Simulation of Galaxy Density Profiles with Velocity-Dependent Self-Interaction

Theoretical High Energy Physics Group

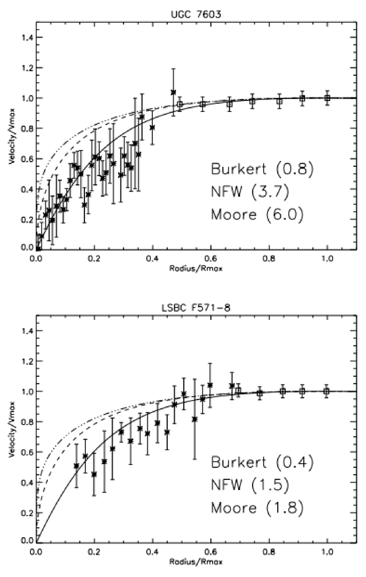
Junho Kang*, Seongsik Kim, Hyun Min Lee

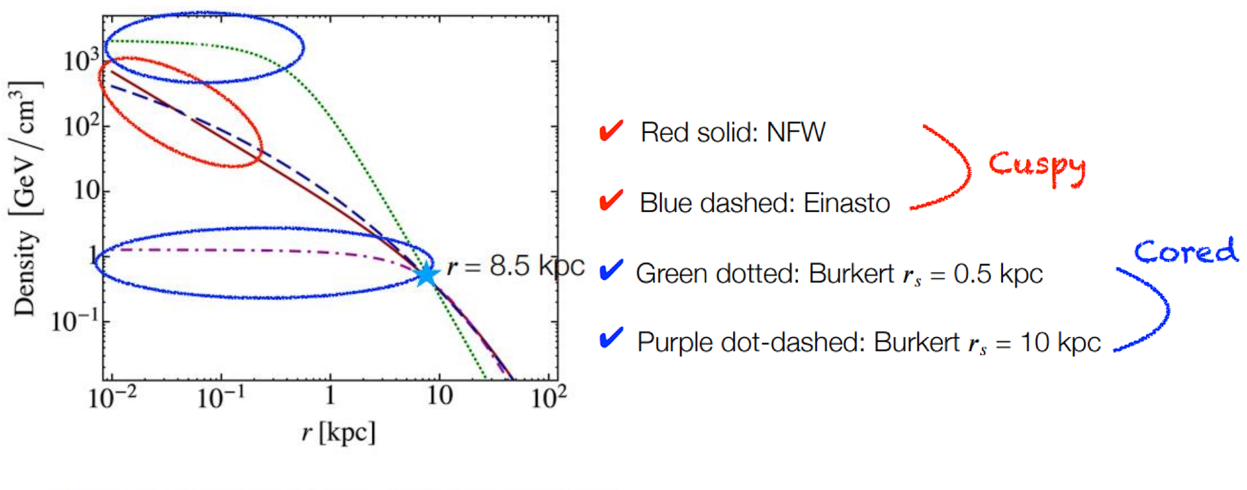
Introduction

The conventional cold dark matter (CDM) model predicts the rotation curves of galaxies, bullet clusters, large scale structures, and the Cosmic Microwave Background power spectrum. However, several cosmological simulations suggest the core density distribution of the dark matter halo is higher and steeper than the observations, which is known as the core-cusp problem.

Several dark matter models are proposed to solve this problem, for example, warm, fuzzy, and Self-interacting dark matter (SIDM) models. In this work, We simulate SIDM with a velocity-dependent cross-section. We also compare the density profiles between CDM, v-indep SIDM, and v-dep SIDM.

Core - Cusp Problem





Cohen, Lisanti, Pierce, Slatyer, JCAP 1310, 061 (2013)

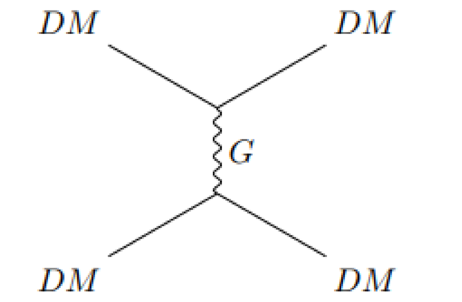
	Rational	Density profile	Halo has…
Observation	Rotation curve of a dwarf galaxy	Pseudo Isothermal profile	Core
Simulation	CDM N-body	NFW profile	Cusp

Pseudo Isothermal profile: $\rho(r) = \rho_0 \left[1 + \left(\frac{r}{R}\right)^2\right]^{-1}$

NFW profile: $\rho(r) = \dfrac{\rho_0}{\left(\frac{r}{R}\right)\left(1+\frac{r}{R}\right)^2}$

The Core-Cusp problem refers to the discrepancy between the dark matter density profile of dwarf galaxies from observations and the density profile predicted by N-body cosmological simulations.

