附录Ⅳ　积　分　表

（一）含有 $ax+b$ 的积分

1. $\int \frac{dx}{ax+b}=\frac{1}{a}\ln|ax+b|+C.$

2. $\int (ax+b)^{\mu}dx=\frac{1}{a(\mu+1)}(ax+b)^{\mu+1}+C\ (\mu\neq -1).$

3. $\int \frac{x}{ax+b}dx=\frac{1}{a^2}(ax+b-b\ln|ax+b|)+C.$

4. $\int \frac{x^2}{ax+b}dx=\frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2-2b(ax+b)+b^2\ln|ax+b|\right]+C.$

5. $\int \frac{dx}{x(ax+b)}=-\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|+C.$

6. $\int \frac{dx}{x^2(ax+b)}=-\frac{1}{bx}+\frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|+C.$

7. $\int \frac{x}{(ax+b)^2}dx=\frac{1}{a^2}\left(\ln|ax+b|+\frac{b}{ax+b}\right)+C.$

8. $\int \frac{x^2}{(ax+b)^2}dx=\frac{1}{a^3}\left(ax+b-2b\ln|ax+b|-\frac{b^2}{ax+b}\right)+C.$

9. $\int \frac{dx}{x(ax+b)^2}=\frac{1}{b(ax+b)}-\frac{1}{b^2}\ln\left|\frac{ax+b}{x}\right|+C.$

（二）含有 $\sqrt{ax+b}$ 的积分

10. $\int \sqrt{ax+b}\,dx=\frac{2}{3a}\sqrt{(ax+b)^3}+C.$

11. $\int x\sqrt{ax+b}\,dx=\frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3}+C.$

12. $\int x^2\sqrt{ax+b}\,dx=\frac{2}{105a^3}(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3}+C.$

13. $\int \frac{x}{\sqrt{ax+b}}dx=\frac{2}{3a^2}(ax-2b)\sqrt{ax+b}+C.$

14. $\int \frac{x^2}{\sqrt{ax+b}}dx=\frac{2}{15a^3}(3a^2x^2-4abx+8b^2)\sqrt{ax+b}+C.$

15. $\int \frac{dx}{x\sqrt{ax+b}}=\begin{cases}\frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right|+C & (b>0),\\ \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}}+C & (b<0).\end{cases}$

16. $\int \frac{dx}{x^2\sqrt{ax+b}}=-\frac{\sqrt{ax+b}}{bx}-\frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}.$

17. $\int \frac{\sqrt{ax+b}}{x}dx=2\sqrt{ax+b}+b\int \frac{dx}{x\sqrt{ax+b}}.$

18. $\int \frac{\sqrt{ax+b}}{x^2}dx=-\frac{\sqrt{ax+b}}{x}+\frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}.$

（三）含有 $x^2\pm a^2$ 的积分

19. $\int \frac{dx}{x^2+a^2}=\frac{1}{a}\arctan\frac{x}{a}+C.$

20. $\int \frac{dx}{(x^2+a^2)^n}=\frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}}+\frac{2n-3}{2(n-1)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}.$

21. $\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C.$

（四）含有 ax^2+b $(a>0)$ 的积分

22. $\int \frac{dx}{ax^2+b}=\begin{cases}\frac{1}{\sqrt{ab}}\arctan\sqrt{\frac{a}{b}}x+C & (b>0),\\ \frac{1}{2\sqrt{-ab}}\ln\left|\frac{\sqrt{a}x-\sqrt{-b}}{\sqrt{a}x+\sqrt{-b}}\right|+C & (b<0).\end{cases}$

23. $\int \frac{x}{ax^2+b}dx=\frac{1}{2a}\ln|ax^2+b|+C.$

24. $\int \frac{x^2}{ax^2+b}dx=\frac{x}{a}-\frac{b}{a}\int \frac{dx}{ax^2+b}.$

25. $\int \frac{dx}{x(ax^2+b)}=\frac{1}{2b}\ln\frac{x^2}{|ax^2+b|}+C.$

26. $\int \frac{dx}{x^2(ax^2+b)}=-\frac{1}{bx}-\frac{a}{b}\int \frac{dx}{ax^2+b}.$

27. $\int \frac{dx}{x^3(ax^2+b)}=\frac{a}{2b^2}\ln\frac{|ax^2+b|}{x^2}-\frac{1}{2bx^2}+C.$

28. $\int \frac{\mathrm{d}x}{(ax^2+b)^2}=\frac{x}{2b(ax^2+b)}+\frac{1}{2b}\int \frac{\mathrm{d}x}{ax^2+b}$.

