

# CE 311K: Introduction to Computer Methods

Krishna Kumar

University of Texas at Austin

*krishnak@utexas.edu*

August 11, 2019

# Overview

## 1 Simulations

## 2 Numerical solution

- Numerical solution of a sliding block

## 3 Aspects of languages

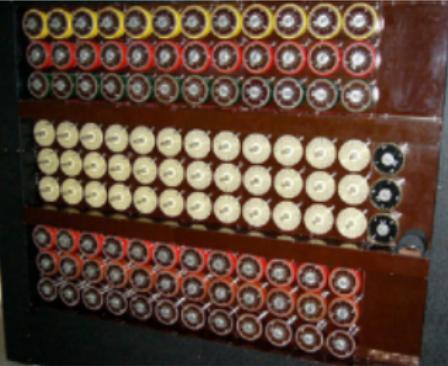
# On computable numbers



Alan Turing



3-Rotor Enigma



The Bombe

Enigma has 150,738,274,937,250 possible states. (credit: Rutherford journal)

## └ On computable numbers



Alan Turing      3-Rotor Enigma      The Bombe  
Enigma has 150,738,274,937,250 possible states. (credit: Rutherford journal)

Turing began to consider whether a method or process could be devised that could decide whether a given mathematical assertion was provable. Turing analyzed the methodical process, focusing on logical instructions, the action of the mind, and a machine that could be embodied as a physical form. Turing developed the proof that automatic computation cannot solve all mathematical problems. This concept became known as the Turing machine, which has become the foundation of the modern theory of computation and computability.

# The Prophet and the Pioneer Computer Scientists



Ada Lovelace, circa 1838 (credit:  
Science Museum, California)



Grace Hopper (credit: unknown)

# CE 311K: Intro to Computer Methods

## └ The Prophet and the Pioneer Computer Scientists



Ada Lovelace, circa 1838 (credit:  
Science Museum, California)



Grace Hopper (credit: unknown)

Lovelace speculated that the Engine 'might act upon other things besides number... the Engine might compose elaborate and scientific pieces of music of any degree of complexity or extent'. The idea of a machine that could manipulate symbols in accordance with rules and that number could represent entities other than quantity mark the fundamental transition from calculation to computation.

Hopper is the pioneer computer scientist, known for creating COBOL – the first computing language.

# To infinity and beyond...



Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, credit: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS).

② Could you guess the storage size requirements?



Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, credit: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS).

ⓘ Could you guess the storage size requirements?

## CE 311K: Intro to Computer Methods

2019-08-11

### └ To infinity and beyond...

Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, with 11,000 lines of code running on **72 kilobytes** of computer memory, credits: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS) and the HDD of **5 petabytes** of data.

If we assume a typical MP3 song as 4MB, we can fit 56 Apollo guidance code in a song, while the 5 PetaBytes of data can hold 250 million songs (there are roughly 97 million songs in the world.)

# To infinity and beyond...



M87 galaxy black hole (JPL)



Gargantua black hole, Interstellar (Warner Bros)

## └ To infinity and beyond...



Scientists have obtained the first image of a black hole, using Event Horizon Telescope observations of the center of the galaxy M87. The image shows a bright ring formed as light bends in the intense gravity around a black hole that is 6.5 billion times more massive than the Sun. The task of observing this super massive black hole is extremely hard, the size ratio is same as observing an orange on the surface of moon from the Earth.

