# The Holt-Winters Approach to Exponential Smoothing: 50 Years Old and Going Strong

Paul Goodwin

**PREVIEW.** *Holt-Winters (HW) is the label we frequently give to a set of procedures that form the core of the exponential-smoothing family of forecasting methods. The basic structures were provided by C.C. Holt in 1957 and his student Peter Winters in 1960. Those of you unfamiliar with exponential smoothing should look at the brief tutorial on the next page.*

*In this column, Paul Goodwin discusses recent research that extends the application of the HW method to deal with some important issues faced by the business forecaster.*

## THE LEGACY OF HOLT-WINTERS

Many companies use the Holt-Winters (HW) method to produce short-term demand forecasts when their sales data contain a trend and a seasonal pattern. Fifty years old this year, the method is popular because it is simple, has low data-storage requirements, and is easily automated. It also has the advantage of being able to adapt to changes in trends and seasonal patterns in sales when they occur. This means that slowdowns or speed-ups in demand, or changing consumer behavior at Christmas or in the summer, can all be accommodated. It achieves this by updating its estimates of these patterns as soon as each new sales figure arrives. The tutorial on the next page describes the basic features of the HW approach to exponential smoothing and provides useful references for further insights into this methodology.

Over the years, the Holt-Winters method has been adapted for use in several important situations not originally examined by its creators. More specifically, researchers have recently looked at three issues:

- How can we stop the method from being unduly influenced by sales figures that are unusually high or low (i.e., outliers)?
- Is the method useful when there are several different seasonal patterns in sales (such as when demand has hourly, daily, and monthly cycles mixed together)?
- How can we obtain reliable prediction intervals from the method? For example, we might want the method to give us a range for next month's sales so that there is a 90% chance that sales will fall within this range. But how can we ensure that we really do have a 90% chance of capturing sales within the range?

## UNUSUAL SALES LEVELS (OUTLIERS)

Unusual sales levels, resulting perhaps from strikes, freak weather conditions, and sales promotions, can cause problems for the

## A Brief Tutorial on the Holt-Winters Method

Assume that we require monthly sales forecasts. To produce a forecast, the Holt-Winters (HW) method needs to estimate up to three components of a forecasting equation:

**1.** The current underlying level of sales. This is the level that remains after we have deseasonalized the sales and attempted to remove the effect of random factors (noise).

**2.** The current trend in our sales. This is the change in the underlying level that we expect to occur between now and next month. For example, if we estimate our current level is 500 units and we expect this to be 505 units next month, then our estimated trend is +5 units.

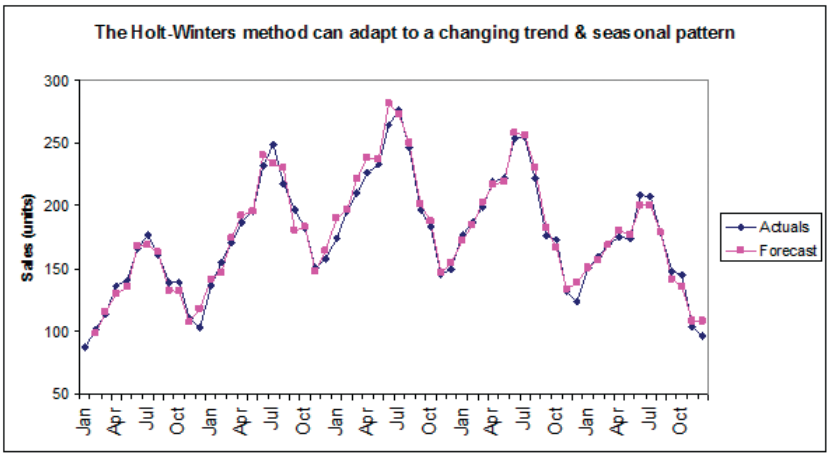
**3.** The seasonal index for the month we are forecasting. Let's say our estimate is 1.2; this means that we expect our sales in this month to be 20% above that month's underlying level, showing that our product tends to sell relatively well at that time of year.

Suppose we are in January and we want a sales forecast for March, two months later. HW estimates that our current level is 500, our trend is 5, and March has a seasonal index of 1.2. The forecast for the level in March will be:

**[Level (500) + 2* Trend (10)] * Seasonal (1.2) = 612 units**

As soon as a new sales figure arrives, HW updates its estimates of the level, trend, and seasonal index for that month. It does this by taking a weighted average of the previous estimates of the component's value and the value suggested by the new sales figure. The weights used are called the *smoothing constants*.

For each component (level, trend, seasonal) there is a smoothing constant that falls between zero and one. Larger smoothing constants mean more weight is placed on the value suggested by the new sales figure and less on the previous estimate. This means that the method will adapt more quickly to genuine changes in the sales pattern, but it might also overreact to freak sales figures. The graph shows how HW forecasts can effectively track trends and seasonal patterns.




And the original articles are listed in the references.

Holt-Winters method. First, the updated estimates of the underlying level, trend, or seasonal pattern are likely to be distorted by the unusual value. An atypically high sales figure this month may cause us to overestimate the rate at which demand for our product is increasing, so that next month's forecast will be too high. Second, the optimized values of the *smoothing constants*, which determine how sensitive our model is to the latest data points, may be distorted by the unusual values. Typically, optimization by our statistical program involves identifying the values of the smoothing constants that lead to the best forecasts on past sales data. If this process is tweaked by the unusual value, we then face the danger that our forecasts will react too slowly or too rapidly to genuine changes in the pattern of sales when they occur.