（五）含有 ax^2+bx+c（$a>0$）的积分

29. $\int \frac{\mathrm{d}x}{ax^2+bx+c}=\begin{cases}\frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}}+C & (b^2<4ac),\\ \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right|+C & (b^2>4ac).\end{cases}$

30. $\int \frac{x}{ax^2+bx+c}\mathrm{d}x=\frac{1}{2a}\ln|ax^2+bx+c|-\frac{b}{2a}\int \frac{\mathrm{d}x}{ax^2+bx+c}$.

（六）含有$\sqrt{x^2+a^2}$（$a>0$）的积分

31. $\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}}=\operatorname{arsh}\frac{x}{a}+C_1=\ln(x+\sqrt{x^2+a^2})+C$.

32. $\int \frac{\mathrm{d}x}{\sqrt{(x^2+a^2)^3}}=\frac{x}{a^2\sqrt{x^2+a^2}}+C$.

33. $\int \frac{x}{\sqrt{x^2+a^2}}\mathrm{d}x=\sqrt{x^2+a^2}+C$.

34. $\int \frac{x}{\sqrt{(x^2+a^2)^3}}\mathrm{d}x=-\frac{1}{\sqrt{x^2+a^2}}+C$.

35. $\int \frac{x^2}{\sqrt{x^2+a^2}}\mathrm{d}x=\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})+C$.

36. $\int \frac{x^2}{\sqrt{(x^2+a^2)^3}}\mathrm{d}x=-\frac{x}{\sqrt{x^2+a^2}}+\ln(x+\sqrt{x^2+a^2})+C$.

37. $\int \frac{\mathrm{d}x}{x\sqrt{x^2+a^2}}=\frac{1}{a}\ln\frac{\sqrt{x^2+a^2}-a}{|x|}+C$.

38. $\int \frac{\mathrm{d}x}{x^2\sqrt{x^2+a^2}}=-\frac{\sqrt{x^2+a^2}}{a^2x}+C$.

39. $\int \sqrt{x^2+a^2}\,\mathrm{d}x=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})+C$.

40. $\int \sqrt{(x^2+a^2)^3}\,\mathrm{d}x=\frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2}+\frac{3}{8}a^4\ln(x+\sqrt{x^2+a^2})+C$.

41. $\int x\sqrt{x^2+a^2}\,\mathrm{d}x=\frac{1}{3}\sqrt{(x^2+a^2)^3}+C$.

42. $\int x^2\sqrt{x^2+a^2}\,\mathrm{d}x=\frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2}-\frac{a^4}{8}\ln(x+\sqrt{x^2+a^2})+C$.

43. $\int \frac{\sqrt{x^2+a^2}}{x}\mathrm{d}x=\sqrt{x^2+a^2}+a\ln\frac{\sqrt{x^2+a^2}-a}{|x|}+C$.

44. $\int \frac{\sqrt{x^2+a^2}}{x^2}\mathrm{d}x=-\frac{\sqrt{x^2+a^2}}{x}+\ln(x+\sqrt{x^2+a^2})+C$.

（七）含有$\sqrt{x^2-a^2}$（$a>0$）的积分

45. $\int \frac{\mathrm{d}x}{\sqrt{x^2-a^2}}=\frac{x}{|x|}\operatorname{arch}\frac{|x|}{a}+C_1=\ln|x+\sqrt{x^2-a^2}|+C$.

46. $\int \frac{\mathrm{d}x}{\sqrt{(x^2-a^2)^3}}=-\frac{x}{a^2\sqrt{x^2-a^2}}+C$.

47. $\int \frac{x}{\sqrt{x^2-a^2}}\mathrm{d}x=\sqrt{x^2-a^2}+C$.

48. $\int \frac{x}{\sqrt{(x^2-a^2)^3}}\mathrm{d}x=-\frac{1}{\sqrt{x^2-a^2}}+C$.

49. $\int \frac{x^2}{\sqrt{x^2-a^2}}\mathrm{d}x=\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}|+C$.

