CE 311K: Introduction to Computer Methods

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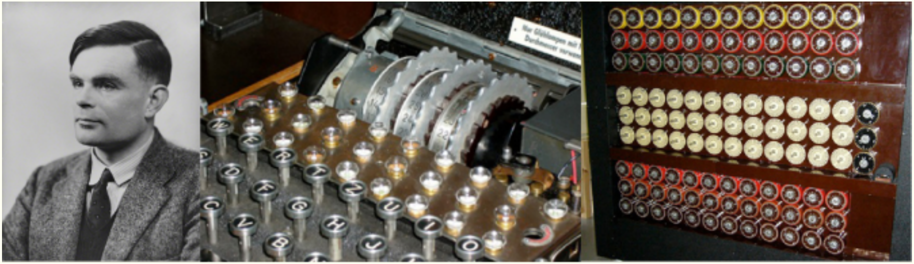
Overview

1. Simulations
2. Numerical solution
   - Numerical solution of a sliding block

On computable numbers



Alan Turing 3-Rotor Enigma The Bombe

Enigma has 150,738,274,937,250 possible states. (credit: Rutherford journal)

On computable numbers

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Turing began to consider whether a method or process could be devised that could decide whether a given mathematical assertion was provable. Turing analyzed the methodical process, focusing on logical instructions, the action of the mind, and a machine that could be embodied as a physical form. Turing developed the proof that automatic computation cannot solve all mathematical problems. This concept became known as the Turing machine, which has become the foundation of the modern theory of computation and computability.

The Prophet and the Pioneer Computer Scientists

Ada Lovelace, circa 1838 (credit: Science Museum, California)

Grace Hopper (credit: unknown)

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Lovelace speculated that the Engine 'might act upon other things besides number... the Engine might compose elaborate and scientific pieces of music of any degree of complexity or extent'. The idea of a machine that could manipulate symbols in accordance with rules and that number could represent entities other than quantity mark the fundamental transition from calculation to computation.

Hopper is the pioneer computer scientist, known for creating COBOL – the first computing language.

Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, credit: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS).

? Could you guess the storage size requirements?

To infinity and beyond...

Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, credit: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS).

ⓘ Could you guess the storage size requirements?

Margaret Hamilton next to a stack of the Apollo Guidance Computer source code (1969, with 11,000 lines of code running on **72 kilobytes** of computer memory, credits: MIT Museum) and Katie Bouman who developed the algorithm for creating the first-ever image of black hole (2019, credits: PBS) and the HDD of **5 petabytes** of data.

If we assume a typical MP3 song as 4MB, we can fit 56 Apollo guidance code in a song, while the 5 PetaBytes of data can hold 250 million songs (there are roughly 97 million songs in the world.)

To infinity and beyond...

M87 galaxy black hole (JPL)

Gargantua black hole, Interstellar (Warner Bros)

To infinity and beyond...

Scientists have obtained the first image of a black hole, using Event Horizon Telescope observations of the center of the galaxy M87. The image shows a bright ring formed as light bends in the intense gravity around a black hole that is 6.5 billion times more massive than the Sun. The task of observing this super massive black hole is extremely hard, the size ratio is same as observing an orange on the surface of moon from the Earth.

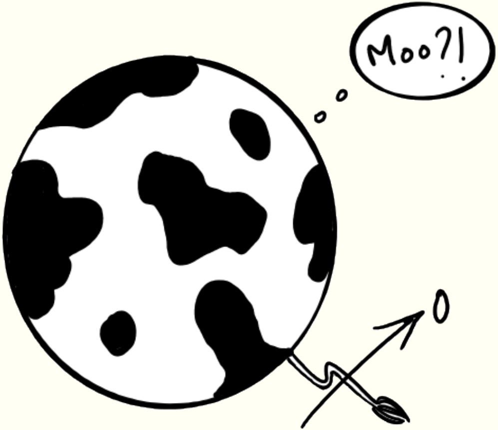
Disney's Frozen: Modeling snow

How to bury Anna under the snow?

How to animate like Disney?

How to achieve the snow simulation?

Modeling the real world: Spherical Cow



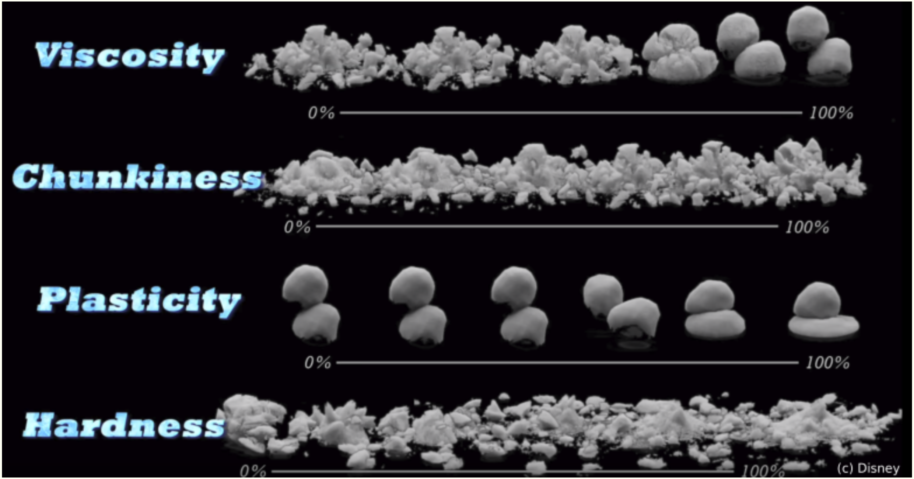
Consider a spherical cow of radius 'R' and a uniform density 'ρ'...

