Causal Strategic Learning under Competitive Selection

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1. Causal Strategic Learning (CSL)

Example of causal strategic learning (CSL):

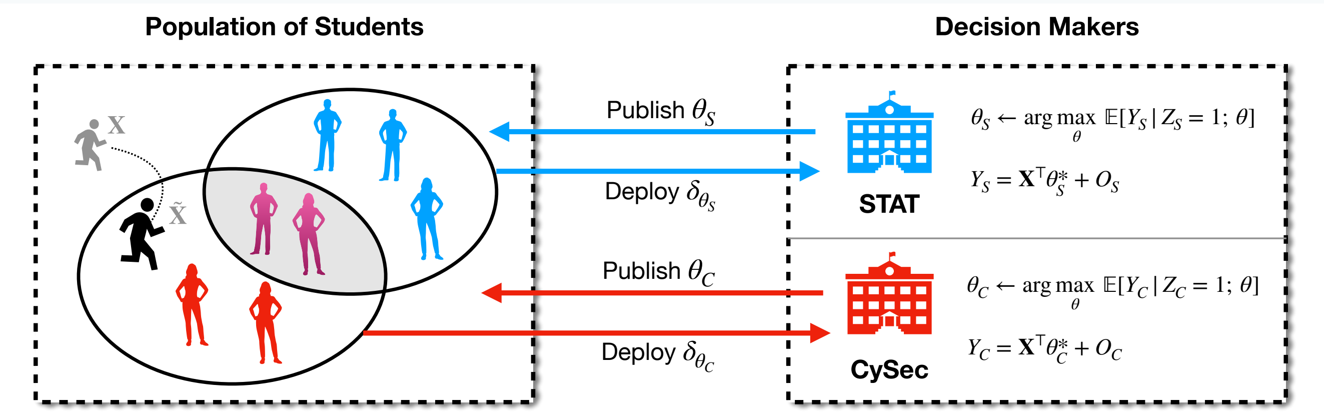
$$\text{College GPA} := \begin{bmatrix}\text{HS GPA}\\ \text{SAT}\end{bmatrix}^\top \begin{bmatrix}0.9\\ 0\end{bmatrix} + \text{Noise}$$

$$\text{Predicted GPA} := \begin{bmatrix}\text{HS GPA}\\ \text{SAT}\end{bmatrix}^\top \begin{bmatrix}0.7\\ 0.001\end{bmatrix}$$

- ***Gaming***: Changing SAT affects the prediction but not the true outcome,
- **Genuine Improvement**: Changing HS GPA also affects the true outcome.

→ Learning the true causal mechanism for better decision making,
(and there are complex interactions between decision makers and strategic agents).

2. CSL under Competitive Selection (An Example)



- 2 departments select from the same pool of students,
- They deploy their own selection rules δ_S and δ_C, parameterised by $\boldsymbol{\theta}_S$ and $\boldsymbol{\theta}_C$,
- Each wants to maximise the future GPA of their **own** students,
- Students modify $\mathbf{X}$ to $\tilde{\mathbf{X}}$ to maximise admission chance.

3. Impact of Competitive Selection

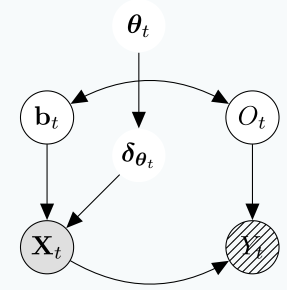


Figure 1: Prior work

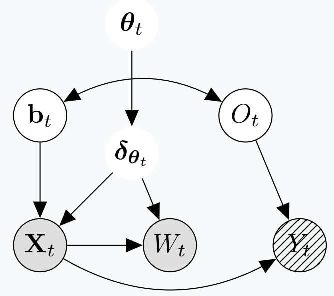


Figure 2: Ours with Selection

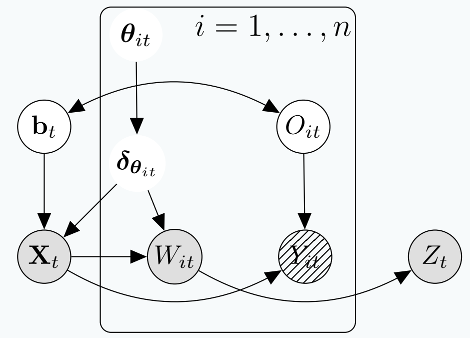


Figure 3: Ours with Competitive Selection

In Fig. (2),
- cond. dependencies, $\boldsymbol{\theta}_t \not\perp \{\mathbf{b}_t, O_t\} \mid W_t$ → harder to solve $\max_{\boldsymbol{\theta}_t} \mathbb{E}[Y_t \mid W_t = 1; \boldsymbol{\theta}_t]$;
- biased data, $\{\boldsymbol{\theta}_t, \mathbf{x}_t, \{y_t | w_t = 1\}\}_{t=1}^T$ → harder to infer causation.

