Backtracking, also called depth-*rst search, will be the focus of the present paper. Backtracking can be used in programs that enumerate all solutions to a given set of constra ints.Backtrac k is the technique of dancing links in computer science. Backtracking programs often know the value of ximplicitly as a byproduct of their normal operation. The program log ic that traverses lists can often be run i n reverse.Scott*s pentomino problem is a special case of the exact cover problem. Each row contains a 1 in the column id entifying the piece, and there are e xactly 1568 such rows. We can name the twelve columns F I L P N T U VW X Y Z.Backtracking is the process of traversing the tree in preord er, *depth *rst* Backtracking involves choosing a row, r, such that A[r, c] = 1 (nondeterministically) and deleting a row from matrix A. The nondeterministic choice of rmeans that the algorithm essentially clones itself into independent subalgorithms.The search trees for Scott*s pentominophthalproblem then have only 10,421 nodes (X at 23) and 12,900 nodes (X at 24) The algorithm Xis to represent each 1 in the Aas adata object x with a list header and column object. The size is the number of 1s in the column, and the name is a symbolic identi*er for printing the answers.The operation of covering column cis is more interesting: It removes cfrom the headerlist and removes all rows in c*s own list from the other column lists they are in. Operation (1), which I mentioned at the outset of this paper, is used here to remove objects in both the horizontal and vertical directions.In Figure 2 we show how search (0) affects the data of (3) Column A is covered by removing both of its rows from their other columns. The structure now takes the form of Figure 3. The outer level routin e,search (0), will proceed to transform Figure 4.The running time of algorithm DL X is essentially proportional to the number of times it applies operation (1) to remove an ob ject from a list. A total of 28 updates are performed during the solution of (3) if we repeatedly choose the shortest column.A backtrack program usually spends most of its time on only a f ew levels of the searchtree. Figure 5 shows the search tree f or the case X = 23 of Dana

Scott*s pentomino problem.The Sheuristic multiplies the total number of memory accesses by a factor of approximately (14 ×3,617,723)/(8×17,818,752)*36% in this example. Extra work on the lower levels has reduce d the need for hard work at the higher levels.Algorithm is faster than Scott*s 8 ×8*2×2 problem, because the backtrack tree is larger and there are 2339 essentially di*erent so lutions [21] The linked-list approach of algorit hm DLX performs a total of 13,691,897 updates, or about 192 million memory accesses. This solves the problem in less than a minute on an IBM 7094.Algorithm DLX begins to outperform other pentomino-placin g procedures in problems where the search tree has many levels. It knows no di*erence, because pieces and cells are simply columns of the given inpu t matrix. The solutions fall into four classes, dep ending on the behavior at the four corners.In the late 1950s, T. H. O*Beirne introduced a pleasantvariation on polyominoes by substituting triangles for squ ares. The twelve hexiamonds were independently dis covered by J. E. Reeve and J. A. Tyrell. They found more than forty ways to arrange th em into a 6×6 rhombus.O*Beirne was fascinated by the fact that seven of the twelve hexiamonds have di*erent shapes when they are *ipped over. In November of 1959, after three months of tri als, he found a solution; and two years later he challenged the readers of New Scientist to match this feat. A solution to the hexiamond problem is maximally symmetric if it has the highest horizontal or vertical symmetry score.Figure 8 shows solutions to O*Beirne*s hexiamond hexagon problem, with the small hexagon at various distances from the center of the large one.There are 46 ways to pack one-sided pentominoes into a 3 ×30 rectangle. The Monte Carlo estimation procedure of [24] sug gests that about 19 quadrillion updates will be needed. If that estimate is correct, I could have the result in a few months.The cells of the hexiamond-hexagon problem can be colored bl ack and white in a similar fashion. Instead of making pieces by joining squares or triangles together, Brian Barwell considered making them from lin e segments or sticks. The tetrasticks are especially interestin g from a recreational standpoint.Filling a 5×5 grid with 15 of the 16 tetrasticks, we must leave out either the H, the J, the L, the N, or the Y. We also need secondary columns I xytorepresent interior junction points (x, y) Each row represents a possible placing of a piece.There are ten one-sided welded tetrasticks if we add the mirror images of the unsymme trical pieces. Only three solutions are possible, i ncluding two perfectly symmetric solutions shown. I*ve decided n ot to show the third solution, which has the X piece in the middle.The Nqueens problem is actually a special case of the generali zed cover problem in the previous section. For example, the 4 queens problem is just t he task of covering eightprimary columns (R0 ,R1,R2,R3,F0,F1,F2,F3) given the sixteen rows.Consider the eight queens problem. There are 8 possible ways to occupy each rank and each *le. It is better to place quee ns near the middle of the board because central positions rule out more po ssibilities.The order in which header nodes are linked together at the sta rt of algorithm DLX can have a signi*cant e*ect on the running time. Algorithm DLX, which uses dancing links to implement the *na t-genicural* algorithm for exact cover problems, can run even faster than other algori thms.In this paper I have used the exact cover problem to illustrat e the versatility of dancing links. The approach works nicely with the Waltz *ltering algorithm. I believe that a terpsichorean technique is s igni*cantly better than the current state at every level. The implementation of algorithm DLX that I used when prepari ng this paper is on webpage http://www-cs-faculty.stanford.edu/~knuth/ grotesqueprograms.html.845 Combinations Puzzles: 845 Interestingly Combinations (Taiwan: R.O.C. Patent 66009). This puzzle, which is available from www.puzzletts.com , actually has only 83 solutions. It carries aChinese title, *Dr. Dragon*s Intelligence Pro*t System.*Richard K. Guy, *O*Beirne*s Hexiamond,* in The Mathemagician and Pied Puzzler. *Packing a square with Y-pentomin oes,* Journal of RecreationalMathematics 7(1974), 229*229.Alfred Wassermann of Universit¨ at B ayreuth covered the Aztec diamond of Figure 15 with one-si ded tetrasticks. He subsequently enumerated the 10,440,433 solutions to the 9×10 one-sided pentominoproblem.