Modeling snow

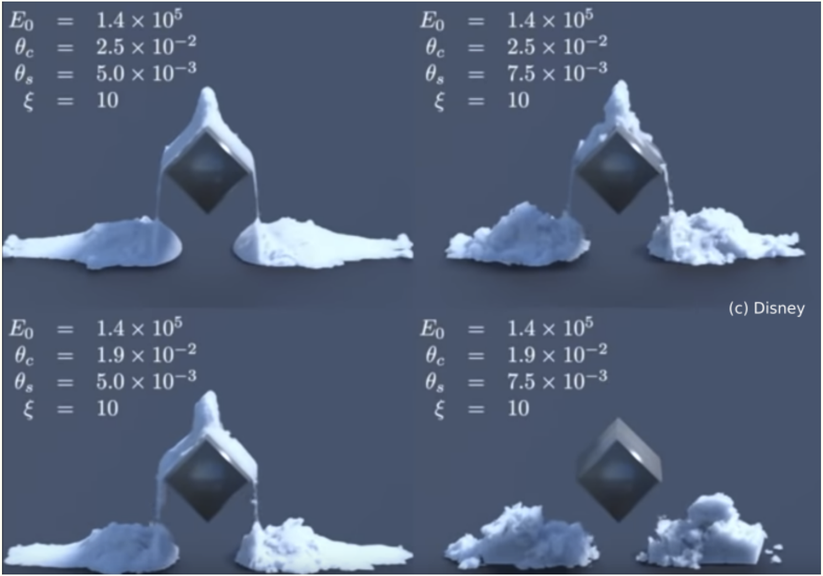
How to animate like Disney: Effect of snow quantity

What type of snow?

Snow properties



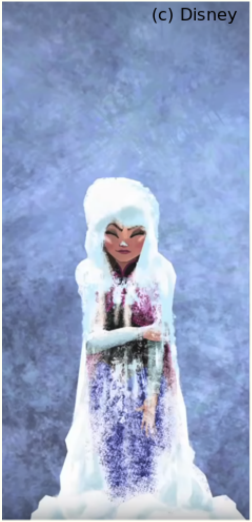
Snow material parameters



How to model snow?

- Representation of snow: Each snow flake is modeled as a particle
- Physics: our model or parameters of how the snow behaves
- Solver: How the snow would move and interact with the objects in the scene.
- Algorithm: A sequence of logical steps required to perform a specific task.
- Implementation and verification of our snow model.

This is a **simulation**.

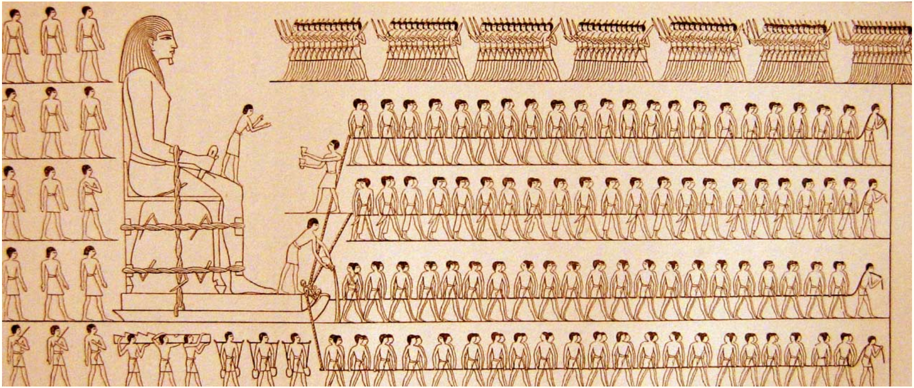


A simulation is an approximate imitation of the operation of a process or system. The first step is to create model, which represents the system's key characteristics, such as its behavior, functions and abstract or physical properties. The model represents the system itself, whereas the simulation represents its operation over time.

There are several numerical techniques that can be used to solve a mathematical problem. They differ in accuracy, length of calulations, and difficulty in programming.

