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New heuristic algorithms for discrete competitive location problems with binary and partially binary customer behavior

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ABSTRACT

We consider discrete location problems for an entering firm which competes with other established firms in a market where customers are spatially separated. In these problems, a given number of facility locations must be selected among a finite set of potential locations. The formulation and resolution of this type of problem depend on customers' behavior. The attraction for a facility depends on its characteristics and the distance between the facility and the customer. In this paper we study the location problem for the so-called Binary and Partially Binary Rules, in which the full demand of a customer is served by the most attractive facility, or by all the competing firms but patronizing only one facility of each firm, the one with the maximum attraction in the firm. Two new heuristic algorithms based on ranking of potential locations are proposed to deal with this sort of location problems. The proposed algorithms are compared with a classical genetic algorithm for a set of real geographical coordinates and population data of municipalities in Spain.

1. Introduction

The location of facilities is a strategic decision for a firm that competes with other firms to provide goods or services to the customers in a given geographical area. Different location models and solution procedures have been proposed to cope with these problems which vary depending on the ingredients to be considered, such as location space, facility attraction, customer patronizing behavior, demand function, decision variables, etc. (see for instance survey papers [1–4]).

Most of the models in the literature deal with the location problem for an entering firm that must compete for the market share in a certain region where other firms are already offering the same goods or service. The entering firm is aimed at determination of the optimal locations for the new facilities with respect to maximization of the market share or profit, taking into account the patronizing behavior of customers. Traditionally it is assumed that the customers choose the nearest facility to be served, but, in addition to the distance, the customer can take into account some characteristics of the facilities for its choice.

Some variants of the attraction model, proposed by Huff [5], have been used as customers' choice rules in order to estimate the market share captured by the competing facilities. In this type of models, the attraction of a facility is measured by a parameter, called the facility quality, divided by a non-negative non-descending function of the distance between the customer and the facility. The quality of each facility depends on the characteristics of the facility. The most common customer choice rules are the ones called proportional and binary (see [6]). Following the proportional rule the customers patronize all the facilities in proportion to facility attraction (see for instance [7–9]). In the case of binary rule the customer patronizes the most attractive facility (see [10–14]).

In this paper we will consider a different rule of customers' choice – the Partially Binary Rule – for which the location problem has been little studied in the location literature. In this case, several firms are presented in the market and each firm may own more than one facility. The full demand of a customer is served by all the competing firms, but patronizing only one facility of each firm – the one with maximum attraction – being the demand split among those facilities proportionally to its attraction (see [15]). Some papers refer to this customer choice rule, but it is not studied in depth. In [16] six scenarios resulting from the combination of three customer choice rules (binary, partially binary, and proportional) with two types of services (essential and unessential) are considered on networks, where known discretization results about the existence of a solution for the set of nodes are extended, and recently, Biesinger et al. [17] study the same six scenarios but on discrete space for the leader–follower problem, proposing formulations as MILP problems for both the leader and the follower problems.

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Due to the difficulty of these problems, in this paper two heuristic algorithms are proposed, which could be used also to solve other discrete competitive location problems. To check performance of the proposed heuristic algorithms, it is necessary to know the optimal solution of the problems in order to compare it with the solution given by a heuristic algorithm. The performance of the proposed heuristic algorithms will be justified by solving the location problem with the binary and partially binary rules, since both problems can be formulated as Integer Linear Programming (ILP) problems, and the optimal solutions can be obtained using standard optimization software (CPLEX, Gurobi, Mosek, Xpress-MP, others), at least for small size problems.

The reminder of the paper is organized as follows: Section 2 consists of description of the location problems, Section 3 is devoted to presentation of the new heuristic algorithms, and Section 4 includes the description and discussion of the experimental investigation of the proposed algorithms; finally, conclusions are presented in Section 5.

2. Discrete location models

An entering firm wants to set up new facilities in a geographical region where similar facilities of other competing firms are already present. There is a set of spatially separated markets and customers are aggregated to geographic demand points in order to make the problem computationally tractable (see [19] for demand aggregation). It is assumed that customers' demand is fixed and known.