# Disney's Frozen: Modeling snow



# How to bury Anna under the snow?

(c) Disney

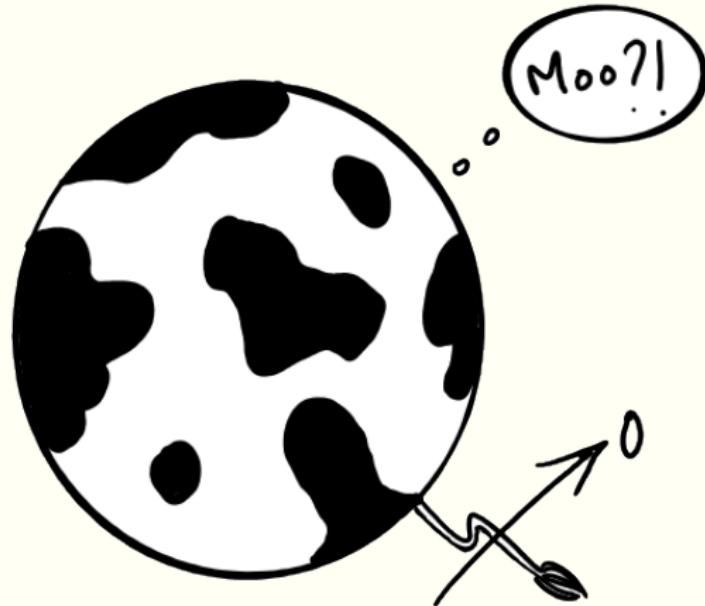


# How to animate like Disney?



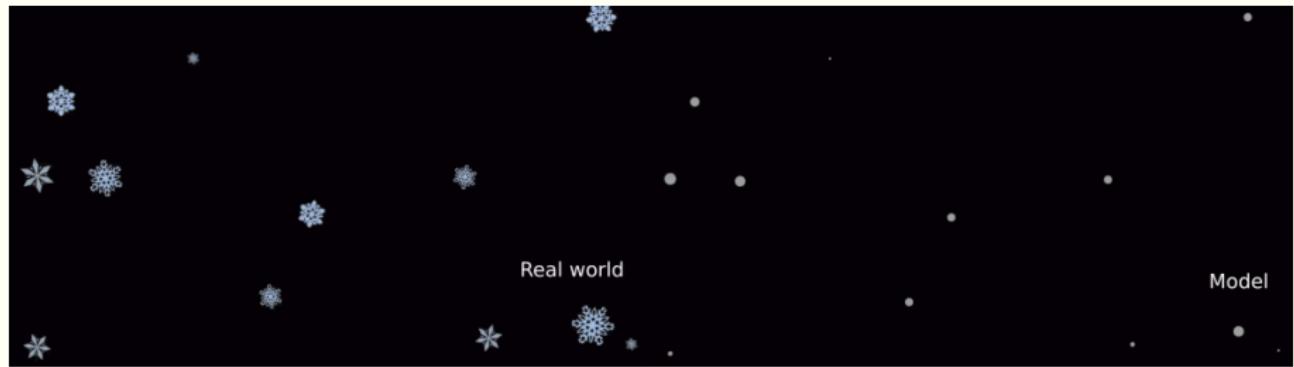
⌚ How to achieve the snow simulation?

# Modeling the real world: Spherical Cow



Consider a spherical cow of radius ' $R$ '  
and a uniform density ' $\rho$ '...