Velocity-dependent Dark Matter



$$\mathcal{L}_{\rm int} = -\frac{c_{\rm DM}}{\Lambda} G^{\mu\nu} T^{\rm DM}_{\mu\nu}$$

There are several possible models for velocity-dependent dark matter.

One example is the interaction of a massive spin-2 particle with dark matter, which introduces an energy-momentum term into the Lagrangian, making the cross-section dependent on velocity.

In the low energy collision, effective range theory allows us a model-independent description for velocity-dependent cross-section. It is given by

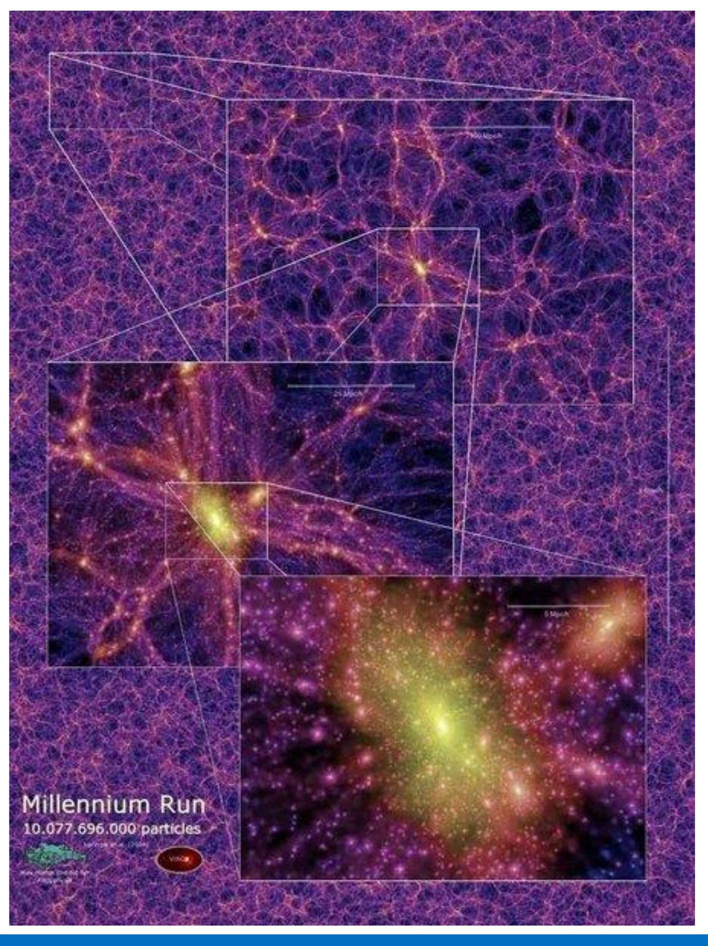
$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0 \approx \frac{4\pi a^2}{1 + k^2\left(a^2 - a r_e\right) + \frac{1}{4}a^2 r_e^2 k^4}$$

(k : momentum, δ_0 : phase shift, a : scattering length, r_e : effective range)

We consider special case $r_e = \frac{a}{2}$ in this work.
Then we get an expression of the following form,