The following general notation is used:

Indices:

i, I	index and set of demand points (customers)
k, K	index and set of firms
j, J_k	index and sets of existing facilities of each firm k

Data:

w_i	demand at i
d_{ij}	distance between demand point i and facility j
a_{ij}	attraction that demand point i feels for facility j
$a_i(J_k)$	maximum attraction that i feels for facilities of the existing firm k, $a_i(J_k) = \max\{a_{ij}: j \in J_k\}$
L	a set of candidate locations for the new facilities
s	a number of new facilities to be located

Variables:

X	a set of locations for the new facilities

Next we will review the location problem with the binary choice rule for which an ILP formulation is available and then we will consider the location problem with the partially binary choice rule.

2.1. Model with the binary rule

Following the binary rule of customers' choice, the full demand of a customer is satisfied by only one facility – the one with maximum attraction – but it may occur that there are more than one facility with maximum attraction. If all the tied facilities are owned by the entering firm, then the firm captures the full demand of the customer, but if a part of the tied facilities are owned by its competitors, it is assumed that the entering firm captures a fixed proportion of customer's demand. Finally, if none of the tied facilities are owned by the entering firm, then no demand is captured from the customer.

Let us define the following sets:

$$I^{>} = \{i \in I: a_i(X) > \max\{a_i(J_k): k \in K\}\}$$

and

$$I^{=} = \{i \in I: a_i(X) = \max\{a_i(J_k): k \in K\}\}$$

where $a_i(X) = \max\{a_{ij}: j \in X\}$ is the maximum attraction that point i feels for the new facilities.

The market share captured by the entering firm for the binary rule is:

$$M_b(X) = \sum_{i \in I^{>}} w_i + \sum_{i \in I^{=}} \theta_i w_i \tag{1}$$

where θ_i is the proportion of demand captured from the customer i. The location problem is:

$$(P_b): \max\{M_b(X): |X| = s, X \subset L\} \tag{2}$$

The latter problem can be formulated as an ILP problem considering the following sets and variables:

$$L_i^{>} = \{j \in L: a_{ij} > \max\{a_i(J_k): k \in K\}\}$$

$$L_i^{=} = \{j \in L: a_{ij} = \max\{a_i(J_k): k \in K\}\}$$

$$I^* = \{i \in I: L_i^{>} \cup L_i^{=} \neq \emptyset\}$$

$$x_j = \begin{cases} 1 & \text{if a new facility is located at } j \\ 0 & \text{otherwise} \end{cases} \qquad j \in L$$

$$y_i = \begin{cases} 1 & \text{if the customer } i \text{ is fully captured by the entering firm} \\ 0 & \text{otherwise} \end{cases} \qquad i \in I^*$$

$$z_i = \begin{cases} 1 & \text{if the customer } i \text{ is partially captured by the entering firm} \\ 0 & \text{otherwise} \end{cases} \qquad i \in I^*$$

Then, the location problem (P_b) is equivalent to:

$$\begin{aligned} \max \quad & \textstyle\sum_{i \in I^*} w_i y_i + \sum_{i \in I^*} \theta_i w_i z_i \\ \text{s.t.} \quad & y_i + z_i \leq 1, \; i \in I^* \\ & y_i \leq \textstyle\sum_{j \in L_i} > x_j, \; \forall\, i \in I^* \\ & z_i \leq \textstyle\sum_{j \in L_i} = x_j, \; \forall\, i \in I^* \\ & \textstyle\sum_{j \in L} x_j = s \\ & x_j \in \{0, 1\}, \; \forall\, j \in L, \; y_i, z_i \geq 0, \; \forall\, i \in I^* \end{aligned}$$

This problem can be solved by standard ILP software, at least for small size problems.

2.2. Partially binary model

In this case, the full demand of the customer is served by all firms, but the customers patronize only one facility from each firm – the one with the maximum attraction. Then the demand is split between those facilities in proportion with their attraction. In this model it is not necessary to consider the possibility of ties because they are irrelevant when it comes to obtaining the total market share captured by the entering firm. It is required to know the number of firms operating on the market, and the number of facilities owned by each firm. This information lets us know the number of facilities to split the demand of each customer, and it also lets us evaluate the proportion of customer demand that will get the most attractive facility of each firm.