# Modeling snow



# How to animate like Disney: Effect of snow quantity

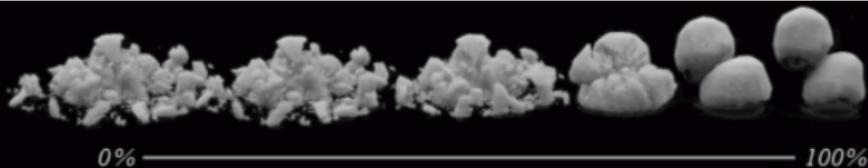


# What type of snow?

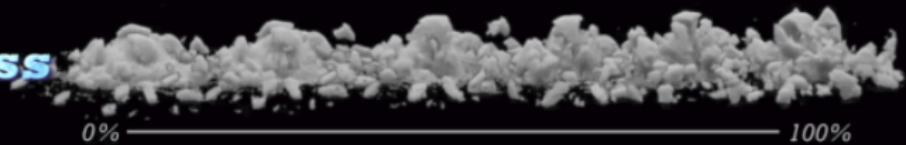


# Snow properties

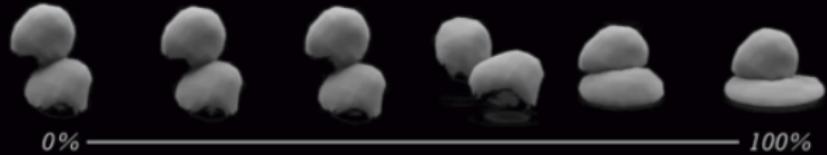
**Viscosity**



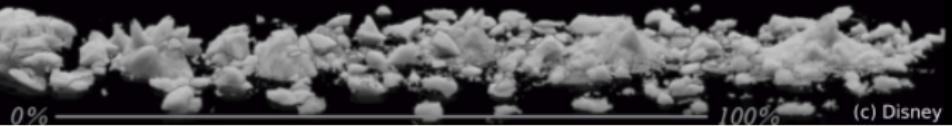
**Chunkiness**



**Plasticity**



**Hardness**



# Snow material parameters

$$\begin{aligned}E_0 &= 1.4 \times 10^5 \\ \theta_c &= 2.5 \times 10^{-2} \\ \theta_s &= 5.0 \times 10^{-3} \\ \xi &= 10\end{aligned}$$



$$\begin{aligned}E_0 &= 1.4 \times 10^5 \\ \theta_c &= 2.5 \times 10^{-2} \\ \theta_s &= 7.5 \times 10^{-3} \\ \xi &= 10\end{aligned}$$



(c) Disney

$$\begin{aligned}E_0 &= 1.4 \times 10^5 \\ \theta_c &= 1.9 \times 10^{-2} \\ \theta_s &= 5.0 \times 10^{-3} \\ \xi &= 10\end{aligned}$$



$$\begin{aligned}E_0 &= 1.4 \times 10^5 \\ \theta_c &= 1.9 \times 10^{-2} \\ \theta_s &= 7.5 \times 10^{-3} \\ \xi &= 10\end{aligned}$$



# How to model snow?

- Representation of snow: Each snow flake is modeled as a particle
- Physics: our model or parameters of how the snow behaves
- Solver: How the snow would move and interact with the objects in the scene.
- Algorithm: A sequence of logical steps required to perform a specific task.
- Implementation and verification of our snow model.

💡 This is a **simulation**.



## └ Simulations

## └ How to model snow?

- Representation of snow: Each snowflake is modeled as a particle
- Physics: our model or parameters of how the snow behaves
- Solver: How the snow would move and interact with the objects in the scene.
- Algorithm: A sequence of logical steps required to perform a specific task.
- Implementation and verification of our snow model.

⌚ This is a simulation.

