other half of the story of gravity. That grip on mass enforces the law of conservation of momenergy. Enforces it automatically. Enforces this conservation in a subtle, clever, hidden way. Enforces it via the principle that the 2-boundary of a 3-boundary of a 4-D spacetime region is zero!

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

In that way the grip of spacetime, there and likewise anywhere, forever bars any creation - or destruction - of momenergy anywhere in spacetime.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The importance of Schwarzschild geometry derives from the tendency of matter to agglomerate into masses that are spherical or nearly so. Its simplicity derives from its static character and its spherical symmetry.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The shape of the Schwarzschild curved space geometry follows a remarably simple curve, discoverable by astonishingly simple reasoning. It is generated by drawing a *parabola* about its *directrix*, another name for the axis of revolution. By this means is created a curved sheet called a paraboloid.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Aging incurred between the stone's takeoff and its return evidently registers the competition between two influences: first, the age lengthening effect of being up there and, second, the age shortening effect of traveling there and back.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

In Newtonian theory, the shape of the energy hill [in the two-body problem] is a miracle. It guarantees that the time for one in-and-out oscillation always agrees exactly with the time for one full 360-degree or 2-pi turn of the orbit. [In geometrodynamics] it takes more than the time for one 360-degree turn to go from outer extremity of the orbit to inner extremity and return. The curve does not close.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Second, however far this traveling region of spacetime curvature has progressed, there the measure of the deformation it makes in spacetime geometry has fallen off in proportion, not to the inverse square of the distance of travel away from the source, but to the *inverse first power* of this distance.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Then I encountered in J. Arthur Thompson the statement that the electric force in an electromagnetic wave points, not straight out from the center of attraction, but transverse to that direction, and with a strength proportional to the inverse first power of the distance.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Almost every feature of an electromagnetic wave transcribes itself into a corresponding feature of a gravity wave.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

In other words, if Alpha plunges down a boomerang shaft straight from the North Pole and at Earth's center smashes into a like mass Beta zooming straight up from the South Pole, observer's on Earth's surface will find the resulting pulse of the gravitational radiation strongest in the temperate latitudes of the northern and southern hemispheres but zero at the poles and zero at the equator. One final feature about the response of the two test masses is important. Does their line of separation lie in the plane of the two reference directions [the line connecting the poles, and the line connecting the test particles to the center of the Earth]? Or at right angles to that plane? In the two cases the accelerations are equal in magnitude but opposite in sign.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Radiation demands more, a fourth time rate of change of reduced quadrupole moment: that is, a steady rate of change of rate of change of rate of change of rate of change of mass nonsphericity!

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

...the decisive measure of source strength is not the quadrupole moment itself, bit its fourth time rate of change!

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Other stars contain both matter and mass. In contrast, the black hole is disembodied mass, mass without matter.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The advent of the term *black hole* in 1967 was terminologically trivial but psychologically powerful.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Angular momentum, for example, can take on only one or another integer multiple of a basic unit of angular momentum. ...In geometric units, where we express both mass and time in meters, this elementary quantum of angular momentum turns out to have the dimension of an area...(the square of the so-called Planck length). This tiny area is an elementary quantum of area....[The entropy of a black hole], a pure number, the so-called Bekenstein number, is exactly the surface area of the horizon of the black hole, divided by 4 times the elementary quantum of area.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Bekenstein went on to explain that the surface area of a black hole is not only analogous to entropy, it is entropy, and the surface gravity of a black hole (measured, for example, by the downward acceleration of a rock as it crosses the horizon) is not only analogous to temperature, it is temperature. A black hole is not totally cold!

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Of all methods to convert bulk matter into energy, no one has ever seen evidence for a more effective process than accretion into a black hole, and no one has ever come up with a more feasible scheme for one.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

In the expanding universe, *only* the distance between one cluster of galaxies and another expands. Atoms don't expand. Meter sticks don't expand. We don't expand.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

Such an expansion had already been predicted by Alexander Friedmann in 1922 on the foundation of Einstein's original simple theory.

- John Wheeler, *A Journey Into Gravity and Spacetime (1990)*

The spatial separation of events is the same for all [inertial] observers... [false; he confuses event pairs with stick lengths.]

- W. S. C. Williams, *Introducing Special Relativity (2002)*

There is one aspect of aberration that can be a source of confusion. Light is often observed by eye, either directly or through a telescope, for example. The line-of-sight, or the direction of pointing of the telescope, is directly opposite to that of propagation of the detected photons. The formulae derived and given in Figure 6.3 contain angles defined by the direction of propagation with respect to the boost, not by the line of sight. [Two forms of the aberration formula.]

- W. S. C. Williams, *Introducing Special Relativity (2002)*

The journal *Nature* was the forum for a controversy about time dilation and the clock paradox in 1956. The matter reappeared, again in *Nature*, in 1974-5, sustained by one fierce critic of special relativity.

- W. S. C. Williams, *Introducing Special Relativity (2002)*

However, we have seen that nearly half a century passed before those who doubted Einstein fell silent, at least in the scientific literature.