The market share captured by the entering firm for the partially binary rule is:

$$M_{pb}(X) = \sum_{i \in I} w_i \frac{a_i(X)}{a_i(X) + \sum_{k \in K} a_i(J_k)} \tag{3}$$

and the location problem is:

$$(P_{pb}): \max\{M_{pb}(X): |X| = s, X \in L\} \tag{4}$$

This problem is a non-linear model, but recently Biesinger et al. [17] have proposed a linear transformation of this model where only two firms are considered, the leader and the follower. That linear transformation follows the idea suggested by Kochetov et al. [20] to obtain a linear formulation of the model when the proportional customer choice rule is considered. In this paper, we have updated the linear transformation to more than two competing firms.

Three new kinds of variables are introduced:

$$y_{ij} = \begin{cases} 1 & \text{if } j \text{ is the most attractive new facility} \\ & \text{to customer } i \\ 0 & \text{otherwise} \end{cases} \qquad i \in I, j \in L$$

$$q_i = \frac{1}{\sum_{j \in L} a_{ij} y_{ij} + \sum_{k \in K} a_i(J_k)} \qquad i \in I$$

$$v_{ij} = w_i q_i a_{ij} y_{ij} \qquad i \in I, j \in L$$

where variables q_i represent the portion of demand at point i captured by the entering firm per unit of attraction felt by a customer at i towards the most attractive new facility for that customer, and v_{ij} is the amount of demand that a possible new facility at location j supplies to customer i. The linearized model of (P_{pb}) is the following:

$$\begin{aligned} \max \quad & \textstyle\sum_{i \in I} \sum_{j \in L} v_{ij} \\ \text{s. t.} \quad & \textstyle\sum_{j \in L} v_{ij} + w_i q_i \sum_{k \in K} a_i(J_k) = w_i, \ \forall\, i \in I \\ & v_{ij} \le w_i y_{ij}, \ \forall\, i \in I, \forall\, j \in L \\ & v_{ij} \le w_i a_{ij} q_i, \ \forall\, i \in I, \forall\, j \in L \\ & w_i a_{ij} q_i \le v_{ij} + W(1 - y_{ij}), \ \forall\, j \in L \\ & y_{ij} \le x_j, \ \forall\, i \in I, \forall\, j \in L \\ & \textstyle\sum_{j \in L} y_{ij} = 1, \ \forall\, i \in I \\ & \textstyle\sum_{j \in L} x_j = s \\ & x_j \in \{0, 1\}, \ \forall\, j \in L, \ q_i \ge 0, \ \forall\, i \in I \\ & v_{ij} \ge 0, \ y_{ij} \in \{0, 1\}, \ \forall\, i \in I, \forall\, j \in L \end{aligned}$$

where $W = \max_{i \in I} w_i \cdot \max_{i \in I, j \in L} a_{ij} \cdot \max_{i \in I} \frac{1}{\sum_{k \in K} a_i(J_k)}$. Thus, this problem can also be solved by standard ILP software, at least for small size problems.

3. Ranking-based discrete optimization algorithm

Two heuristic algorithms have been developed to solve the location problems described in Section 2. The algorithms are based on a single agent random search in the neighborhood of the best solution found so far. Like most of the random search algorithms, the proposed ones do not guarantee determination of the optimal solution, but rather its approximation. On the other hand the proposed algorithms do not require to have analytical expression of the objective function of the problem being solved – the only requirement is to have availability to evaluate objective value at any possible solution.

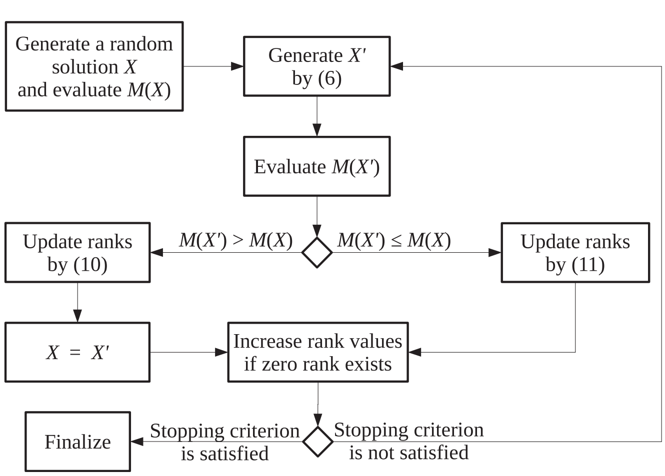

Fig. 1. Scheme of RDOA.