50. $\int \frac{x^2}{\sqrt{(x^2-a^2)^3}}\mathrm{d}x=-\frac{x}{\sqrt{x^2-a^2}}+\ln|x+\sqrt{x^2-a^2}|+C$.

51. $\int \frac{\mathrm{d}x}{x\sqrt{x^2-a^2}}=\frac{1}{a}\arccos\frac{a}{|x|}+C$.

52. $\int \frac{\mathrm{d}x}{x^2\sqrt{x^2-a^2}}=\frac{\sqrt{x^2-a^2}}{a^2x}+C$.

53. $\int \sqrt{x^2-a^2}\,\mathrm{d}x=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}|+C$.

54. $\int \sqrt{(x^2-a^2)^3}\,\mathrm{d}x=\frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2}+\frac{3}{8}a^4\ln|x+\sqrt{x^2-a^2}|+C$.

55. $\int x\sqrt{x^2-a^2}\,\mathrm{d}x=\frac{1}{3}\sqrt{(x^2-a^2)^3}+C$.

56. $\int x^2\sqrt{x^2-a^2}\,\mathrm{d}x=\frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2}-\frac{a^4}{8}\ln|x+\sqrt{x^2-a^2}|+C$.

57. $\int \frac{\sqrt{x^2-a^2}}{x}\mathrm{d}x=\sqrt{x^2-a^2}-a\arccos\frac{a}{|x|}+C$.

58. $\int \frac{\sqrt{x^2-a^2}}{x^2}\mathrm{d}x=-\frac{\sqrt{x^2-a^2}}{x}+\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

（八）含有$\sqrt{a^2-x^2}$（$a>0$）的积分

59. $\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C.$

60. $\int \frac{\mathrm{d}x}{\sqrt{(a^2-x^2)^3}}=\frac{x}{a^2\sqrt{a^2-x^2}}+C.$

61. $\int \frac{x}{\sqrt{a^2-x^2}}\mathrm{d}x=-\sqrt{a^2-x^2}+C.$

62. $\int \frac{x}{\sqrt{(a^2-x^2)^3}}\mathrm{d}x=\frac{1}{\sqrt{a^2-x^2}}+C.$

63. $\int \frac{x^2}{\sqrt{a^2-x^2}}\mathrm{d}x=-\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C.$

64. $\int \frac{x^2}{\sqrt{(a^2-x^2)^3}}\mathrm{d}x=\frac{x}{\sqrt{a^2-x^2}}-\arcsin\frac{x}{a}+C.$

65. $\int \frac{\mathrm{d}x}{x\sqrt{a^2-x^2}}=\frac{1}{a}\ln\frac{a-\sqrt{a^2-x^2}}{|x|}+C.$

66. $\int \frac{\mathrm{d}x}{x^2\sqrt{a^2-x^2}}=-\frac{\sqrt{a^2-x^2}}{a^2x}+C.$

67. $\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C.$

68. $\int \sqrt{(a^2-x^2)^3}\,\mathrm{d}x=\frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2}+\frac{3}{8}a^4\arcsin\frac{x}{a}+C.$

69. $\int x\sqrt{a^2-x^2}\,\mathrm{d}x=-\frac{1}{3}\sqrt{(a^2-x^2)^3}+C.$

70. $\int x^2\sqrt{a^2-x^2}\,\mathrm{d}x=\frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2}+\frac{a^4}{8}\arcsin\frac{x}{a}+C.$

71. $\int \frac{\sqrt{a^2-x^2}}{x}\mathrm{d}x=\sqrt{a^2-x^2}+a\ln\frac{a-\sqrt{a^2-x^2}}{|x|}+C.$

72. $\int \frac{\sqrt{a^2-x^2}}{x^2}\mathrm{d}x=-\frac{\sqrt{a^2-x^2}}{x}-\arcsin\frac{x}{a}+C.$

（九）含有$\sqrt{\pm ax^2+bx+c}$（$a>0$）的积分

73. $\int \frac{\mathrm{d}x}{\sqrt{ax^2+bx+c}}=\frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right|+C.$

74. $\int \sqrt{ax^2+bx+c}\,\mathrm{d}x=\frac{2ax+b}{4a}\sqrt{ax^2+bx+c}+\frac{4ac-b^2}{8\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right|+C.$

