1 Nyquist for finite signals

Suppose we have a finite duration signal:

$$\mathbf{X}^\top = [x[0], x[1], \ldots x[T-1]] \tag{1}$$

and we wish to decimate it as illustrated in Figure 1.

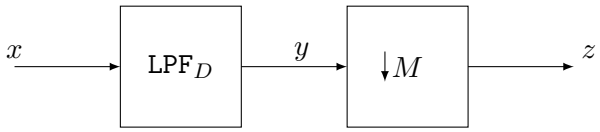

Figure 1: The decimation process.

To do this and avoid aliasing, the low-pass filter's ($\texttt{LPF}_D$) stop-band must start at π/M. Assuming this filter is an FIR filter of length L with impulse response $h_{\texttt{LPF}_D}$ then we get

$$y[t] = h_{\texttt{LPF}_D}[t] \star x[t] \tag{2}$$

where $\star$ indicates linear convolution. This makes y of length $T + L - 1$.

The decimator then creates

$$z[t] = y[tM] \tag{3}$$

so that z will be of length $\frac{T+L-1}{M}$.