Ranking-based Discrete Optimization Algorithm (RDOA), for which scheme is given in Fig. 1, begins with an initial variable – a subset of candidate locations

$$X = \{x_1, x_2, \ldots, x_s\} \tag{5}$$

which is considered as the best known (but not necessarily optimal) solution of the problem being solved. A new solution X' is generated by in turn taking location from X and changing them to another one with probability $\pi_c = 1/s$, thus ensuring that a single facility will be changed in average.

The new facility l, in case of change, is selected from $L' = L \backslash (X \cup X')$ – the set of all possible locations without those, which already forms solution X'. Mathematically, values of solution X' can be described as

$$x'_i = \begin{cases} l \in L \backslash (X \cup X'), & \text{if } \xi_i < 1/s \\ x_i, & \text{otherwise} \end{cases} \tag{6}$$

where ξ_i is a random number uniformly generated over the interval [0, 1], and $i = 1, 2, \ldots, s$.

Each candidate location $l_i \in L$, has different probability to be selected, which is evaluated using one of the two strategies. The first strategy is based on the rank r_i of the candidate location l_i, and probability $\pi^r{}_i$ to select l_i is calculated by

$$\pi_i^r = \frac{r_i}{\sum_{j=1}^{|L|} r_j} \tag{7}$$

One can see from the equation that a larger value of the rank r_i leads to a larger probability to select l_i, therefore larger rank values are assigned to more promising candidate locations. The procedure of assignment of rank values to the candidate locations will be discussed hereinafter.

The second strategy is specially adopted for solution of the facility location problems and is based on the rank r_i and a geographical distance $d(l_i, x_{\prime z})$ between the candidate location l_i and the location $x_{\prime z}$; here $x_{\prime z}$ is a location already forming solution X' and is expected to be changed ($z = 1, 2, \ldots, s$). It is assumed that probability to select the candidate location l_i to change $x_{\prime z}$ is (1) proportional to the value of the rank r_i and (2) simultaneously inversely proportional to $d(l_i, x_{\prime z})$ – closer candidate facilities have more chances to be selected. Mathematically the probability can be expressed as

$$\pi_i^{rd} = \frac{r_i}{d(l_i, x_{\prime z}) \sum_{j=1}^{|L|} \frac{r_j}{d(l_j, x_{\prime z})}} \tag{8}$$

Depending on which strategy is used to evaluate the probability, the algorithms are denoted by RDOA and RDOA-D, respectively.

At the beginning of both algorithms all candidate solutions have unit ranks:

$$r_i = 1, \quad \text{for all } i = 1, 2, \ldots, |L| \tag{9}$$

Later ranks are automatically adjusted according to the success and failures in improvement of the best known solution by selecting a particular candidate location. If the newly generated solution X' improves the best solution found so far X, i.e. $M(X') > M(X)$, where $M(\cdot)$ stands for the objective function, then

(1) the ranks of all candidate locations in X' are increased by one, and
(2) the ranks of all candidate locations that form X, but do not form X' are reduced by one.

Mathematically it can be described as

$$r_i = \begin{cases} r_i + 1, & \text{if } l_i \in X' \\ r_i - 1, & \text{if } l_i \in X \backslash X' \\ r_i, & \text{otherwise} \end{cases} \tag{10}$$

Otherwise, if $M(X') \le M(X)$, then the ranks of all candidate locations which form unsuccessfully generated solution X', but do not form the best known solution X, are reduced by one; mathematically,

$$r_i = \begin{cases} r_i - 1, & \text{if } l_i \in X' \backslash X \\ r_i, & \text{otherwise} \end{cases} \tag{11}$$

If any of the ranks reaches zero, then ranks are increased by one.

If the new solution X' improves X, then X is changed by X'. Such a process is continued till a stopping criterion, which is usually based on the number of function evaluations, is satisfied.

4. Experimental investigation

The proposed algorithms RDOA and RDOA-D have been experimentally investigated by solving a competitive facility location problem for an entering firm considering different rules for customers' choice: the binary and the partially binary rules (see Section 2). The database with real geographical data of coordinates and population of 6960 municipalities (which will be considered as demand points) in Spain has been used for the investigation; see Fig. 2 for the illustration of the geographical data. The distances between demand points have been calculated in kilometers using great circle principle – Haversine distance [21], and the attraction that demand point i feels for facility j has been taken as $a_{ij} = \frac{1}{1 + d_{ij}}$.