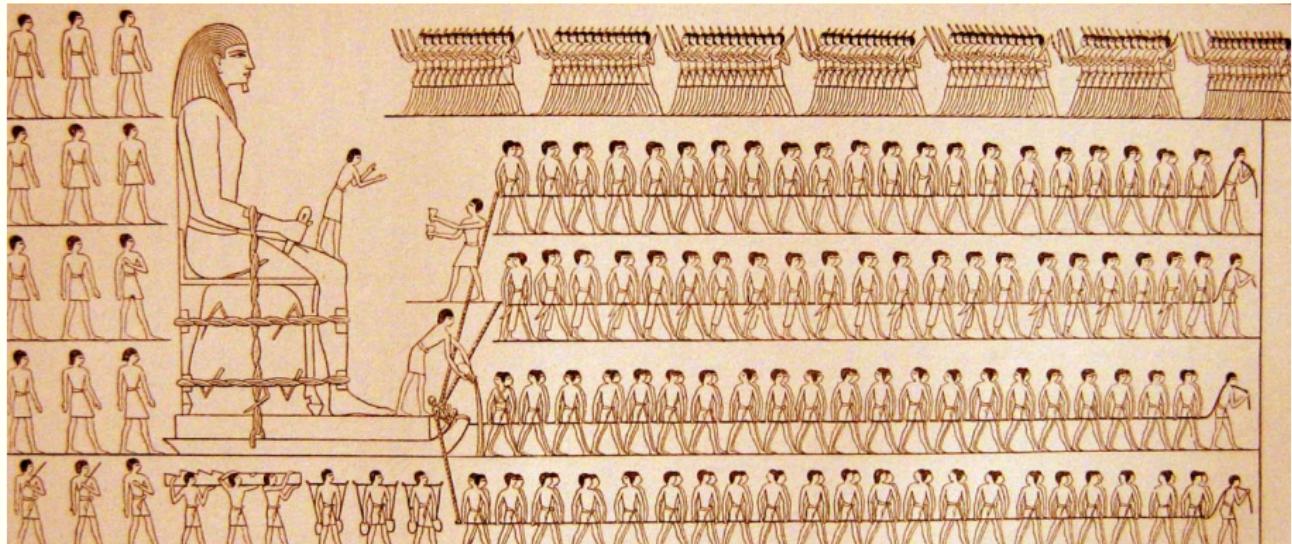


A simulation is an approximate imitation of the operation of a process or system. The first step is to create model, which represents the system's key characteristics, such as its behavior, functions and abstract or physical properties. The model represents the system itself, whereas the simulation represents its operation over time.

There are several numerical techniques that can be used to solve a mathematical problem. They differ in accuracy, length of calculations, and difficulty in programming.

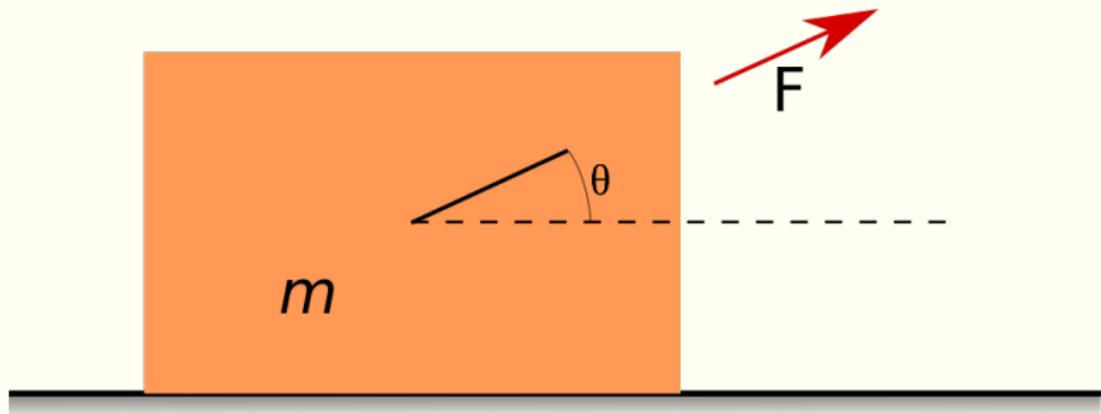
Since numerical solutions are an approximation and the computer program that executes the numerical method might have errors (or bugs), a numerical solution needs to be examined closely.