$$\sigma(v) = \sigma_0\left(1 + \left(\frac{v}{w}\right)^2\right)^{-2}$$

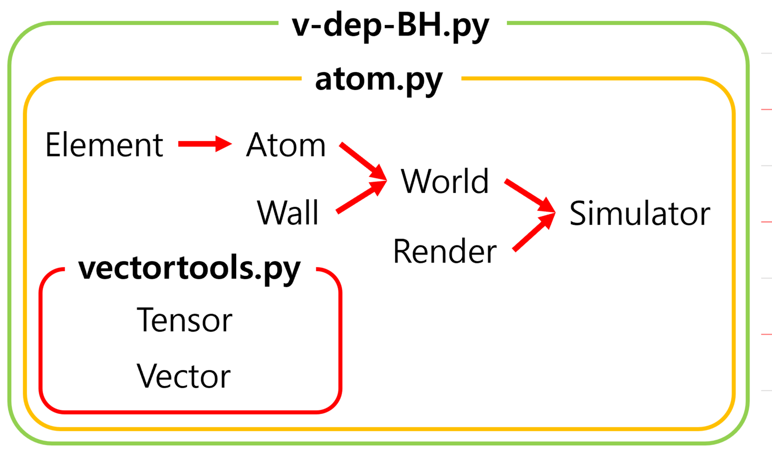
N-body Simulation

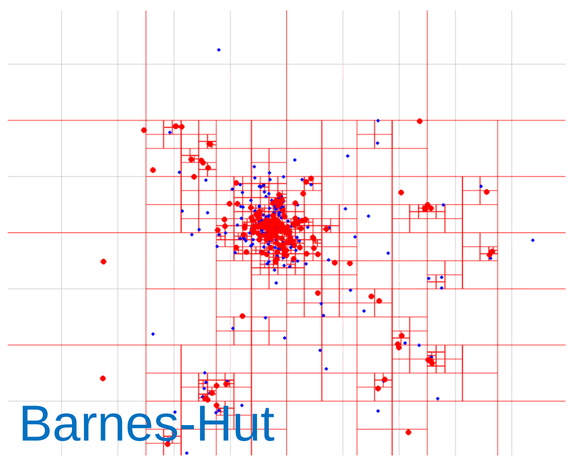


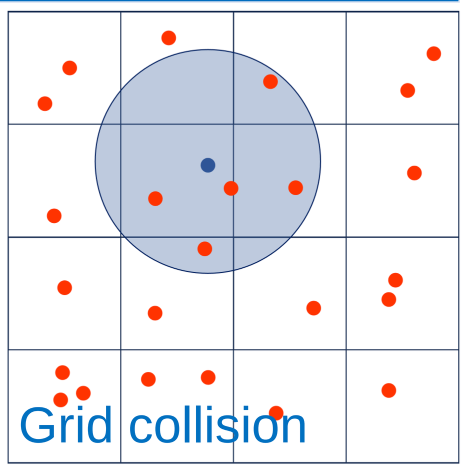
Simulation for system of N particles interacting gravitational force.

- Methods:
  - Direct N method
  - Barnes-Hut algorithm(Tree code)
  - Particle-mesh code
  - P3M and PM-Tree code
  - and more…
- Applications:
  - Solar system
  - Star cluster
  - Dark matter halo
  - Large-scale structure of the universe
  - and more…

Implement of Dark Matter Simulation



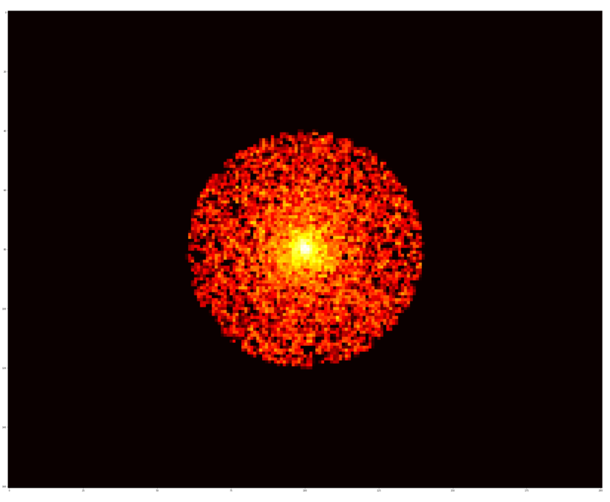




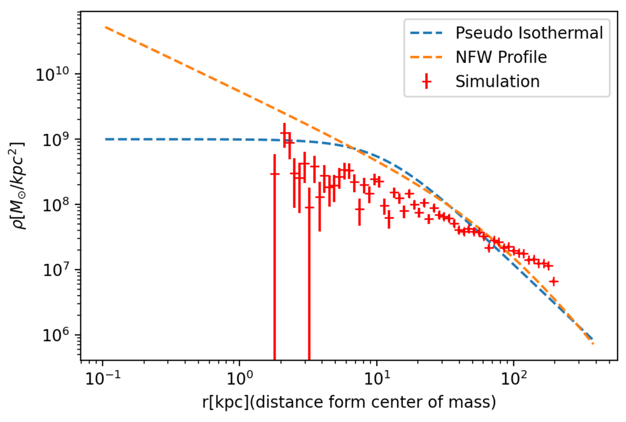
The code is based on Python. This eases managing simulation code for any kind of dark matter.