75. $\int \frac{x}{\sqrt{ax^2+bx+c}}\mathrm{d}x=\frac{1}{a}\sqrt{ax^2+bx+c}-\frac{b}{2\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right|+C.$

76. $\int \frac{\mathrm{d}x}{\sqrt{c+bx-ax^2}}=\frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$

77. $\int \sqrt{c+bx-ax^2}\,\mathrm{d}x=\frac{2ax-b}{4a}\sqrt{c+bx-ax^2}+\frac{b^2+4ac}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$

78. $\int \frac{x}{\sqrt{c+bx-ax^2}}\mathrm{d}x=-\frac{1}{a}\sqrt{c+bx-ax^2}+\frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$

（十）含有$\sqrt{\pm\frac{x-a}{x-b}}$或$\sqrt{(x-a)(b-x)}$的积分

79. $\int \sqrt{\frac{x-a}{x-b}}\,\mathrm{d}x=(x-b)\sqrt{\frac{x-a}{x-b}}+(b-a)\ln\left(\sqrt{|x-a|}+\sqrt{|x-b|}\right)+C.$

80. $\int \sqrt{\frac{x-a}{b-x}}\,\mathrm{d}x=(x-b)\sqrt{\frac{x-a}{b-x}}+(b-a)\arcsin\sqrt{\frac{x-a}{b-a}}+C.$

81. $\int \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}}=2\arcsin\sqrt{\frac{x-a}{b-a}}+C\ (a<b).$

82. $\int \sqrt{(x-a)(b-x)}\,\mathrm{d}x=\frac{2x-a-b}{4}\sqrt{(x-a)(b-x)}+\frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-a}}+C\ (a<b).$

（十一）含有三角函数的积分

83. $\int \sin x\mathrm{d}x=-\cos x+C.$

84. $\int \cos x\mathrm{d}x=\sin x+C.$

85. $\int \tan x\mathrm{d}x=-\ln|\cos x|+C.$

86. $\int \cot x\mathrm{d}x=\ln|\sin x|+C.$

87. $\int \sec x\mathrm{d}x=\ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+C=\ln|\sec x+\tan x|+C.$

88. $\int \csc x\mathrm{d}x=\ln\left|\tan\frac{x}{2}\right|+C=\ln|\csc x-\cot x|+C.$

89. $\int \sec^2 x\mathrm{d}x=\tan x+C.$

90. $\int \csc^2 x\mathrm{d}x=-\cot x+C.$

91. $\int \sec x\tan x\mathrm{d}x=\sec x+C.$

92. $\int \csc x\cot x\mathrm{d}x=-\csc x+C.$

93. $\int \sin^2 x\mathrm{d}x=\frac{x}{2}-\frac{1}{4}\sin 2x+C.$

94. $\int \cos^2 x\mathrm{d}x=\frac{x}{2}+\frac{1}{4}\sin 2x+C.$

95. $\int \sin^n x\mathrm{d}x=-\frac{1}{n}\sin^{n-1}x\cos x+\frac{n-1}{n}\int \sin^{n-2}x\mathrm{d}x.$

96. $\int \cos^n x\mathrm{d}x=\frac{1}{n}\cos^{n-1}x\sin x+\frac{n-1}{n}\int \cos^{n-2}x\mathrm{d}x.$

97. $\int \frac{\mathrm{d}x}{\sin^n x}=-\frac{1}{n-1}\cdot\frac{\cos x}{\sin^{n-1}x}+\frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^{n-2}x}.$

98. $\int \frac{\mathrm{d}x}{\cos^n x}=\frac{1}{n-1}\cdot\frac{\sin x}{\cos^{n-1}x}+\frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\cos^{n-2}x}.$

99. $\int \cos^m x\sin^n x\mathrm{d}x=\frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x+\frac{m-1}{m+n}\int \cos^{m-2}x\sin^n x\mathrm{d}x$

$$=-\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x+\frac{n-1}{m+n}\int \cos^m x\sin^{n-2}x\mathrm{d}x.$$

100. $\int \sin ax\cos bx\mathrm{d}x=-\frac{1}{2(a+b)}\cos(a+b)x-\frac{1}{2(a-b)}\cos(a-b)x+C.$

