

# Proof Corrections

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Items are enumerated as in the faxed document.

## 1 Main article errors

1. Remove the word “the”
2. Replace “significant recent progress” with “significant progress recently”
3. Replace “Earlier this year” with “Last year”
4. Two comments on Equation (2)
  - Break this equation (into multiple lines) at the =
  - Replace  $\varepsilon_t$  with  $\varepsilon_{c,t}$
5. Replace “tth” with “t-th”
6. It seems as if in  $[\text{Ca}^{2+}]_b$ , the “b” is bold. It should not be.
7. Replace “referred to as” with “called”
8. Replace “i)” and “ii)” with “(i)” and “(ii)”, respectively
9. Replace  $\stackrel{def}{=}$  with  $\stackrel{def}{=}$ , ie, the “=” should be in line with the rest of the equation, and the “def” should be just above it in a smaller font. This is a recurring problem.
10. Same as 9
11. Multiple comments on Equation (6)
  - Same as 9
  - Replace stuff after = with
$$\begin{cases} \mathcal{N}([\text{Ca}^{2+}]_t; \hat{\mu}(n_t), \sigma_c^2 \Delta) (p\Delta) & \text{if } n_t = 1 \\ \mathcal{N}([\text{Ca}^{2+}]_t; \hat{\mu}(n_t), \sigma_c^2 \Delta) (1 - p\Delta) & \text{otherwise,} \end{cases}$$
  - Try to fit within a single column, given the above substitution
12. Replace “Now the goal is to efficiently estimate  $P_{\theta}(\mathbf{H}_t | \mathbf{O}_{1:T}) = P_{\theta}(n_t, [\text{Ca}^{2+}]_t | F_{1:T})$  for all  $t$ , the posterior distribution of the hidden signals, given all the observations.” with “ Now the goal is to efficiently estimate  $P_{\theta}(\mathbf{H}_t | \mathbf{O}_{1:T}) = P_{\theta}(n_t, [\text{Ca}^{2+}]_t | F_{1:T})$ , the posterior distribution of the hidden signals, given all the observations, for all  $t$ . ”
13. Replace “ $P_{\theta}(\mathbf{H}_t | \mathbf{O}_{1:t})$ ” with “ $P_{\theta}(\mathbf{H}_t | \mathbf{O}_{1:t})$ ”, ie, only the  $\theta$  gets subscripted.

14. In Equation (9), after the final  $=$ , it should read:

$$P_{\theta}(F_t | [\text{Ca}^{2+}]_t^{(i)}) P_{\theta}([\text{Ca}^{2+}]_t^{(i)} | [\text{Ca}^{2+}]_{t-1}^{(i)}, n_t^{(i)}) P_{\theta}(n_t^{(i)}) / Z,$$

The only acceptable places to put line breaks are between the “)”’s and “P”’s.

15. Replace “ $\{n_t, [\text{Ca}^{2+}]_t\}$ ” with “ $n_t$  or  $[\text{Ca}^{2+}]_t$ ”

16. Two remarks about Equation (12)

- Replace

$$\frac{\text{argmax}}{\theta}$$

with

$$\text{argmax}_{\theta}$$

(note that  $\text{argmax}$  goes in line with the rest of the text, and  $\theta$  goes below it in a smaller font, similar to 9).

- Note that the  $\theta$  should be in bold.
- The “,” should be after the final “)”, not before

17. The “ $\theta'$ ” should be “ $\theta'$ ”, ie, bold.

18. Replace “variance” with “noise term”

19. Remove “Main Result.”

20. Remove sentence “Note that... of generality.”

21. See 9

22. Replace “ $(q(\mathbf{H}_t) = P_{\theta}(\mathbf{H}_t^{(i)} | \mathbf{H}_{t-1}^{(i)}, \mathbf{O}_t))$ ” with “ $, q(\mathbf{H}_t) = P_{\theta}(\mathbf{H}_t^{(i)} | \mathbf{H}_{t-1}^{(i)}, \mathbf{O}_t),$ ”

23. Replace “ $(n_{1:T})$ ” with “ $, n_{1:T}$ ” and “ $(\tau)$ ” with “ $, \tau,$ ”

24. Remove “Saturated simulation.”

25. Replace “ $\xi = 4 \times 10^{-4}$  a.u.” with “ $\xi = 4 \times 10^{-4}$   $\mu\text{A}/\text{photon}$ , and replace  $\sigma_F = 10^{-4}$  a.u.” with “ $\sigma_F = 10^{-4}$   $\mu\text{A}$ ”