# What is the optimal angle to pull the statue?



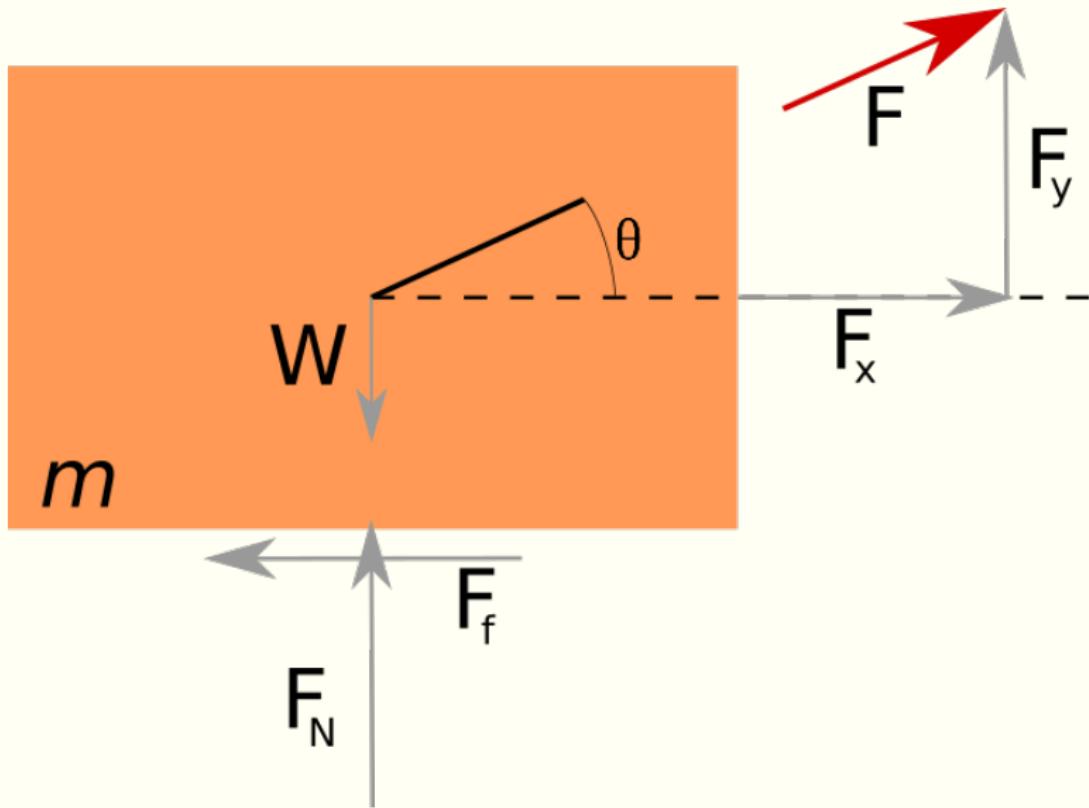
A wall painting from the tomb of Djehutihotep (credit: [martinhumanities.com](http://martinhumanities.com))

# Numerical solution of a sliding block: Approximation



What is the optimal angle to pull the block applying the least amount of force?

# Numerical solution of a sliding block: Forces



# Numerical solution of a sliding block: Forces

$$F_x = F \cos \theta \quad \& \quad F_y = F \sin \theta$$

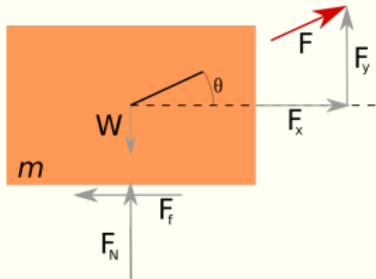
$$F_f = \mu \cdot F_N = \mu \cdot W - \mu F_y = \mu mg - \mu F \sin \theta$$

Vertical forces  $\sum F_{vert} \uparrow: F_y + F_N - W = 0$

$$F_N = \mu mg - F \sin \theta$$

Horizontal forces  $\sum F_{hor} \rightarrow: F_x + F_f = 0$

$$F \cos \theta - \mu mg + \mu F \sin \theta = 0$$



$$F = \frac{\mu \cdot mg}{(\cos \theta + \mu \sin \theta)}$$

## Numerical solution of a sliding block: Compute force

- Given  $W = 25kN(2500 \text{ kg})$ ,  $\theta = 45^\circ$  and  $\mu = 0.75$  ( $35^\circ$ ):

$$F = \frac{0.75 \times 25}{\cos(45) + 0.75 \sin(45)} = 15.15 \text{ kN.}$$

- Given  $F = 17.5kN(2500 \text{ kg})$  and  $\mu = 0.75$ , what's  $\theta$ ?