Performance of an algorithm has been evaluated by the quality of the best solution X found by the algorithm, which is expressed by the ratio

$$Q(X) = \frac{M(X)}{M(X_O)} \tag{12}$$

where X_O is the optimal solution of the particular problem instance. A larger value of $Q(X)$ means a better quality of the solution X, and the largest possible value $Q(X) = 1$ means that $X \sim X_O$.

The best solution found by each algorithm has been approximated using 10,000 function evaluations for each instance. Due to stochastic nature of RDOA and RDOA-D, the algorithms have been run for 100 independent runs and average results have been considered. The results obtained by RDOA and RDOA-D have been compared with the results obtained by the Genetic Algorithm (GA) [22,23] with the population of 100 individuals, uniform crossover with the rate of 0.8 and mutation rate of $1/s$; GA has been run for 100 generations, thus performing 10,000 function evaluations in total. All experiments have been run in a PC with Pentium IV Processor, 3.2 GHz and 3 GB RAM.

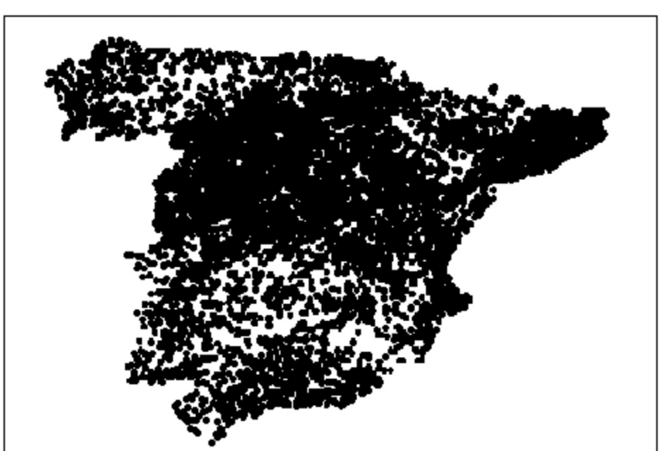

Fig. 2. 6960 municipalities in Spain.

4.1. Results for the binary model

It was considered that there are 3 firms already in the market, each of which has the same number of preexisting facilities located in the most populated demand points; see Table 1 for indices of pre-existing facilities of two problem instances (5 or 10 preexisting facilities each).

Different numbers $s \in \{5, 10\}$ of facilities expected to locate as well as different sizes $|L| \in \{500, 1000, 5000\}$ of the candidate locations' set have been investigated for both instances of pre-existing facilities. We will denote an instance of the location problem by FLP$(\cdot,\cdot,\cdot)$, where the first parameter stands for the number of pre-existing facilities, the second parameter – for the number of new facilities expected to locate, and the last one – for the size of L. Following the latter notation, all problem instances used in the investigation can be listed as

$$FLP(5, 5, 500),\quad FLP(5, 10, 500),\quad FLP(5, 10, 1000),\quad FLP(5, 10, 5000),$$
$$FLP(10, 5, 500),\quad FLP(10, 10, 500),\quad FLP(10, 10, 1000),\quad FLP(10, 10, 5000).$$

For these instances, the optimal solution X_O has been determined with a deterministic algorithm by using Xpress [18].

Results of the approximation of the optimal solution for the facility location problem with binary rule for customers' choice are presented in Table 2, where the first column identifies the instance of the problem, the next three columns present average quality of the best solution obtained by different algorithms, and the last column presents the indices of candidate locations which form the optimal solution.

One can see from the table that both versions of RDOA notably outperform GA. In general, it can be observed from the table that RDOA outperforms GA, and inclusion of geographical distance factor when evaluating the probability to select a particular candidate location has significant advantage independent of the problem instance used in the investigation – the quality of the solution is always higher than 0.98.