26. Remove “In vitro bursts.”

27. Replace “i)” with “(i)”

28. Remove “Real data saturation”

29. Replace and “ii)” with “(ii)”

30. Add the word “its” between “zero with” and “time constant”

31. Remove “Array of inferences.”

32. Remove “Generalized linear model particle filter smoother.”

33. Remove “In vitro data superresolution”

34. Replace “PFS both improves inference accuracy over the optimal linear method and provides” with “PFS both (i) improves inference accuracy over the optimal linear method and (ii) provides”

35. Replace “ $\{\alpha, \beta, \sigma_F\}$ ” with “ $\{\alpha, \beta, \xi, \sigma_F\}$ ”

36. Replace “i)” with “(i)” and “ii)” with “(ii)”

## 2 Appendix errors

1. Two things
  - See 9
  - Break at =
2. Break into two lines if possible
3. Try to into two lines
4. Replace “ $\{[\text{Ca}^{2+}]_{t-1}\}^{(i)}$ ” with “ $[\text{Ca}^{2+}]_{t-1}^{(i)}$ ”
5. Equation (34) should be rewritten as below. Note the location of the line break

$$g'(x) = \left( \frac{k_d(\beta - F_t)}{F_t - \beta - \alpha} \right)^{1/n} \frac{nk_d(\beta - F_t)}{F - \beta - \alpha} \left( -\frac{k_d}{F - \beta - \alpha} - \frac{k_d(\beta - F_t)}{(F_t - \beta - \alpha)^2} \right).$$

6. Replace “are function of all the parameters:  $\alpha, \beta, \xi, \sigma_F, n$  and  $k_d$ ” with “are function of  $\{\alpha, \beta, \xi, \sigma_F, n, k_d\}$ ”
7. Line break after = sign
8. Two things
  - See 9
  - Replace cent sign with  $\mathbb{Z}$
9. Replace ' with  $\mathbb{Z}$
10. Remove “Laplace . . . distribution.”
11. Replace “The circles” with “The filled circles”
12. Line beginning with  $q_{\theta}^S$  should not be indented
13. (10) and (11) should not be links to references, they are sequences
14. Remove “Sampling strategies:” and capitalize “the”
15. (10) is a sequence, not a reference, no link
16. Remove “Mixture approximation:” and capitalize “appropriate”
17. “Computing. . .” should be a one level down in the heading hierarchy
18. This equation should not have a line break where the second line break is. I suggest:

$$P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_{v-1}) = \sum_{n=0,1} a_{n,v-1} \int P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_v) P_{\theta}([\text{Ca}^{2+}]_v \mid [\text{Ca}^{2+}]_{v-1}, n_v = n) d[\text{Ca}^{2+}]_v,$$

or

$$P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_{v-1}) = \sum_{n=0,1} a_{n,v-1} \int P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_v) P_{\theta}([\text{Ca}^{2+}]_v \mid [\text{Ca}^{2+}]_{v-1}, n_v = n) d[\text{Ca}^{2+}]_v,$$

19. “ $a_{1, \quad v-1}$ ” should be “ $a_{1,v-1}$ ”, ie, there seems to be a space between the comma and the ‘v’, which should not be present. same is true for  $a_{0,v-1}$ . This problem repeats in several places

20. Indenting is totally inappropriate. Please use:

$$P_{\theta}([\text{Ca}^{2+}]_v \mid [\text{Ca}^{2+}]_{v-1}, n_v) = \mathcal{N}([\text{Ca}^{2+}]_v; [\text{Ca}^{2+}]_{v-1} - \Delta/\tau([\text{Ca}^{2+}]_{v-1} - [\text{Ca}^{2+}]_b) + An_v, \sigma_c^2 \Delta),$$

21. Line break is again inappropriate. replace with

$$\begin{aligned} & \int P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_v) P_{\theta}([\text{Ca}^{2+}]_v \mid [\text{Ca}^{2+}]_{v-1}, n_v = n) d[\text{Ca}^{2+}]_v \\ &= \int \mathcal{N}([\text{Ca}^{2+}]_v; \tilde{\mu}_v, \tilde{\sigma}_v^2) \times \mathcal{N}([\text{Ca}^{2+}]_v; [\text{Ca}^{2+}]_{v-1} - \Delta/\tau([\text{Ca}^{2+}]_{v-1} - [\text{Ca}^{2+}]_b) + An_v, \sigma_c^2 \Delta). \end{aligned}$$

22. Replace  $\chi$  with  $C_v$

23. Replace  $\chi$  with  $C_v$

24. Line break should be:

$$P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_{v-1}) = \sum_{n=0,1} a_{n,v-1} \mathcal{N}([\text{Ca}^{2+}]_{v-1}; \tilde{\mu}_v^S(n), (\tilde{\sigma}_v^S)^2),$$

25. Line break inappropriate. Replace with:

$$P_{\theta}^{NL}(F_v \mid [\text{Ca}^{2+}]_{t-1}) = \sum_{n=0,1} a_{n,t-1} \sum_{m=1}^{2^{v-t}} a_{mt} \mathcal{N}([\text{Ca}^{2+}]_{t-1}; \tilde{\mu}_{mt}^S(n), (\tilde{\sigma}_t^S)^2),$$