$$\text{Try } \theta = 60^\circ : F = \frac{0.75 \times 25}{\cos(60) + 0.75 \sin(60)} = 16.31 \text{ kN.}$$

$$\text{Try } \theta = 70^\circ : F = \frac{0.75 \times 25}{\cos(70) + 0.75 \sin(70)} = 17.91 \text{ kN.}$$

$$\text{Try } \theta = 65^\circ : F = \frac{0.75 \times 25}{\cos(65) + 0.75 \sin(65)} = 17.00 \text{ kN.}$$

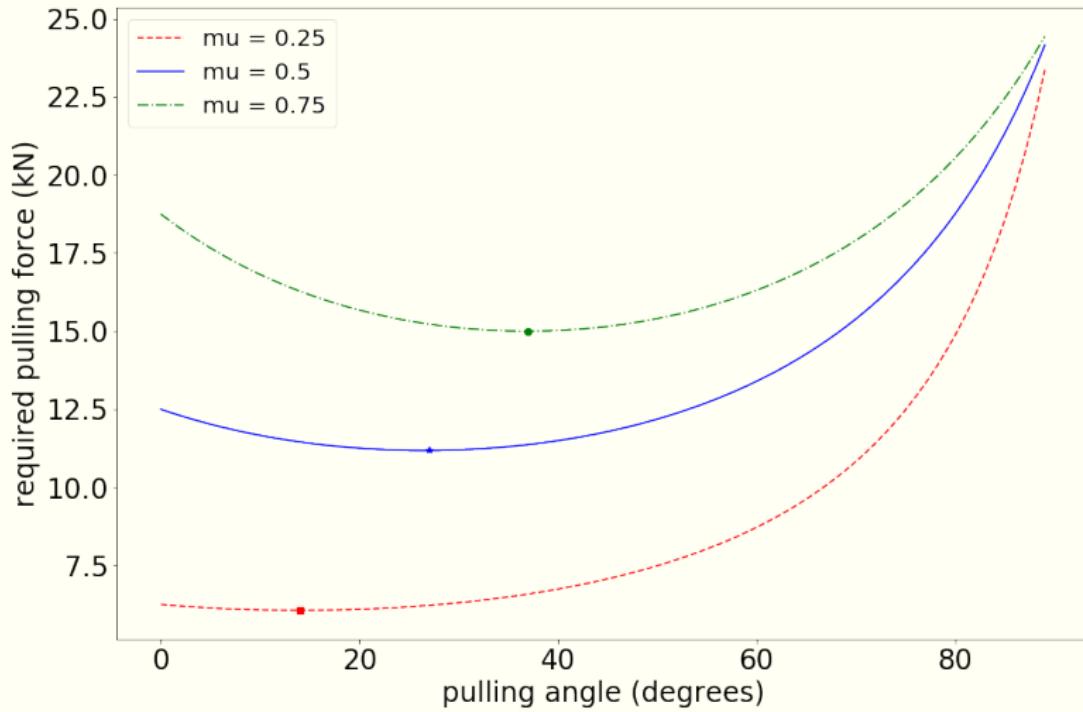
$$\text{Try } \theta = 67,5^\circ : F = \frac{0.75 \times 25}{\cos(67.5) + 0.75 \sin(67.5)} = 17.43 \text{ kN.}$$

This is **bisection method!**

## Q What are the characteristics of a numerical solution?

- A numerical recipe is a *sequence of simple steps*
- *Flow of control* as each step is executed.
- Yields an *approximate* numerical answer (a finite number) for the problem
- These solutions can be very accurate
- Most answers are determined in an iterative approach (numerical method: mathematical / computer-aided technique) until a desired minimum/acceptable accuracy is obtained
- Typically, a finite set of iterations (steps) are used in the numerical method to obtain a solution. A means of determining *when to stop*.