The algorithms have been also compared by their performance profiles when solving the instance FLP(10,10,5000) – the instance with 10 facilities expected to locate and the largest set of candidate locations. The performance profiles, illustrated in Fig. 3, show that average quality of the best solution found by RDOA-D after 1000 function evaluations approximately equals to the average quality of the best solution found by RDOA, and approximately equals to the quality of the best solution found by GA after 10,000 function evaluations. That firmly justifies the advantage of RDOA-D against RDOA and GA when solving FLP(10,10,5000) with the binary rule of customers' choice. The dashed lines above and below a graph indicate the confidence interval with confidence level 0.05, and show that RDOA-D produces the solution with notably smaller confidence interval, compared with solutions produced by RDOA and GA.

4.2. Results for the partially binary model

Due to the complexity of the reformulation of the model as a linear programming problem presented in Section 2, Xpress has not been able to solve the instances used in the previous subsection for

Table 1
Indices of pre-existing facilities for each firm.

Firm	Indices of demand points	
	5 preexisting facilities	10 preexisting facilities
J_1	1, 4, 7, 10, 13	1, 4, 7, 10, 13, 16, 19, 22, 25, 28
J_2	2, 5, 8, 11, 14	2, 5, 8, 11, 14, 17, 20, 23, 26, 29
J_3	3, 6, 9, 12, 15	3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Table 2
Average quality values of the best solution, found by different algorithms and the optimal solution found by Xpress considering binary rule of customers' choice.

Instance	GA	RDOA	RDOA-D	The optimal solution
FLP(5,5,500)	0.987	0.999	0.999	1, 61, 178, 264, 393
FLP(5,10,500)	0.960	0.992	0.998	1, 3, 61, 133, 178, 264, 324, 345, 393, 398
FLP(5,10,1000)	0.946	0.982	0.998	1, 3, 61, 178, 264, 324, 345, 393, 398, 702
FLP(5,10,5000)	0.880	0.930	0.989	1, 3, 61, 178, 393, 398, 2039, 2521, 3610, 3745
FLP(10,5,500)	0.984	0.998	0.999	1, 3, 234, 264, 373
FLP(10,10,500)	0.958	0.987	0.993	1, 2, 3, 114, 121, 234, 264, 298, 367, 373
FLP(10,10,1000)	0.943	0.980	0.995	1, 2, 3, 114, 234, 264, 298, 367, 373, 906
FLP(10,10,5000)	0.885	0.944	0.988	1, 2, 3, 234, 298, 367, 373, 723, 1137, 2039

the lack of memory, so to check whether the proposed heuristics are able to find the optimal solution, a new set of problem instances has been run using the maximum size of the parameters that Xpress had been able to solve.

It was considered that there are 3 firms already in the market, each of which has the same number of pre-existing facilities located in the most populated demand points as in the previous test problems, but in this case, the indices of pre-existing facilities are 3 or 5 each (see Table 3 for the case of 3 pre-existing facilities each).

Different numbers $s \in \{5, 10\}$ of facilities expected to locate as well as different sizes $|L| \in \{30, 40, 50\}$ of the candidate locations' set have been investigated for both instances of pre-existing facilities, and $|I| = 589$, municipalities with a population greater than or equal to 10,000 inhabitants. All problem instances used in this investigation can be listed as

$$\begin{array}{lll} FLP(3, 5, 30), & FLP(3, 5, 40), & FLP(3, 5, 50), \\ FLP(3, 10, 30), & FLP(3, 10, 40), & FLP(3, 10, 50), \\ FLP(5, 5, 30), & FLP(5, 5, 40), & FLP(5, 5, 50), \\ FLP(5, 10, 30), & FLP(5, 10, 40), & FLP(5, 10, 50). \end{array}$$

The proposed algorithms reached the optimal solution for all instances within several thousands of function evaluations, which were performed with in a couple of seconds. This would validate the reliability of the proposed algorithms whereas further experiments (on more complex instances) would show the limitations of the algorithms. The optimal results are shown in Table 4.