26. “Superresolution sampling spikes” should be one level down in hierarchy, ie, it should be indented

27. Replace  $n_t^{(i)}$  with  $n_t^{(i)}$ , ie, the  $(i)$  should be directly above  $t$ , not off to the side. this happens in 3 places on this line, and elsewhere

28. “Superresolution...” should be indented, just as 26

29. See 27

30. See 27 (twice on this line)

31. Line break is inappropriate, move it to after =

32. Same as 19

33. Same as 19

34. Two comments on this equation

- Replace  $\frac{\text{argmax}}{\{b, \mathbf{k}, \boldsymbol{\omega}\}}$  with

$$\frac{\text{argmax}}{\{b, \mathbf{k}, \boldsymbol{\omega}\}}$$

in both instances (ie, argmax goes in line with the rest,  $\{b, \mathbf{k}, \boldsymbol{\omega}\}$  goes under in a smaller font

- (61) should be on the bottom line, not the middle one

35. Replace  $\tau, A, [\text{Ca}^{2+}]_b, \sigma_c$  with

$$\underset{\tau, A, [\text{Ca}^{2+}]_b \geq 0}{\text{argmax}}$$

. Note a few differences

- (a)  $\text{argmax}$  is inline with rest of text, the other stuff is below
- (b)  $\sigma_c$  has been replaced with  $\leq$

36. Again,  $\text{argmin}$  should go inline,  $x > 0$  should go underneath. equation should look like

$$\hat{\mathbf{x}} = \underset{x > 0}{\text{argmin}} \frac{1}{2} \mathbf{x}' \mathbf{Q} \mathbf{x} + \mathbf{L}' \mathbf{x},$$

37. Remove  $\times$

38. Same deal again with the  $\text{argmin}$ , equation should look like:

$$\hat{\mathbf{x}} = \frac{1}{2} \underset{\mathbf{x}_p \geq 0, \forall p}{\text{argmin}} \sum_{t=1}^T \sum_{i,j=1}^N J_{t,t-1}^{(i,j)} \left\| \mathbf{C}_t^{(i,j)} \mathbf{x} + d_t^{(i,j)} \right\|_2^2,$$

39. Again with the  $\text{argmax}$ , (69a) should look like

$$\hat{\sigma}_c^2 = \underset{\sigma_c^2}{\text{argmax}} \sum_{t=1}^T \sum_{i,j=1}^N J_{t,t-1}^{(i,j)} \left( -\frac{1}{2} \ln(2\pi\sigma_c^2\Delta) - \frac{1}{2} \frac{([\text{Ca}^{2+}]_t^{(i)} - \mu_{t,t-1}^{(i,j)})^2}{\sigma_c^2\Delta} \right)$$

40. Same with  $\text{argmin}$ , equation should look like

$$\{\hat{\alpha}, \hat{\beta}\} = \underset{\alpha, \beta \geq 0}{\text{argmin}} \sum_{t=1}^T \sum_{i=1}^N \frac{(F_t - \alpha S([\text{Ca}^{2+}]_t) - \beta)^2}{\xi S([\text{Ca}^{2+}]_t) + \sigma_F} + \ln(\xi S([\text{Ca}^{2+}]_t) + \sigma_F),$$

41. Another problem with  $\text{argmin}$ , equation should look like

$$\{\hat{\xi}, \hat{\sigma}_F\} = \underset{\xi, \sigma_F \geq 0}{\text{argmin}} (r_t - \xi S([\text{Ca}^{2+}]_t) - \sigma_F)^2,$$

### 3 response to inquiries

1. ok.
2. ok.
3. ok.
4. See [Yuste and Konnerth, 2006]
5. See [Tay et al., 2007]
6. See [Yasuda et al., 2004]
7. See [Borst and Abarbanel, 2007]
8. See [Sasaki et al., 2008]
9. See [Smith et al., 2001]
10. See [Dempster et al., 1977]
11. See [Samejima et al., 2004]
12. This is correct. NIPS is a conference proceedings.
13. Please replace with [Huys and Paninski, 2009]
14. Maybe also look for Transactions of the ASME, as in [Kalman, 1960]
15. Please replace with [Huys and Paninski, 2009]
16. Journal title is “Network: Computation in Neural Systems”, as suggested by [Paninski et al., 2004]
17. See [Vogelstein et al., 2009]
18. See [Pillow et al., 2008]
19. See [Shumway and Stoffer, 2006]
20. Replace “To sample conditioned on the next observation” with “To sample  $\{n_t, [\text{Ca}^{2+}]_t\}$  conditioned on the next observation”

## References

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