# Numerical solution of a sliding block: Friction angles



# Q What does a computer do?

- Fundamentally:
  - performs *calculations* - a billion calculations per second.
  - Remembers results 100s of gigabytes of storage.
- Computers only know and do what you tell them!
- **Fixed program** computer (calculator)
- **Stored program** computer - machine stores and executes instructions



IBM Bluegene/P supercomputer (credit: unknown)

## CE 311K: Intro to Computer Methods

## └ Aspects of languages

## └ Q What does a computer do?

## Q What does a computer do?

- Fundamentally:
  - performs calculations - a billion calculations per second.
  - Remembers results 100s of gigabytes of storage.
- Computers only know and do what you tell them!
- Fixed program computer (calculator)
- Stored program computer - machine stores and executes instructions



IBM BlueGene/P supercomputer (credit: unknown)

A single rack of IBM Bluegene/P in 2007 could do 5Terra flops as fast as the current iPhones X.

UT Austin's Frontera can do 38 PetaFlops. In contrast a human brain can do 1,000 petaflops or 10 quadrillion operations per second (i.e., 1 exaflop or a billion - billion calculations).

# Aspects of languages: Primitive constructs

- Turing showed that you can compute anything using **6 primitives**.
- Anything computable in one language is computable in any other programming language.
- *English:* words
- *Programming languages:* numbers, strings, simple operators



English word cloud (credit: Michael Twardos)

A word cloud visualization of common Python reserved words and operators. The words are arranged in a grid-like structure. From top-left to bottom-right, the words are: float, \*\*, \* <= >, bool, string, int, /, NoneType, +, ==, and =. This represents the core set of primitive constructs available in the Python language.

Python word cloud  
(credit: unknown)

## CE 311K: Intro to Computer Methods

## └ Aspects of languages

## └ Aspects of languages: Primitive constructs

## Aspects of languages: Primitive constructs

- Turing showed that you can compute anything using **6 primitives**.
- Anything computable in one language is computable in any other programming language.
- English words
- Programming languages: numbers, strings, simple operators



English word cloud (credit: Michael Twarode)

```
float **  
* as :bool  
int  
= --  
+  
-
```

Python word cloud (credit: unknown)

In fact, Alan Turing, who was a really great computer scientist, he showed that you can compute anything using the six primitives. And the six primitives are move left, move right, read, write, scan, and do nothing. Using those six instructions and the piece of tape, he showed that you can compute anything. And using those six instructions, programming languages came about that created a more convenient set of primitives. You don't have to program in only these six commands.

## CE 311K: Intro to Computer Methods

## └ Aspects of languages

## └ Aspects of languages: Primitive constructs

## Aspects of languages: Primitive constructs

- Turing showed that you can compute anything using **6 primitives**.
- Anything computable in one language is computable in any other programming language.
- English words
- Programming languages: numbers, strings, simple operators



# Aspects of languages: Syntax and Static semantics

## Syntax

- *English*
  - “boy dog cat”: syntactically invalid
  - “boy hugs cat”: syntactically valid
- *Python*
  - 5 "Hi": syntactically invalid
  - 3.2 \* 7: syntactically valid

**Static semantics** is when syntactically valid strings have meaning:

- English: “I are hungry”: syntactically valid, but static semantic error
- Python: 5 + "Hi": syntactically valid, but static semantic error

# Aspects of languages: Semantics

Semantics is the meaning associated with a syntactically correct string of symbols with no static semantic errors:

- *English*: can have many meanings “Flying planes can be dangerous”.
- *programming languages*: have only one meaning but may not be what programmer intended

# Where things can go wrong

- **syntactic errors:** - common and easily caught
- **static semantic errors:**
  - some languages check for these before running program
  - can cause unpredictable behavior
- no semantic errors **but different meaning than what programmer intended**
  - program crashes, stops running
  - program runs forever
  - program gives an answer but different than expected