After verifying the effectiveness of the proposed algorithms, all the algorithms have been applied to solve the same instances of the location problem of the previous subsection, but with the different rule of customers' choice – the partially binary rule. As Xpress has

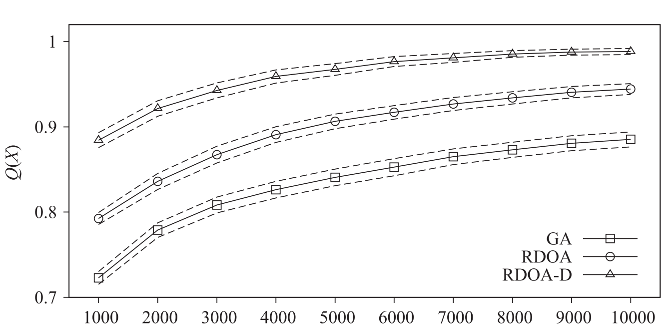

Fig. 3. Performance profiles of the algorithms when solving the facility location problem with the binary rule of customers' choice.

Table 3
Indices of 3 pre-existing facilities for each firm.

Firm	Indices of demand points
J_1	1, 4, 7
J_2	2, 5, 8
J_3	3, 6, 9

not been able to find the optimal solutions to these instances, the best solution found in all experiments (300 for each instance) has been considered as the optimal solution X_O. This has been done taking into account the fact that RDOA-D determines the optimal solution in most of the runs and for all instances of the problem with the binary rule. Moreover the best solution for the problem with the partially binary rule has been determined by RDOA-D in most of the runs, which suggests us to consider the best solution found in all experiments as the optimal solution for a particular instance of the problem. Results of the investigation are presented in Table 5, where meaning of the columns remain the same as in Table 2.

One can see from the table that the results obtained considering the partially binary rule are similar to those obtained when considering the binary rule, though solutions referring to the best market share are different. It can be also realized from the table that for the location problem with the partially binary rule, the proposed ranking based algorithms outperform GA and that inclusion of geographical distance factor has notable advantage as for the problem with the binary rule.

Fig. 4 shows the performance profiles for GA, RDOA, and RDOA-D when solving the same problem instance as in the case of the binary rule as this instance appeared to be one of the most complicated in the case of the partially binary rule as well. Performance profiles show that the location problem with partially binary rule is more complicated for all algorithms, especially at the early stage of the algorithms – the curves are lower comparing with those in Fig. 3. On the other hand, RDOA-D has more prominent advantage against RDOA – average quality of X obtained by RDOA-D after 2000 function evaluations approximately equals to average quality of X obtained by RDOA after 10,000. In the case of the partially binary rule, one can see from Fig. 4, the average quality of X obtained by RDOA-D after 2000 function evaluations approximately equals to average quality of X obtained by RDOA after 9000. The confidence intervals indicated by the dashed lines show that 95% of the solutions produced by RDOA-D are very close to the optimal solution – the probability that the quality of the solution X produced by RDOA-D will be $Q(X) = 0.98 \pm 0.0016$ is 0.95. The same probability is that the quality of the solution produced by

Table 4
Optimal solutions obtained by Xpress and all proposed algorithms.

Instance	Optimal solution	Objective value
F(3,5,30)	1, 2, 3, 4, 18	11,079,384.66
F(3,5,40)	1, 2, 3, 4, 18	11,079,384.66
F(3,5,50)	Out of memory	
F(3,10,30)	1, 2, 3, 4, 5, 6, 7, 8, 10, 18	13,911,636.65
F(3,10,40)	1, 2, 3, 4, 5, 6, 7, 8, 10, 18	13,911,636.65
F(3,10,50)	Out of memory	
F(5,5,30)	1, 2, 3, 4, 8	9,607,635.61
F(5,5,40)	1, 2, 3, 4, 8	9,607,635.61
F(5,5,50)	Out of memory	
F(5,10,30)	1, 2, 3, 4, 5, 6, 7, 8, 18, 23	12,153,054.66
F(5,10,40)	1, 2, 3, 4, 5, 6, 7, 8, 18, 23	12,153,054.66
F(5,10,50)	Out of memory	

Table 5
Average quality values of the best solution, found by different algorithms and the best solution found in all experiments for the problem instances with partially binary rule of customers' choice.