- N-body Simulation algorithm
  - Direct N-body method: $t \sim O(N^2)$
  - Barnes-Hut algorithm: $t \sim O(N \log N)$ ✔
- Collision detection algorithm
  - Direct N collision detection: $t \sim O(N^2)$
  - Grid collision detection: $t \sim O(N^2/\mathrm{nm})$ ✔

Simulation Setup

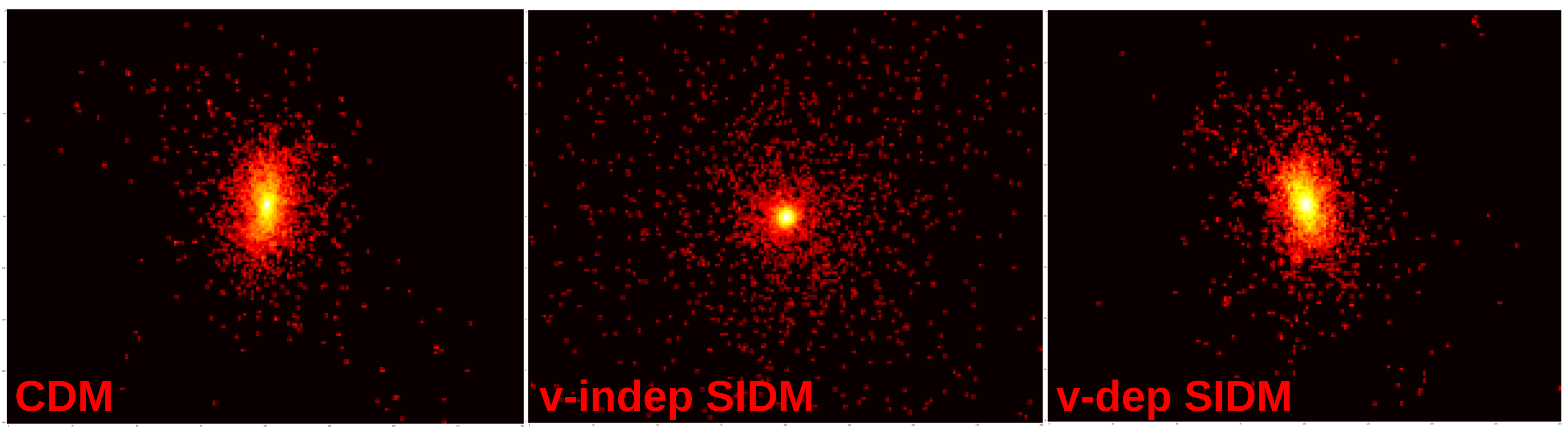


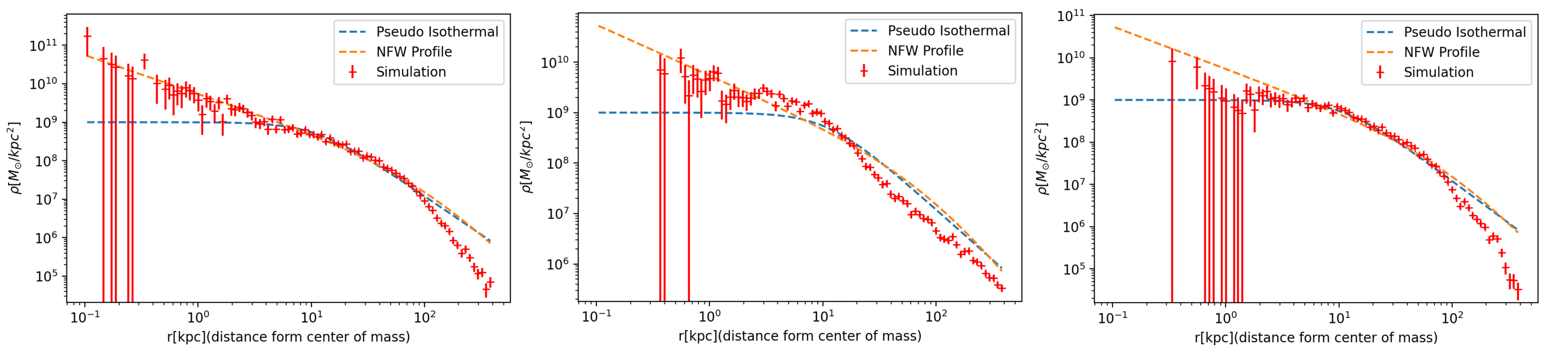
Disk radius:	200 kpc
Initial Density profile:	$\rho \propto \frac{1}{r}$
Initial Velocity:	5 kpc/Gyr & Random direction
Total mass:	$2.5 \times 10^{12}\ M_\odot$
Particle Number:	5000
Unit time step:	0.01 Gyr
Total simulation time:	5 Gyr