In Fig. (3), $p(Y_{jt} \mid Z_t = j)$ depends on $p(Z_t = j \mid \mathbf{X}_t)$ which depends on $\{p(W_{it} \mid \mathbf{X}_t)\}_{i=1}^n$
→ interference from rival decision makers.

4. An Optimal Solution

Table 1: Our assumptions on agents' behaviour.

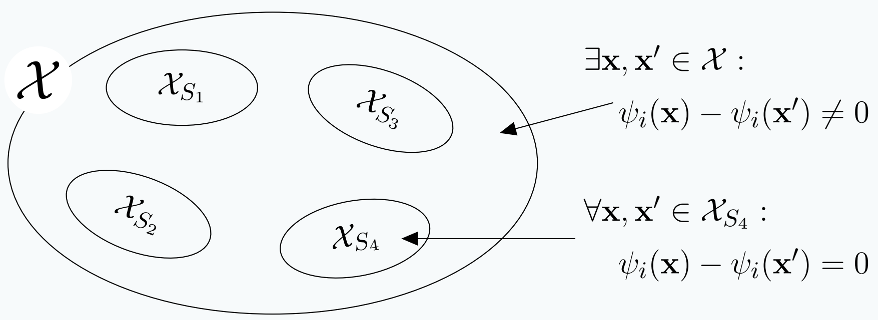
	base covariates	strategic response	preference factors	compliance model
is included	✓	✓	✓	✓
is heterogeneous	✓	✗	✗	✗

We provide:
- the optimal solution for the case where interactions result in a linear function of $\boldsymbol{\theta}_{it}$,
- conditions that can be imposed by the government to safeguard social welfare.

$$\begin{aligned}\arg\max_{\boldsymbol{\theta}_{it}} \mathbb{E}\left[Y_{it} \mid Z_t = i;\ \boldsymbol{\theta}^{\text{all}}\right] &= \arg\max_{\boldsymbol{\theta}_{it}} \left\{\mathbb{E}\left[\text{Base} \mid Z_t = i;\ \boldsymbol{\theta}^{\text{all}}\right] + \mathbb{E}\left[\text{Improvement} \mid Z_t = i; \boldsymbol{\theta}^{\text{all}}\right]\right\}\\ &= \arg\max_{\boldsymbol{\theta}_{it}} \left\{ \left(A_i^\top \boldsymbol{\theta}_{it} + B_i + h_i\left(\boldsymbol{\theta}_t^{\text{all}\setminus i}\right)\right) + \left(\boldsymbol{\theta}_{it}^\top \gamma_{it} \mathcal{E}\mathcal{E}^\top \boldsymbol{\theta}_i^* + C_i\right) \right\}\\ &= \frac{A_i + \gamma_{it}\mathcal{E}\mathcal{E}^\top \boldsymbol{\theta}_i^*}{\|A_i + \gamma_{it}\mathcal{E}\mathcal{E}^\top \boldsymbol{\theta}_i^*\|}\end{aligned}$$

Ideally,
- $A_i = k \times \mathcal{E}\mathcal{E}^\top \boldsymbol{\theta}_i^*$ for some $k > 0$,
- and more...

5. Inferring Causal Parameters



$$Y_{it} = \mathbf{X}_t^\top \boldsymbol{\theta}_i^* + \psi_i(\mathbf{X}_t) \Rightarrow \frac{dY_{it}}{d\mathbf{X}_t} = \boldsymbol{\theta}_i^* + \frac{d(\psi_i(\mathbf{X}_t))}{d\mathbf{X}_t} \quad ✗ \tag{1}$$

$$\exists \mathcal{X}_S : \Delta Y_{it} = (\Delta \mathbf{X}_t)^\top \boldsymbol{\theta}_i^* + \underbrace{\Delta\psi_i(\mathbf{X}_t)}_{=0} \Rightarrow \frac{d(\Delta Y_{it})}{d(\Delta \mathbf{X}_t)} = \boldsymbol{\theta}_i^* \quad ✓ \tag{2}$$

A cooperative protocol for all decision makers to partition $\mathbf{X}_t \mid Z_t$ correctly!

All variables here are conditioned on Z_t, but not shown here for the sake of simplicity.

6. Simulation