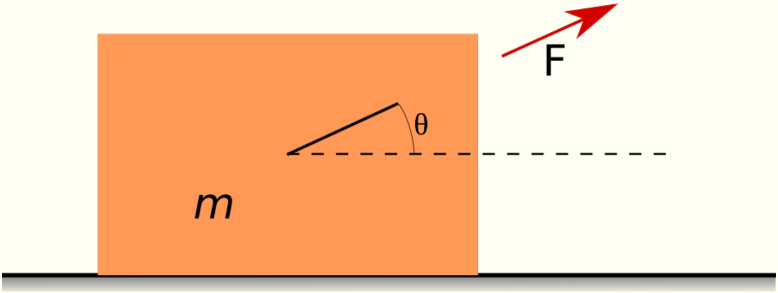
Since numerical solutions are an approximation and the computer program that executes the numerical method might have errors (or bugs), a numerical solution needs to be examined closely.

What is the optimal angle to pull the statue?



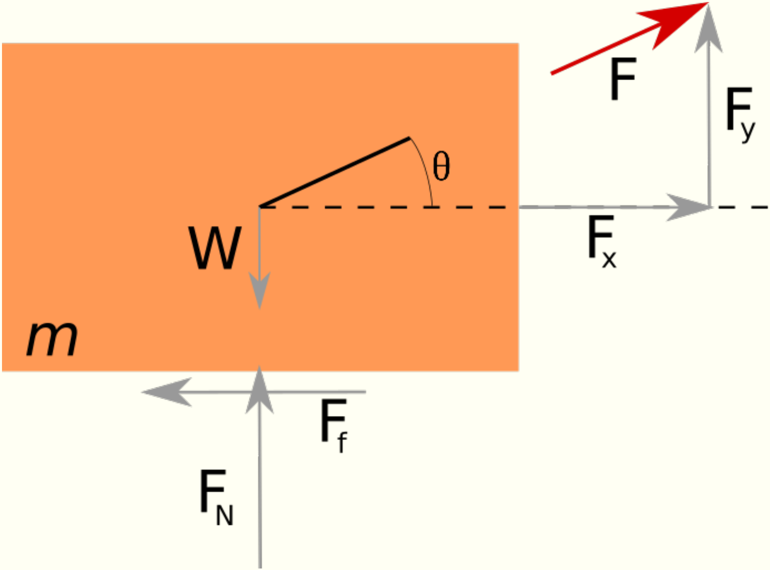
A wall painting from the tomb of Djehutihotep (credit: martinhumanities.com)

Numerical solution of a sliding block: Approximation



What is the optimal angle to pull the block applying the least amount of force?

Numerical solution of a sliding block: Forces



Numerical solution of a sliding block: Forces

$$F_x = F\cos\theta \quad \& \quad F_y = F\sin\theta$$

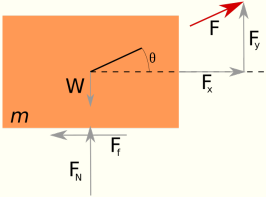
$$F_f = \mu \cdot F_N = \mu \cdot W - \mu F_y = \mu mg - \mu F\sin\theta$$

Vertical forces $\sum F_{vert} \uparrow$: $F_y + F_N - W = 0$

$$F_N = \mu mg - F\sin\theta$$

Horizontal forces $\sum F_{hor} \rightarrow$: $F_x + F_f = 0$

$$F\cos\theta - \mu mg + \mu F\sin\theta = 0$$



$$\boxed{F = \frac{\mu \cdot mg}{(\cos\theta + \mu\sin\theta)}}$$

Numerical solution of a sliding block: Compute force

- Given $W = 25kN(2500\,\text{kg})$, $\theta = 45^o$ and $\mu = 0.75$ (35^o):

$$F = \frac{0.75 \times 25}{\cos(45) + 0.75\sin(45)} = 15.15\,\text{kN}.$$

- Given $F = 17.5kN(2500\,\text{kg})$ and $\mu = 0.75$, what's θ?

$$Try\ \theta = 60^o: \ F = \frac{0.75 \times 25}{\cos(60) + 0.75\sin(60)} = 16.31\,\text{kN}.$$

$$Try\ \theta = 70^o: \ F = \frac{0.75 \times 25}{\cos(70) + 0.75\sin(70)} = 17.91\,\text{kN}.$$

$$Try\ \theta = 65^o: \ F = \frac{0.75 \times 25}{\cos(65) + 0.75\sin(65)} = 17.00\,\text{kN}.$$

$$Try\ \theta = 67,5^o: \ F = \frac{0.75 \times 25}{\cos(67.5) + 0.75\sin(67.5)} = 17.43\,\text{kN}.$$

This is **bisection method!**

What are the characteristics of a numerical solution?

- Yields an *approximate* numerical answer (a finite number) for the problem
- These solutions can be very accurate
- Most answers are determined in an iterative approach (numerical method: mathematical / computer-aided technique) until a desired minimum/acceptable accuracy is obtained
- Typically, a finite set of iterations (steps) are used in the numerical method to obtain a solution

Numerical solution of a sliding block: Friction angles