Instance	GA	RDOA	RDOA-D	The best solution
FLP(5,5,500)	0.984	0.998	1.000	1, 2, 3, 4, 8
FLP(5,10,500)	0.965	0.992	0.998	1, 2, 3, 4, 5, 6, 7, 8, 18, 221
FLP(5,10,1000)	0.946	0.978	0.996	1, 2, 3, 4, 5, 6, 7, 8, 18, 221
FLP(5,10,5000)	0.896	0.928	0.988	1, 2, 3, 4, 5, 6, 7, 8, 18, 221
FLP(10,5,500)	0.987	0.993	1.000	1, 2, 3, 4, 221
FLP(10,10,500)	0.964	0.992	0.999	1, 2, 3, 4, 5, 6, 7, 8, 15, 221
FLP(10,10,1000)	0.946	0.980	0.995	1, 2, 3, 4, 5, 6, 7, 8, 15, 221
FLP(10,10,5000)	0.882	0.916	0.983	1, 2, 3, 4, 5, 6, 7, 8, 15, 221

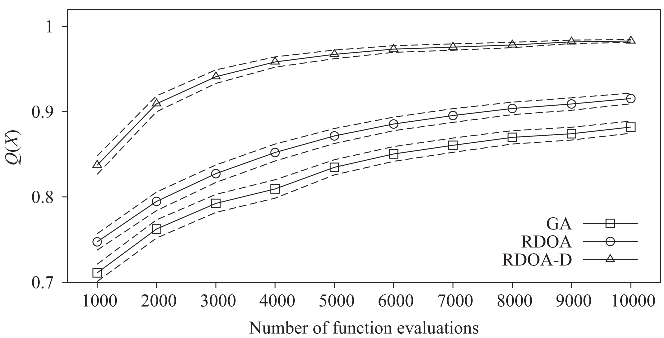

Fig. 4. Performance profiles of the algorithms when solving the facility location problem with the partially binary rule of customers' choice.

RDOA will be 0.92 ± 0.0063, and the quality of the solution produced by GA will be 0.88 ± 0.0071.

4.3. Interchange of the optimal solutions

Because of the similar results, we wonder if the optimal solution of binary case (let us denote it by X_b) could be used as a solution for the partially binary case (denoted by X_{pb}) and vice versa. To compare $M_b(X_b)$ with $M_b(X_{pb})$, the percentage of average error has been evaluated by

$$E_b = \frac{M_b(X_b) - M_b(X_{pb})}{M_b(X_b)} \times 100. \tag{13}$$

Analogously for $M_{pb}(X_{pb})$ with $M_{pb}(X_b)$:

$$E_{pb} = \frac{M_{pb}(X_{pb}) - M_{pb}(X_b)}{M_{pb}(X_{pb})} \times 100. \tag{14}$$

The results are shown in the next table.

In all cases, exchange of the optimal solution of a model to the optimal solution of the other model leads to worse results in the objective function, as it was expected. Since the degree of difference between the objective values is on interval [13.29, 22.39], both models should be applied independently (see Table 6).

Table 6
The percentage difference of the market share caused by interchange of the optimal solutions.

Instance	E_b	E_{pb}
FLP(5,5,500)	14.55	20.66
FLP(5,10,500)	21.96	18.16
FLP(5,10,1000)	22.18	18.18
FLP(5,10,5000)	22.39	17.62
FLP(10,5,500)	15.45	18.38
FLP(10,10,500)	14.67	13.29
FLP(10,10,1000)	14.70	13.31
FLP(10,10,5000)	15.90	15.19

5. Conclusions

In this paper two competitive facility location models have been considered, one when the classical binary choice rule is used for customer behavior and another one – for the partially binary rule. Two heuristic algorithms have been proposed to solve the location problems, one of which is based on the ranking of possible locations and the other one – based on both the ranking and the distance between candidate locations. To check the performance of the proposed algorithms, we have used real geographical coordinates and population data of 6960 municipalities in Spain, and have compared the solutions generated with the best feasible solution found, and in the particular case of the binary model and for small size instances in the partially binary model, with the optimal solution given by the exact approach using Xpress. Computational experiments prove that both algorithms are always better than GA, and RDOA-D obtains always the best solution for all parameters of the problem. The quality of obtained solutions with RDOA-D is around 0.98 for all cases when binary customer choice rule is considered, and when the partially binary rule of customers' behavior is used, the quality is 1.00 for small size instances, and is around 0.99 for bigger size instances.

In conclusion, both heuristic algorithms proposed here, RDOA and RDOA-D, can be a good option to solve competitive facility location problems when an entering firm wants to locate new facilities in the market, and customer choice rule is binary or partially binary.

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