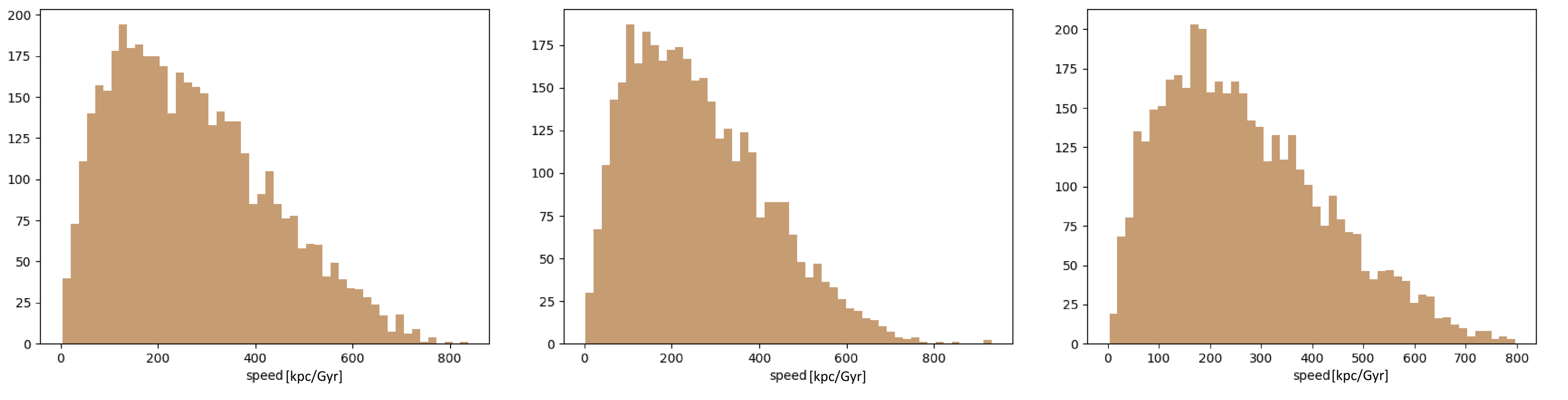


	CDM	v-indep SIDM	v-dep SIDM
$\frac{\sigma_0}{m}$	$0\mathrm{cm}^2/\mathrm{g}$	$2.17\mathrm{cm}^2/\mathrm{g}$	$2.17\mathrm{cm}^2/\mathrm{g}$
w	-	∞	$180\mathrm{km/s}$

Simulation Results







Conclusion

- We find dark matter with velocity dependence can solve core-cusp problem. But it does not change the shape of dark halo significantly, unlike the conventional SIDM.
- The density profile differs from the conventional density profile at large distance. This is due to dimension issues in the simulation.

References

- Marco Cirelli et al (2011). PPPC 4 DM ID: a poor particle physicist cookbook for dark matter indirect detection. Journal of Cosmology and Astroparticle Physics, 2011(03), 051–051.
- Sean Tulin, & Hai-Bo Yu (2018). Dark matter self-interactions and small scale structure. Physics Reports, 730, 1–57.
- Xiaoyong Chu, Camilo Garcia-Cely, & Hitoshi Murayama (2020). A practical and consistent parametrization of dark matter self-interactions. Journal of Cosmology and Astroparticle Physics, 2020(06), 043–043.
- Marchesini, D et al (2002). Hα Rotation Curves: The Soft Core Question. The Astrophysical Journal, 575(2), 801–813.
- Yoo-Jin Kang, & Hyun Min Lee (2021). Effective theory for self-interacting dark matter and massive spin-2 mediators. Journal of Physics G: Nuclear and Particle Physics, 48(4), 045002.
- Moritz S. Fischer et al (2023). Cosmological and idealised simulations of dark matter haloes with velocity-dependent, rare and frequent self-interactions.

중앙대학교 물리학과
Department of Physics, Chung-Ang University