- W. S. C. Williams, *Introducing Special Relativity (2002)*

...infinity is not suitable for the human mind. In theoretical physics, we must have cutoffs.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

For me (and of course also for many other theoretical physicists), the most puzzling aspect of Einstein gravity is its ability to alter the causal structure of spacetime completely.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

...in some alternative history of physics, the existence of a spin-1/2 particle could have been another famous prediction of special relativity, like E=mc^2.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

I am particularly respectful of, perhaps even awed by, the action principle. It is truly amazing that, while many phenomenological theories cannot be derived from an action, all the fundamental interactions we know - gravity, strong, weak, and electromagnetic - can be.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The two older physicists [Lorentz and Poincaré] were not able to abandon the perfectly sensible notion that if there is a wave, something must be waving....So they had the ether as a dynamic variable. Einstein simply trashed the ether and asserted that nothing could also wave.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The horizon is an inherently nonlocal concept.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

In spacetime, space and time are unified, but time... appears to be first among equals.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Symmetry and the action principle constitute the two great themes of theoretical physics.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

In teaching physics, I sometimes feel, with only slight exaggeration, that students are confused by bad notation almost as much as by the concepts.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Take a look at the 19th century literature, before the use of indices became widespread. I am always amazed by the fact that, for example, Maxwell could see through the morass of the electromagnetic equations written out component by component.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

A tensor is something that transforms like a tensor.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The central message here is that coordinates do not have intrinsic geometric significance.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

I like to think of [displacement and gradient] as the two "primeval" vectors.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

This is Noether's Theorem, proved in that momentous year for physics 1915: for every transformation that leaves the Lagrangian unchanged, there is a conserved quantity.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Mysteriously, all of fundamental physics is governed by the action, from which the equations of motion follow.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The twin "paradox" is resolved by pointing out that it is not a paradox at all. In ordinary space, nobody claims that the lengths of different paths connecting two points are necessarily the same... The lengths of different paths connecting two points in spacetime can of course be different.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Although simultaneity fails, causality still holds.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Then the action does not change under the transformation in (2), known as a gauge transformation. We have discovered a hidden symmetry of the action, called gauge invariance. Strictly speaking, a gauge symmetry of the type discussed here is not a symmetry, but rather a redundancy in the description.... [the potential field] A(x) contains degrees of freedom that are not physical....

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Knowing this, can we deduce Maxwell's theory and hence, the facts of electromagnetism, without ever stepping inside a laboratory? To a large extent, we can! The requirement of Lorentz invariance is a powerful constraint on Nature.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Einstein taught us to deduce physics from symmetry, instead of symmetry from physics.... Instead of laboriously distilling this theory from a motley collection of experimental facts and then extracting a symmetry, he formulated a symmetry empowering him to to write down his theory of gravity in one fell swoop.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

I regard Einstein's understanding of how symmetry dictates design as one of the truly profound insights in the history of physics. Fundamental physics is now conducted largely according to Einstein's schema rather than that of 19th century physics.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The equivalence principle is a statement about physics in a small region of spacetime.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

More accurately, gravity is equivalent to the curvature of spacetime, or gravity and the curvature of spacetime are really the same thing.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The equivalence principle is not a statement about symmetry.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The scalar curvature is the unique scalar we can form out of the metric and two powers of derivatives.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

Any process not explicitly forbidden will occur, even though the probability of the process actually occurring may be very small. Thus, in the quantum world, Einstein is no longer allowed to remove the cosmological constant from his theory.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The puzzle for quantum field theorists is not whether lambda is present in the action, but why observationally it has the value it does.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

...in Einstein gravity, the term "matter" is often used in an extended sense to include everything else besides gravity, such as the electromagnetic field, which we normally do not think of as matter.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

I must emphasize, once again, that the gravitational field is [g with covariant indices], not [g with contravariant indices]. The reason is that from the very beginning we defined coordinates to carry upper indices... Thus, particles couple to [g with covariant indices], not [g with contravariant indices].

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

The nonlinearity of Einstein's field equations renders exact solutions rather unlikely, except in situations endowed with a high degree of symmetry.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

... the gravitational field is trying to extremize the scalar curvature.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

As this discussion makes clear, it is really advantageous to have radial light rays always move along 45-degree lines. To this, Roger Penrose added another attractive feature of having the range for the coordinates be finite. The resulting diagram is known as a [Carter-Penrose] diagram and is extraordinarily useful for seeing the causal structure of the spacetime.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

It is as if the entropy of a black hole were to reside completely on its surface. Indeed, imagine laying down a grid on the surface of a black hole. Somehow, each Planck-sized cell contains one unit of entropy. This mysterious property of black holes, which represents one of the deepest puzzles in theoretical physics, led 't Hooft and Susskind separately to formulate the so-called holographic principle.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*

It is worth emphasizing that frame dragging is not a mysterious effect associated somehow only with black holes, as some people confusedly think. It is a general relativistic effect, which does not occur in Newtonian gravity, generated by any massive body. In particular, the Earth drags the spacetime frame around it.

- Anthony Zee, *Einstein Gravity in a Nutshell (2013)*