101. $\int \sin ax\sin bx\mathrm{d}x=-\frac{1}{2(a+b)}\sin(a+b)x+\frac{1}{2(a-b)}\sin(a-b)x+C.$

102. $\int \cos ax\cos bx\mathrm{d}x=\frac{1}{2(a+b)}\sin(a+b)x+\frac{1}{2(a-b)}\sin(a-b)x+C.$

103. $\int \frac{\mathrm{d}x}{a+b\sin x}=\frac{2}{\sqrt{a^2-b^2}}\arctan\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}}+C\ (a^2>b^2).$

104. $\int \frac{\mathrm{d}x}{a+b\sin x}=\frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}\right|+C\ (a^2<b^2).$

105. $\int \frac{\mathrm{d}x}{a+b\cos x}=\frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)+C\ (a^2>b^2).$

106. $\int \frac{\mathrm{d}x}{a+b\cos x}=\frac{1}{a+b}\sqrt{\frac{a+b}{b-a}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{b-a}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{b-a}}}\right|+C\ (a^2<b^2).$

107. $\int \frac{\mathrm{d}x}{a^2\cos^2 x+b^2\sin^2 x}=\frac{1}{ab}\arctan\left(\frac{b}{a}\tan x\right)+C.$

108. $\int \frac{\mathrm{d}x}{a^2\cos^2 x-b^2\sin^2 x}=\frac{1}{2ab}\ln\left|\frac{b\tan x+a}{b\tan x-a}\right|+C.$

109. $\int x\sin ax\mathrm{d}x=\frac{1}{a^2}\sin ax-\frac{1}{a}x\cos ax+C.$

110. $\int x^2\sin ax\mathrm{d}x=-\frac{1}{a}x^2\cos ax+\frac{2}{a^2}x\sin ax+\frac{2}{a^3}\cos ax+C.$

111. $\int x\cos ax\mathrm{d}x=\frac{1}{a^2}\cos ax+\frac{1}{a}x\sin ax+C.$

112. $\int x^2\cos ax\mathrm{d}x=\frac{1}{a}x^2\sin ax+\frac{2}{a^2}x\cos ax-\frac{2}{a^3}\sin ax+C.$

（十二）含有反三角函数的积分（其中 $a>0$）

113. $\int \arcsin\frac{x}{a}\mathrm{d}x=x\arcsin\frac{x}{a}+\sqrt{a^2-x^2}+C.$

114. $\int x\arcsin\frac{x}{a}\mathrm{d}x=\left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arcsin\frac{x}{a}+\frac{x}{4}\sqrt{a^2-x^2}+C.$

115. $\int x^2\arcsin\frac{x}{a}\mathrm{d}x=\frac{x^3}{3}\arcsin\frac{x}{a}+\frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}+C.$

116. $\int \arccos\frac{x}{a}\mathrm{d}x=x\arccos\frac{x}{a}-\sqrt{a^2-x^2}+C.$

117. $\int x\arccos\frac{x}{a}\mathrm{d}x=\left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arccos\frac{x}{a}-\frac{x}{4}\sqrt{a^2-x^2}+C.$

118. $\int x^2\arccos\frac{x}{a}\mathrm{d}x=\frac{x^3}{3}\arccos\frac{x}{a}-\frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}+C.$

119. $\int \arctan\frac{x}{a}\mathrm{d}x=x\arctan\frac{x}{a}-\frac{a}{2}\ln(a^2+x^2)+C.$

120. $\int x\arctan\frac{x}{a}\mathrm{d}x=\frac{1}{2}(a^2+x^2)\arctan\frac{x}{a}-\frac{a}{2}x+C.$

121. $\int x^2\arctan\frac{x}{a}\mathrm{d}x=\frac{x^3}{3}\arctan\frac{x}{a}-\frac{a}{6}x^2+\frac{a^3}{6}\ln(a^2+x^2)+C.$

（十三）含有指数函数的积分

122. $\int a^x\mathrm{d}x=\frac{1}{\ln a}a^x+C.$

123. $\int \mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a}\mathrm{e}^{ax}+C.$

124. $\int x\mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a^2}(ax-1)\mathrm{e}^{ax}+C.$

125. $\int x^n\mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a}x^n\mathrm{e}^{ax}-\frac{n}{a}\int x^{n-1}\mathrm{e}^{ax}\mathrm{d}x.$