## Key Points

- While Holt-Winters remains a mainstay approach to business forecasting, it has recently been extended to deal with three problem areas.
- One is the presence of unusual values (outliers). Left unattended, outliers can distort HW forecasts.
- Another is the prevalence of multiple seasonal cycles, such as a combination of day-of-week patterns and month-of-year patterns. Traditional HW could account for only a single seasonal pattern.
- Third is the need for prediction intervals, which affect safety-stock calculations, among other things. Traditional HW intervals in use tend to be too narrow, misleading us into thinking our forecasts are more precise than they really turn out to be.

To counter these problems, Sarah Gelper and her colleagues have developed an easily implemented mechanism that automatically identifies outliers and downgrades their influence (Gelper, Fried, and Croux, 2010). When a forecast has an absolute one-step-ahead error that is so large it exceeds a threshold, the sales value that we were trying to forecast is considered an outlier. It is then automatically replaced by a "cleaned" sales figure. This cleaned value is just sufficient for it to avoid being considered an outlier. The conventional forecasting method is then applied to the cleaned series.

To illustrate, suppose that forecast errors roughly follow a bell-shaped normal distribution with a mean of zero and a standard deviation of 30. Any error that is greater than +60 or less than -60 (i.e., two standard deviations) could be taken as a signal that the sales value is an outlier. If, in a given month, we have actual sales of 700 units but we only forecast 400 units, we have a huge error of 300 units, easily exceeding the threshold of 60. We would then replace the sales of 700 by cleaned sales equal to the forecast plus the threshold (i.e., 400 + 60 =460 units). Thus, the unusually large sales are replaced by more "moderate" sales that are just on the boundary of being declared an outlier.

When the researchers tested their *Robust Holt-Winters method* on both simulated and real data, both of which contained outliers, they found that it gave more accurate forecasts than both classical Holt-Winters and other methods that have been suggested for dealing with the outlier problem. It also performed well when outliers were not present. Their paper suggests how the standard deviation of forecast errors should be estimated when outliers are present in sales series, and how to select smoothing constants under these conditions. Manual cleaning of data can be a time-consuming chore for forecasters. The great attraction of Robust Holt-Winters is that the cleaning can be done automatically by software.

### MULTIPLE SEASONAL CYCLES

The Holt-Winters method was designed to handle data where there is a conventional seasonal cycle across the course of a year, such as monthly seasonality. However, many series have *multiple cycles*: the demand for electricity will have hourly (patterns across the hours of a day), daily (patterns across the days of the week), and monthly cycles. Similar patterns occur in the number of calls received by call centers or the workload faced by hospitals.

James Taylor (Taylor 2010) has extended the conventional HW method to deal with double and triple seasonal cycles. His extensions simply involve an additional smoothing equation and smoothing constant for each extra cycle. Taylor tested these methods on half-hourly electricity demand data by producing half-hourly forecast up to one day ahead. He found that the triple-cycle method was more accurate than versions of Holt-Winters with fewer cycles. Its accuracy was also similar to a more complex autoregressive moving average (ARMA) model that was also designed to model the three "seasonal" cycles.

## PREDICTION INTERVALS

Despite the half-century that has elapsed since the introduction of Holt-Winters, it is only recently that the statistical properties of the method have been explored (e.g. see Hyndman and colleagues, 2005). Knowing about these properties should help us to obtain more reliable ways of estimating prediction intervals.

Usually, prediction intervals turn out to be too narrow, so that, for example, 90% prediction intervals capture actual sales far less than 90% of the time. In essence, the intervals are underestimating the amount of uncertainty we have about the future. This is partly because we cannot be sure that our forecasting model is the true model for our data. In practice, we will be uncertain that the values of the smoothing constants we are using are the correct ones. Also, to begin the process of applying Holt-Winters to data, we have to determine initial values for the underlying sales level and the trend and seasonal pattern. Again, we cannot be sure that our estimates of these initial values are correct.

José D. Bermúdez and his fellow researchers have suggested a method that they claim leads to accurate prediction intervals (Bermúdez, Segura, and Vercheri, 2010). The method uses a Bayesian framework that allows the uncertainty surrounding the smoothing constants and the initial values to be represented as probability distributions. These distributions are then updated in the light of each new sales figure, allowing the calculation of the prediction intervals to take into account the uncertainty we have about our model at any point in time.

The proposed method is relatively complex, but it gave impressive results when tested on the 3003 series used in the M3 competition (Makridakis and Hibon, 2000). For example, for monthly series, on average, 91% of actual observations fell within the 90% prediction intervals. For quarterly series, the corresponding figure was 88%. The researchers did not directly compare the accuracy of their prediction intervals with those obtained through alternative methods, but they quote plenty of evidence to suggest that such intervals would be too narrow.

## CONCLUSION

Fifty years on, researchers are still finding ways to improve the Holt-Winters method and to extend the conditions where it can be applied. This continued interest is a testament to the method's ability to produce reliable forecasts without sacrificing simplicity or transparency. Who would bet against it still being an important part of the forecaster's toolbox 50 years from now?

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