126. $\int xa^x\mathrm{d}x=\frac{x}{\ln a}a^x-\frac{1}{(\ln a)^2}a^x+C.$

127. $\int x^na^x\mathrm{d}x=\frac{1}{\ln a}x^na^x-\frac{n}{\ln a}\int x^{n-1}a^x\mathrm{d}x.$

128. $\int \mathrm{e}^{ax}\sin bx\mathrm{d}x=\frac{1}{a^2+b^2}\mathrm{e}^{ax}(a\sin bx-b\cos bx)+C.$

129. $\int \mathrm{e}^{ax}\cos bx\mathrm{d}x=\frac{1}{a^2+b^2}\mathrm{e}^{ax}(b\sin bx+a\cos bx)+C.$

130. $\int \mathrm{e}^{ax}\sin^n bx\mathrm{d}x=\frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\sin^{n-1}bx(a\sin bx-nb\cos bx)+\frac{n(n-1)b^2}{a^2+b^2n^2}\int \mathrm{e}^{ax}\sin^{n-2}bx\mathrm{d}x.$

131. $\int \mathrm{e}^{ax}\cos^n bx\mathrm{d}x=\frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\cos^{n-1}bx(a\cos bx+nb\sin bx)+\frac{n(n-1)b^2}{a^2+b^2n^2}\int \mathrm{e}^{ax}\cos^{n-2}bx\mathrm{d}x.$

（十四）含有对数函数的积分

132. $\int \ln x\mathrm{d}x=x\ln x-x+C.$

133. $\int\frac{\mathrm{d}x}{x\ln x}=\ln|\ln x|+C.$

134. $\int x^n\ln x\mathrm{d}x=\frac{1}{n+1}x^{n+1}\left(\ln x-\frac{1}{n+1}\right)+C.$

135. $\int(\ln x)^n\mathrm{d}x=x(\ln x)^n-n\int(\ln x)^{n-1}\mathrm{d}x.$

136. $\int x^m(\ln x)^n\mathrm{d}x=\frac{1}{m+1}x^{m+1}(\ln x)^n-\frac{n}{m+1}\int x^m(\ln x)^{n-1}\mathrm{d}x.$

（十五）含有双曲函数的积分

137. $\int \mathrm{sh}\,x\mathrm{d}x=\mathrm{ch}\,x+C.$

138. $\int \mathrm{ch}\,x\mathrm{d}x=\mathrm{sh}\,x+C.$

139. $\int \mathrm{th}\,x\mathrm{d}x=\ln\mathrm{ch}\,x+C.$

140. $\int \mathrm{sh}^2x\mathrm{d}x=-\frac{x}{2}+\frac{1}{4}\mathrm{sh}\,2x+C.$

141. $\int \mathrm{ch}^2x\mathrm{d}x=\frac{x}{2}+\frac{1}{4}\mathrm{sh}\,2x+C.$

（十六）定积分

142. $\int_{-\pi}^{\pi}\cos nx\mathrm{d}x=\int_{-\pi}^{\pi}\sin nx\mathrm{d}x=0.$

143. $\int_{-\pi}^{\pi}\cos mx\sin nx\mathrm{d}x=0.$

144. $\int_{-\pi}^{\pi}\cos mx\cos nx\mathrm{d}x=\begin{cases}0, & m\neq n,\\ \pi, & m=n.\end{cases}$

145. $\int_{-\pi}^{\pi}\sin mx\sin nx\mathrm{d}x=\begin{cases}0, & m\neq n,\\ \pi, & m=n.\end{cases}$

146. $\int_0^{\pi}\sin mx\sin nx\mathrm{d}x=\int_0^{\pi}\cos mx\cos nx\mathrm{d}x=\begin{cases}0, & m\neq n,\\ \frac{\pi}{2}, & m=n.\end{cases}$

147. $I_n=\int_0^{\frac{\pi}{2}}\sin^nx\mathrm{d}x=\int_0^{\frac{\pi}{2}}\cos^nx\mathrm{d}x,$

$$I_n=\frac{n-1}{n}I_{n-2}$$

$$=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{4}{5}\cdot\frac{2}{3}\ (n\text{ 为大于 1 的正奇数}),I_1=1,\\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\ (n\text{ 为正偶数}),I_0=\frac{\pi}{2}.\end{cases}$$